Basics of Mechanical Engineering-1

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Week 06

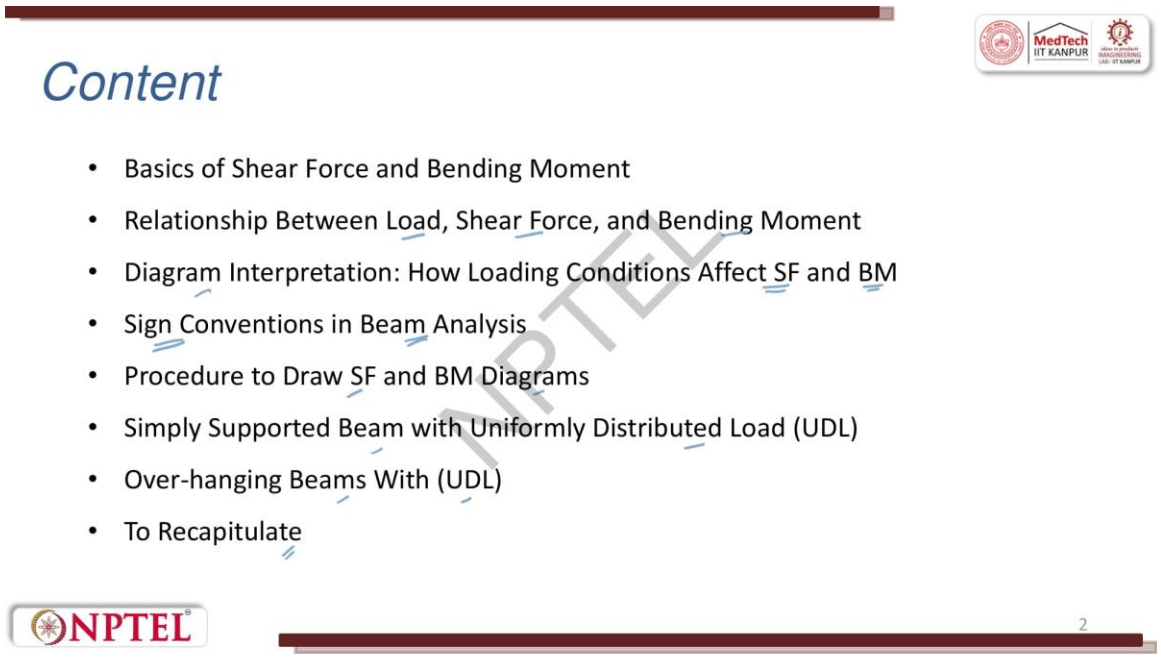
Lecture 25

Shear Force and Bending Moment Diagram (Part 1 of 3)

Welcome to the next lecture. Here we are going to discuss about Shear Force and Bending Diagram. Shear force, when there are two forces acting and what is the consequences of it. For example, when you have a deck of cards, you try to do shearing. In the top, when you try to move the card, so it is nothing but a shearing operation.

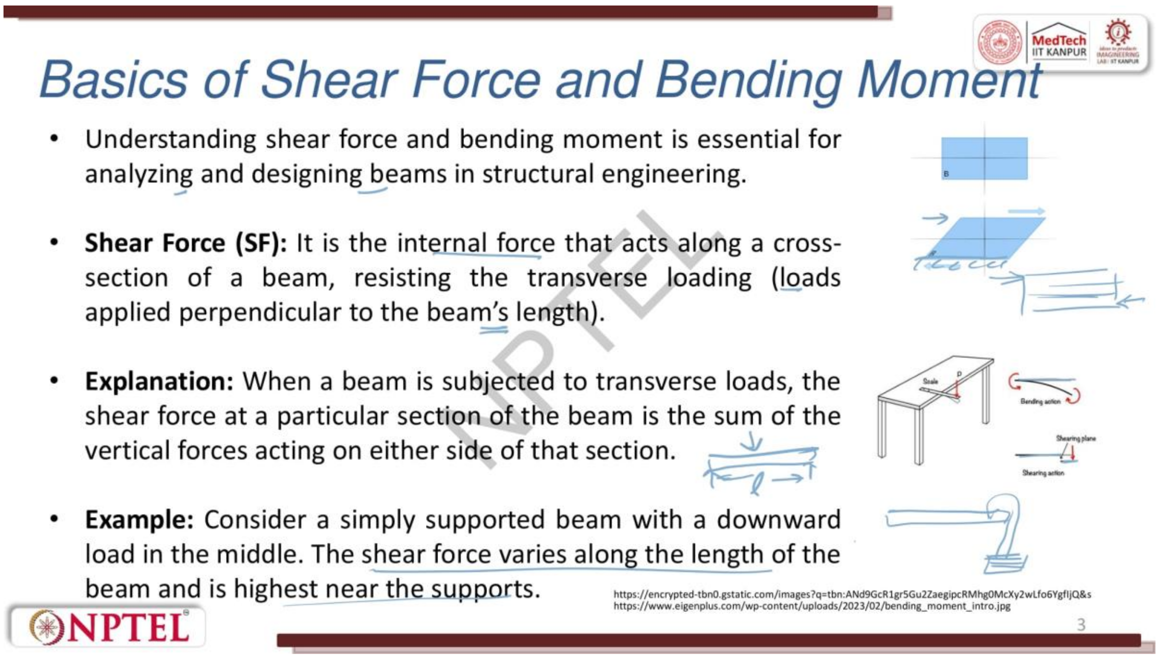
A similar force is involved in real time when you are constructing a building. Inside the building, what happens to the column? If you are doing metal cutting, again shear force comes into existence. Next topic is Bending Moment. Moment is the force applied on a particular point or on a particular area.

We try to look into that. These two topics are very important. So this will try to give you a feel what are the forces and how are we supposed to counter them while building structures like house, bridges. temples and high-rise buildings. So they always look at Shear Force and Bending Moment Diagram.



The content of this lecture is first we will try to see the Shear Force and Bending Moment. Then we will try to see the Relationship between Load, Shear Force and Bending Moment. Next, how is it interpreted using a diagrammatic form? How loading conditions affect shear force and bending moment? The sign conversion is very important both in Mohr circle and when we are doing shear force and bending moment. Convention in Beam Analysis.

