Basics of Mechanical Engineering-1

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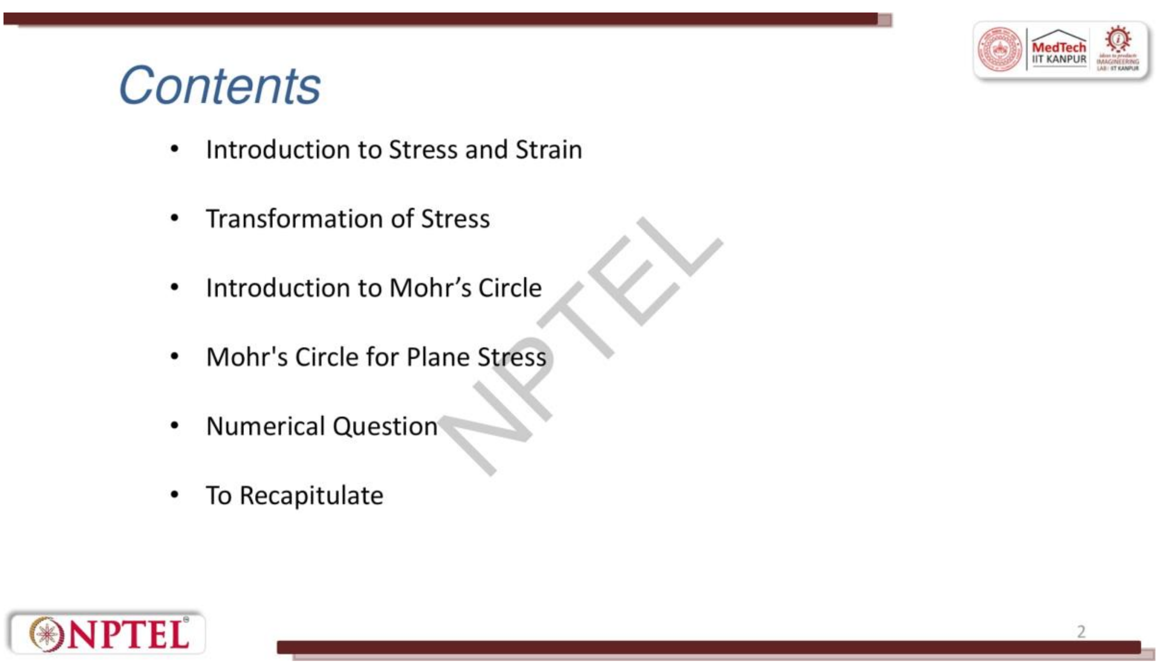
Week 06

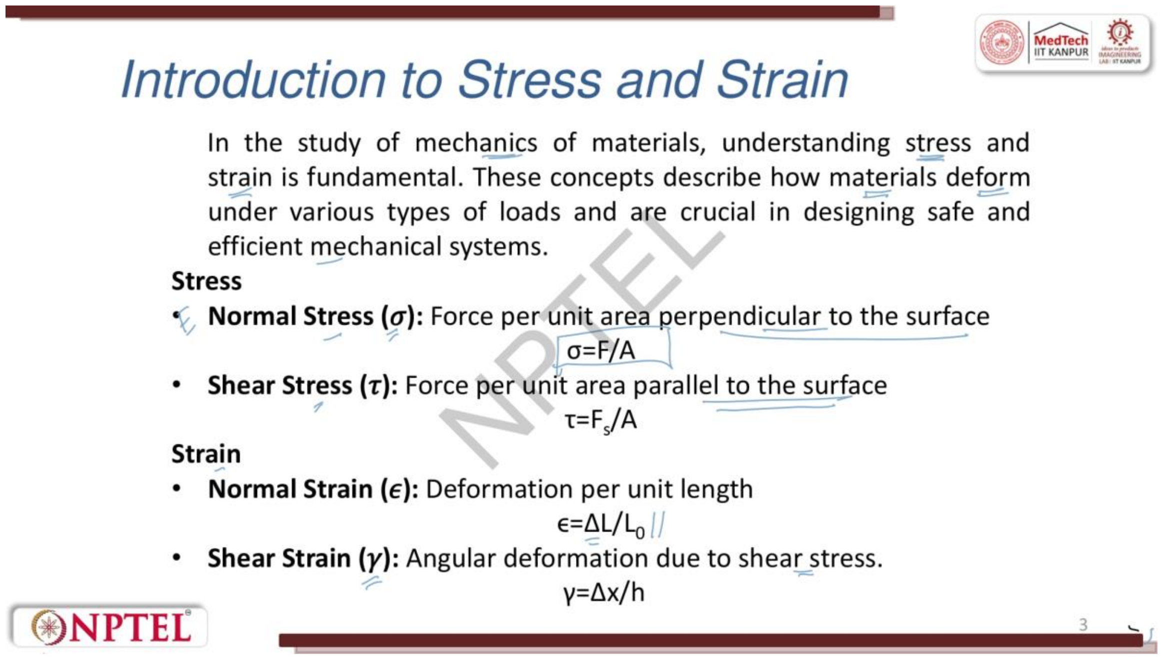
Lecture 24

Mohr's Circle

Welcome to the lecture on Mohr's circle. This is a very important topic in this course where we are trying to understand the stress acting on a body.



In this lecture we will try to have the content as Introduction to Stress and Strain, Transformation of Stress, Introduction to Mohr's Circle, Mohr's Circle for Plane Stress Condition, then a Numerical Question we will try to solve and finally we will try to have a recap.



So, Introduction to Stress and Strain. In the study of mechanisms of material, understanding stress and strain is very fundamental.

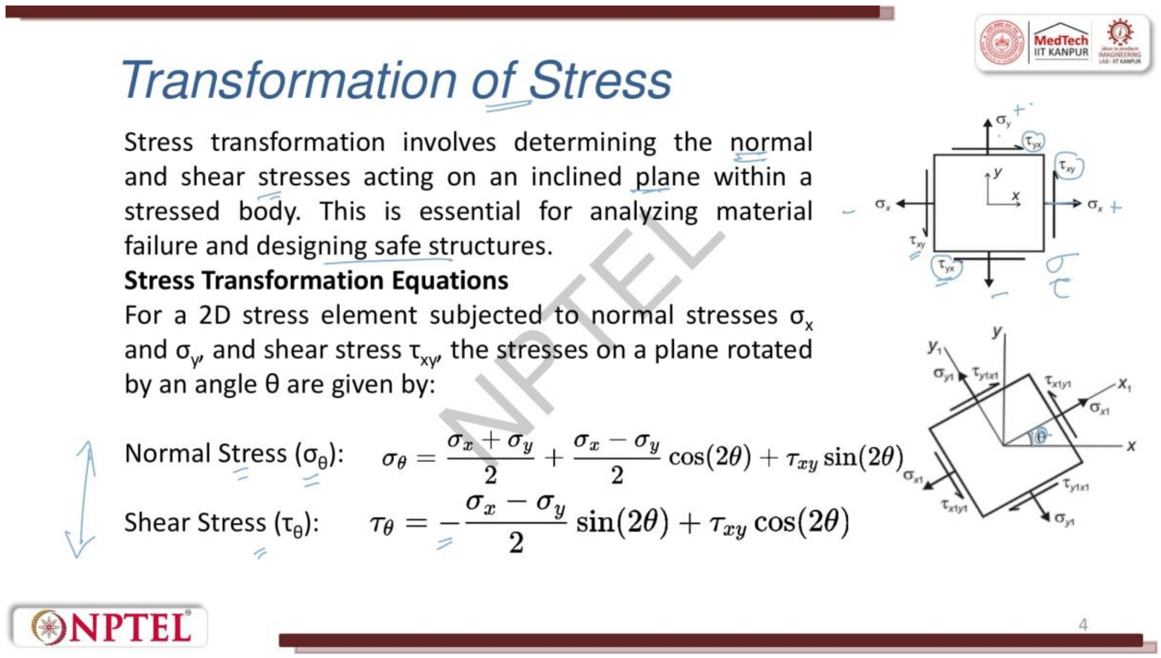
These concepts describe how material deforms under various types of loads. The loads can be tensile, compressive, bending, shear, torsion or combination which are very crucial in designing safe and efficient mechanical systems.

