

# **Basics of Mechanical Engineering-1**

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**Week 05**

**Lecture 21**

**Buckling of Columns**

The next lecture for discussion is going to be Buckling of columns. What are columns? Suppose you have a flat plate and then you have something which is used for spacing between two plates. So, then this is called as a column. And here, there might be loads which are coming into existence.

So, there can be compression load, there can be tensile load, there can be shear load. So, now our discussion will be more focused towards Buckling of columns. Cylinder and column are the simple primitive structures wherein which the load analysis will be generally carried out. If you look at a building, suppose you are staying in the ground floor and somebody is staying in the first floor. Between these two people, structurally you have a member which tries to maintain the space as well as take care of all the loading conditions that is done by a column.

So predominantly the column will take a compression load when you talk about a flat or a housing or a building. So here there will be more discussion on Buckling of columns. Buckling means you are compressing; it is trying to deform its existing shape.

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## Contents



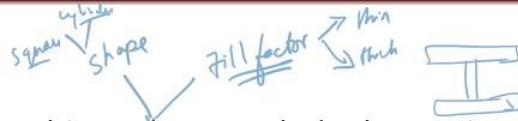
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So, in this lecture, we will try to have a small introduction, followed by it will be on Columns, then Slenderness, then slenderness Ratio, Euler's formula for Buckling, then Effective Length Factor, Critical Buckling Load concepts, Column Failure modes. Application and case study.

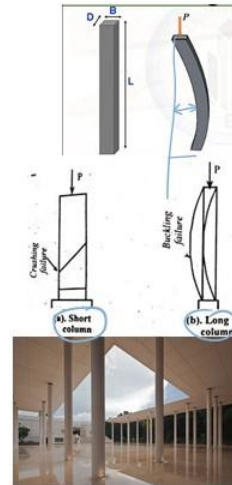
We will try to have two different case studies. One will be on machine firms and the other one will be a generic one. And finally, we will try to have a recap of this lecture.

# Introduction



Importance of studying columns and slenderness in engineering:

- Columns are fundamental structural elements that carry compressive loads.
- They are crucial in a wide range of engineering applications, from buildings and bridges to machinery and infrastructure.
- Understanding columns and their behavior under various conditions is essential for designing safe and efficient structures.



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Why is Column important? The importance of studying columns and very important term, slenderness in engineering.

Columns are fundamental structural elements that carry compressive load. As I told you, you can have a load on the top, you can have a holding at the bottom. In between, there is a column. Column can be square, extruded. or it can be a hollow square extruded, or it can be a cylinder, or it can be a solid cylinder. It can be a hollow cylinder; it can be a solid cylinder.

So, the column and slenderness is very important. They are crucial in the wide range of engineering application. From building to bridges to machinery to infrastructure. So, what did I tell you here? The column can be of two types I said.

Its shape is one and the fill factor is the other. Fill factor in turn takes you towards a thin cylinder, thick cylinder. Shape can take you to a square cross-section area or a cylindrical cross-section area, depending upon your requirements. So, many at times it is square. When you talk about bridges, the cylinder which is drawn or the column which is done is cylindrical in nature.

So, they are crucial in wide range of engineering applications from building to bridges to machinery to infrastructure. Understanding the column and its behavior under various conditions is essential for designing a safe and efficient structure. And sometimes when

there is a natural disaster, the buckling happens and the column tries to take some amount of deflection. So, this is called as Buckling. Deviation from the existing location.

So, buckling will come into existence. You can have a short column or a long column. Short column sometimes might lead you to shear. Long columns might lead you to bending. So, when it tries to bend, you can always understand some amount of deflection is taken.

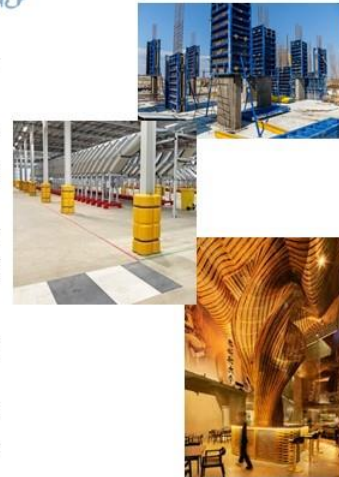
## Introduction

Element  $\left\{ \begin{array}{l} \text{Design/shape/parameter} \\ \text{material parameter} \end{array} \right.$



Key reasons why studying columns and slenderness is important in engineering:

- 1. Structural Integrity:** Columns must be designed to withstand loads without collapsing. Understanding their behavior helps prevent structural failures.
- 2. Material Efficiency:** Proper design can lead to more efficient use of materials, reducing costs and environmental impact.
- 3. Safety:** Ensuring columns can support loads safely is critical for the safety of buildings and structures.
- 4. Innovation:** Knowledge of column behavior can lead to innovative designs and applications in modern engineering.



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The key reasons why we study column and slenderness is important engineering is structural integrity. The columns must be designed to withstand load without collapsing. When you build tall structures, today we talk about 140, 150, 200, 400 meters, so a tall structure. When we are trying to have a hanger for aeroplane workshop or when you are trying to have a hanger for automotive industry assembly line, so there also they have these columns. Column must be designed to withstand loads without collapsing.

Understanding their behavior helps in preventing structural failures. Material efficiency, when we do a proper design, the material choice can be done and the material will be used 100%. So, when we try to take any element in manufacturing or in engineering, so we will have two things. When we try to look at all the applied mechanics formula, we will have design-based or shape-based formula or parameters we will have and then we

will have material-based parameters. When we do a proper choice of shape and do a proper design, we always can choose a proper material and choose conservative material for building structures.