Then how to draw shear force and bending moment diagram. What is the difference between simply supported beam with UDL (Uniformly Load Distributed)? Then it is overhanging beam with UDL and finally we will try to have a recap.



Basics of Shear Force and Bending Moment. Understanding shear force and bending moment is essential for analyzing and designing beams in structural engineering.

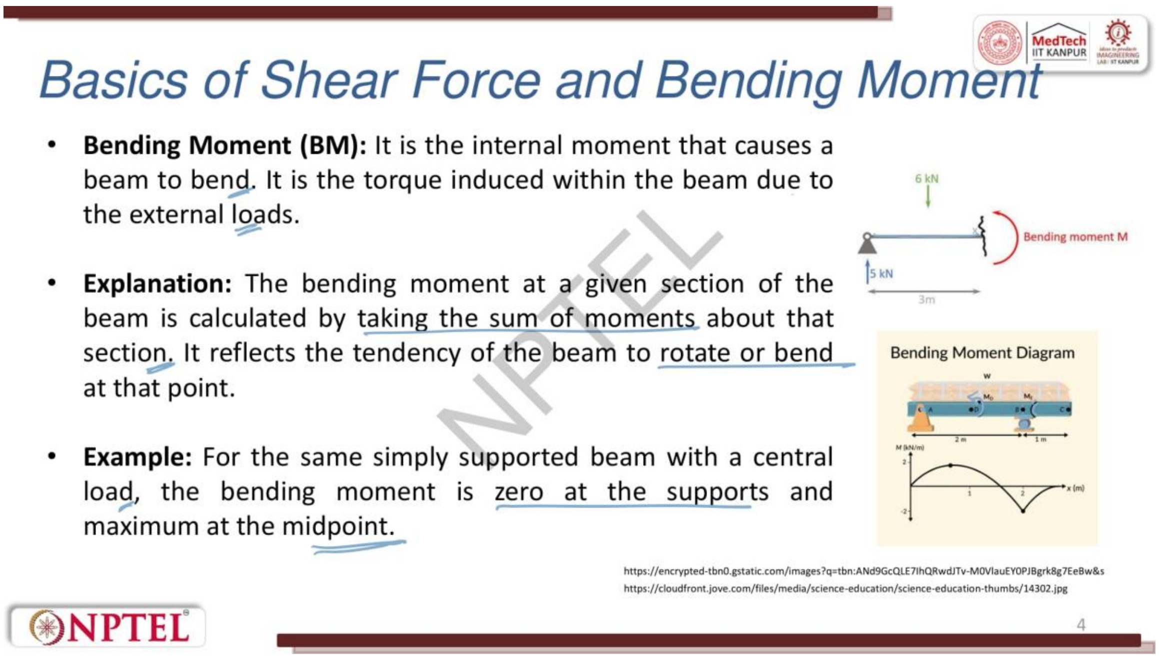
So as I told you this is an element or a box where in which you try to fix the bottom and then you try to pull or push in the direction. So what happens is a shearing force. Shear force is the internal force that acts along a cross section of a beam resisting the transverse load. The loads applied perpendicular to the beam length. So we have an element, we have it is grounded and then we have a force which pushes it.

So this force is called as the shear force. In the deck of cards as I told you, you will have a force which comes from here, you have a force which comes from the opposite side. The explanation for it, when a beam is subjected to transverse load, the shear force at a particular section of the beam is the sum of the vertical force acting on either sides of the cross section. So you try to take a scale. Fix it on top of your table.

Now move the scale having some free hang. So that makes it a cantilever. So now this is the scale and now you are trying to apply load from the top. If you want to have a real life situation, you can think of a scale which is loaded with a weight. Based upon the weight, the scale deflects.

So this action is called as Bending Action. And the shear plane is what it is this and then you have a small shear in this plane, there is a shearing action which happens. Consider a simply supported beam with a downward load in the middle. The shear force varies along the length of the beam and is highest near the support. This is very important.

When you are trying to have a simply supported beam with a downward load at the middle; the shear force varies along the length, this is along the length L of the beam and it is highest near the supports.



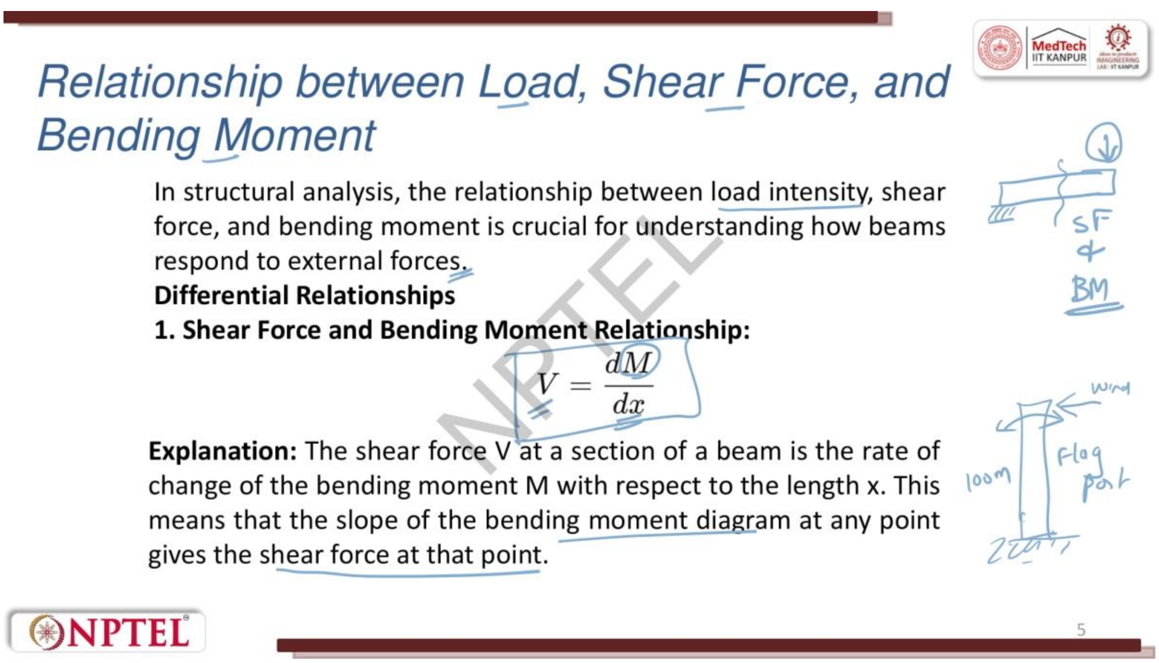
Bending Moment, it is also an internal moment. So if you go to the previous case, we had shear force (L). We had shear force also an internal force acting along the cross section of the beam. When we are trying to do bending moment, it is the internal moment that causes a beam to bend.