The Stress is always defined as force per unit area. This is engineering stress, normal stress, engineering stress or only stress which is represented as σ. It is nothing but force per unit area perpendicular to the surface.

This is very important. If there is an angle, then that cosθ term comes into effect. Please keep that in mind. The shear stress, which is exhaustively used while metal cutting. The stress is defined as τ, that is also represented as force per unit area parallel to the surface.

So, normal stress was perpendicular, then shear stress is parallel to the surface. So, $\tau = F_s/A$.

Normal strain. Which is nothing but deformation per unit length, which is represented as $\epsilon = \Delta L/L_0$, which is the original length. When we define Shear strain, because we saw shear stress, Shear strain is represented as γ and which is nothing but angular deformation due to shear stress. So $\gamma = \Delta x/h$.



So, how do you do Transformation of Stress? We will try to discuss in this slide. The stress transformation involves determining the normal and shear stress acting on an inclined plane. So, this is a plane.

So, you are trying to do at an inclination of θ. This is essential for analyzing material, failure and designing safe structure. When you represent this in a unit element, you will have a square, then you will have delta x in one direction and maybe this is positive, this is negative, delta y in positive and then negative. Then where is τ? So τ is the shear which happens here.

So you will have four τ_s represented which is τ_x going towards y is τ_{xy} and then when you are moving from y to x it is τ_{yx}. When you are moving from y to x, it is τ_{yx} and then in this direction, you will have τ_{xy}. So here you see σ and you also see τ. The suffix x, y is very important. When we look at shear transformation equation for a 2D stress element, subjected to normal stress σ_x and σ_y, shear stress is nothing but τ_{xy}. The stresses on a plane rotated by an angle θ are always given as normal stress is

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} + Cos(2\theta) + \tau_{xy} \sin(2\theta)$$

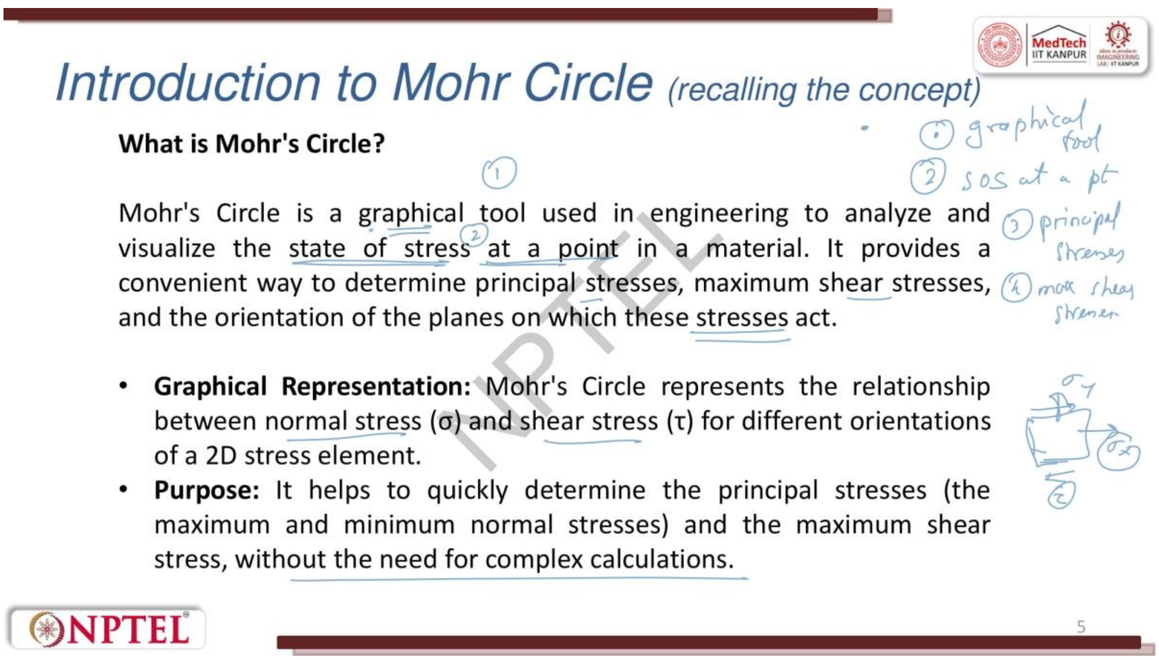
This is for a normal stress.

When you are looking at shear stress,

$$\sigma_\theta = -\frac{\sigma_x - \sigma_y}{2} + Sin(2\theta) + \tau_{xy}\, \text{Cos}(2\theta)$$

These two formulas are very important. Since we have a lot of syllabus has to be covered. We are not getting into how did we derive this. We are just looking at the formula here.

So, here when the stress element is rotated by an angle θ, we try to get the normal stress, τ_θ and σ_θ. If it is not rotated by θ, then these cos terms, sin terms will disappear.



So, let us recall the concept of Mohr's circle which we studied in between very briefly in a lecture. What is Mohr's circle? Mohr's circle is a graphical tool.