So, material efficiency, proper design can lead to more efficient use of materials, reducing the cost and environmental impact. Safety ensures column can support load safely is critical for safety of the building and structures. Innovation can happen. Knowledge of column behavior can lead to innovative design and application in modern engineering. So, you can have simple material, but just by playing with the shape and making thin columns and assembling columns on each other, we can try to have a very strong structure. So, understanding different types of columns and the concept of slender ratio is important.

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## Introduction



**Objective:** Understanding Different Types of Columns and the Concept of Slenderness Ratio.

The primary objectives of this section are:

- 1. Identify Different Types of Columns:** Learn about the various types of columns used in engineering, including their characteristics and applications.
- 2. Understand the Concept of Slenderness Ratio:** Grasp the significance of the slenderness ratio in determining the stability and performance of columns under load.




So, the basic objective of this section is identified different types of columns. So, there are different types of columns. We will try to see how is it used in engineering and including the characteristics and application. The concept of Slenderness Ratio is also very important.


Grasp the significance of the slenderness ratio is in determining the stability and the performance of the column. One is shape, as I told you, type of columns. The other one is

the slenderness ratio. This is how a typical column is constructed in the residential area or in buildings. So, you will have a square cross section and the square cross section is supported by supporting structures and then what they do is they try to pour cement into it and once it is cured, they release it and make a column.


So, this column again if you see the cross section, you will have lot of reinforcement of steel and this is a matrix. This is matrix and these are steel reinforcement. You can see it in your house or nearby construction area.

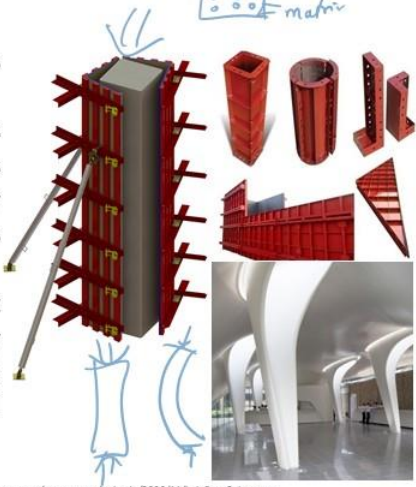
## Columns







- Columns are vertical structural elements that primarily resist axial compressive loads.
- They play a crucial role in transferring loads from the structure above to the foundations below, ensuring that the weight is properly distributed and the structural integrity is maintained.
- Columns must be designed to withstand not only the compressive forces but also any bending or buckling that may occur due to lateral loads or imperfections in material and construction.







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Columns are vertical structural elements that primarily resist axial compressive load. You can also use columns as I told you between two floors you can use a column.

Then you can have the complete building resting on columns. These are all called as Pile Foundation resting on columns. You can have that. And then the other thing is you can also have it on a bridge where you can have columns. In bridge when you have columns, it is trying to transfer a load and these loads are dynamic.

So, they play a crucial role in transferring loads from the structure above to the foundation below. Ensuring the weight is properly distributed and the structural integrity is obtained. So then generally what we do is assume that we have a flat floor. You can have multiple columns. So now these columns try to distribute the load.

When you try to have a column and when you try to compress it, it tries to buckle down. This is called as Bending or Buckling. So, the material will get deformed. So, a straight line now will become small. So, the distance will reduce.

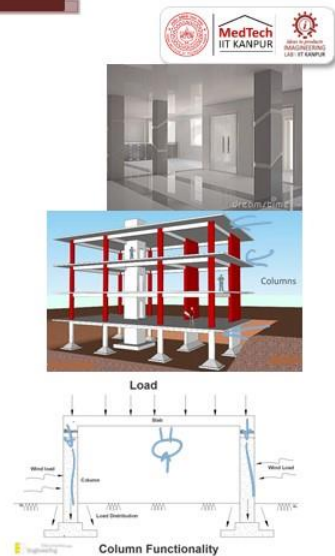
The column must be designed to withstand not only the compressive force but also any bending or buckling that may occur due to a lateral load or imperfection in material and construction.

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## Columns – Importance

### 1. Buildings:

- **Support Floors and Roofs:** Columns provide essential support to the horizontal elements of a building, such as floors and roofs, by bearing their loads and transferring them to the foundation.
- **Load Distribution:** They help distribute the weight of the building evenly, preventing localized failure and ensuring overall stability.



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<https://engineeringdiscoveries.com/wp-content/uploads/2019/08/Column-Function-1024x627.png>



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In buildings, support floor and roof is done by a column. Column provides the essential. These are all, the red ones are all columns. The columns provide essential support to the horizontal element.

This is the flat is a horizontal element. Horizontal element of the building such as floor and roof by bearing their load and transferring them to the foundation. The load distribution they help in distributing the weight of the building evenly towards the foundation preventing a localized failure and ensuring overall stability. So, you will have this is like a crane whatever you see. So, a gantry crane, right.

So, it is almost like that. So, you have a load which is there and then that this is a column right and then that rests on a foundation. So, when it moves very fast, this foundation should take care of this of the deflection plus the weight with which the crane grabs tries

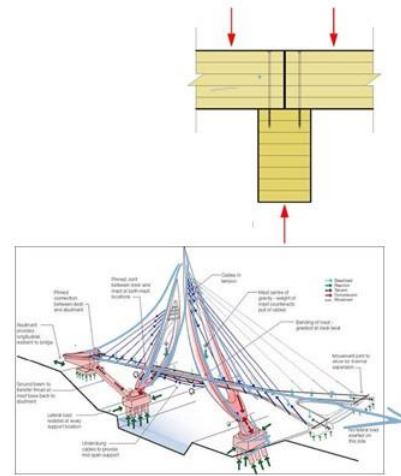
to take care of the deflection. So, when the deflection has to be avoided the load on this will be evenly distributed.

