Basics of Mechanical Engineering-1

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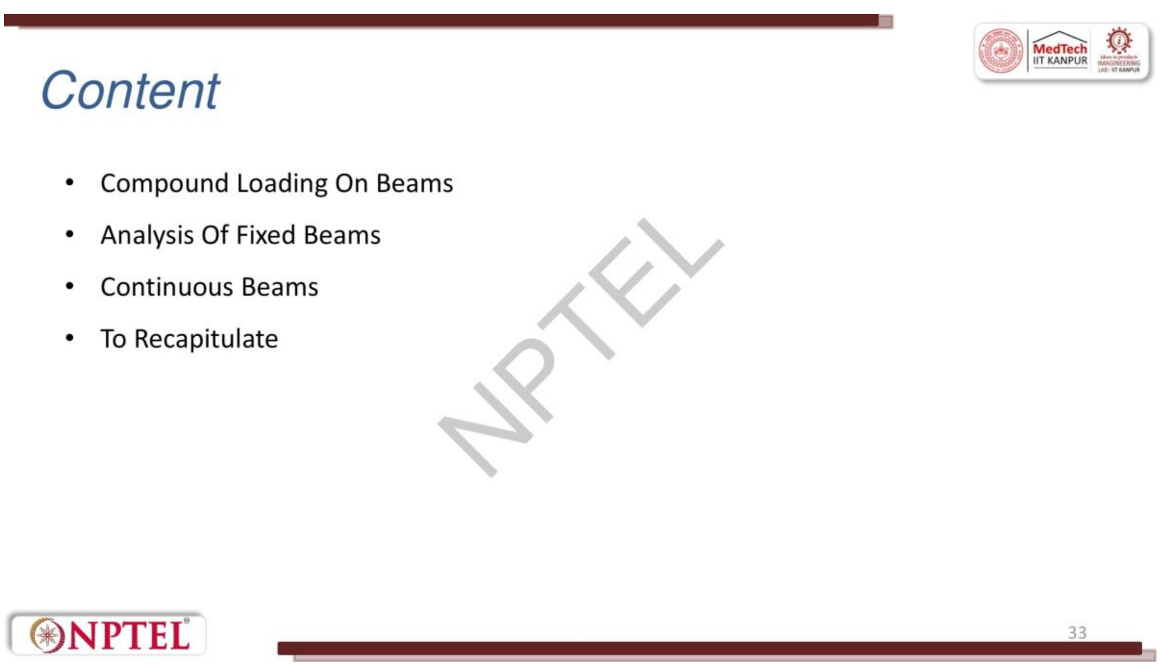
Week 06

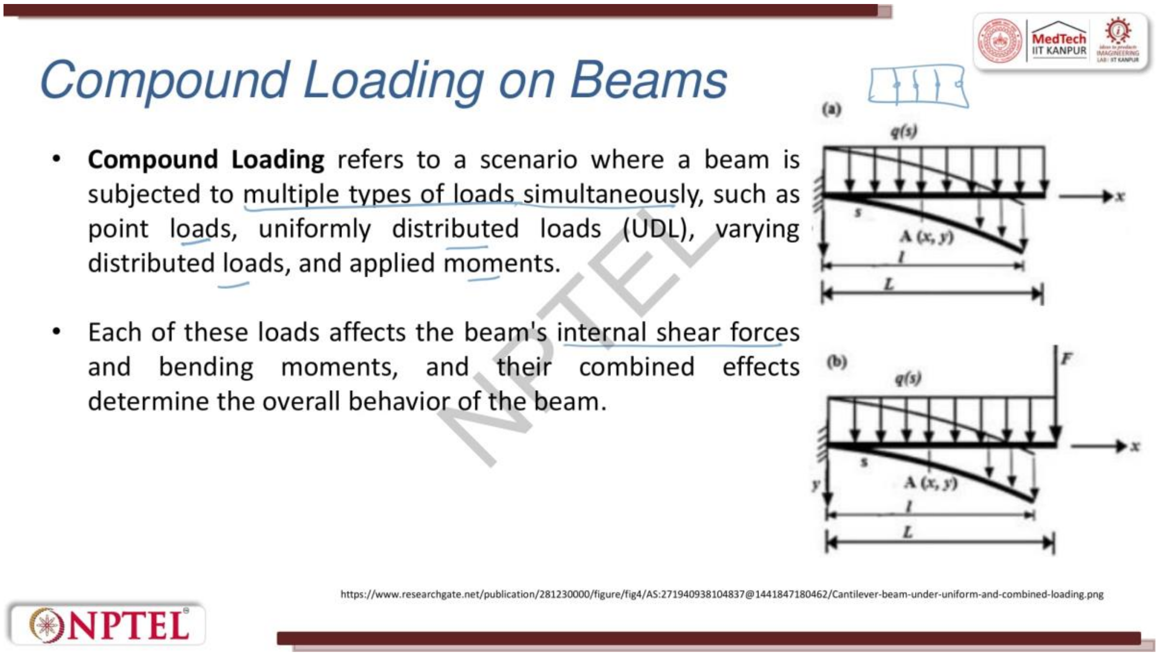
Lecture 27

Shear Force and Bending Moment Diagram (Part 3 of 3)

Welcome to the next lecture on shear force and bending moment. We have gone through a detailed analysis of finding out the relationship between Load, Shear Force and Bending Moment. We have also gone through how do you solve a problem, when there is a concentrated load, when there is a uniform distributed load and a combination of these two. With that understanding, let us keep moving to understand shear force and bending moment.



In this lecture, we will try to cover Compound Loading on Beams, Analysis of Fixed Beams, Continuous Beams. Finally, we will try to have a recap.



What are compound loading on beams? Compound Loading refers to a scenario when a beam is subjected to multiple types of loads simultaneously, such as point load, UDL, varying distributed load and applied moment. So, you see here, we are trying to have point load, UDL, varying distributed load and applied load. So you can have two of them, three of them or four of them.

You can have combinations of this and interestingly you can also have weightage factors. For each of them, what is its influence on the overall performance which you feel? Each of these loads affects the beam's internal shear force and bending moment and their combined effect determines the overall behavior of the beam. You can always try to divide this into four problems. And when you try to do it as four different problems, when you try to combine, you will have to take care of combining.

The other way around is you try to do solving all these things in one shot. But you should have a feel that the internal shear force and bending moment will be affected when there is a compound loading.

Compound Loading on Beams

Effects on Beam Behaviour:

Superposition Principle:

- The principle of superposition is used to analyze compound loading.
- The effects of each load type (point load, UDL, moment) on the shear force and bending moment are calculated individually.
- The resultant shear force and bending moment at any section of the beam are then obtained by summing these individual effects.