It is the torque induced when the beam due to the external load. So you have a beam or a rod and it is supported. You have a cross section, right. And then you are trying to vibrate it, so there is a bending moment which happens. The explanation is, the bending moment at a given section of the beam is calculated by taking the sum of the moments about the section.

It reflects the tendency of the beam to rotate or bend at the point. For example, you have a scale or a I cross section or a flat plate which is fixed at one point and it is placed on top of a roller on the other side. So, there is a moment which happens at the points. So, that we calculated and we represented in this diagram. So, what is the Bending Moment?

It is nothing but a sum of moments about a section. It reflects the tendency of the beam to rotate or bend. So when I support it here, then if I support it here and I support it here, now if I want a load to happen this way, because of the resistance it can try to travel in this direction. So here what happens is there is a rotation or bending. It can happen this way or it can happen this way.

So for the same simply supported beam with the central load, the bending moment is zero at the supports and maximum at the midpoints. Now, are you able to differentiate shear force and bending moment? Shear force is that, consider a simply supported beam with a downward load in the middle, the shear force varies along the length and it is highest near the supports. Whereas, when we talk about bending moment for the same simply supported beam with the central load, the bending moment is 0 at the supports and maximum at the midpoints.



What is the relationship between Load, Shear Force and the Bending Moment? This is what comes to your mind. I have a structure or I have a flat plate or I have a scale. Now in this scale, I try to apply load. So now I try to fix this point and I try to apply load. So now I should understand what is the relationship between load applied, shear force and bending moment.

In structural analysis, the relationship between load intensity, the magnitude of the load applied, shear force and bending moment is crucial for understanding how beams respond to the external force. It can so happen it can bend and stay there. So, you rotate it. Now, you have a vertical rod and then you have a fixed one. Now, there is a wind velocity.

So, based upon the wind velocity, the rod can deflect. Assume that this is a flag post, tall may be 100 meter. So, now it is erected. Now, when the wind velocity is there, how is the wind velocity going to respond? So, for this, now the magnitude of the wind velocity is important.

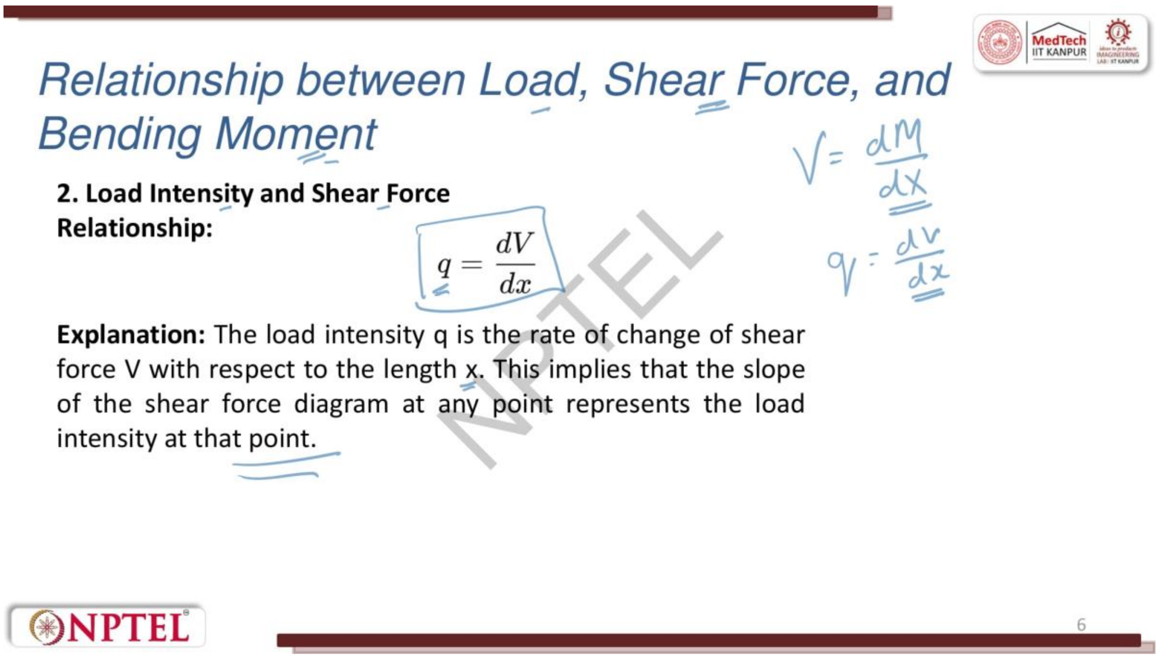
Now the deflection will happen. So it is grouted at the base only. So now depending upon the deflection, the base grouting will be made stronger and stronger. So the differential relationship is the shear force and bending moment relationship is

$$V = \frac{dM}{dx}$$

Where V is the shear force at a cross section of the beam is the rate of change of moment M with respect to the length x. It is a rate of change, rate of change of moment with respect to the length x.

This means that the slope of the bending moment diagram at any point gives the shear force at that point. Please note it down. This is very important. The means that the slope of the bending moment diagram. We will draw a bending moment diagram for every structure.

That we will discuss it in this lecture itself. When we try to get the slope of a bending moment diagram, then we get the shear force at that point. So that is why we represent $V = \frac{dM}{dx}$, rate of change of moment along the length. So what is the relationship between load, shear force and bending moment?