## Columns – Importance



### 2. Bridges:

- **Bear Deck Loads:** In bridge construction, columns, often referred to as piers, bear the loads from the bridge deck, including the weight of vehicles and other traffic.
- **Transfer Loads to Ground:** These piers transfer the loads to the ground or to deep foundations, maintaining the structural integrity of the bridge.



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When we talk about bridges interestingly so bridges the load there is a vehicle as far as buildings are concerned, there is not much of dynamics. People will be there.

The density, when there are a lot many people in one area, the density of that area increases. Maybe all of them jump, that is another story. But here, in bridges, it is a completely different story. On Bridges, the car will be speeding, the vehicles will have its own dynamic loading condition and there will be service condition like wind coming into existence. So, bridge design is little more challenging compared to a building design.

So, the bridge deck load in bridge construction, the column offered refers to as a pierce, bears the load from the bridge deck which includes the weight of the vehicle and other traffic. When we talk about transfer loads to ground, these pillars transfer the load to the ground or to deep foundation maintaining the structural integrity of the bridge. So, you can see here this is a bridge, you can see there, here is a bridge which goes and this bridge is supported. These are all supports and these supports are in turn grouted or which is held to a foundation.

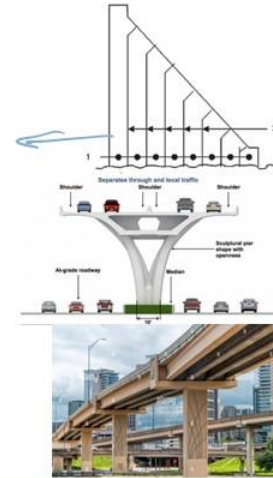


## Columns – Importance



### 3. Other Structures:

- **Towers and Dams:** Columns are integral in the construction of towers, providing vertical support and stability. In dams, they help in withstanding water pressure and other loads.
- **Infrastructure Projects:** Various infrastructure projects, such as elevated highways and parking structures, rely on columns to support and distribute loads effectively.



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Towers and dams, the columns are integral in the construction of towers, providing vertical support and stability. In dams, they help in withstanding the water pressure and other loads.

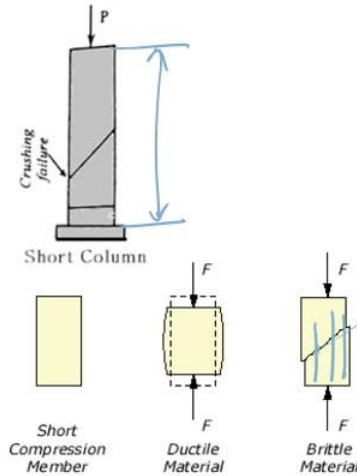
So, these are all columns. You can see all these things are columns. So, 1, 2, 3, 4, 5, 6 these are all columns and when we are trying to talk about bridge. So, today we talk about modular bridge metros are constructed all across the country using modular concept and they have overhanging bridges. So, it will always be a column will be there.

Infrastructural projects, various infrastructure projects such as elevated highway and parking structures rely on column to support and distribute the load effectively.

# Columns – Types

## 1. Short Columns

- Short columns are columns whose length is relatively small compared to their cross-sectional dimensions.
- These columns typically fail due to material crushing under axial compressive loads rather than buckling.
- The design and analysis of short columns focus primarily on the compressive strength of the material used.



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[www.civildat.com/wp-content/uploads/2020/04/Untitled-design-40.jpg](http://www.civildat.com/wp-content/uploads/2020/04/Untitled-design-40.jpg)

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Now let us look into Types of Columns. There are two Types of Columns. One is called a Short column, the other one is called as Long column. The Short columns are columns whose length is comparatively smaller to their cross-section dimension.

It will be something like a barrel. These columns typically fail due to material crushing under axial compressive force rather than buckling. So, a short compression member, when you apply forces, it tries to buckle. So, when it tries to buckle, first it tries to bulge, then after bulging, it tries to buckle. So Short columns are columns where the length is relatively smaller to their cross-sectional dimensions.

The design and analysis of short columns focuses primarily on the compressive strength of the material used. So, if there is a column, then when you apply load, if it is a ductile material, it undergoes bulging. And this is common for a ductile material. When there is a brittle material, when there are forces, it shears. So that is why what we do is we reinforce the columns.

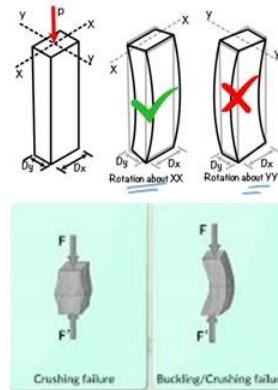
So even when it shears, this reinforcement tries to hold the matrix material which has got sheared.

## Columns – Types



### Characteristics of Short Column:

- **Low slenderness ratio:** The slenderness ratio is the ratio of the column's effective length to its radius of gyration. A low slenderness ratio indicates that the column is stout and less likely to buckle under axial loads.
- **Predominant failure mode is crushing or yielding:** Due to their low slenderness ratio, short columns primarily fail by material crushing or yielding.
- **Common in lower stories of buildings where buckling is less likely:** Short columns are often used in the lower stories of buildings where the height of the columns is relatively short compared to the upper stories.



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The Characteristics for a Short column. Low Slender Ratio. The slender ratio is the ratio of the column's effective length to its radius of gyration. A low slenderness ratio indicates that the column is stout and less likely to buckle under axial loads.

