Basics of Mechanical Engineering-1

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Week 07

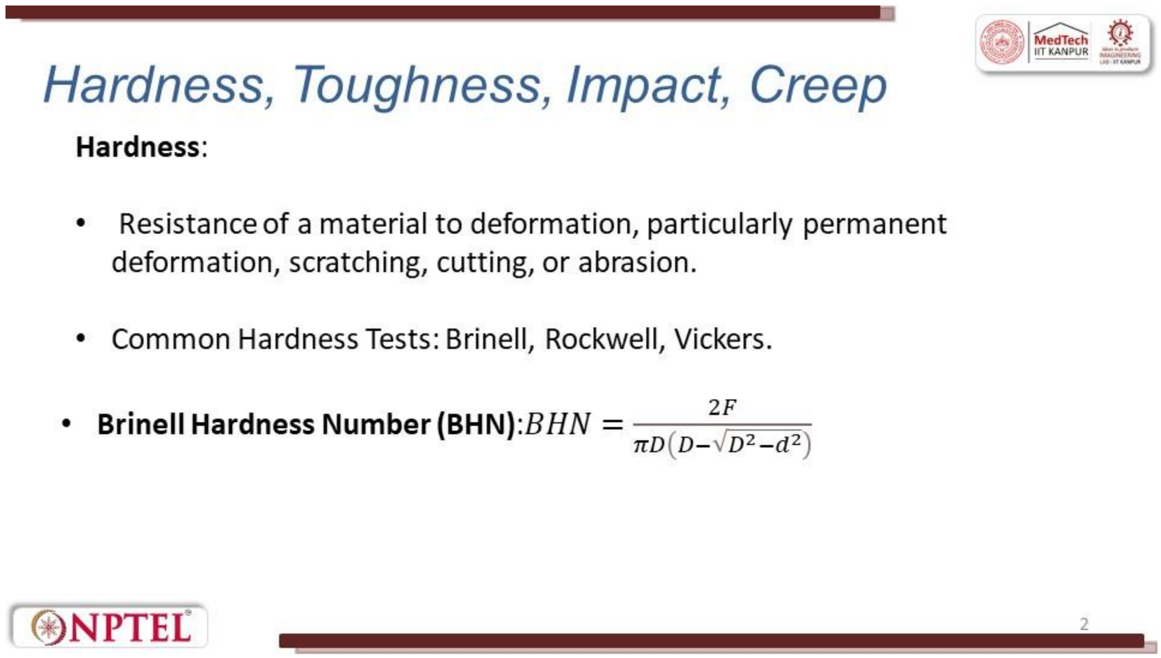
Lecture 29

Tutorial-3 (Part 1 of 2)

Hello friends, welcome back to the course Basics of Mechanical Engineering 1. Professor Ramkumar has covered a lot of topics regarding the basics in mechanical engineering, starting from units and dimensions. We talked about laws of motion, scalars and vectors, all the basic elements which are employed in mechanical engineering applications. And we also covered the topics in solid mechanics. We covered majorly the mechanical properties such as stress strain, residual stresses.

Then we talked about stress strain curve. We talked about principal stresses. And I think the principal stresses, I have also covered tutorial, which was tutorial 1 and 2. This tutorial 3 would be covering remaining part of soil mechanics that is Hardness, Toughness, Impact, Creep as one of the sets. Then we will talk about Static and Fatigue Loading and Critical Loads.

Some numerical problems would also be discussed upon these. Then Stresses in Cylinders and Spheres and also Buckling of Columns would be covered and then we will also try to see some problem statements on how to draw Mohr circle to understand or find the forces acting upon any body in different directions. Let me start it from the Hardness, Toughness, Impact and Creep.



The definition of hardness if you call it is resistance of a material to deformation, just to recall quickly. So particularly permanent deformation is there, scratching is there, cutting is there, abrasion is there.

Resistance to any permanent deformation is hardness. The common hardness tests which are there in the laboratory are Brinell, Rockwell, Vickers hardness test and BHN that is Brinell Hardness Number. $BHN = \frac{2F}{\pi D\left(D-\sqrt{D^2-d^2}\right)}$

I will come to the notations and everything in the forthcoming slides.

Hardness, Toughness, Impact, Creep

Creep:

- Time-dependent permanent deformation under constant load or stress.
- **Creep Rate**: $\dot{\epsilon} = A\sigma^{n}e^{-\frac{Q}{RT}}$
- Where: $\dot{\epsilon}$ = creep rate, σ = applied stress, Q = activation energy, R = gas constant, T = absolute temperature.