The relationship between load intensity and shear force is

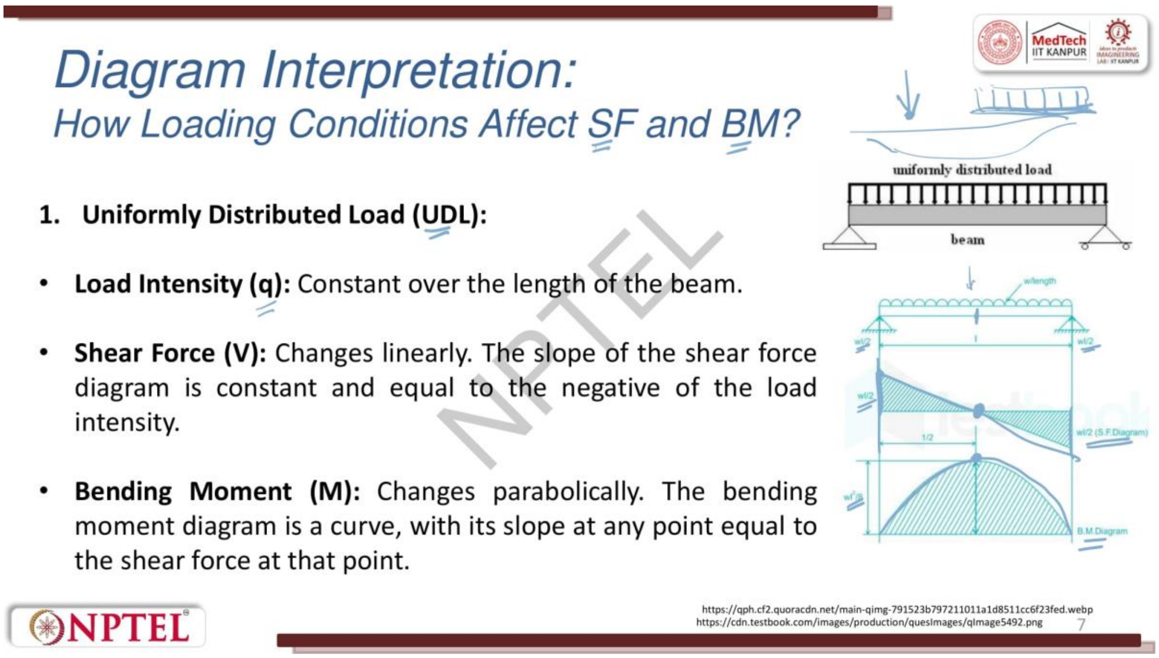
$$q = \frac{dV}{dx}$$

What is q? q is the load intensity, is the rate of change of shear force V with respect to x. This implies that the slope of the shear force diagram at any given point represents the load intensity at that point. So it is very clear there is a relationship between shear force and bending moment. So that is nothing but $V = \frac{dM}{dx}$. Rate of change of moment with respect to x gives the shear force.

Now we are trying to see what is the relationship between load versus shear force. So here we write the change of shear force V with respect to x. We will try to give the load intensity at a point. So how are the diagram represented with shear force and bending moment? So how loading conditions affect shear force and bending moment? UDL (Uniformly Distributed Load).

You have a beam, this beam is supported at the bottom and then on top of the beam you are trying to apply a Uniform Distributed Load. So, this is a point concentrated load. Now, this load is spread. So, this is sitting on a point. Now, I convert this sitting on a point into an area.

For example, let us assume you have a sharp knife to cut an apple. You are now trying to take the same apple, use a fork to pierce it. So now you see the pressure is applied over the area. Here it was concentrated at a point. So UDL is uniformly distributed around the load. And if you are going in slightly advanced studies, we can try to convert this UDL into Gaussian distribution and see what is the effect.



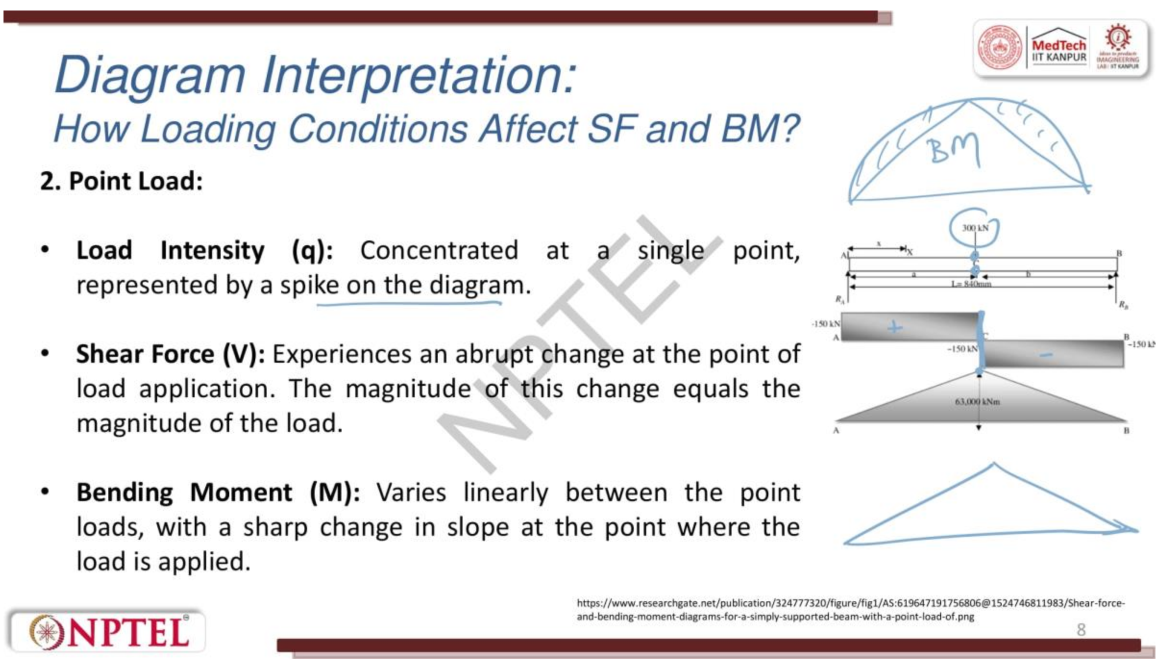
In a generalized form, we call it as Uniform Distributed Load. The load intensity q is a constant over the length beam. So why? The load whatever is getting applied is q over an area.

So that is what it is said, constant intensity. The shear force V changes linearly So for this if you want to draw a shear force diagram, you can see here, at the supports you will have maximum load and then at the center, it will try to change the slope of the shear force diagram is constant and is equal to negative of the load intensity it starts from here. The load and then it comes to at the midpoint it comes to 0 and then it crossovers here. So at the midpoint you will have a 0 or the slope of the curve gets attached to a 0 point.