So, this is a column and here when you try to apply load, you see here it buckles and then this is rotation about x and this is rotation about y. This should not happen. So, slenderness ratio is the ratio of the column's effective length to its radius of gyration. Predominant failure mode is crushing or yielding. Due to their low slenderness ratio, short columns primarily get crushed or yielded. So crushing is always this.

Load is applied, it crushes. Common in lower stories of buildings where buckling is less likely to happen. Short columns are often used in lower stories of the building where the height of the column is relatively short compared to the upper store. So, wherever it is there, we use short columns.

## Columns – Types

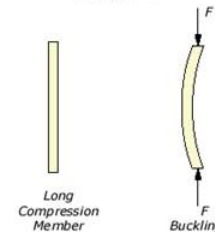
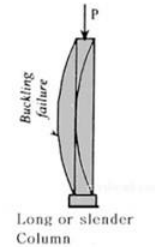


### 2. Long Columns

- Long columns have a length that is significantly larger compared to their cross-sectional dimensions, making them susceptible to buckling under axial loads.
- The slenderness ratio is a key factor in their stability, and the design must account for the potential of buckling.
- A long or slender column is the one whose ratio of effective length to its least lateral dimension is not less than 12. Then it is termed as a long column.

$$(l_e/b) > 12$$

$$l_{eff}/b > 12$$



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[www.dynalife.com/formulae/yallid\\_mechanics/columns/images/CompressionMember\\_Long.gif](http://www.dynalife.com/formulae/yallid_mechanics/columns/images/CompressionMember_Long.gif)

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Long columns are as the name specifies clearly, has a length that is significantly larger than compared to the cross-section dimension.

Making them susceptible to buckling under axial load. So, it is a long compression member. When it is done it buckles. So, the slender ratio is a key factor in their stability. And design must account for the potential of buckling.

In short columns, it will always crush. In long columns, predominant failure is always buckling. A long or a slender column is the one whose ratio of effective length to its least lateral dimension is not less than 12. Then, it is referred as long columns. So, what are we talking about?

This is the length. We are talking about the dimension here. So, a long or a slender column is the one whose ratio of effective length to its least lateral dimension is not less than 12. Then, the term is called as Long columns.

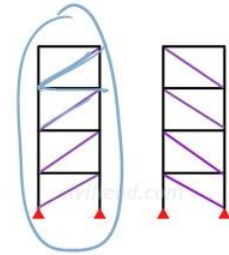
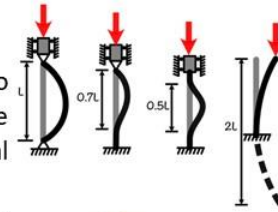
$$(l_e/b) > 12$$

## Columns – Types



### Characteristics:

- **High slenderness ratio:** A high slenderness ratio indicates that a column is tall and thin, making it more prone to lateral deformation or buckling under axial loads.
- **Predominant failure mode is buckling:** Long columns with a high slenderness ratio primarily fail by buckling, where lateral deformation under compressive load leads to sudden, catastrophic failure.
- **Require careful consideration of lateral supports and bracing** To prevent buckling, long columns need lateral supports, bracing, or increased cross-sectional dimensions to enhance stability and load-carrying capacity.



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The Characteristics. High slender ratio. A high slenderness ratio indicates that a column is tall and thin, making it more prone to lateral deformation or buckling under axial load. In certain cases, we would like to have such columns. So, high slender ratio indicates that the column is tall and thin. The Predominant failure mode is buckling.

Long columns with high slender ratio primarily fails by buckling, where lateral deformation under compression load leads to sudden or catastrophic failure. So, you can see in this, there are columns laterally, vertically as well as at an angle and horizontally also it is there. Requires careful consideration of lateral support and bracing. To prevent buckling, long columns need lateral support. These are all lateral supports brazing or increased cross section dimension to enhance stability and load carrying capability.

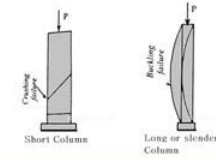
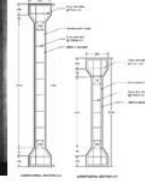
## Columns – Example



**Concrete Columns:** These are widely used in building construction and can be reinforced with steel bars or wrapped with helical reinforcement to enhance their strength and ductility.

**Short Concrete Columns:** Typically found in the lower stories of buildings. These are designed to bear significant axial loads and resist crushing. They have low risk of buckling due to shorter height.

**Long Concrete Columns:** Used in multi-story buildings where height increases buckling risk. They require additional reinforcement to prevent instability. They may need lateral bracing or increased cross-sectional dimensions for enhanced stability.



[www.researchgate.net/figure/Schematic-diagrams-of-long-column-and-short-column-showing-reinforcement-details\\_fig1\\_366538156](http://www.researchgate.net/figure/Schematic-diagrams-of-long-column-and-short-column-showing-reinforcement-details_fig1_366538156)  
[www.civilload.com/wp-content/uploads/2020/04/long-column-vs-short-column-min-1.png](http://www.civilload.com/wp-content/uploads/2020/04/long-column-vs-short-column-min-1.png)  
<https://housing.com/news/wp-content/uploads/2023/04/Reinforced-concrete-column-design-02.png>



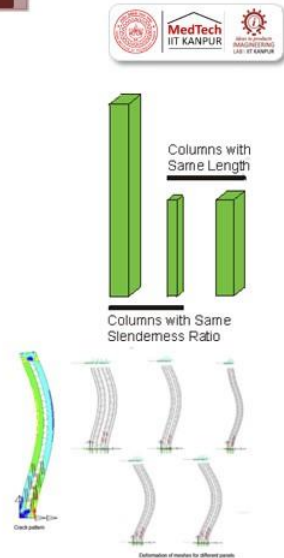
Some of the examples of columns: concrete columns, they are widely used in buildings construction and can be reinforced with steel bars or wrapped with helical reinforcement to enhance their strength and ductility. Today, what we are doing is we are also trying to have columns where in which we wrap with composite tapes at the portion where after design calculation, we see there is going to be a crack or there may be a crack. We try to wrap it around and reinforce. So, the other way around is we try to have a good wettability between the steel wire and the concrete. So, the concrete columns are widely used in buildings.