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So, also Creep, it is time-dependent permanent deformation under the constant load and stress. This was discussed like you put clothes for a dry on a metal wire, the metal wire while having the load for the whole day, the weight of the clothes is not that heavy, but the load is there for 8 hours.

And this 8 hours each day for 6 months, you will see the wire getting loosened. So, that is creep load is happening there. So, creep rate is epsilon dot is A sigma power n, then the negative exponential of Q by R and T. So, where epsilon naught is creep rate, sigma is applied stress, Q is activation energy, R is gas constant, T is absolute temperature.

Hardness, Toughness, Impact, Creep

Problem Statement: *A steel sample is subjected to a Brinell hardness test using a 10 mm diameter ball and a load of 3,000 kgf. The diameter of the indentation is 4 mm. Calculate the Brinell Hardness Number (BHN).*

Solution:

Solution :

Given : $D = 10$ mm , $F = 3{,}000$ kgf , $d = 4$ mm

$$BHN = \frac{2 \times 3000}{\pi \times 10 \times \left(10 - \sqrt{10^2 - 4^2}\right)}$$

$$= \frac{6000}{\pi \times 10 \times \left(10 - \sqrt{84}\right)}$$

$$= \frac{6000}{\pi \times 10 \times 0.83} \approx 230 \text{ BHN}$$

Ans. BHN = 228.76

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So, let me try to see some numerical statements and keep on discussing the relationships there. A steel sample is subjected to Brinell Hardness test using a 10 mm diameter ball and a load of 300 kgf. The diameter of indentation is 4 mm. Calculate Brinell Hardness Number.

I will also discuss in the next week the laboratory demonstrations on the different tests which are there in solid mechanics, the tensile test, compression test and also, we will try to talk about test in hardness and there also we will see how do we calculate, how do we actually conduct the test to understand the Brinell Hardness or other hardness numbers and we will see how do the relations hold good. So, here BHN is given.

Solution :

Given : $D = 10$ mm , $F = 3{,}000$ kgf , $d = 4$ mm

$$BHN = \frac{2 \times 3000}{\pi \times 10 \times \left(10 - \sqrt{10^2 - 4^2}\right)}$$

$$= \frac{6000}{\pi \times 10 \times \left(10 - \sqrt{84}\right)}$$

$$= \frac{6000}{\pi \times 10 \times 0.83} \approx 230 \text{ BHN}$$

Ans. BHN = 228.76

So, this is how do we calculate the BHN value. We also try to see the test on different materials. This is the steel sample, it is telling and BHN number has come this value.

Hardness, Toughness, Impact, Creep

Problem Statement: *A notched specimen is subjected to an impact test, and the energy absorbed is 20 J. The cross-sectional area at the notch is 0.01 m². Calculate the impact toughness.*

Solution:

Given: Energy absorbed = 20 J
cross-sectional area = $0.01 m^2$

$$\text{Impact toughness} = \frac{\text{Energy absorbed}}{\text{Cross-sectional area}}$$

$$= \frac{20 J}{0.01 m^2} = 2000 J/m^2$$

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Now, another problem statement that is given regarding toughness. A notch specimen is subjected to an impact test and the energy absorbed is 20 joules. The cross-sectional area of the notch is $0.01m^2$. Calculate the impact toughness.

Given: Energy absorbed = 20 J
cross-sectional area = $0.01 m^2$

$$\text{Impact toughness} = \frac{\text{Energy absorbed}}{\text{Cross-sectional area}}$$

$$= \frac{20 J}{0.01 m^2} = 2000 J/m^2$$

Hardness, Toughness, Impact, Creep

Problem Statement: *A cylindrical specimen of diameter 10 mm is subjected to a tensile force of 10,000 N. Calculate the stress and determine the material's toughness if the area under the stress-strain curve is 300 MPa.*

Solution:

Given : Diameter = 10 mm = 0.01 m

Force = 10,000 N

Cross-section Area $A = \pi\left(\frac{d}{2}\right)^2 = \pi\left(\frac{0.01}{2}\right)^2$

$= 7.854 \times 10^{-5} m^2$

Stress $\sigma = \frac{F}{A} = \frac{10000}{7.854 \times 10^{-5}} \approx 127.4$ MPa

toughness = 300 MPa

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A cylindrical specimen of diameter 10 meter is subjected to this load. So, we need to calculate the stress and determine the material's toughness if area under the stress strain curve is 300 megapascal.