When we try to draw the bending moment, we get the bending moment in a parabolic form. So here you can see the bending moment changes parabolically. The bending moment diagram is a curve. It is a parabola, so it is a curve. With its slope at any point equal to the shear force whatever we draw.

So here it is very clear this is Shear Force Diagram, this is Bending Moment Diagram. So, this is exactly half L/2. So, at the L/2 point it crossovers. So, here you can see w here it is wL/2. So, it is wL/2.

So, here it will be wL/2 will be the shear force, it will come to 0 then go to wL/2. And here it will be w. So, it is a parabola. So, the highest point will be $wL^2/3$. So, this will be the magnitude.

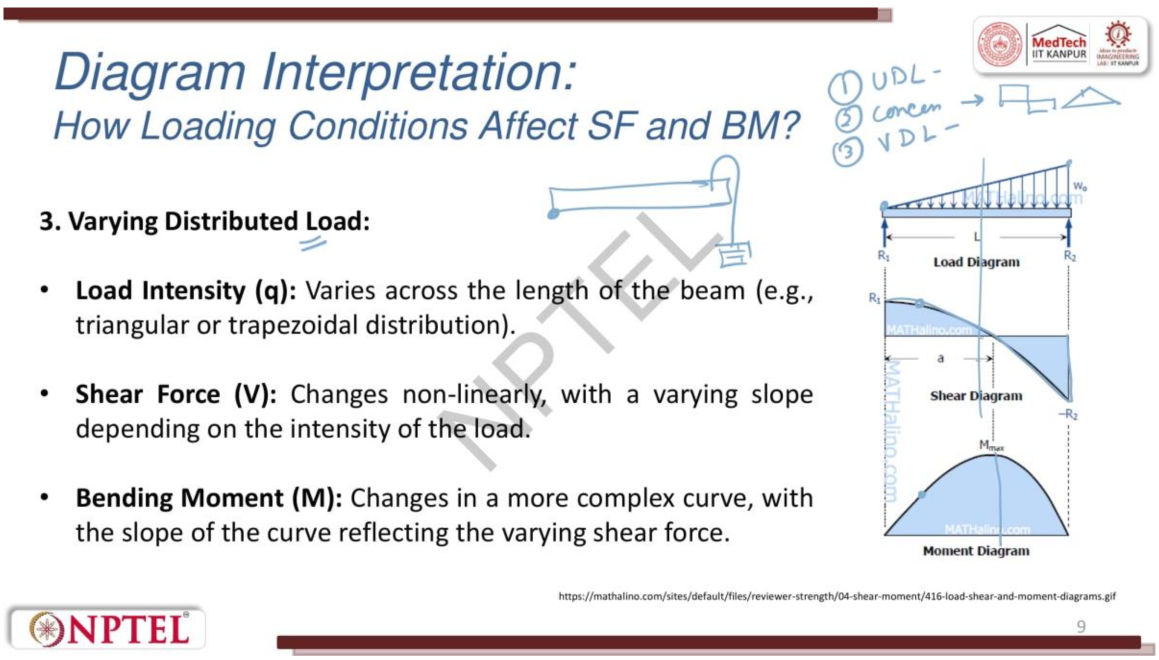


The next one is the Point Load. So, first we saw UDL (Uniformly Distributed Load). The next one what we see is a Point Load. Point load means the load is concentrated at a given point, a single point representing a spike on the diagram. So here the experience of the beam with respect to a point load, what will happen is there will be a sudden change in the shear force. So experiences an abrupt change at a point of the load application.

The magnitude of this change equals the magnitude of the load. So whatever load you apply, there is a change from if you take positive to negative. So whatever is the load, there will be a change in the load. And interestingly, exactly at that point, you try to get the maximum bending moment. When it is UDL, you got a bending moment like a curve.

When it is point contact, this is bending moment. When it is point contact, you try to get it as a triangle. So, in some of the case, so it is also interesting. You can convert this bending moment, the uniform UDL into a simply supported for calculation simplicity. So you can see here this will be the additional area, but you can try to generalize it if you want.

When there is a complex situation, you can try to do that. There is a condition where you can also have varying load. That means to say 0 at the extreme end and maximum at a point. For example, if you have a scale and if you try to load it with some weight. So what happens is at this point the load will be almost zero. When you keep going towards the end there will have a maximum load.



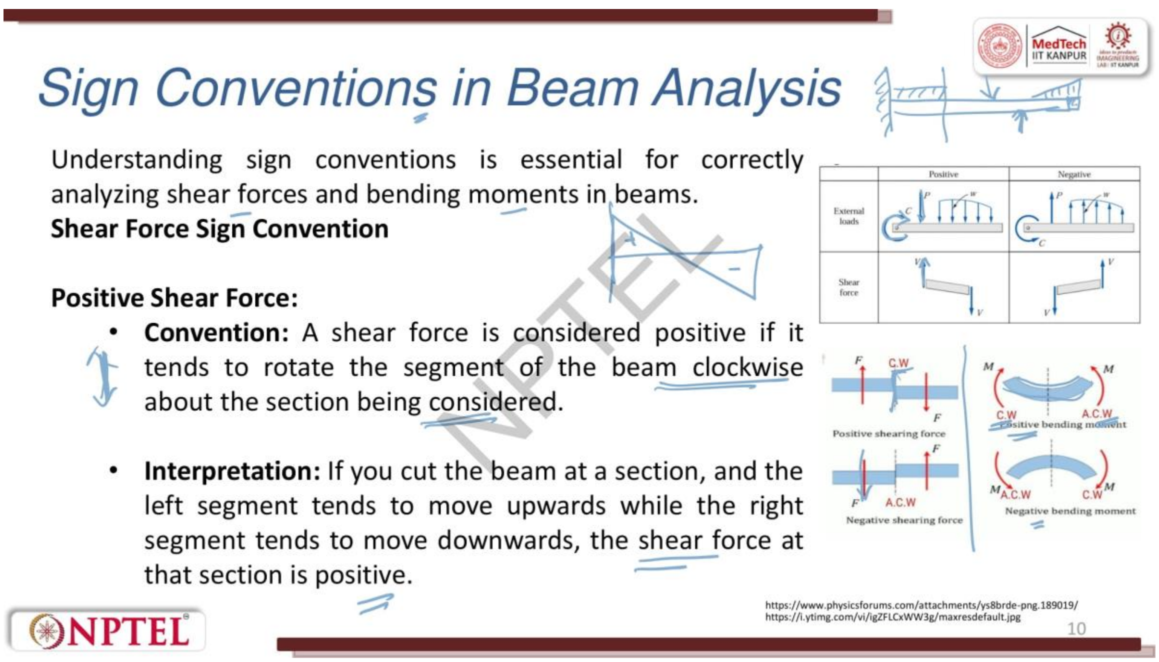
So this one is called as Varying Distributed Load. And when you discuss about Varying Distributed Load, you can see that varies across the length of the beam. Example a triangle or a trapezoidal distribution. If you try to take the shear force V which is again a non-linear form it expresses.