Short concrete columns and Long concrete columns typically found in the lower stories of the building is Short concrete columns. They are designed to bear significant axial load and resist crushing. Whereas in Long, it is used in multi-story building where the height increases the buckling risk. They require additional reinforcement to prevent instability, so they try to increase the cross-section area of the given column.

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## Slenderness

- Slenderness refers to the tendency of a structural column to buckle under compressive loads.
- It is a measure of how slender or slender a column is relative to its cross-sectional dimensions, influencing its stability and load-carrying capacity.



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Slenderness is referred to the tendency of a structural column to buckle under compression load.

It is a measure of how slender or slender a column is relative to its cross-sectional dimension. Influencing is stability and load carrying capacity. So, you can see here, column with same length, you can have slender ratio is changed. So, depending upon the load, you can try to change the slenderness ratio.

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## Effects of Slenderness Ratio

### Influence on Buckling

- **Increased Susceptibility:** As the slenderness ratio increases, the column becomes more slender relative to its cross-sectional dimensions.
- This increased slenderness heightens the column's susceptibility to buckling.
- Columns with higher slenderness ratios are more likely to experience lateral deformation under compressive loads, leading to potential instability and failure.



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The effectiveness of slenderness ratio influences the buckling. It increased susceptibility. As the slenderness ratio increases, the column becomes slenderer relative to the cross-section dimension. This increased slender heightens, the column's susceptibility to buckling. The column with high slenderness ratio are more likely to experience lateral deformation under compression load leading to instability.

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## Effects of Slenderness Ratio



### Critical Slenderness Ratio

- **Distinction Between Column Types:** The critical slenderness ratio serves as a boundary between short and long columns.
- Columns with a slenderness ratio below this critical value are classified as short columns and typically fail by crushing or material yielding.
- In contrast, columns with a slenderness ratio above the critical value are classified as long columns, where buckling becomes the predominant failure mode.
- Proper design and analysis are crucial to address these differences and ensure structural safety.



What is the criticalness of the slenderness ratio? The distinction between the two column types. The Critical Slenderness Ratio serves as a boundary between short and long columns. Columns with a slenderness ratio below this critical value are classified as short columns and typically fail by crushing. And in contrast, the column with slenderness ratio above the critical value is classified as long columns where buckling becomes the predominant failure.

So, crushing or buckling, you have to decide. And again, when something is getting buckled, you can always try to support. and try to reduce the buckling. The proper design and analysis are crucial to address this difference and ensure structural safety.



# Effective Length Factor



## Boundary Conditions

The effective length factor ( $K$ ) depends on the boundary conditions of the column ends. These conditions determine how much the column can deflect and consequently, its buckling behavior. Common boundary conditions include:

- **Fixed-Fixed:** Both ends are restrained from rotating and translating.
- **Pinned-Pinned:** Both ends are free to rotate but restrained from translating.
- **Fixed-Free:** One end is fixed while the other is free to move and rotate.
- **Fixed-Pinned:** One end is fixed while the other is free to rotate but not translate.



There are boundary conditions for effective length factor. So, the effective length factor depends on the boundary condition of the column ends. These columns determine how much the column can deflect and consequently its buckling behavior. The common Boundary Conditions are Fixed Boundary conditions, Pinned boundary conditions. So, it is Fixed-Fixed boundary condition, Pinned-Pinned boundary condition, Fixed-Free boundary condition, Fixed-Pinned boundary conditions. So, it is not the slenderness alone, it is also the boundary with which it is there, that also is important.

When we talk about fixed-fixed or both ends are restrained from rotating and translating. Suppose I try to take a column which has both sides fixed. So, now I cannot rotate. I cannot rotate, I cannot translate. So, if you have a Fixed-Fixed boundary condition, it is going to refrain you from rotating and translating.

When we have Pinned-Pinned, both ends are free to rotate, but restrained from translating alone. So, Pinned-Pinned means I am trying to pin it at the top portion, pin it at the bottom portion. It gives you a rotation but never allows you to translate. So, this is the next boundary condition. Both ends are free to rotate, but restrained from the translating.

The next one is Fixed-Free. One end is fixed, while the other end is free to move and rotate. So, other end is free to rotate and move. So, the bottom edge is fixed. So, that is fixed free.

One end is fixed, the other end can rotate, translate. The Fixed and Pinned is one end is fixed, the other end is only free to rotate but not translate. So how beautifully the boundary conditions are laid. One is both ends fixed, no rotation, no translation. The other thing is both ends fixed, rotation is allowed.

The third one is fixed free is bottom fixed. It can translate and rotate. Fixed pin is it can translate and it is free to rotate. It is not able to translate.

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## Effective Length Factor



The effective length ( $L_{eff}$ ) of a column is the length used in buckling calculations and is given by:

$$L_{eff} = K \times L$$

where:

- K = Effective Length Factor
- L = Actual length of the column

The effective length factors (K) for boundary conditions are as follows:

- **Fixed-Fixed:**  $K = 0.5$
- **Pinned-Pinned:**  $K = 1.0$
- **Fixed-Free:**  $K = 2.0$
- **Fixed-Pinned:**  $K = 0.7$

These factors account for how the end conditions impact the column's effective length, influencing its buckling characteristics.