Given : Diameter = 10 mm = 0.01 m

Force = 10,000 N

Cross-section Area $A = \pi\left(\frac{d}{2}\right)^2 = \pi\left(\frac{0.01}{2}\right)^2$

$= 7.854 \times 10^{-5} m^2$

Stress $\sigma = \frac{F}{A} = \frac{10000}{7.854 \times 10^{-5}} \approx 127.4$ MPa

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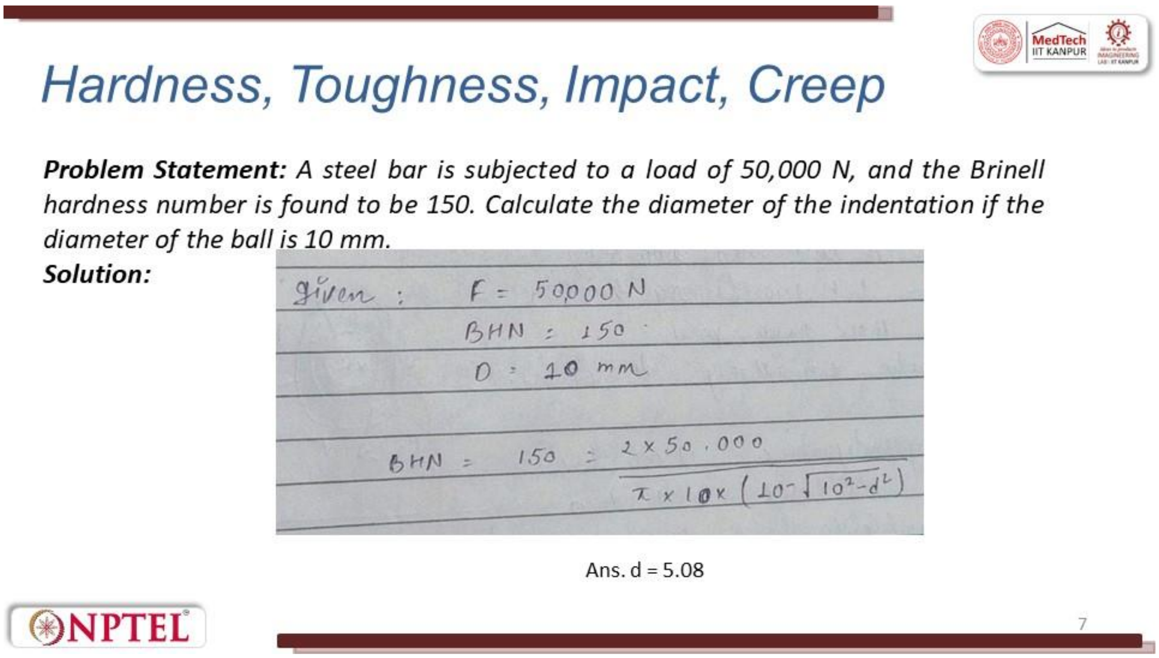
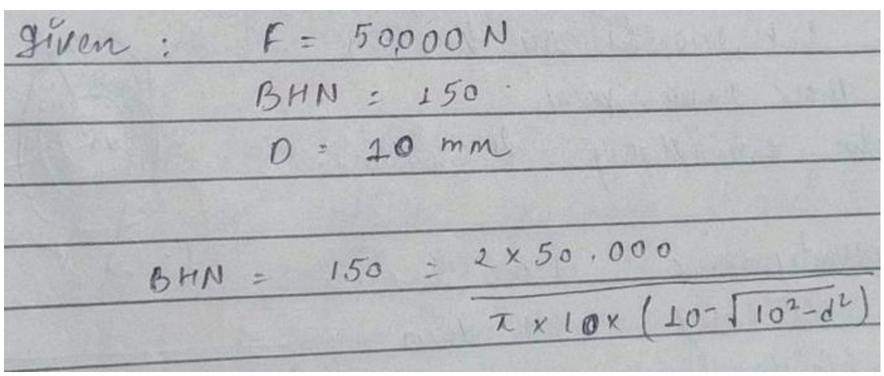


So, similarly let us do another numerical in BHN. It is very similar to the problem statement we did above.