With a varying slope depending upon the intensity of the load. And it need not be at the mid. It need not be at the mid. The mid where the crossover happens of the shear force it can be offsetted. Exactly where it is getting offsetted, you will try to get the maximum bending moment.

So it will not be like a parabola, it will be like a shift parabola. So this is where you get the Bending Moment diagram. The changes is a more complex curve with the slope of the curve reflecting with the varying shear force. So whatever you try to take a slope here, you try to get the shear force. Whatever you try to take a slope here, you try to get the load.

So that is how it works. So now, till now we have seen three conditions. One is UDL. The second one is concentrated load. The third one is varying distributed load.

Three conditions we have seen. Okay. So in concentration load, the bending moment will be like a triangle and here it will be an abrupt change. In UDL, you will try to have gradual change. When we talk about VDL, it is Varying Distributed Load, you try to see the offset.



The next important topic for discussion is going to be the sign conversion in beam analysis. It is very essential for you to understand correctly what are the signs which are used while analyzing the shear force and the bending moment. As I told you, when you try to have a shear force like this thing. So, if you take positive, negative. So, how did this come positive and why is it considered as positive? Why is it considered as negative?

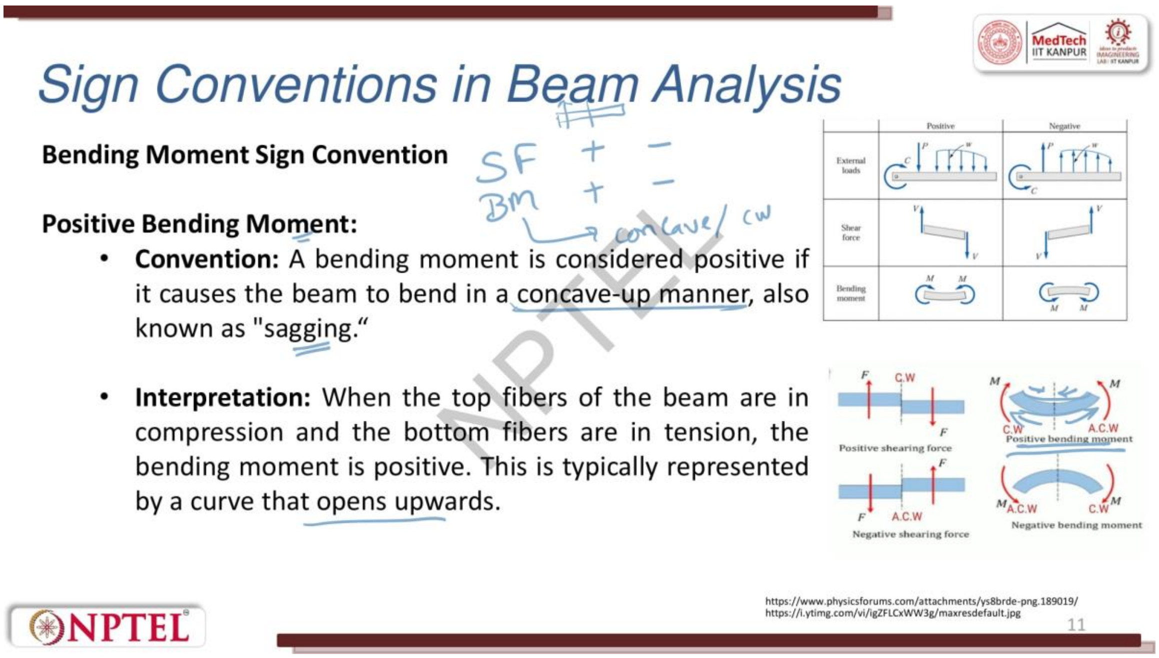
And second thing, whatever convention you follow, it has to be uniform during the complete process. For example, counterclockwise you take it positive, then all along you should have counterclockwise positive. If you have it clockwise, you have it. But there is a sign convention which is followed. I would request you to follow the sign convention.

And when it is a simple structure, you will have only varying distributed load, you will have only the load at the extreme ends, point load, right. So, but in reality what can happen is you can have a beam which is fixed, wherein which still here you can have UDL, then you can have a point load and then from here you can have a varying load and here you can support it. So, here you will keep it as 3. So, in one beam itself you can have all the 3 loads coming into existence. If that is the case then we should follow the sign conventions properly.

Positive shear force conversion. A shear force is considered positive if it tends to rotate the segment of the beam clockwise about the cross section being considered. This is positive shear force. So once you have a positive shear force, you can try to get the loads, load intensity, how is it getting distributed? The interpretation is, if you cut the beam at the section and the left segment tends to move upward while the right segment tends to move downward, the shear force at that section is positive.

For example, when we try to see positive and negative shear force, when the external loads are applied, like C or P or like R right on a beam. So, now, the shear force will be positive at the point P and at the point V you can see there, right. So, here so when we try to take a section of it, you see that the F goes up and the F goes down. So, you have in a clockwise direction when you move. So, then it is called as positive shear force.