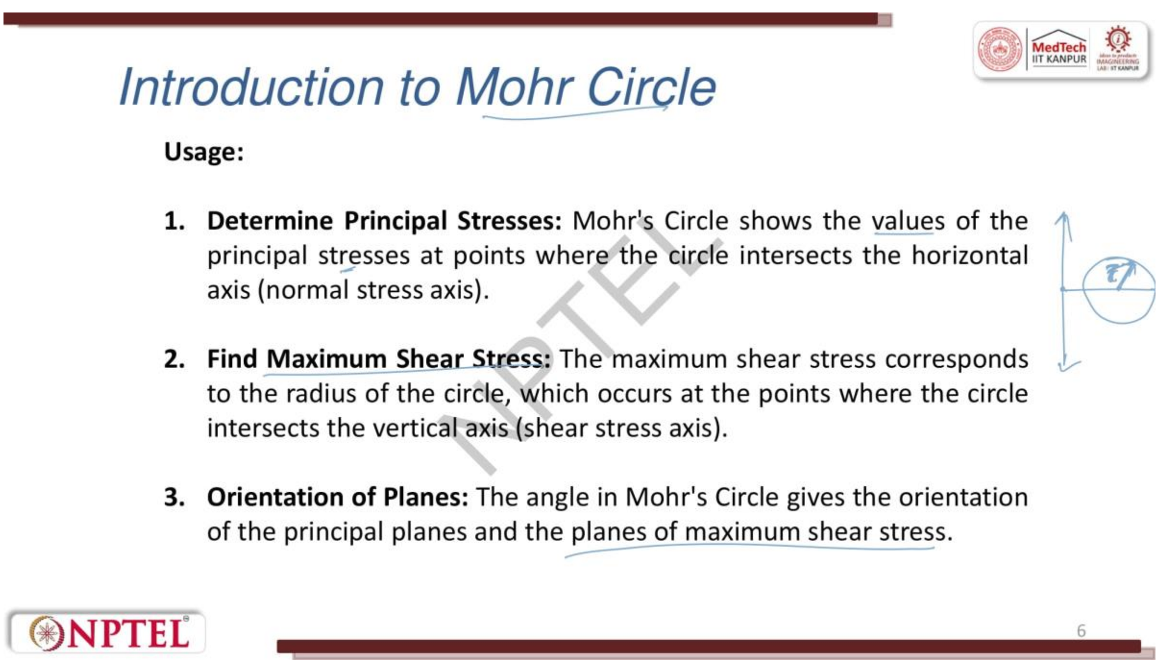
Keep that in mind. It is a graphical tool used in engineering to analyze and visualize the state of stress. So Mohr circle is a graphical tool and state of to analyze and visualize the state of stress at a point in a material. This is important. So graphical tool state of stress in at a given point.

In a material, it provides a convenient way to determine principal stresses, maximum shear stress and its orientation of the plane on which the stresses act. So, the important points are going to be graphical tool. Second thing, it tries to find out the state of stress at a point. And then what are you trying to do? It conveniently tries to tell the principal stress(es) and maximum shear stress.

Stress. Why is it important? Because this will try to give us the feeling what is the stress acting on a point and accordingly we will try to design or add material, delete material to meet out to the service condition. The graphical representation. Mohr's circle represents the relationship between normal stress and shear stress for different orientation of a 2D stress element. So, the square whatever we did is the element and we try to take it σ_x, σ_y, then we try to say a τ term.

So, all these things are there, right? So, it tries to give the relationship between σ and τ. It helps to quickly determine the principal stress and maximum shear stress without doing any complex calculation. So this graphically represents and again it is a first principle. So Mohr's circle was good, had been working very nicely for 2D elements.

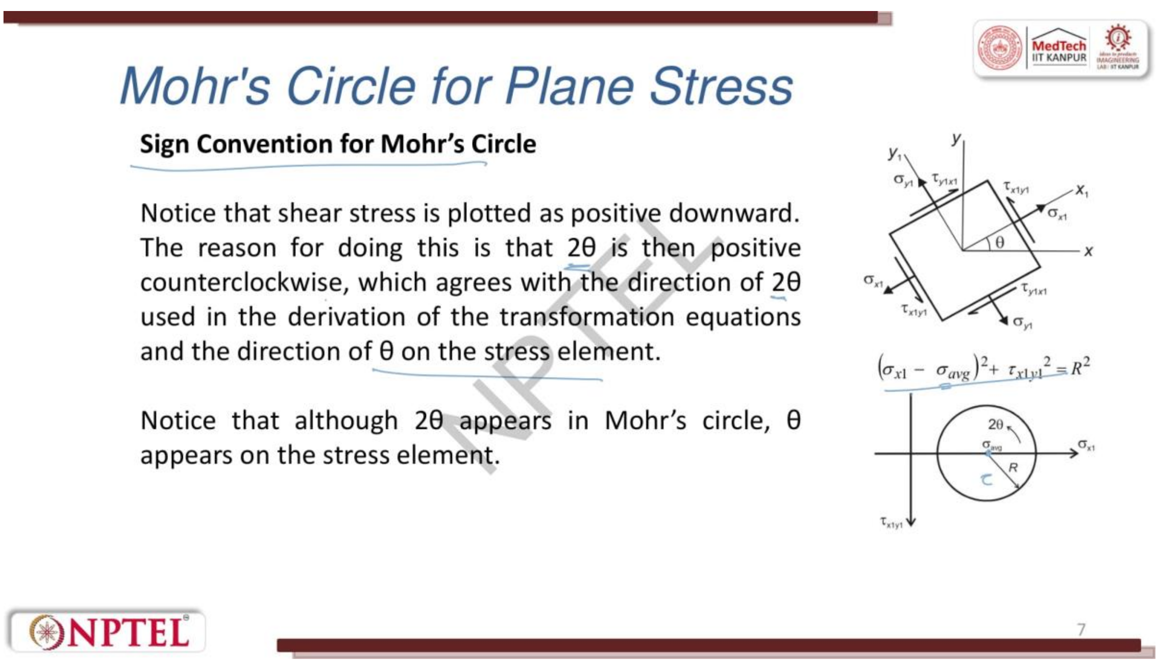
People have extrapolated and moved towards 3D elements and there also they do Mohr circle analysis with some modifications. But it is very good for a 2D representation graphically, the relationship between principal stress and maximum shear stress.



So what is its usage? It helps in determining the principal stresses. Mohr circle shows the values of the principal stress at a point where the circle intersects the horizontal axis.

So, basically what we will do is we will try to have a vertical plane, a horizontal plane and then since it is said as a circle, you will try to draw a circle. Now, when you try to draw a circle, you will have to know the radius. So, all these things will come from the relationship between principal stress and shear stress. It helps in finding maximum shear stress, right? The maximum shear stress corresponds to the radius of the circle τ.

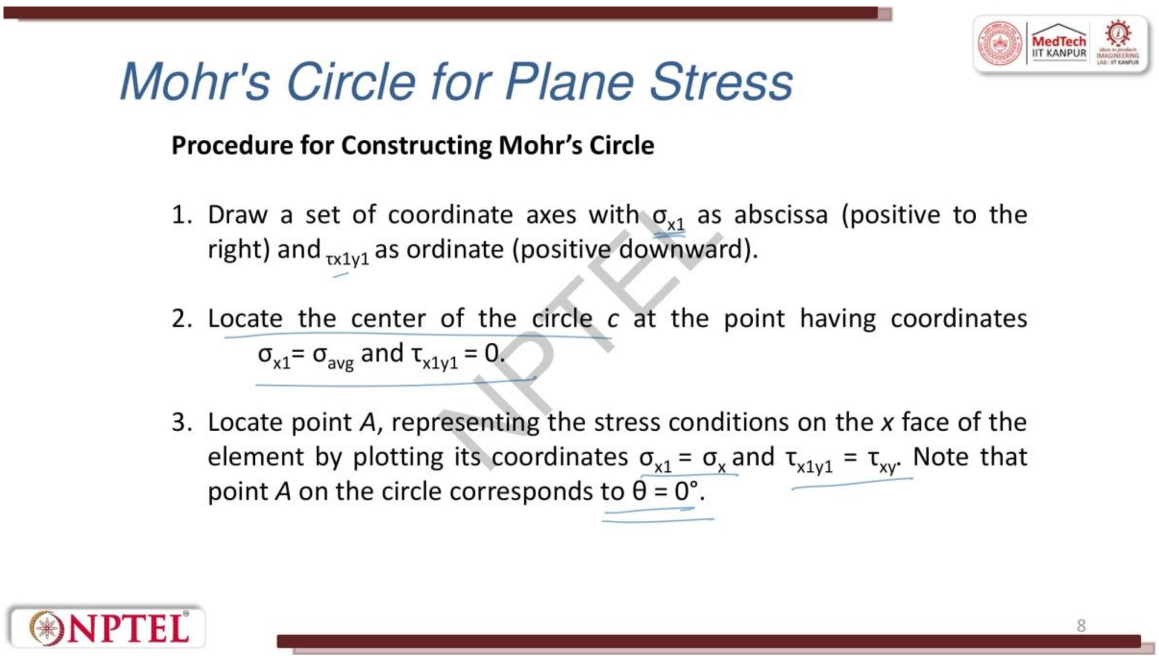
So, which occurs at a point where the circle intersects the vertical line. So, it is here, but I am just, in the representation, I am saying r is here. Orientation of the plane, the angle in Mohr circle gives the orientation of the shear plane and the plane of maximum shear stress. So by this time you would have understood Mohr circle is a graphical representation. It is a 2D which is going to give me a relationship between normal stress or principal stress with maximum shear stress.