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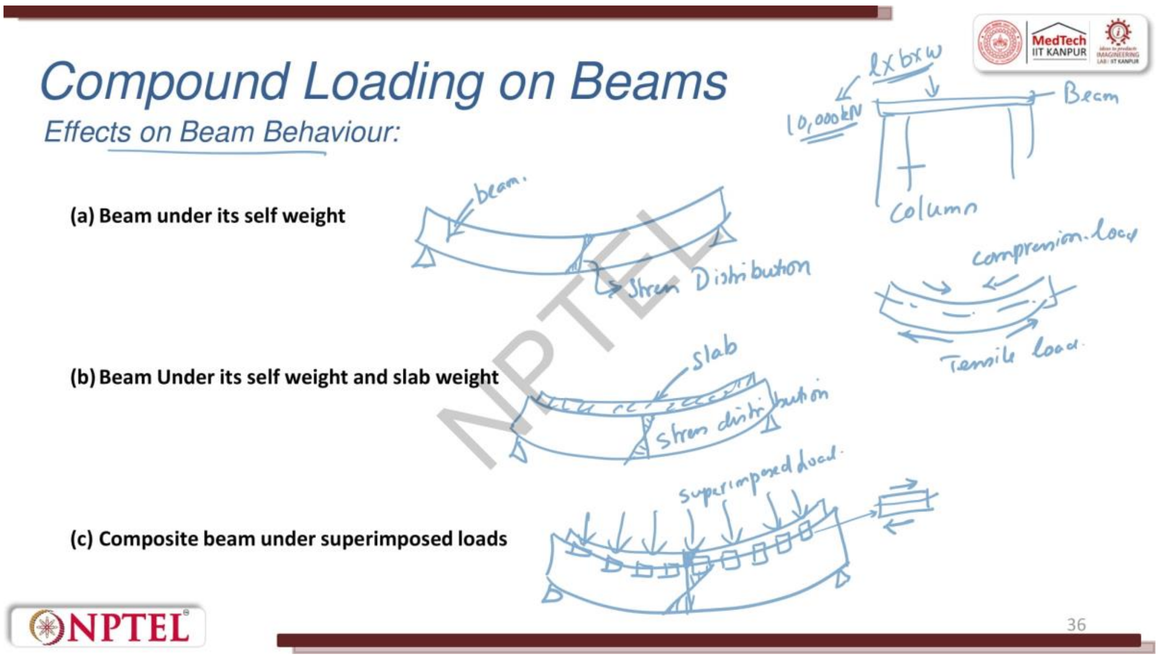
So for that we have a superposition principle. The principle of superposition is used to analyze compound loading. The effects of each load type on the shear force and bending moment are calculated individually.

The resultant shear force and bending moment at any cross section of the beam are then obtained by summing their individual effects. This is how we try to solve a complex problem. We try to split the problem into small fragments and then we try to add them all and finally find out what is their summation influence on the output. So, the resultant shear force and bending moment at any section of the beam are then obtained by summing these individual effects.



So, let us see the effect of beam behavior when the beam is under its self weight. See, when we are constructing big buildings, we will have columns and we will have beams. These are called as Column, these are called as Beam. The beam will also have its dimension L into B into w which in turn if you see will be of round about 10,000 kilo Newton's weight in certain buildings. So, you should understand the beam also has its own weight. So, because of this weight, there can be a sagging.

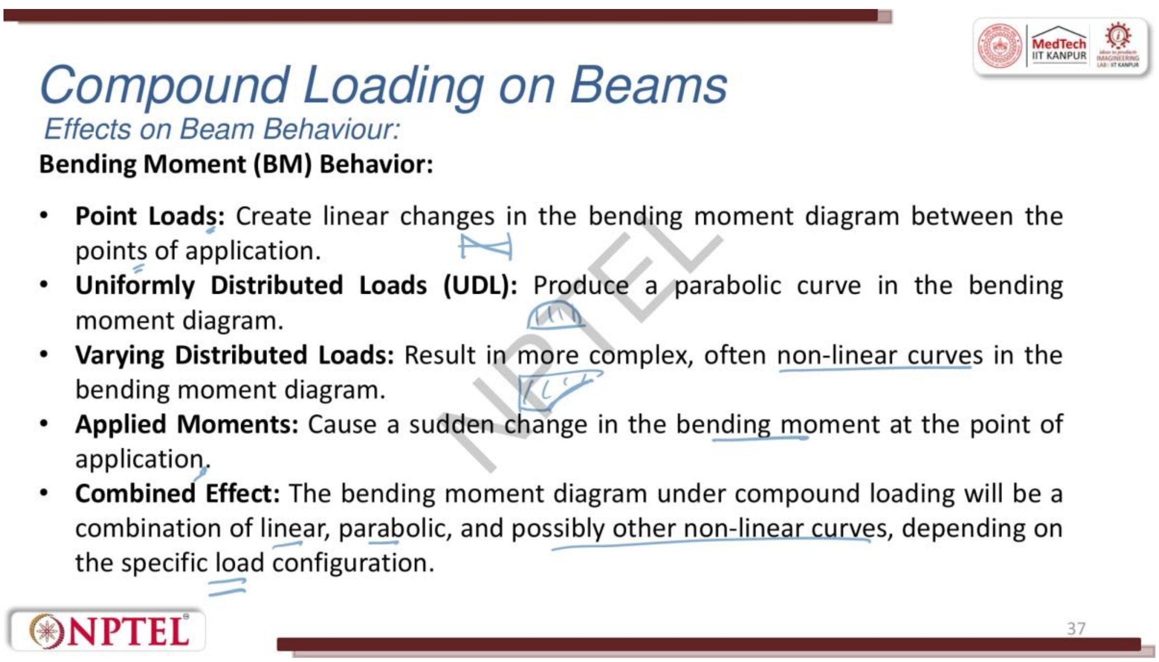
So, we are trying to look at that beam under its self-weight. So, here you have a beam which is supported and you have its Stress distribution. This is called as Stress Distribution. Right. And this is a beam. Okay. The beam under itself weight and slab weight.

So what happens? You have a beam. Then on top of the beam, you have a slab. This is a slab. Flooring, right.

There is another thing which is on top of the beam. So you can see here. This is the stress distribution, the magnitude increases. And as I told you in the previous lectures, that when there is a beam, the top fiber will undergo compression and the bottom fellow will undergo tension. Compression load and tensile load.

Here you will have your neutral axis. Then let us see a Composite beam under superimposed loading. So here you will have a beam. That beam will have a; and then you will have a weight attached to it. And exactly here you will have, if I try to take elements like this, you will have the loads going like this.

So this is called a Superimposed load. And if you see the stress distribution, you will have the stress distribution independent. so if I want you to make this portion will have a stress distribution different. So these are the elements which we are doing. And these are the different types. How a beam behaves under self weight? The beam is under its weight and slab weight. And composite beam under superimposed loading.

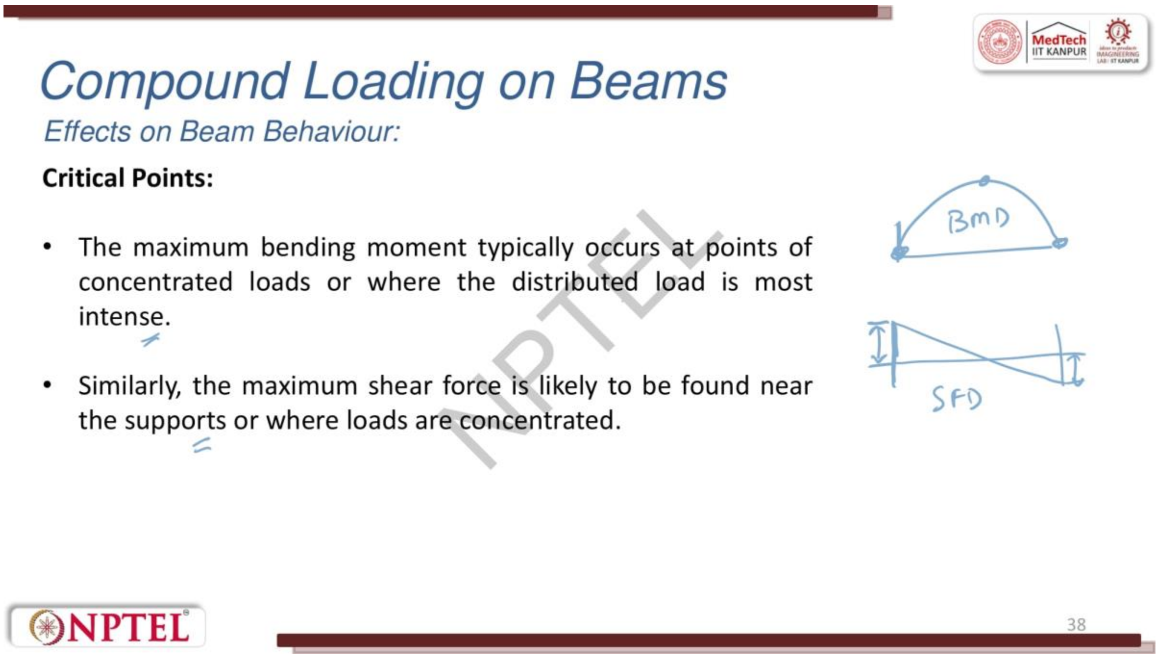


Bending Moment Behavior. Point load create linear change in the bending moment diagram between the point of application.