So, the Effective Length Factor is another important parameter. Till now, we saw what is column, short column, long column, slenderness ratio. How do you have a varying slenderness ratio even for a fixed one? Then we went into critical value. Then we have moved into Effective Length Factors. While going through, we also saw the different boundary conditions.

If you understand this, when you do any engineering, let it be machinery engineering, let it be mechanical engineering, civil engineering, you will enjoy it. This subject is not focused only to mechanical, civil and chemical and other also. It is a common engineering course. An engineer should know this basic sense.

So that's what we are trying to go through in the introduction of mechanical engineering. So, the effectiveness length ( $L_{eff}$ ) of a column is the length used in buckling calculations and it is given by

$$L_{eff} = K \times L$$

What is K? K is the effectiveness length factor and L is the actual length of the column. This is the actual length. K is the effectiveness.

So, the K generally for varying boundary conditions takes to take different values. If it is fixed to fixed condition the K value is 0.5. When it is Pinned-Pinned, it is 1. Fixed and Free, it is 2. And Fixed and Pinned, it is 0.7.

These factors account for how the end conditions impact the column effective length which influences its buckling characteristics.

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## Numerical Example - Slenderness Ratio



Calculate the slenderness ratio for a steel column with the following properties:

- Actual length ( $L$ ) = 6 meters
- Least radius of gyration ( $r_{min}$ ) = 50 mm

$$\text{Slenderness ratio } (\lambda) = \frac{L}{r_{min}} = \frac{6000}{50} = 120$$

$$\therefore \text{Slenderness ratio } (\lambda) = 120$$



Let us try to solve a simple problem. So, here we are supposed to calculate the slenderness ratio for a steam column with the following properties. So, how do we calculate the slenderness ratio?

The slenderness ratio ( $\lambda$ ) is given by:  $\frac{L}{r_{min}}$

Given: Length (L) = 6 m = 6000 mm

Least radius of gyration ( $r_{\min}$ ) = 50 mm

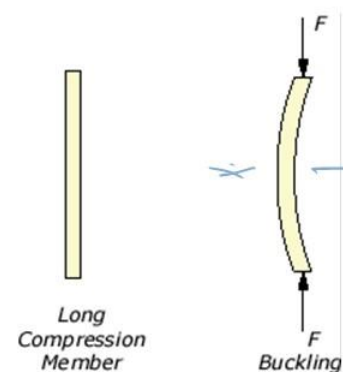
$$\text{Calculation: } \lambda = \frac{L}{r_{\min} \frac{1}{50}} = 120$$

∴ Slenderness ratio of the given steel column = 120. So, this is how you calculate the slenderness ratio.

## Critical Buckling Load Concept



- The **critical buckling load** is the maximum axial load that a column can sustain before it becomes unstable and buckles.
- This load marks the threshold between stable compression and catastrophic failure due to lateral deformation.



<https://qph.cf2.quoracdn.net/main-qimg-9317e4fb46fb4309975bfc3d1e32b883>

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Critical Buckling Load Concepts. The Critical Buckling Load is the maximum axial load that a column can sustain before it becomes unstable or it can start buckling or it buckles. So, compression load, so it is buckling. This load marks the threshold between the stable compression and the catastrophic failure due to lateral deformation. This is lateral deformation. Critical buckling load, at what load it buckles, so that I try to identify and then start working on it.

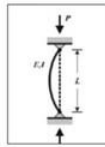
# Critical Buckling Load Concept



## Importance

Understanding the critical buckling load is crucial for ensuring the safety and stability of structural designs. It allows engineers to:

- **Predict Failure:** Anticipate the load at which a column will buckle, preventing unexpected collapses.
- **Design Safely:** Specify appropriate materials, dimensions, and support conditions to avoid buckling under expected loads.
- **Optimize Structures:** Balance material use and structural efficiency while maintaining safety margins.



"—": Original Shape  
"- - -": Buckled Shape



Buckled Steel Column



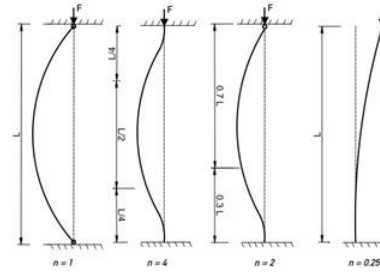
Buckled R.C.C. Column



<https://pining.com/564x29f6e71/296e717c9bb03b1455b56a0e23889b39.jpg>  
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So, the importance of Critical Buckling Load concept is and it helps in understanding the critical buckling load is crucial to ensure the safety and the stability of structural design. It allows the engineer to predict failure, design safe and optimize structure. When I say predict failure, it anticipates the load at which the column will buckle. When I say design safely, it is the appropriate material choice and the supporting conditions which is required. Optimum structure, it balances the material used and structural efficiency while maintaining safety margins.

## End fixity Coefficient



The following table shows the values of end fixity coefficient (C) for various end conditions:

S.No.	End Conditions	End fixity coefficient (C)
1.	Both ends hinged	1
2.	Both ends fixed	4
3.	One end fixed and other hinged	2
4.	One end fixed and other free	0.25

So, the following table shows the value of end fixity coefficient C for various end conditions. Both hinged, it is 1; both ends fixed it is 4; one end fixed and other end hinged it is 2; one end fixed and other end free it is 0.25. These are all coefficients which has come by several calculations and engineering thought process.