When we try to do the Mohr circle, the most important thing is the sign convention. This is where people always try to do simple mistakes. So, it is a good idea to follow the sign convention and make it a standard practice when you even start solving the problem to note down the signs. Notice that the shear stress is plotted as positive downward. The reason for this is that 2θ is then positive counterclockwise, which agrees with the direction of 2θ used in the derivation of the transformation equation and the direction θ on a shear element.

So, downward θ. Notice that although 2θ appears in Mohr circle, θ appears on the shear element. So you can see here, this is what we were discussing till now. You will have a vertical axis, you will have a horizontal axis, you will try to fix the σ point and then you will try to have radius as τ, then the 2θ goes in this direction. So, it can be expressed in a equation form.

So, which is nothing but $(\sigma_{x1} - \sigma_{avg})^2 + \tau_{x1y1}{}^2 = R^2$. From this you can try to find out what is stress average.



The procedure for constructing the Mohr's circle. We will see the procedure and next we will try to draw. First step is draw a set of coordinate axis with σ_{x1} as abscissa, τ_{x1y1} as ordinates.

Then locate the center of the circle C at the point having coordinates $\sigma_{x1} = \sigma_{avg}$ and $\tau_{x1y1} = 0$. So, how to locate the center? We follow this. Locate a point 'A' representing the stress conditions on the x axis of the element by plotting its coordinate $\sigma_{x1} = \sigma_x$ and $\tau_{x1y1} = \tau_{xy}$. Note that the point 'A' on the circle corresponds to $\theta = 0°$.

Mohr's Circle for Plane Stress

Procedure for Constructing Mohr's Circle

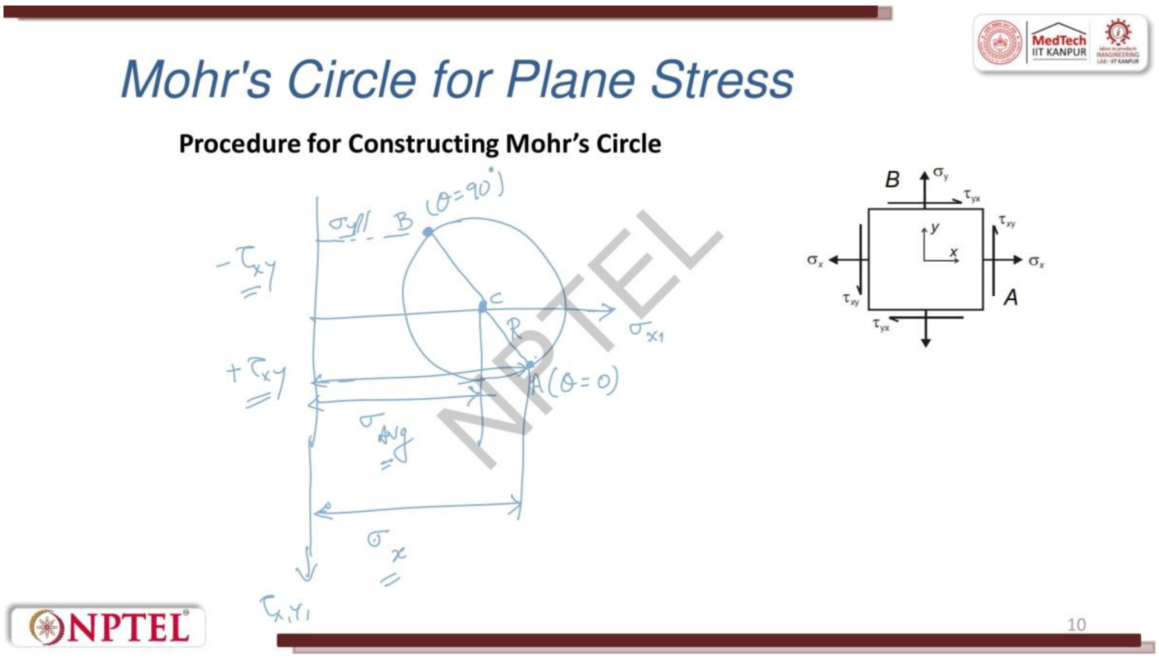
4. Locate point *B*, representing the stress conditions on the *y* face of the element by plotting its coordinates $\sigma_{x1} = \sigma_y$ and $\tau_{x1y1} = -\tau_{xy}$. Note that point *B* on the circle corresponds to $\theta = 90°$.

5. Draw a line from point *A* to point *B*, a diameter of the circle passing through point *c*. Points *A* and *B* (representing stresses on planes at 90° to each other) are at opposite ends of the diameter (and therefore 180° apart on the circle).

6. Using point *c* as the center, draw Mohr's circle through points *A* and *B*. This circle has radius *R*.

Now, let us locate the point 'B'. Point 'B' represents the stress condition on the y-phase of the element by plotting its coordinate $\sigma_{x1} = \sigma_y$ and $\tau_{x1y1} = -\tau_{xy}$. So, note the point B on the circle corresponds to $\theta = 90°$. So that is why this minus comes into existence. Then draw a line from point A to point B, a diameter of the circle passing through the point C. Point A and B are at opposite ends of the diameter.

So the point A and B which represents the stress on the plane at 90 degrees to each other and are at opposite ends of the diameter therefore 180 degrees apart on the circle. Using point C as the center, draw a Mohr circle through point A and B. The circle has a radius of R.