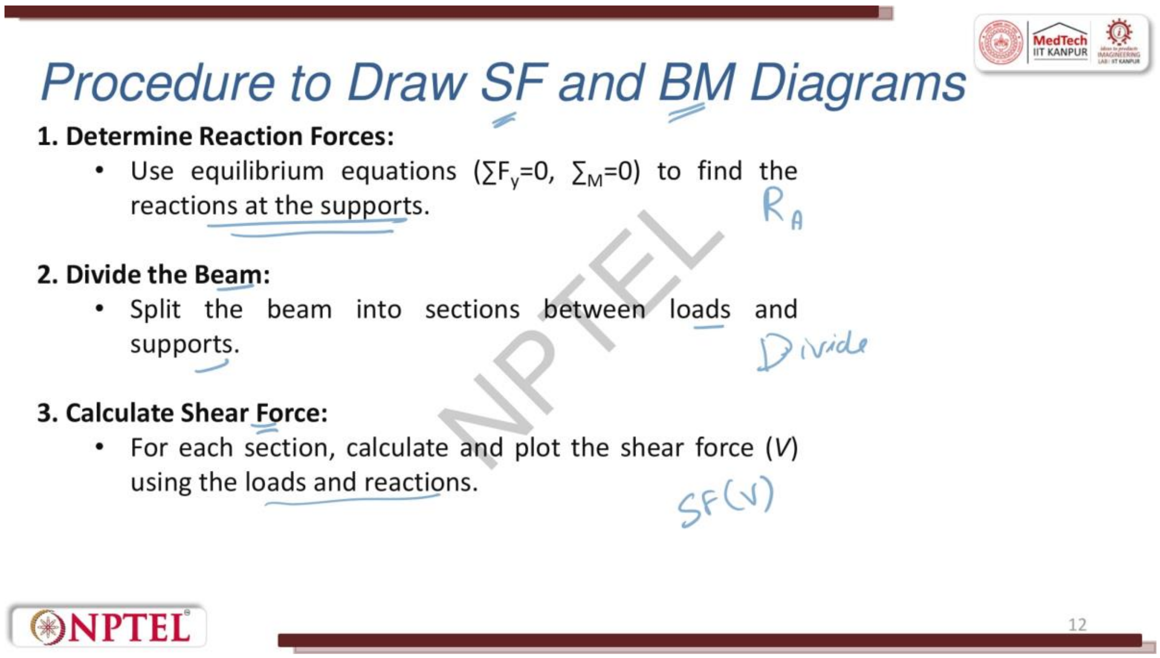
When you go back to the interpretation. If you cut the beam at the cross section and the left segment tends to move downward. When it is trying to move downward while the right segment moves backward. Upward the shear force is said to be negative, so you can also try to see the the moment which happens this is a beam and then we are trying to take a moment. So this is clockwise and this is counterclockwise.



So here if you try to take it, so this is we represent that as positive bending moment and this is represented as negative bending moment. So this is very important convention to be followed while trying to solve the problems of shear force and bending moment. so when do you get the positive bending moment your bending moment is considered positive if it causes the beam to bend in a concave up manner, also known as Sagging, it is called as positive. So, this is positive bending moment. It is sagging, right.

So, it is concave. When the top fiber of the beam are in compression and the bottom side are in tensile, the bending moment is positive. This is typical represented by a curve that opens upward. I am sure by this time you will be able to understand SF positive, SF negative, BM positive and BM negative. So BM negative, it will go towards a concave curve.

And here if the force and take a cross section goes up then it is called as Positive Shear Force. The counter of it is negative so it has to be considered in the clockwise direction.



So the Procedure to draw Shear Force and Bending Moment diagram. We will see the procedure and in the next slide we will try to solve a problem wherein which you will have more clarity. Determine the reaction force that will be the step 1.

Use equilibrium equation and try to take ($\sum F_y=0$, $\sum_M=0$) to find the reaction at the supports. This is the first step. Find out reaction forces. Next, divide the beam. Split the beam into sections between loads and support.

So the step number two, first is find out reaction force, then you try to divide the beam. Then calculate the shear force. Shear force is V. Calculate the shear force using the load and reaction.

And after calculating the shear force, you try to calculate the bending moment. Integrate the shear force diagram to find and plot the bending moment of each cross section.

Finally, you apply boundary conditions, ensure the shear force and bending moment values matches the condition at support and free ends. There are only five simple procedure to be followed to draw a SF diagram and a BM diagram. First step is to find out the reaction forces at the supports. Second step, you try to split the beam between loads and supports. The third one is calculate shear force using load and reaction force.

Load and reaction force you will try to do. Then fourth is from the shear force, you try to integrate the shear force diagram to find out and plot the bending moment diagram. The last one is apply boundary conditions and match the conditions at supports and free ends. So, these are the five steps you have to follow for drawing a bending moment and a shear force diagram.

Numerical Problem: Cantilever beam with point load

A cantilever beam of length 2 m carries the point loads as shown in Fig. Draw the shear force and B.M. diagrams for the cantilever beam.

SFD

D +800 N

Shear force remains constant between D & C

At C, the Shear Force = 800 + 500 = 1300 N

Between C & B, S.F is 1300 N

At B, Shear Force = 1300 + 300 = 1600 N

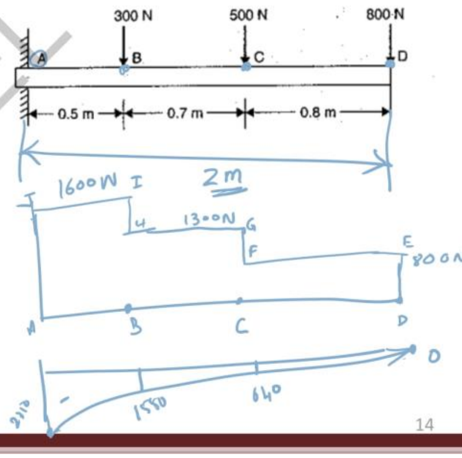
Shear force between B to A = 1600 N.

S.F at D = F_D = 800 N

S.F at C = F_C = 800 + 500 = 1300 N

SF at B = F_B = 1300 + 300 = 1600 N

SF at A = F_A = 1600 N



Numerical Problem: Cantilever beam with point load

B.M.D

At D, C, B, A.

Bending moment at D is Zero.

(i) BM at any section bet C & D at a distance x, and D is given by

$M_x = -800 \times x$ (straight line law).

(ii) BM at C, the value of x = 0.8 m.

∴ BM at C = $M_x = -800 \times 0.8 = -640$ Nm.

(iii) BM at any section between B & C at a distance x from D is given.