When there are loads, then you will have between the points of application. Uniformly Distributed Loads produce a parabolic curve in the bending moment diagram. When you have Varying Distributed Load, results in more complex, often non-linear curves in the bending moment diagram. Applied Moments, they cause a sudden change in the bending moment at the point of application. And finally, the Combined Effect, if you see, the bending moment diagram under compound moments,

loading will be a combination of linear, parabolic and possible other non-linear curves depending on the specific load configuration. So point load will be linear. UDL will be parabolic. We saw in the previous lectures. Varying Distributed Load, you can have a linear combination, something non-linear type, you can have something like this.

Applied is a change in the bending moment at the point of application. And finally, Combined Effect, you have a combination of all these things.



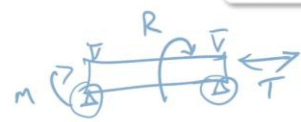
The critical points to note. The maximum bending moment typically occurs at points of concentrated load or where the distribution load is most intense. Similarly, the maximum shear force is likely to be found near the supports where the loads are or where the loads are concentrated.

So two points. So here bending moment, if you try to do for a UDL. So this is the bending moment diagram. If you try to draw the shear force for it, you will have maximum value here and maybe you might cross the minimum and go here. So this is Shear Force diagram.

So I just wanted to say this point. At the zero of bending moment, you have maximum shear forces. That's the only point. We are not looking into the loading condition here. So, similarly the maximum shear force is likely to be found near the supports or where the loads are concentrated.

Analysis of Fixed Beams

Behavior of Fixed Beams Under Loading

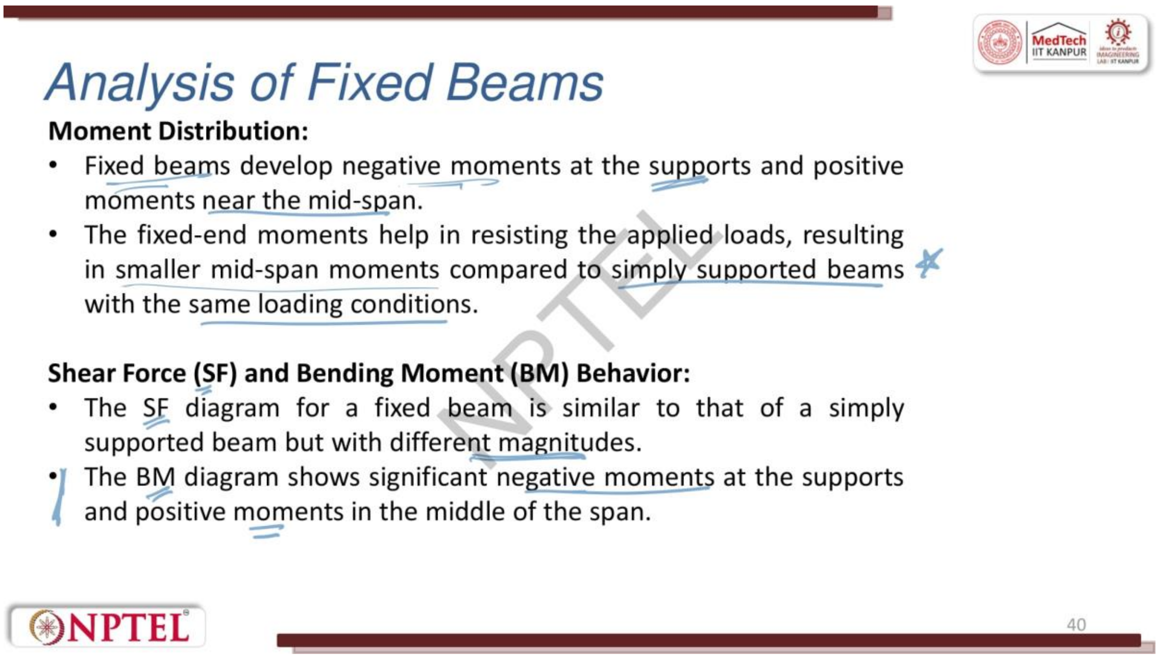


- **Fixed Beams** are structural elements that are restrained at both ends, preventing both rotation and translation.
- This restraint creates moments at the supports in addition to the shear forces.
- The behavior of fixed beams under loading is different from simply supported beams due to these additional moments, which help reduce deflections and moments in the span.

Behavior of Fixed Beam Under Loading. Fixed beams are structural elements that are restrained at both ends, preventing both rotation and translation. Fixed beam, you have a beam which are restraining at both the ends. This is one end, this is the other end, preventing both translation and rotation. This is translation and rotation.

This restraint creates moments at the supports in addition to the shear force. Because of this, there is always a moment which comes under the support. These are M. This restraints creates moments at the supports in addition to the shear force. The behavior of fixed beam under loading is different from simply supported beam due to these additional moments which help reduce deflection and moment in the span. This if you restrict it, right.

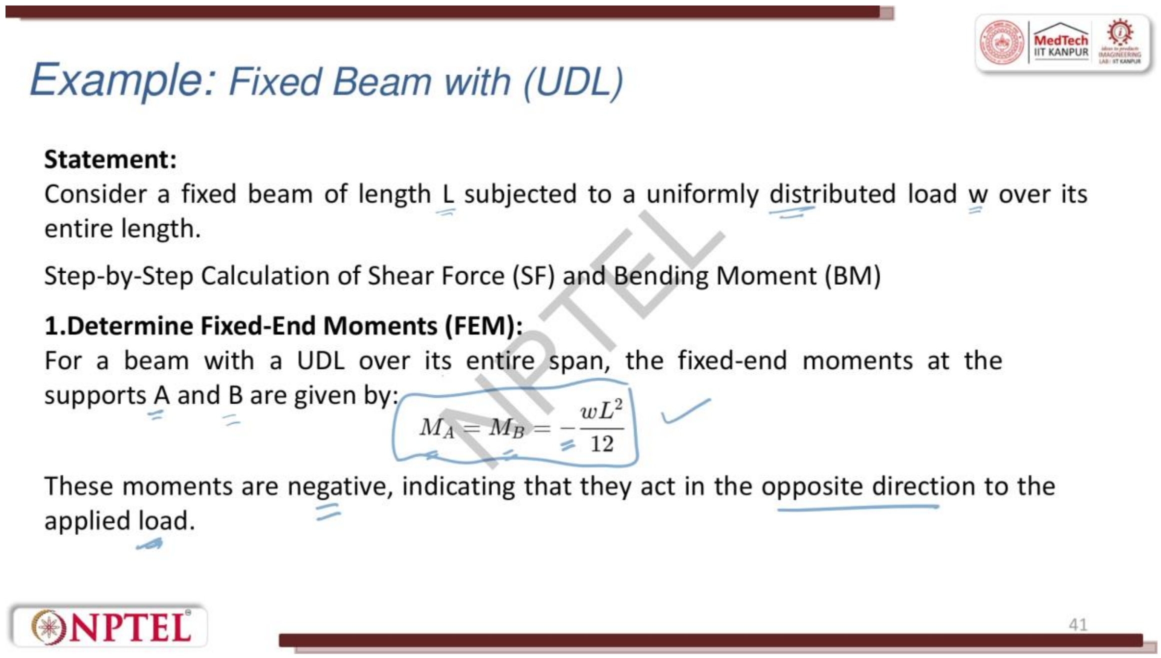
So this is going to give a different behavior because you have an additional moment which helps to reduce the deflection and the moment in the span.