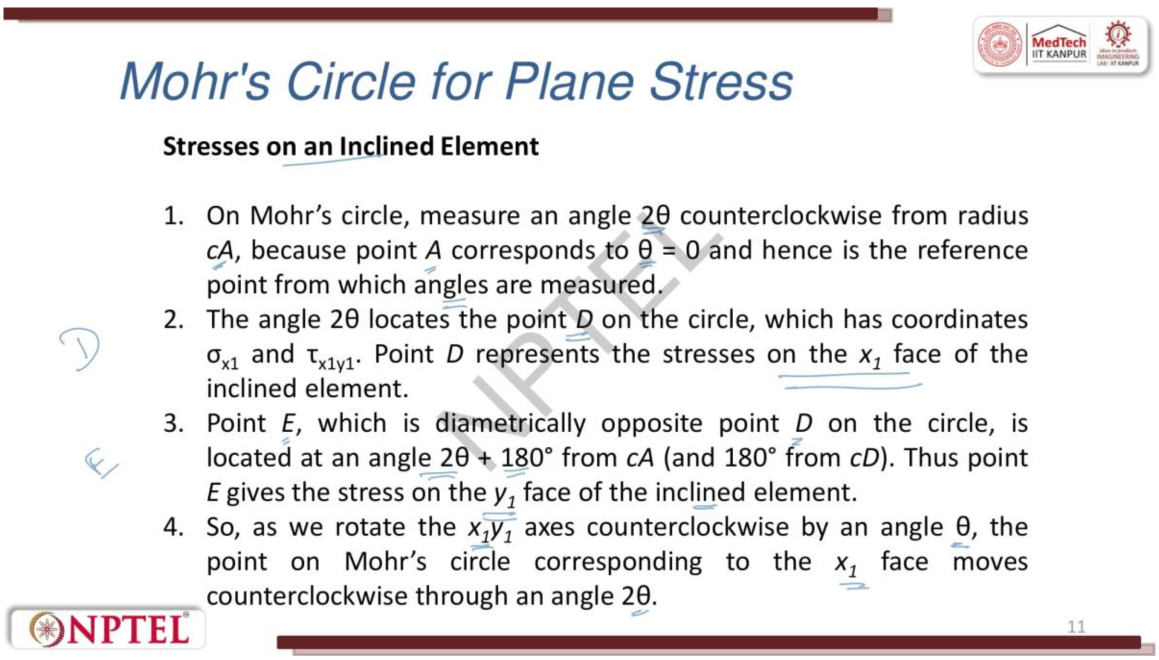
Now, let us try to see how to draw the Mohr circle. Whatever we have seen till now, let us see how do we draw it. So, first we will try to draw the axis R. So, this is going to be $-\tau_{xy}$ and this is going to be τ_{xy}. And along this direction, we are going to have σ_x. So, here we try to find out the σ_{avg}.

So, this is σ average for you. Now with this, what we are trying to do is, we are trying to draw a circle. So the circle, this is point C. So point A and B are there. This is point A, point B. This is the radius which we were talking about. At point A, $\theta = 0$ degree and at point B, $\theta = 90$ degree.

So, the point B will be the x-y axis, the x-axis will be -τ. So, the shift along the σ_x will be σ_y. And point A will be σ_x. And this line will be τ_{x1y1}. So this is how we try to draw a Mohr circle. So the point which is kept as a center along the σ_x is going to be the point C, which is nothing but σ_{avg}.

So from here, this is a center. So with the radius, we try to draw. AC is the radius R, we try to draw a point A and we try to draw a point B. So what is A and B? A and B are nothing but the σ_y point is going to be B and σ_x point is going to be A. So now you join AB passing through the point C. So now C becomes the radius. So, when we try to draw this and the magnitude of A is also defined by τ_{xy}.

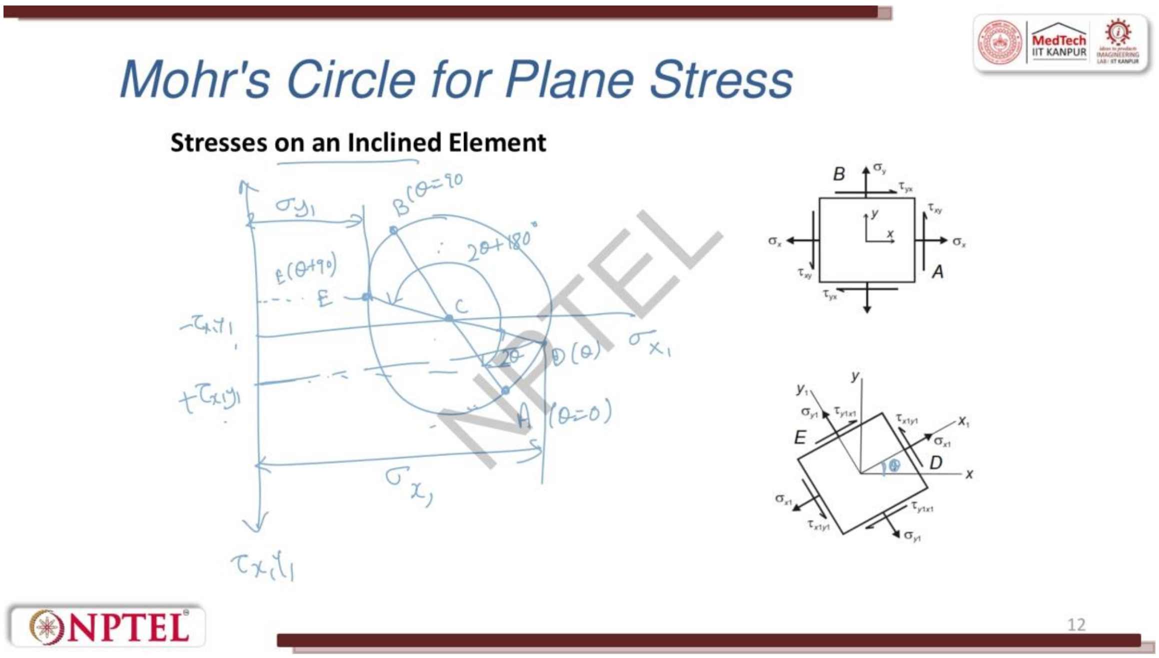
So, this is magnitude for A. So, magnitude A along the τ_{xy} line will be will be τ_{xy} at point A and minus τ_{xy} at point B. The x axis displacement will be for A it is σ_x and for B it is σ_y. I am sure you will understand the procedure. For drawing the Mohr circle, Mohr circle is a graphical representation tool which tries to give the relationship between the principal stress and the maximum shear stress.



The Stresses on an Inclined Plane. On Mohr circle, measure an angle 2θ counterclockwise for a radius cA.

Because A corresponds to $\theta = 0$ and hence is the reference point from which angles are measured. The 2θ located locates the point D on the circle. We will get to that when we try to draw the inclined plane. The angle 2θ locates the point D on the circle which has coordinates σ_{x1}, τ_{x1y1}. The point D represents the stress on x_1 face of the inclined plane.

We try to have a point E. So we are introducing a D point, we are introducing a E point. You will see the drawing when we do it. Point E which is diametrically opposite point of D on the circle is located at an angle 2θ + 180 from cA and 180 from cD. Thus points E gives the stress on the y_1 plane face of the inclined angle. So, now, we rotate x_1y_1 axis counterclockwise by an angle θ. The point on Mohr circle corresponds to the x_1 face moves counterclockwise through an angle 2θ.