## Euler's formula for Buckling

the crippling or buckling load ( $W_{cr}$ ) under various end conditions is given by the following.

$$W_{cr} = \frac{C\pi^2 EI}{l^2} = \frac{C\pi^2 EAK^2}{l^2} = \frac{C\pi^2 EA}{(l/k)^2} \quad \therefore l = Ak$$

E = young's modulus

A = cross-section area.

k = least radius of gyration of the C-S

l = length of column.

C = constant, depends on end conditions of the column.

So, let us now see the Euler's formula for Buckling. So, the theory says the Crippling or Buckling load load ( $W_{cr}$ ) critical under various end conditions is given by the following equation: So,

$$W_{cr} = \frac{C\pi^2 EI}{l^2} = \frac{C\pi^2 E A k^2}{l^2} = \frac{C\pi^2 EA}{\left(\frac{l^2}{k}\right)} \quad (l = A \times k^2)$$

$k$  is the least radius of gyration of the cross section. Then  $l$  is the length of the column and  $C$  is the constant. This constant depends on end conditions of the column, right. So, this is Euler's formula for buckling.

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## Numerical Problem



The maximum load on a petrol engine push rod 300 mm long is 1400 N. It is hollow having outer diameter 1.25 times the inner diameter. Spherical seated bearings are used for the push rod. The modulus of elasticity for the push rod is 210 kN/mm<sup>2</sup>. Find suitable size for the push rod, taking a factor of safety = 2.5.



Now, let us try to solve a numerical problem. Maximum load of a petrol engine push rod 300 mm long is 1400 Newton. It is hollow having outer diameter as 1.2 times the inner diameter. Spherically seated bearing are used for the push rod, spherical seating, that means to say rotation, it is rotating, right. I said when we saw four different types. Fixed-Fixed, Pinned-Pinned, Fixed-Free, Fixed-Pinned.

So, what is that Fixed-Pinned? Fixed-Pinned, one end is fixed while the other end is free to rotate but not translate. So, you should be very careful when we read the statement. The modulus of elasticity  $E$  is given. Find the suitable size of the push rod taking the factor of safety as 2.5 times.

Given =  $l = 300 \text{ mm}$ ,  $W = 1400 \text{ W}$ ,  $D = 1.2 d$ ,  $E = 210 \text{ kN/mm}^2$ ,  $m = 2.5$

∴ Moment of inertia of the push rod

$$I = \frac{\pi}{64} (D^4 - d^4)$$

$$I = \frac{\pi}{64} (91.25d)^4 - d^4$$

$$= 0.07 d^4 \text{ mm}^4.$$

We know the crippling load.

$$W_{cr} = m \times w = 2.5 \times 1400 = 3500 \text{ N}$$

Euler's formula,

$$W_{cr} = \frac{\pi^2 E I}{L^2}$$

$$3500 = \frac{9.87 \times 210 \times 10^3 \times 0.07 d^4}{(300)^2} = 1.6 d^4$$

$$d^4 = \frac{3500}{1.6} = 2.188 \text{ or } d = 6.84 \text{ mm.}$$

$$D = 1.25d$$

$$= 1.25 \times 6.84$$

$$= 8.55 \text{ (Ans.)}$$

So, when you look into the problem, we have asked to find out the suitable size of the push rod taking into factor 2.5. So, we have solved this problem and I am sure this is a straightforward substituting the formulae and trying to get the answer.



## Column Failure Modes

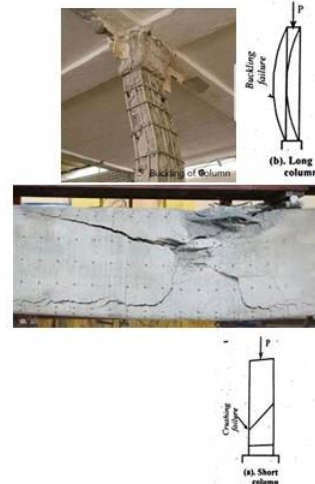
### Types of Failure

#### 1. Buckling

- Lateral deformation of a column under axial load, leading to sudden and often catastrophic failure.
- **Example:** a column bending sideways under compressive load.

#### 2. Crushing

- Failure due to material yielding or compressive crushing when the stress exceeds the material's compressive strength.
- **Example:** a short, stout column with compressive force causing it to crush vertically.



[www.structuralguide.com/wp-content/uploads/2020/04/Buckling-of-Column.jpg](http://www.structuralguide.com/wp-content/uploads/2020/04/Buckling-of-Column.jpg)  
<https://encrypted-tbn0.gstatic.com/images?q=tbn:ANd9GcRWmK8znpZahur8AMN Y1DabVRid8SPuDHBFUA&s>  
[www.engineeringintira.com/wp-content/uploads/2014/12/IMG\\_6355-1024x845.jpg](http://www.engineeringintira.com/wp-content/uploads/2014/12/IMG_6355-1024x845.jpg)

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Moving forward, let us now see into some of the column failures we have already discussed in length. So, types of failure, it can be Buckling failure, it can be a Crushing failure. Buckling failure or lateral deformation of the column under axial load leading to a sudden or often catastrophic failure.

A column bending sideways under compression load is a buckling failure model. So here the material due to material yielding or compression crushing, when the stress exceeds the material's compressive strength, we have a crushing. So generally, which is a common feature in short.

## Column Failure Modes

### 3. Combined Stress

- It occurs when a column experiences both axial load and bending moments, leading to complex stress distributions and potential failure.
- **Diagram:** a column under both compressive load and lateral force causing it to bend and compress simultaneously.



[www.structuralguide.com/wp-content/uploads/2020/04/Column-failure-1.jpg](http://www.structuralguide.com/wp-content/uploads/2020/04/Column-failure-1.jpg)  
[www.structuralguide.com/wp-content/uploads/2020/04/Compression-failure-1.jpg](http://www.structuralguide.com/wp-content/uploads/2020/04/Compression-failure-1.jpg)

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Here Combined Stresses are also possible. It occurs when the column experience both axial load and bending moment leading to a complex stress distribution and potential failure.