Moment Distribution. Fixed beams develop negative moments at the supports and positive moments near the mid-span. Very important point, fixed beam, negative moment at the support and positive at the mid-span. The fixed end moments help in resisting the applied loads resulting in smaller mid-span moments compared to simply supported beams with the same loading condition.

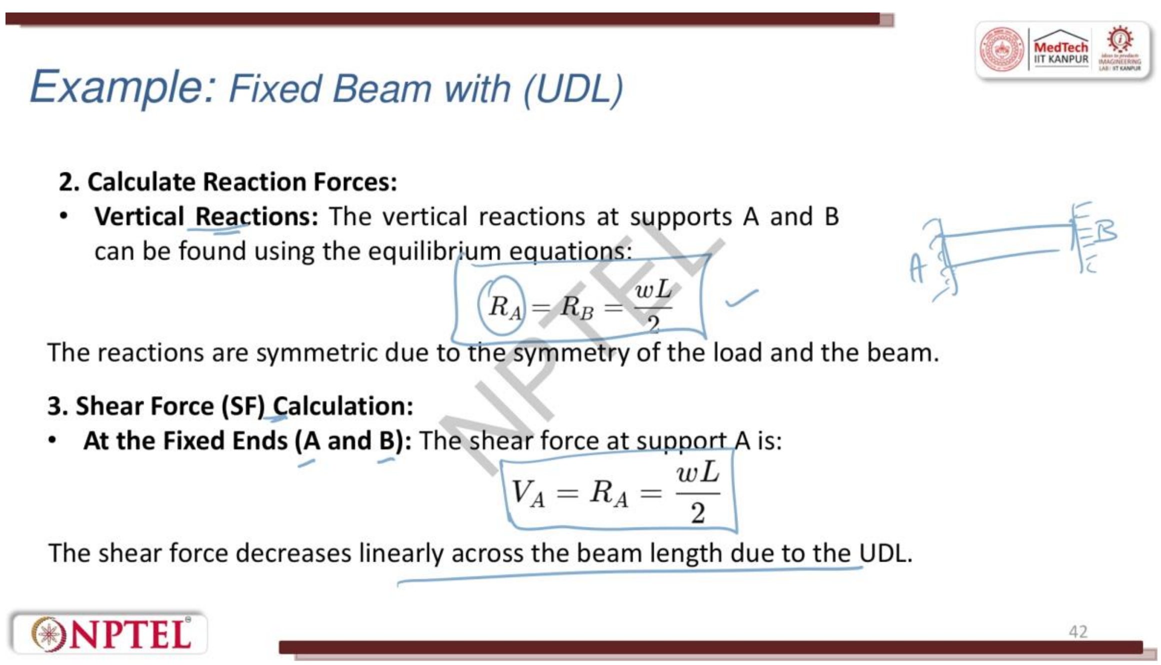
These are some of the points which you have to understand and to keep it in your memory. The fixed end moments help in resisting the applied load. Both ends are fixed resulting in smaller mid-span moment compared to simply supported beam with the same loading condition. The Shear Force and Bending Moment Behavior. The shear force diagram for a fixed beam is similar to that of a simply supported beam but with different magnitudes.

Shear force, there is not much of change. But bending moment diagram shows significant negative moments at the supports and positive moments at the middle of the span. So this point is also very important. When we try to look at the fixed beam, you should understand there will not be much of difference for shear force. But bending moment, there will be a huge difference.



Statement. Consider a fixed beam of length L. Subjected to a UDL, w over its entire length. So now how do you solve this problem? Determine the fixed end moments.

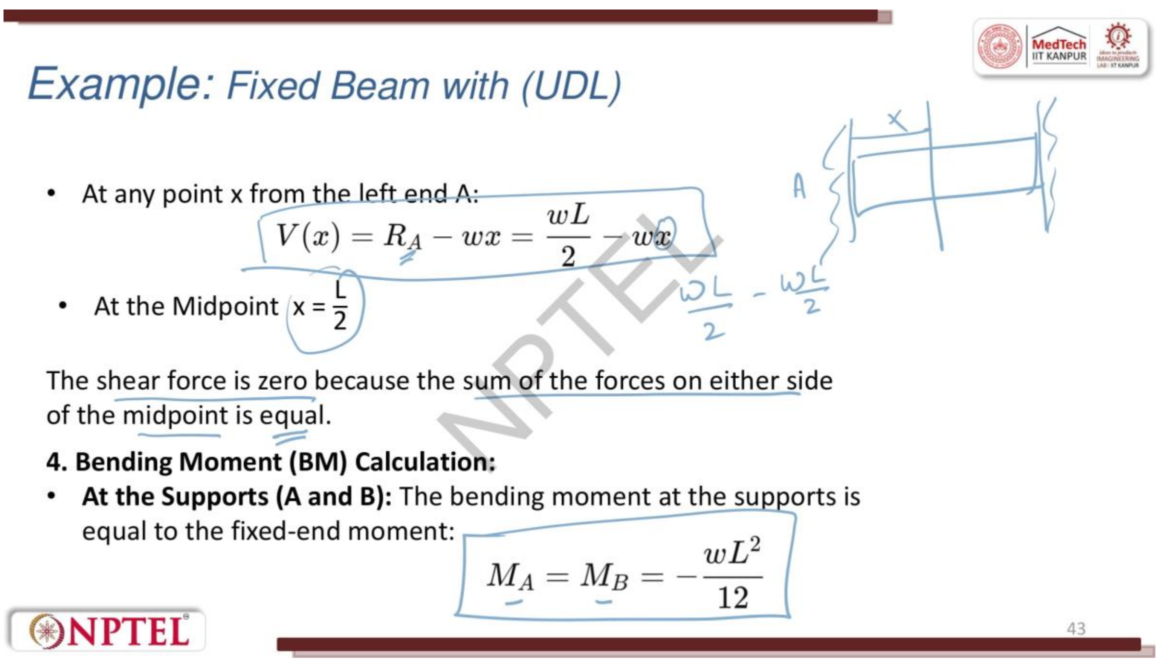
For a beam with a UDL over its entire length span, the fixed end moments at the support A and B are given as $M_A = M_B = -\frac{w \cdot L^2}{12}$.These moments are negative indicating that they act in the opposite direction of the applied load. So, if the load is like this, it acts in the opposite direction.



Next, after calculating the moment, let us calculate the reaction forces. So, the reaction forces in the vertical direction is going to be the vertical reaction at support A and B can be found using the equilibrium equation $R_A = R_B = -\frac{wL}{2}$.