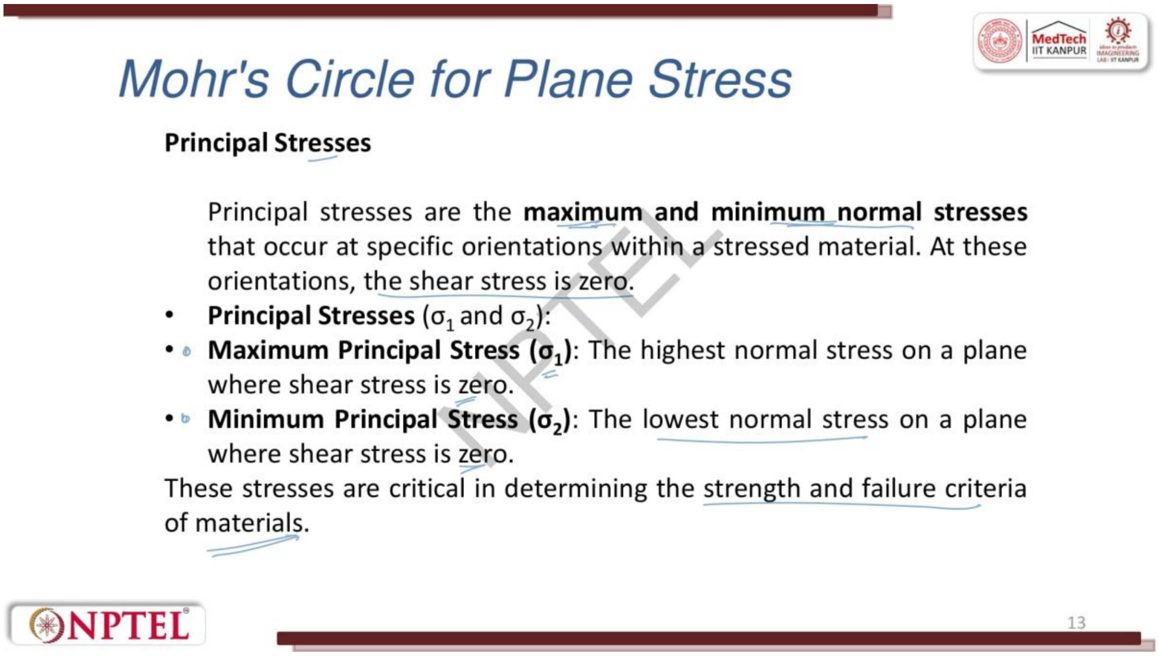


So, how is this? This is the θ, whatever we have drawn. This is the normal one. With here you are rotating at an angle θ.

Now this θ also has to be taken care in our Mohr circles. So if we try to draw. This becomes your τ_{x1y1}. This is σ_{x1}. You have a center which is nothing but σ average then you have a circle you have a point A, so which hits the other end which is point B. Now there is a shift which is happening this is the 2θ shift, and this angle will be 2 θ plus 180 degrees.

Now this point becomes your E point and this point becomes your D point and this is your C. Now if we try to represent, this becomes your σ_{x1} and this becomes your σ_{y1} and you will have a magnitude along this direction, this becomes your minus τ_{x1y1} and this becomes your B, this becomes your plus τ_{x1y1} and here E is, so D is θ, A is θ = 0, D is θ = 90, E is θ + 90, right. So this is the angle shift which is done.

So this is the θ which is shifted. So this is how you try to represent in the Mohr circle of a plane stress, the stress on an inclined element.



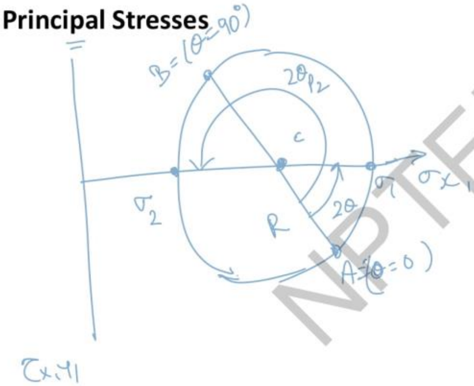
So moving forward, if we wanted to find out for a Principal Stress, the Principal Stresses are the maximum and minimum normal stresses that occur at a specific orientation within a stressed material. At these orientations, the shear stress is always 0. Keep that in mind.

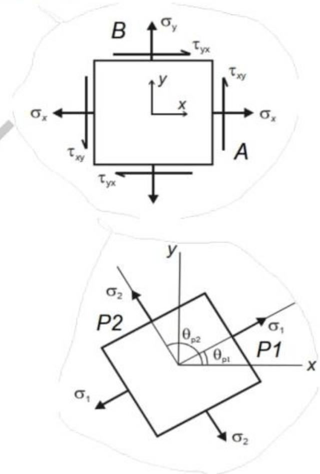
The shear stress will be always 0. So, Principal Stress are the maximum and minimum normal stresses. And at these orientations, the shear stress is 0. Principal stress can be presented as σ_1 and σ_2. Maximum principal stress will be represented as σ_1, the highest normal stress on a plane where the shear stress is 0.

Minimum principal stress is σ_2 where the lowest normal stress on a plane where the shear stress is 0. So both the cases the shear stress should be 0. This is the maximum value and this is the minimum value. These stresses are critical in determining the strength and the failure criteria of the material. So we always look for these two data points.

Mohr's Circle for Plane Stress

Principal Stresses





So, how is it represented in a given figure? This is τ_{x1y1}. This is σ_{x1}. Point C, this is a point where A equal to 0 means $\theta = 0$, where point B, $\theta = 90$ degrees, ok.

And this one is represented as 2θ and this is R. And this will be 2θ where this is σ_2 and this will be σ maximum is σ_1 and this will be σ_2. So, this is σ_{x1}, σ_1, σ_2 and here you will have 2θ, this is $2\theta_{p2}$. So, this is how you represent the Principal Stress in a Mohr circle diagram; Maximum, Minimum Principal Stress σ_1, σ_2 and then it is inclined at an angle 2θ AB and the radius whatever we are trying to do.

Mohr's Circle for Plane Stress

Maximum Shear Stress

- This is the highest shear stress that occurs within a material, typically at an orientation of 45° to the principal stress directions.
- **Calculation:** The maximum shear stress can be found using the principal stresses σ_1 and σ_2