Diagram column under both compressive load and lateral force causing it to bend and compress simultaneously is called the Combined Stress failure.

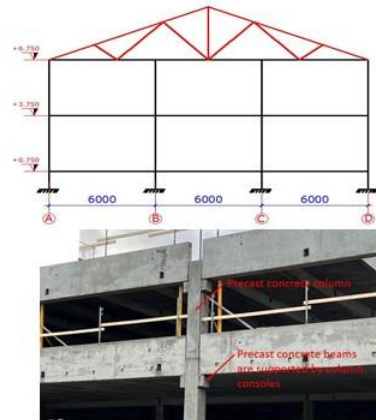
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## Applications and Case Studies



### Mechanical Engineering Applications

- **Machine Frames**  
Columns are integral to the frames of various machines, providing structural stability and support to withstand operational loads.
- **Structural Supports**  
Columns serve as essential elements in the structural framework of buildings, bridges, and other infrastructures, ensuring load transfer and stability.



[https://4.bp.blogspot.com/-8qgD\\_sugfk/NwQ1r1j5SkI/AAAAAAAAADHk/YDWHANc5mu4Y2r1EGwARQM5UhzZlavoWCLcBGA4sJ1600/building%2Bsection.JPG](https://4.bp.blogspot.com/-8qgD_sugfk/NwQ1r1j5SkI/AAAAAAAAADHk/YDWHANc5mu4Y2r1EGwARQM5UhzZlavoWCLcBGA4sJ1600/building%2Bsection.JPG)  
[www.structuralbasics.com/wp-content/uploads/2022/11/Example-of-precast-concrete-column.jpg](http://www.structuralbasics.com/wp-content/uploads/2022/11/Example-of-precast-concrete-column.jpg)



So let us now look into two case studies. So, the first case study is going to be a mechanical engineering application, Machine Frames. Let it be lathe machine, let it be drilling machine, let it be milling machine, let it be your car, let it be anything. So, you will always have a frame.

So, Machine Frame. Columns are integral to the frame of various machines, providing structural stability and support to withstand operational loads. Structural support columns serves as essential element. If you see here, it's a machine frame. So, if you see, column serves as an essential element in the structural framework of building bridges and other infrastructure, ensuring the transfer of load.

## Case Study 1 : Machine Frames



**Scenario:** A large industrial machine frame collapsed due to buckling during operation.

**Cause:** Inadequate consideration of compressive forces and improper material selection.

**Analysis:** The design did not account for the critical buckling load, leading to failure under expected operational loads.

**Solution:** Redesigning the frame with higher-grade materials and performing finite element analysis (FEA) to ensure stability under all loading conditions.

**Lessons Learned:** Always consider compressive forces in design. Use FEA to predict buckling behavior and optimize material selection.



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A large industrial machine frame collapses due to buckling during operation. The cause, inadequate consideration of compressive forces and improper material selection led to the collapse of the frame. Analysis, the design did not account for the critical buckling load leading to the failure under expected operational loads. The solution will be redesigning the frame with high grade material and performing finite element analysis to ensure stability under all loading conditions. The loading condition can be compression, can be buckle, can be shear, can be a combination of this.

Lesson learnt? Always consider compressive force in design. Use finite element to predict the behavior before you get into development.

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## Case Study 1 : Machine Frames



### Tacoma Narrows Bridge Collapse (1940)

**Overview:** The bridge, known as "Galloping Gertie," collapsed due to aeroelastic flutter, a form of dynamic buckling.

**Cause of Failure:** Design did not account for wind-induced oscillations, leading to progressive deformation and failure.

**Lessons Learned:** Understood the importance of dynamic analysis in structural design, especially for bridges and long-span structures.



<https://cdn.britannica.com/21/59921-050-BCBF8853/Collapse-Tacoma-Narrows-Bridge-Washington-state-1940.jpg>  
[www.simscale.com/blog/tacoma-narrows-bridge-collapse/](http://www.simscale.com/blog/tacoma-narrows-bridge-collapse/)



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Tacoma Narrows Bridge collapsed in 1940. The bridge is known as 'Galloping Gertie'. Collapsed due to aerostatic fluttering, a form of dynamic buckling.

The cause for failure is the design did not account for wind-induced oscillation. So, that is what I said, column. When you have a column, there is also a dynamic load. The dynamic load here is a swinging and if people are jumping, it is a cyclic load. So, here in this bridge, they did not take wind-induced oscillation.

So, the lesson learned is understood the importance of dynamic analysis in structural design, especially in bridges and long span structures.

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## To Recapitulate



- What is the purpose of studying columns and slenderness in engineering?
- What are columns and why are they important in structural engineering?
- How are columns classified based on their length and mode of failure?
- What is the slenderness ratio and how is it calculated?
- What is Euler's critical load formula? State parameters it depend upon.
- How do different boundary conditions affect the effective length of a column?
- What is the critical buckling load and why is it important in column design?
- What factors influence the buckling load of a column?
- What are the different failure modes of columns?



To recap what we went through in this lecture, we first saw what is the purpose of studying column and slenderness in the column, what are the different types of columns and its importance, classification of column based on length and modes of failure, how do you calculate the slenderness ratio, Euler's critical load formula, what are the different boundary conditions which are very important when you try to talk about columns? Then what is critical buckling load and its important what factors influence the critical load of a column? And finally, we saw different failure modes.

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Friends, we have used the following books as reference material for preparing this lecture. Thank you very much.