Ans. d = 5.08

Hardness, Toughness, Impact, Creep

Problem Statement: *Calculate the Brinell Hardness Number (BHN) for a material if the diameter of the indentation made by a 5000 kgf load and a 10 mm diameter ball is 3 mm.*

Solution:

Given:

Load: 5000 Kg-f

Ball dia: 10mm (D)

Identation: 3mm (d)

Formula used –

$$BHN = \frac{2F}{\pi D\left(D - \sqrt{D^2 - d^2}\right)}$$

on putting all values.

$$BHN = \frac{2 \times 5000}{\pi \times 10\left(10 - \sqrt{100 - 9}\right)}$$

$$= \frac{2 \times 5000}{\pi \times 10 \times 0.46}$$

$$= \frac{1000}{14.45}$$

$$\boxed{BHN = 691.97}$$

Just to solidify our calculations once again, I will do one more numerical in this direction. BHN is given, load is given 5000 kgf and 10 meter diameter ball is there and dia of indenter is 3 millimeter.

Given:

Load: 5000 Kg-f

Ball dia: 10mm (D)

Identation: 3mm (d)

Formula used –

$$BHN = \frac{2F}{\pi D\left(D - \sqrt{D^2 - d^2}\right)}$$

on putting all values.

$$BHN = \frac{2 \times 5000}{\pi \times 10\left(10 - \sqrt{100 - 9}\right)}$$

$$= \frac{2 \times 5000}{\pi \times 10 \times 0.46}$$

$$= \frac{1000}{14.45}$$

$$\boxed{BHN = 691.97}$$

Hardness, Toughness, Impact, Creep

Problem Statement: *A specimen in a Charpy impact test absorbs 80 J of energy before fracturing. If the cross-sectional area at the notch is 16 mm², determine the impact energy per unit area.*

Solution:

Given : Energy absorbed = 80 J

Cross-sectional Area = 16 mm² = 16×10^{-6} m²

Impact Energy per unit Area = $\frac{\text{Energy absorbed}}{\text{cross-sectional area}}$

$= \frac{80\ J}{16 \times 10^{-6}\ m^2} = 5 \times 10^6\ J/m^2$

Let me also take one more numerical on impact test and we will try to see toughness or we will try to find impact energy per unit area, when it is given that in a Charpy impact test, it absorbs 80 joules of energy before fracturing, the cross-sectional area of the notch is 16 mm square.

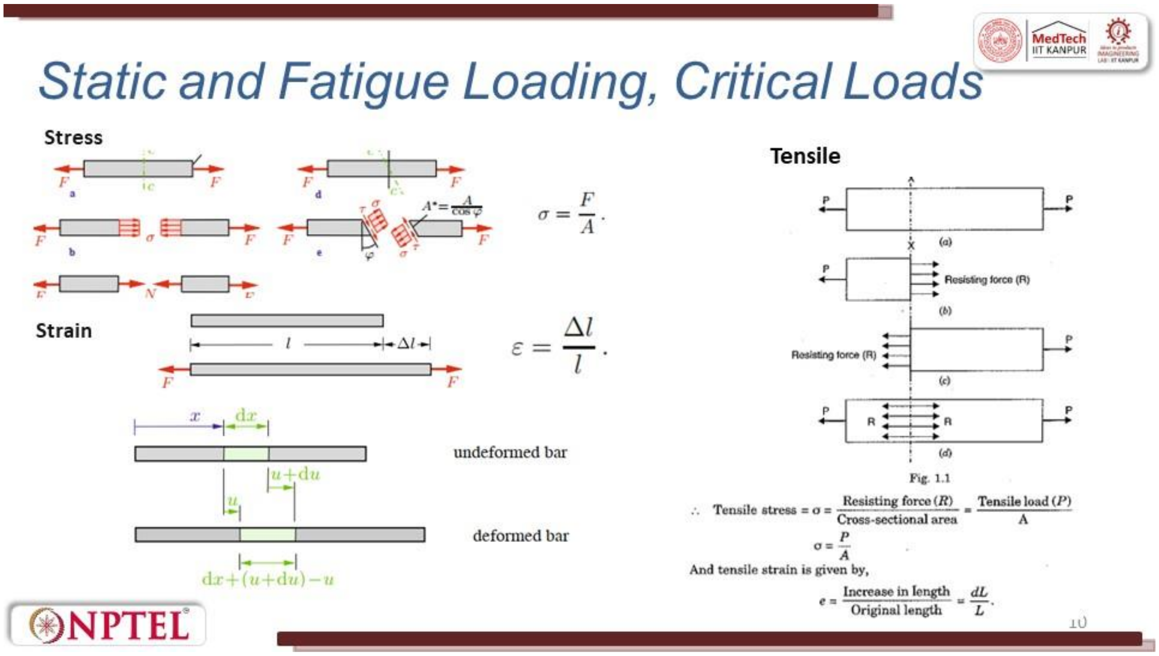
Given : Energy absorbed = 80 J

Cross-sectional Area = 16 mm² = 16×10^{-6} m²

Impact Energy per unit Area = $\frac{\text{Energy absorbed}}{\text{cross-sectional area}}$