$$\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2}$$

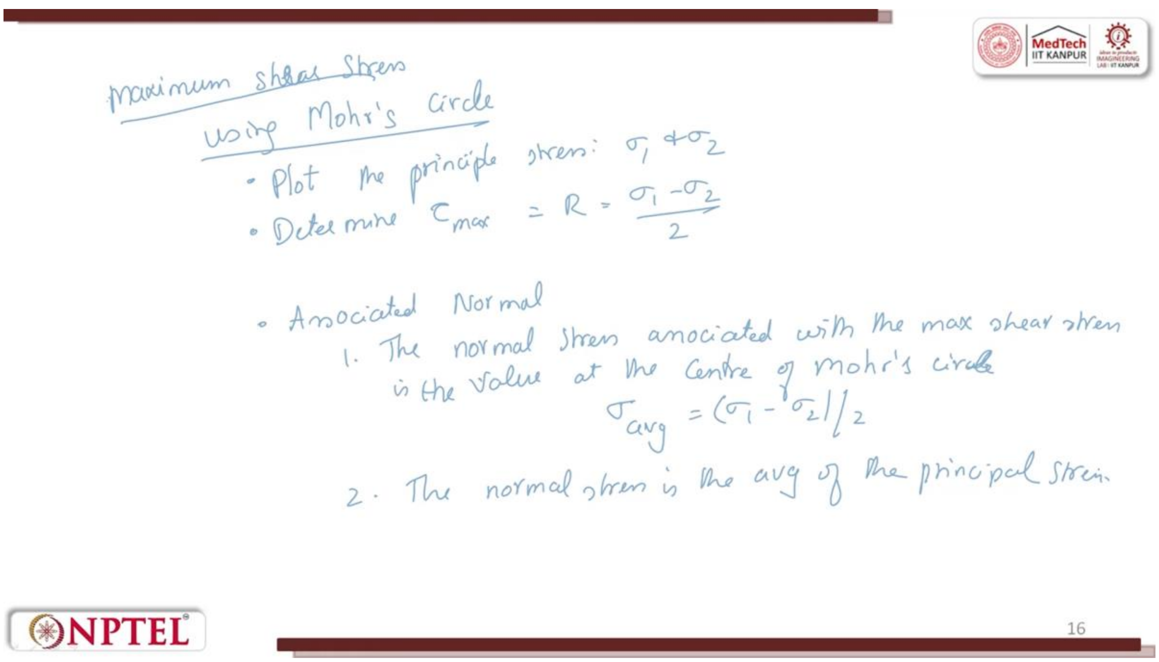
- This occurs on planes that are oriented 45° from the principal stress directions.

The next is Maximum Shear Stress. So we have already found out σ_1 and σ_2 which is minimum principal stress. This is maximum principal stress.

Now we are trying to find out Maximum Shear Stress. This is the highest shear stress that occurs within a material typically at an orientation of 45 degrees on the principal stress direction. So the calculation is done.

The maximum shear stress can be found using the principal stress σ_1 and σ_2.

$So\ \tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$. This occurs on planes that are oriented 45 degrees from the principal stress direction.



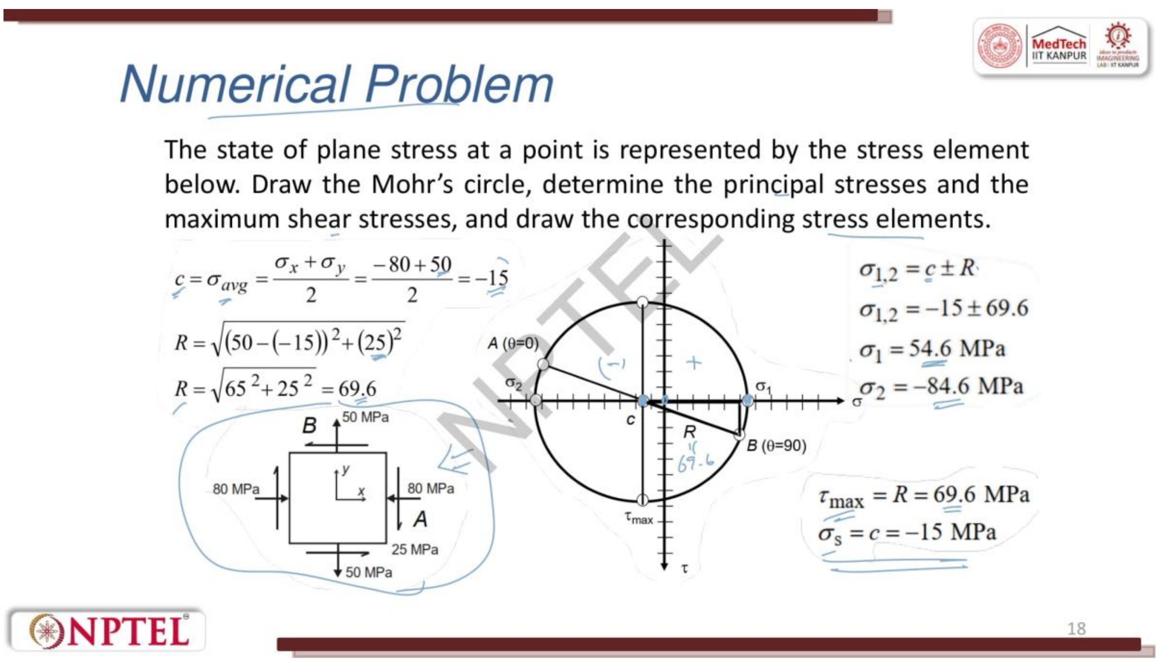
So if we want to find out the maximum shear stress, we use more using Mohr circle first what we do is you plot plot the principle plot the principle stresses. On the Mohr circle, so what are you plotting?

σ 1 and σ 2. Next, you determine τ max, which is nothing but r equal to σ 1 minus σ 2 by 2. So now, the associated normal stress if you want to find out. So, what is associated normal stress? The normal stress associated with the with the max maximum shear stress is the value at the center of Mohr circle, which is nothing but σ average is equal to σ 1 minus σ 2 by 2.

So the normal stress is the average of the principle. So, if you want to find out the maximum shear stress, you have to plot σ 1, σ 2 and then τ max you can find out like this. Associated normal stress, the normal stress associated with the maximum shear stress is the value at the center of the Mohr circle which can be defined like this. So, how do we represent the maximum shear stress in a Mohr circle diagram? This becomes your σ 1 or this becomes σ x. This becomes τ x1y1.

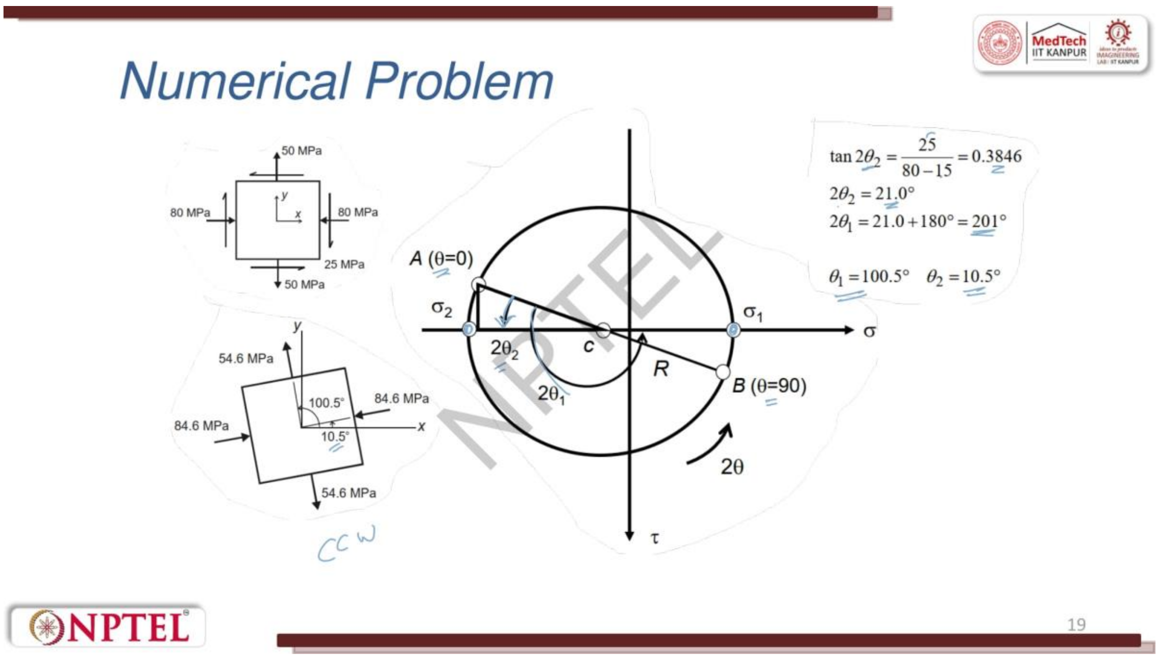
So, you will have a center which is average. So, you draw a circle. This is the τ min. This is the τ min and from here this is the τ max, ok. So, along the point we have A, B. So, θ equal to 0, θ equal to 90 and this one is called as 2 θ.