The reactions are symmetric due to the symmetry of the load and the beam. Now, first what we did, we calculated the moment, then we calculated the reaction force. From here, we will try to move and calculate the shear force. At the fixed end A and B beam, it is fixed, this is A. At the fixed end A and B, the shear forces at support are going to be $V_A = R_A = -\frac{wL}{2}$.

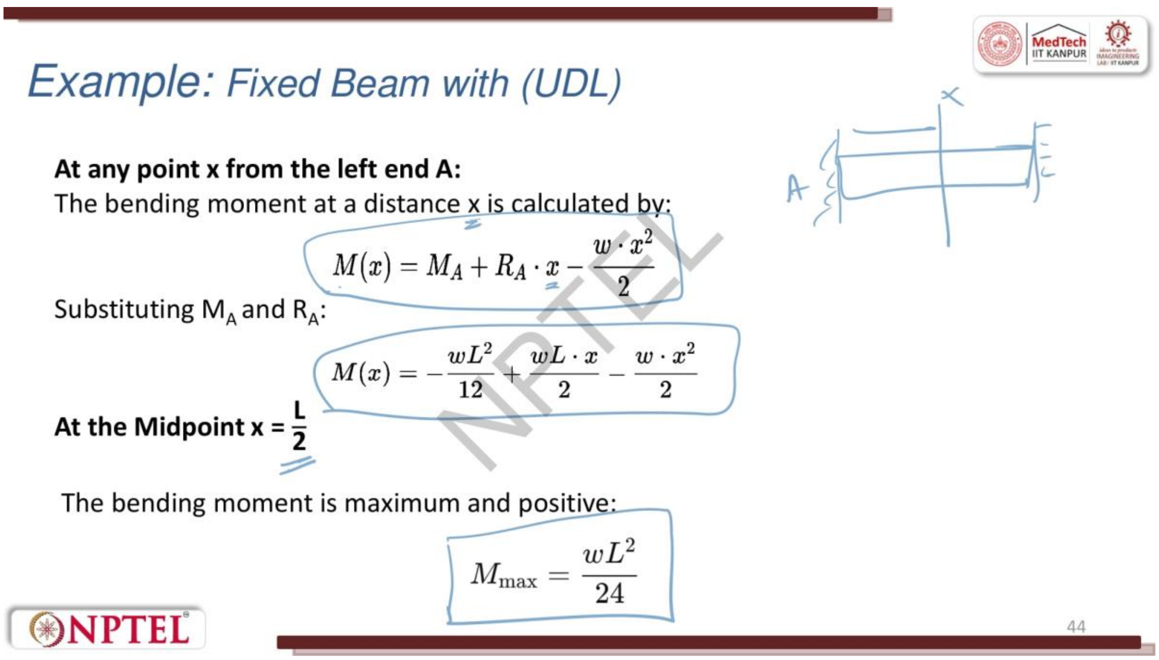
The shear force decreases linearly across the length due to UDL. Because we are talking of a UDL getting applied.



At any point x, this is A, Any point x from the left end A, which can be given as $V(x) = R_A . wx = \frac{wL}{2} - wx$. This R_A, we have already found out it is wL/2.

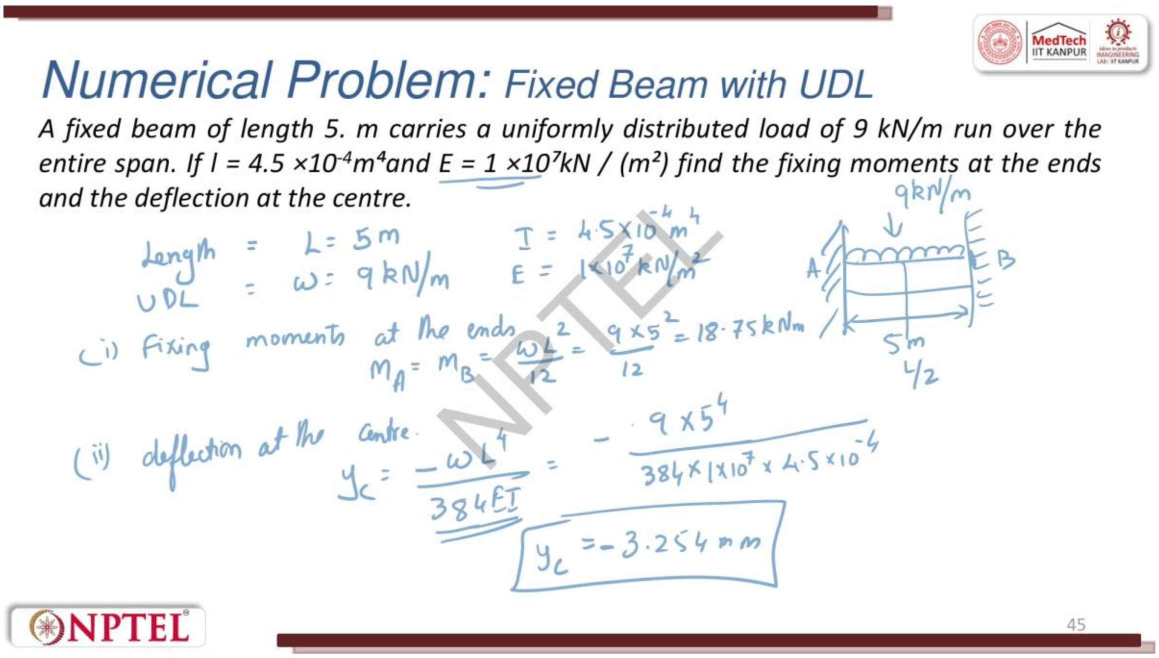
So at the midpoint x = L/2 then you can substitute it and find out what is it. So you will have x = L/2 that will be wL/2 - x = wL/2 (x = L/2). We are substituting this x. The shear force is 0 because the sum of the forces on either sides of the midpoint is equal. The shear force is 0. We have given the reason because the sum of the forces on either sides of the midpoint is equal.

Now you have to move to the fourth step in solving this problem which is nothing but bending moment calculation at the support A and B. The bending moment at the supports is equal to the fixed end moment $M_A = M_B = -\frac{wL^2}{12}$. What is L? L is the length.



At any given point X from the left of A, this is x. So at any point x from the left end of A, the bending moment at the distance x is calculated by $M(x) = M_A + R_A \cdot x - \frac{w \cdot x^2}{2}$.

Substituting M_A and R_A with the previous one, we get $M(x) = \frac{wL^2}{12} + \frac{wL.x}{2} - \frac{w \cdot x^2}{2}$ where x is taken as the midpoint. So the bending moment is maximum and positive at the place where $M_{max} = \frac{wL^2}{24}$.



Now using this, we will try to solve a simple problem. A fixed beam of length 5 meters carries a uniform distributed load of 9 kN per meter run over the entire span. So, a fixed beam, the length is 5 meters and it is entirely done by a UDL which is nothing but 9 kilo Newton per meter.