At C, x = 0.8 and at B, x = 0.8 + 0.7 = 1.5, x can be from 0.8 to 1.5 m.

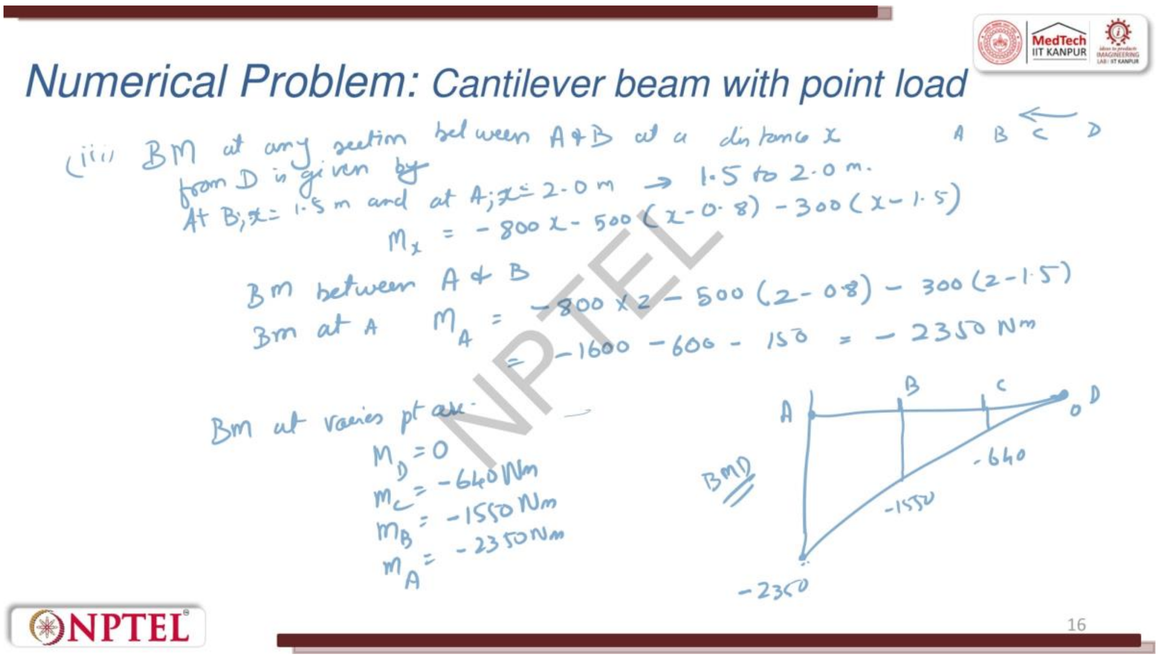
$M_x = -800x - 500(x - 0.8)$ - - - - (1)

BM between B & C also varies by a straight line law

BM at B is obtained by substituting x = 1.5 m in (1)

$M_B = -800 \times 1.5 - 500(1.5 - 0.8)$

$= -1550$ Nm



Now, let us try to solve a simple problem and understand how to draw the shear force and bending moment for a given cantilever beam.

Shear Force Diagram:

The shear force at Din + 800 N. This shear force remains constant between D and C. At C, due to point load, the shear force becomes (800+500) = 1300 N. Between C and B, the shear force remains 1300 N. At B again, the shear force becomes (1300 + 300) = 1600 N. The shear force between B and A remains constant and equal to 1600 N. Hence the shear force at different points will be as given below:

SF at D, F_D = +800N

SF at C, F_C = +800 +500 = +1300N

SF at B, F_B = +800 + 500 + 300 = +1600N

SF at A, F_A = +1600N

The shear force, diagram is shown in Fig. (b) which is drawn as:

Draw a horizontal line AD as the base line. On the base line mark the points B and C below the point loads. Take the ordinate DE = 800N in the upward direction. Draw a line EF parallel to AD. The point F is vertically above C. Take vertical line FG = 500 Through G, draw a horizontal line GH in which point H is vertically above B. Draw vertical line HI =

300N From I, draw a horizontal line IJ. The point J is vertically above A. This completes the shear force diagram.

Bending Moment Diagram:

The bending moment at D is zero:

i. The bending moment at any section between C and D at a distance x and D is given by,

$M_x = -800 \times x$ which follows a straight line law

At C, the value of $_x$ = 0.8 m

∴ B.M. at C, $M_c = -800 \times 0.8 = -640$ Nm.

ii. The B.M. at any section between B and C at a distance x from D is given by

(At C, x = 0.8 and at B, x = 0.8 + 0.7 = 1.5m. Hence here x varies from 0.8 to 105).

$$M_x = -800x - 500(x-0.8)$$

Bending moment between B and C also varies by a straight line law.

B.M. at B is obtained by substituting x = 1.5m in equation … (i),

$$\therefore \quad MB = -800 \times 1.5 - 500\ (1.5 - 0.8)$$

$$= -1200 - 350$$

$$= -1550 \text{ Nm}$$

iii. The B.M. at any section between A and B at a distance x from D is given by

(At B, x = 1.5 and at A, x = 2.0 m. Hence x varies from 1.5 m to 2.0m)

$$M_x = -800x - 500\ (x - 0.8) - 300\ (x - 1.5) \quad \ldots(ii)$$

Bending moment between A and B varies by a straight line law.

B.M. at A is obtained by substituting x = 2.0 m in equation (ii),

$\therefore M_A = -800 \times 2 - 500\ (2 - 0.8) - 300\ (2 - 1.5)$

$= -800 \times 2 - 500 \times 1.2 - 3000 \times 0.5$

= -1600 – 600 – 150

= -2350 Nm

Hence the bending moments at different points will be given below:

M_D = 0

M_C = -640 Nm

M_B = -1550 Nm

M_A = -2350 Nm

And The bending moment is shown in Fig. (c) which is drawn as. Draw a horizontal line AD as a base line and mark the points B and C on this line. Take vertical lines CC' = 640Nm BB' = 1550 Nm and AA' = 2350 Nm in the downward direction. Join points D, C', B' and A' by straight lines. This completes the bending moment diagram.

So by solving this problem, I am sure you will be able to have a hold on how to draw a bending moment diagram and a shear force diagram for a cantilever which is having concentrated loads applying at varying points. Thank you so much.