This is a point C and then you have the σ, you have σ s here. So, if there is an angle of θ which is inclined, so correspondingly the maximum shear stress can be figured out.

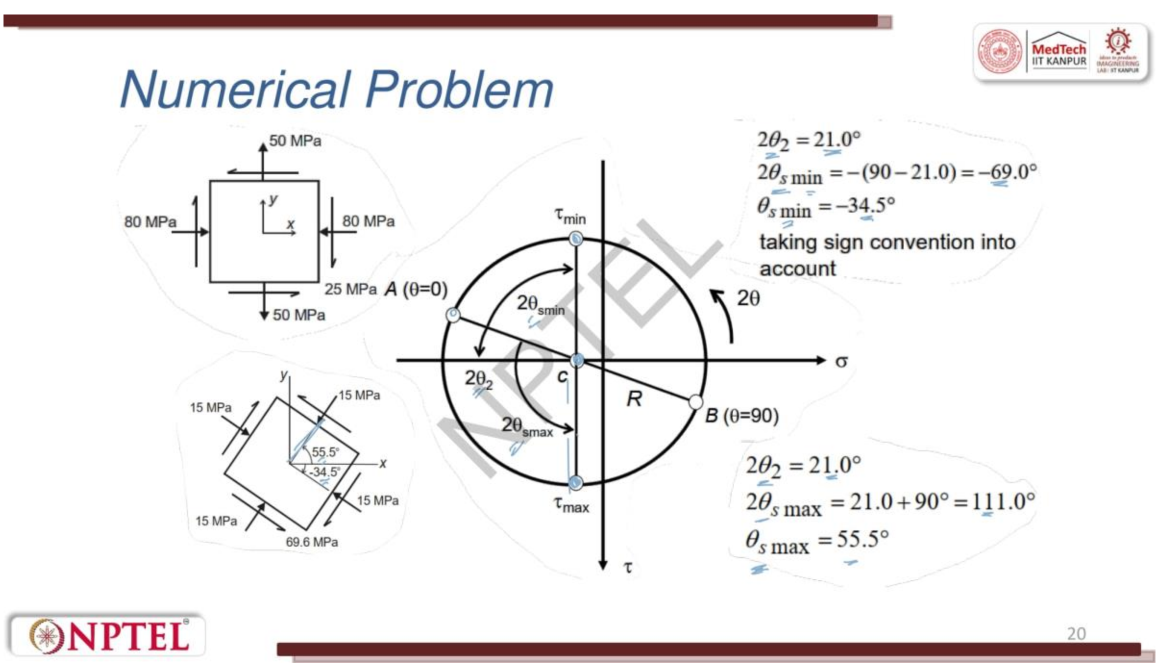


Now, let us try to solve a numerical problem using the concepts whatever we have gone through. The state of plane stress at a given point is represented by the stress element below. Draw the Mohr circle diagram.

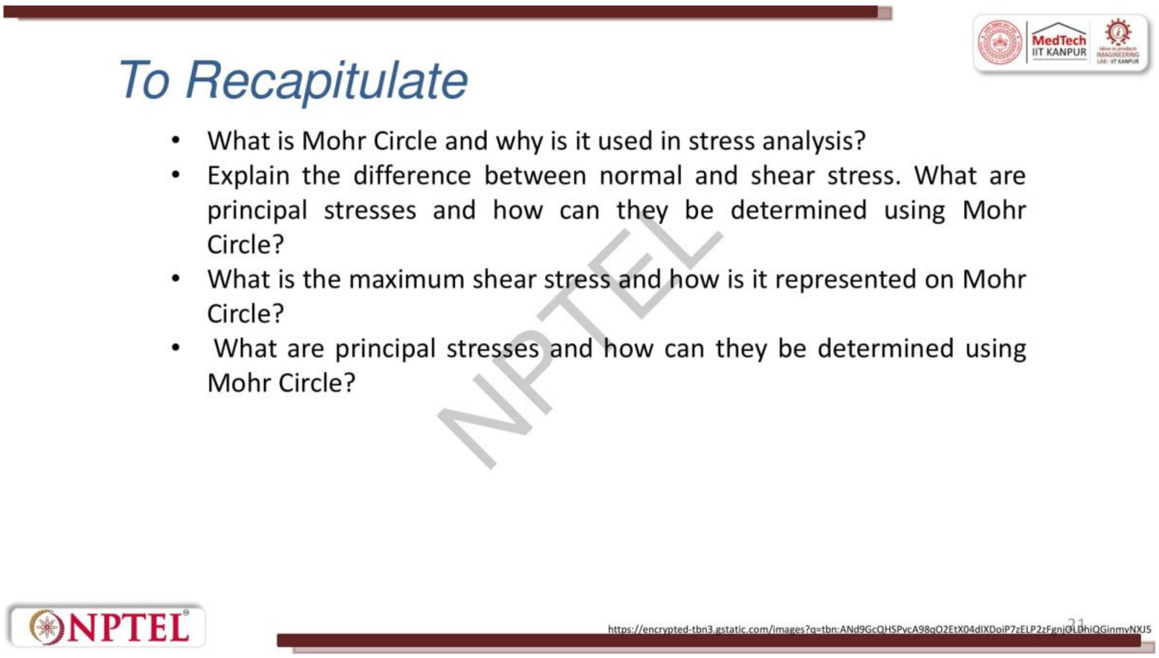
Determine the principal stress and the maximum shear stress and also draw the corresponding stress elements. (Note: See the figure.)



Now if the element is rotated by an angle 10.5 then how do we resolve it? (Note: For more, see the figure.)



So the third case which we are seeing for the same where in which you have same for the same element when the rotation is happening in this direction where it is 34.5 and which is rotated. (Note: For more, see the figure.)



To recap, whatever we have seen in this particular lecture is how to draw Mohr circle. Why is Mohr circle very important? What is the difference between normal stress and shear stress? What are principal stresses?

How can they be determined using Mohr circle? What is maximum shear stress and how it is represented in a Mohr circle? What are principal stresses? How can that be determined in a Mohr circle? This is a very important topic wherein which you are trying to find out on a point what is the relationship between the principal stress and the maximum shear stress.

References

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These are the references which we have used in making this lecture.

Thank you very much.