Solution: Given:

Length, $L = 5m$

U.D.I. $w = 9kN/m$

Value of $I = 4.5 \times 10^{-4}\, m^4$

Value of $E = 1 \times 10^{7}\, KN/m^2$

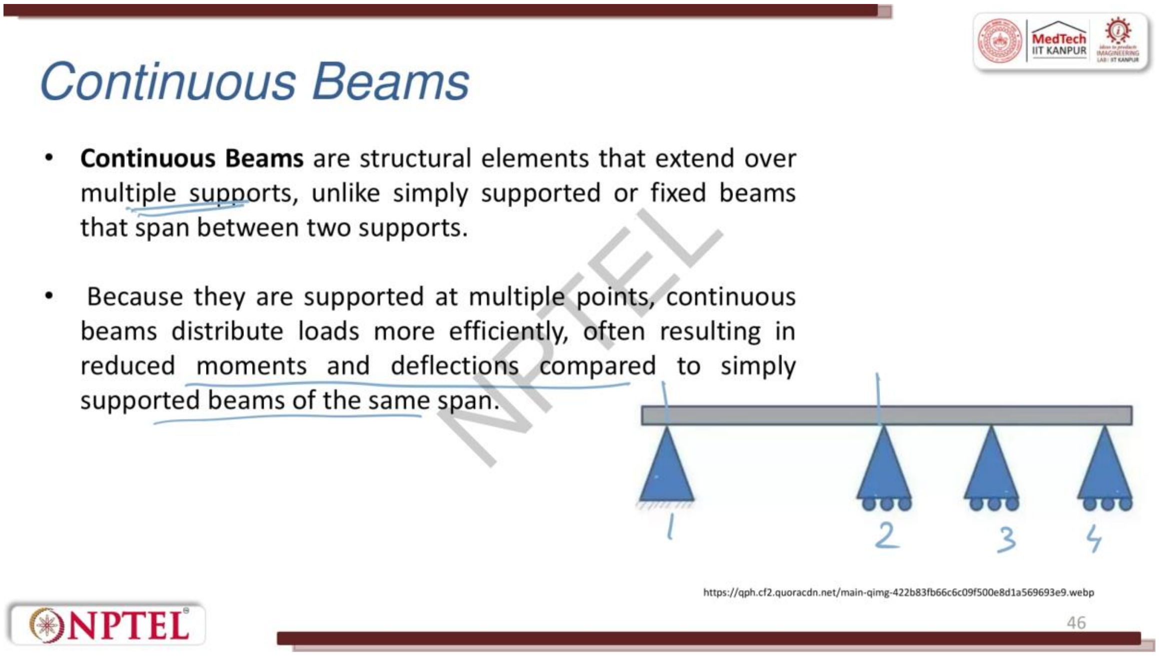
i. The fixing moments at the ends is given by equation (15.9) as

$$M_A = M_B = -\ \frac{w\,.\,L2}{12} = \frac{9 \times 5^2}{12} = 18.75\ \text{kNm.} \quad \text{Ans.}$$

ii. The deflection at the centre is given by equation (15.11) as

$$Y_c = \frac{w \times L4}{384EI} = -\ \frac{9 \times 5^4}{384 \times 1 \times 10^7 \times 4.5 \times 10^{-4}} = 0.003254\ \text{m} = -3.254\ \text{mm.} \quad \text{Ans.}$$

I am sure by solving this problem you would have understood the fixed beam with UDL. Moving further, we will try to see continuous beams.

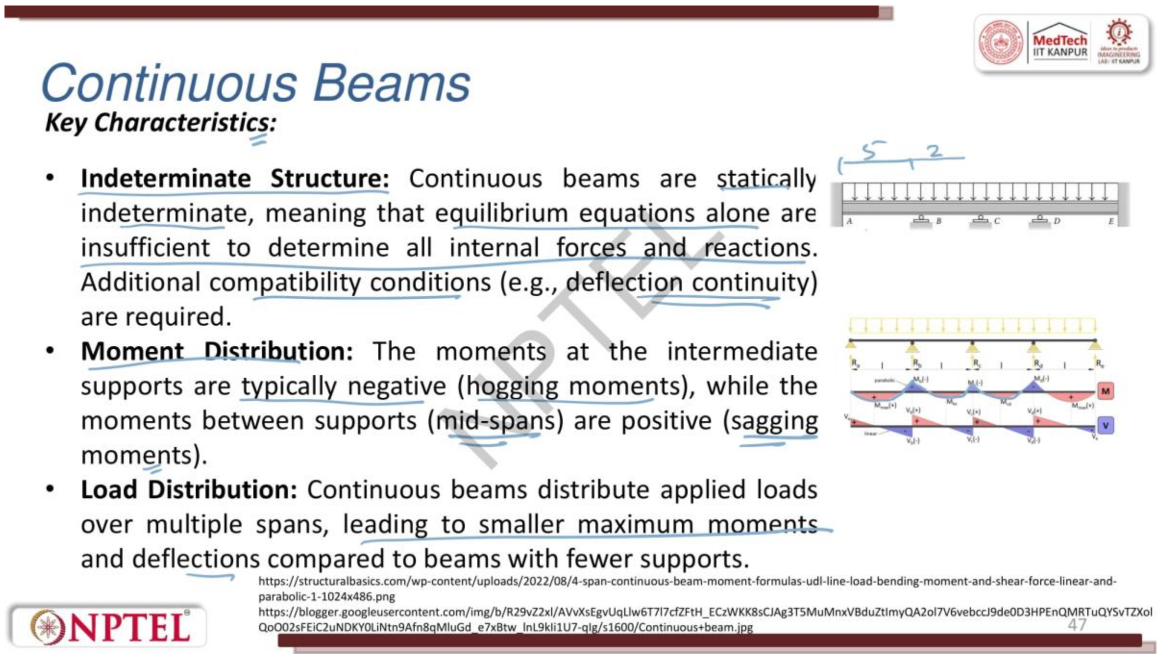


Continuous beams are structural elements that extend over multiple supports. Unlike simply supported or fixed beam, the span between the two supports.

So in simply support what we do is we try to take only one span and then we do. But in continuous beam, we try to have more supports given to support the beam. When you have a long beam, you always try to support at multiple places. So continuous beam are structural element that extend over multiple supports. Because they are supported at multiple points, 1, 2, 3, 4, continuous beam distributes load more efficiently.

Often, resulting in reduced moment and deflection as compared to that of simply supported and other cases. Natural, right. You have a simply supported beam. So, if there is a load and all, only two points, they take the shear force and the bending moment you try to figure out. So, now you have multiple supports.

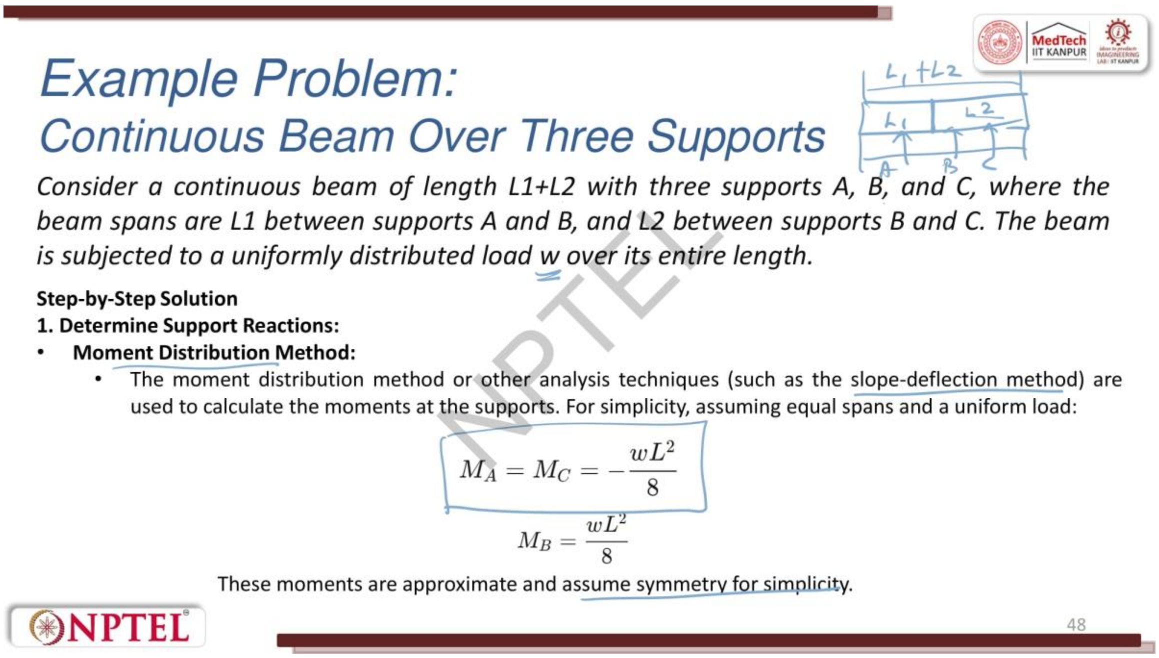
Naturally, the deflection is going to be low and the bending moment is also going to be changed accordingly.



So, what are the key characteristics in continuous beam? So you can have indeterminate structure. Here the continuous beam are statistically indeterminate. Meaning the equilibrium equation alone are insufficient to determine all the internal force and reaction.

So, you have to have additionally compatibility conditions like deflection continuity should be there. For example, from here to here if it is 5 millimeter deflection and from here to here it should be 5 plus whatever it is 2 millimeter deflection. So, that is what deflection continuity has to be maintained in indeterminate structures. Moment Distribution, the moment at the intermediate supports are typically negative while the moment between supports are positive. So here it is hogging moments and while in the moments between supports mid-span are positive sagging moments you will try to have.

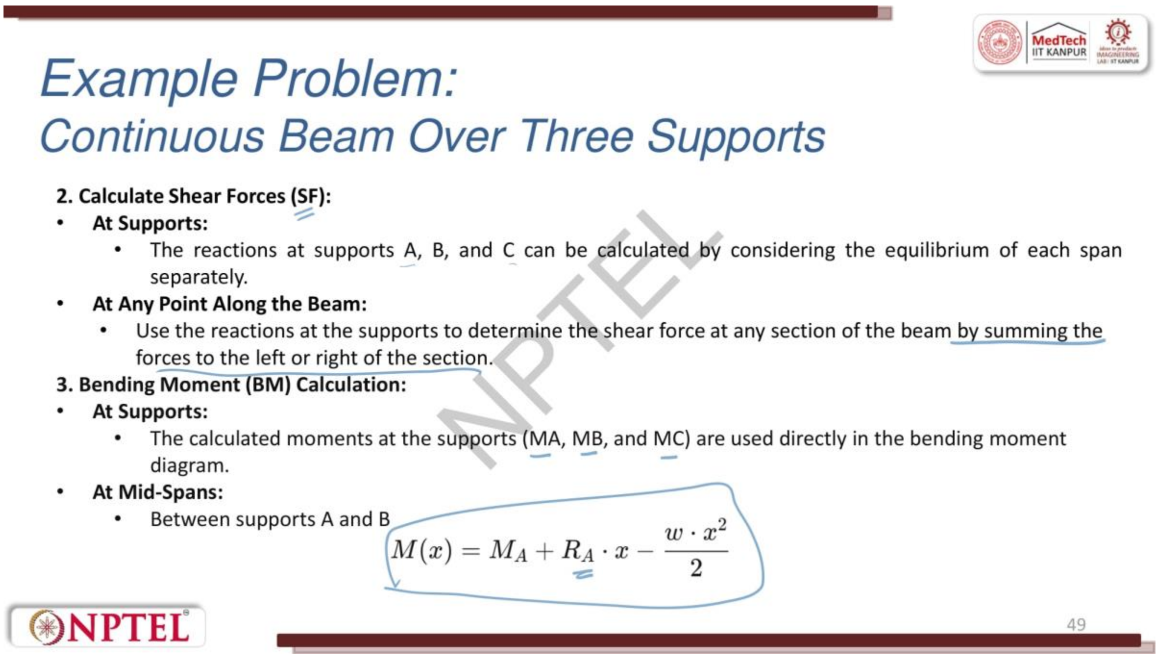
So if you see here this will be and wherever there is a support, there is an increase then again it gets into the negative zone, you go above and then it gets reduced. So moment distribution also is very interesting in these conditions like continuous beam the load distribution. Continuous beam distributes applied loads over multiple spans leading to a smaller maximum momentum and deflection compared to beams with support. The point whatever we have discussed is also brought in here.



So let us take an example problem and see how do we solve it. Consider a continuous beam of length L1 + L2 with three supports A, B, C. So, now you have a beam which is of a length L1 + L2 and then supports A, B and C where the length span are L1 between support A and B and L2 between B and C. The beam is subjected to a uniformly distributed load w over its entire length. So, here I have just said L1, L2 together increase in span. Now, to be more precise, L1 is support A and B, so it is up to here is L1 and B and C is L2. So, what are the steps involved in solving?

So, first we have to determine the support reactions, the moment distribution method has to be followed. The moment distribution method or other analysis techniques such as slope deflection method are used to calculate the moment at the supports. For simplicity, assuming equal span and uniform load $M_A = M_C = -\frac{w \cdot L^2}{8}$ which is written as $M_B = \frac{w \cdot L^2}{8}$. And at the midpoint B, it will be positive. So, $ML^2/8$.

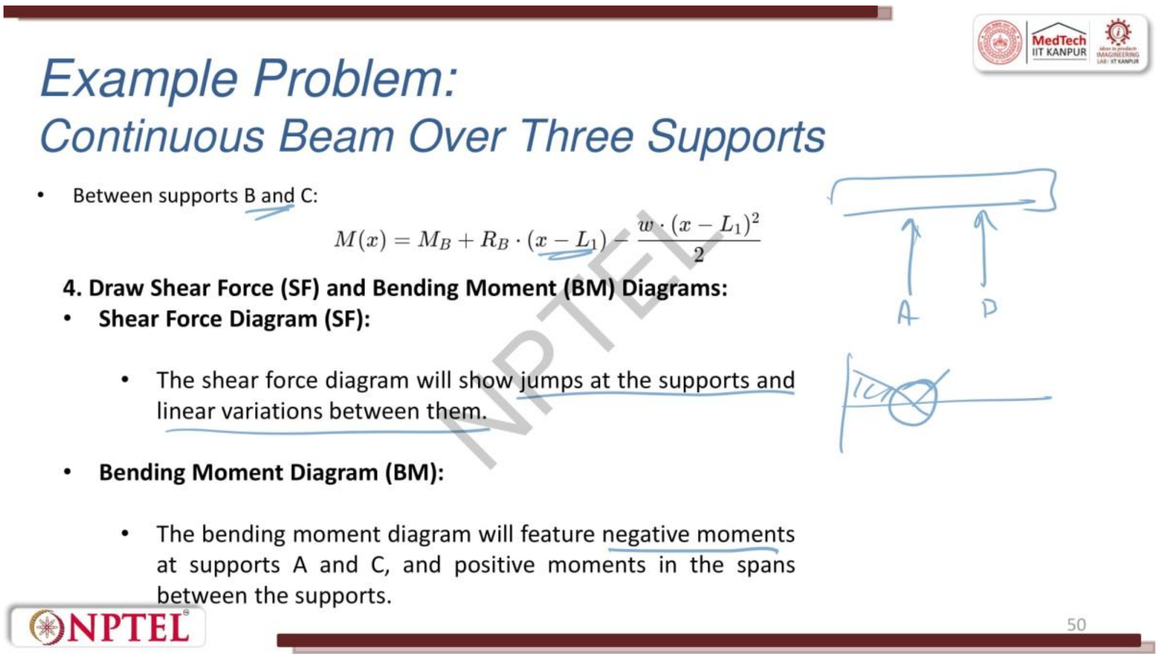
These moments are approximate and assume symmetry for simplicity. But in real time, this itself is complex to be solved.



The second step is you try to calculate the shear force. At the supports, the reaction at support A, B and C can be calculated by considering the equilibrium of each span separately. At any point along the beam, use the reactions at the supports to determine the shear force at any section of the beam by summing the forces to the left or right of the section.

This is very important. Beam by summing the forces to the left or right of the section. So bending moment calculation at supports. The calculated moments at the supports M_A, M_B and M_C are used directly in the bending moment diagram. You have bending, so you will have a force diagram, you have a load diagram, shear force diagram and bending moment.

So bending moment we are calculating. At the mid span between A and B, $M(x) = M_A + R_A \cdot x - \frac{w \cdot x^2}{2}$. x, you can try to replace it with the length. When you go to B, it will be $M(x) = M_B + R_B \cdot (x - L_1) - \frac{w \cdot (x+L1)}{2}$.



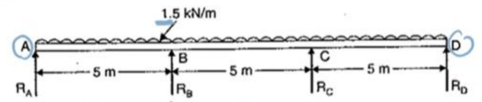
So you will try to get for momentum between B and C. Now you draw the shear force diagram and bending moment diagram.

The shear force diagram will show jumps at the supports and linear variation between them. So it can show jumps at the support. So A, you have A and B There will be a jump at the support. It can go deflection down or it can go high, whatever it is.

When you solve it, you try to get it. So you can see there is a jump. And bending moment diagram will feature negative moments at support A and C and positive moment between the supports.

Numerical Problem: Continuous Beam

A continuous beam ABCD of length 15 m rests on four supports covering 3 equal spans and carries a uniformly distributed load of 1.5 kN/m length. Calculate the moments and reactions at the supports. Draw the S.F. and B.M. diagrams also.



Given

AB $L_1 = 5$ m

BC $L_2 = 5$ m

CD $L_3 = 5$ m

UDL $= w_1 = w_2 = w_3 = 1.5$ kN/m

Ends A & D are simply supported, the support moments at A & D are zero

$M_A = 0$ & $M_D = 0$

from Symmetry $M_B = M_C$

To find the support moments at B & D, we use Clapeyron's theorem for three moments is applied for ABC & for BCD.

Numerical Problem: Continuous Beam

For *ABC*, we get

$$M_A L_1 + 2M_B(L_1 + L_2) + M_C \cdot L_2 = \frac{6a_1\bar{x}_1}{L_1} + \frac{6a_2\bar{x}_2}{L_2}$$

or $$0 \times 5 + 2M_B(5 + 5) + M_C \times 5 = \frac{6a_1\bar{x}_1}{5} + \frac{6a_2\bar{x}_2}{5}$$

or $$20M_B + 5M_C = \frac{6}{5}(a_1\bar{x}_1 + a_2\bar{x}_2) \quad ...(i)$$

Now a_1 = Area of B.M. Diagram due to u.d.l. on *AB* when *AB* is considered as simply supported beam

$$= \frac{2}{3} \times AB \times \text{Altitude of parabola}$$

$$= \frac{2}{3} \times 5 \times \frac{w_1 L_1}{8} = \frac{2}{3} \times 5 \times \frac{1.5 \times 5^2}{8} = 15.625$$

$$\bar{x}_2 = \frac{L_1}{2} = \frac{5}{2} = 2.5 \text{ m}$$

Due to symmetry $a_2 = a_1 = 15.625$ and $\bar{x}_2 = \bar{x}_1 = 2.5$

Substituting these values in equation (*i*), we get

$$20M_B + 5M_C = \frac{6}{5}(15.625 \times 2.5 + 15.625 \times 2.5)$$

$$= \frac{6}{5} \times 2 \times 15.625 \times 2.5 = 93.750$$

or $$20M_B + 5M_B = 93.750 \quad (\because M_B = M_C \text{ due to symmetry})$$

or $$M_B = \frac{93.750}{25} = 3.75 \text{ kNm}$$

$\therefore$ $M_B = M_C =$ **3.75 kNm. Ans.**

Now let us try to solve a simple problem and understand how is the behavior of shear force and bending moment for a continuous beam. So a continuous beam simply supported at A and D which has in between supports B and C. The distances between AB, BC and CD are all equal 5 meters.

The UDL which is applied is 1.5 kilo Newton per meter. So, now let us try to solve this problem for our understanding. So, given is going to be length L1 which is A to B which is 5 meters. Then B to C is L2 which is again 5 meters. Then C to D which is L3 which is again 5 meters.

The UDL is given as w1 equal to w2 equal to w3 which is 1.5 kilo Newton per meter. So, the ends A and D are simply supported. So, the support moment at A and D are 0. So $M_A = 0$ and $M_D = 0$. So from symmetry we try to take $M_B = M_C$.

So to find the support moment at B and D, we use Clapeyron's Theorem. So, the Clapeyron theorem for three moments is applied for A, B, C and for B, C, D.

For ABC, we get

$$M_A L_1 + 2M_B(L_1 + L_2) + M_C \cdot L_2 = \frac{6a_1\bar{x}_1}{L_1} + \frac{6a_2\bar{x}_2}{L_2}$$

or $$0 \times 5 + 2M_B(5+5) + M_C \times 5 = \frac{6a_1\bar{x}_1}{5} + \frac{6a_2\bar{x}_2}{5}$$

or $$20M_B + 5M_C = \frac{6}{5}(a_1\bar{x}_1 + a_2\bar{x}_2) \quad \ldots(i)$$

Now a_1 = Area of B.M. Diagram due to u.d.l. on AB when AB is considered as simply supported beam

$$= \frac{2}{3} \times AB \times \text{Altitude of parabola}$$

$$= \frac{2}{3} \times 5 \times \frac{w_1 L_1}{8} = \frac{2}{3} \times 5 \times \frac{1.5 \times 5^2}{8} = 15.625$$

$$\bar{x}_2 = \frac{L_1}{2} = \frac{5}{2} = 2.5 \text{ m}$$

Due to symmetry $a_2 = a_1 = 15.625$ and $\bar{x}_2 = \bar{x}_1 = 2.5$

Substituting these values in equation (i), we get

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or $$20M_B + 5M_B = 93.750 \quad (\because M_B = M_C \text{ due to symmetry})$$

or $$M_B = \frac{93.750}{25} = 3.75 \text{ kNm}$$

$\therefore$ $M_B = M_C =$ **3.75 kNm. Ans.**

To Recapitulate

- What is a fixed beam?
- Why are fixed beams more rigid than simply supported beams?
- What are continuous beams, and how do they differ from simply supported beams?
- How does the moment distribution method help in analyzing continuous beams?
- What is an influence line diagram, and how is it useful for continuous beams?

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To recap what we have seen in this lecture, we saw what is a fixed beam. Then why are fixed beam more rigid than simply supported beam? What are continuous beam and how do they differ from simply supported beam? How does the moment distribution method help in analyzing the continuous beam and what is the influence of line diagram and how is it useful for continuous beams?

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We have used the following books as reference for preparing the slides. Thank you very much.