$= \frac{80\ J}{16 \times 10^{-6}\ m^2} = 5 \times 10^6\ J/m^2$

Static and Fatigue Loading, Critical Loads



Now, let us move to the Static and Fatigue Loading. So, just to recall the general relationships that we have, this is stress. When force is acting in two opposite directions, you see, or the force could also be applied in the same direction, that is compression.

So, the stress here is force per unit area, where area is given here. So, if it is inclined.

$$A^* = \frac{A}{Cos\varphi}$$

Phi is the angle at which this is breaking. So, this is stress that is there. The force in different directions. If it is the direction of force, exactly force in opposite direction of the force is normal force N.

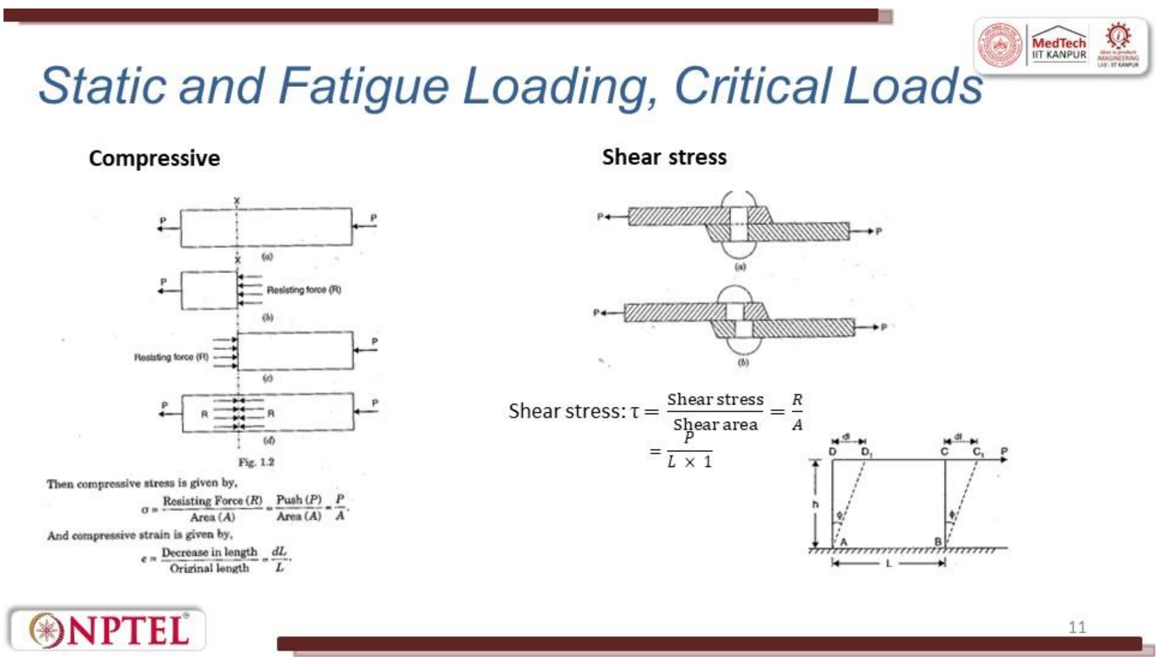
So, strain is change in length per unit original length. So, this is the original length L, small l here and this is the change in length delta l. So, the total force here is F acting in both the directions, in the left and the right directions. So, force per unit radius over stress as it is given. Now, strain could also be there for two conditions, undeformed bar and deformed bar.

For undeformed bar, you should say the x has changed, the strain is there and dx is the increase in length. When the bar is also deformed, this is deformation value u. So, when u or beyond u, it is u + du. Here, the total change in length becomes dx + u + du - u, right.

Similarly, to recall the concepts on tensile loading, you see that tensile loading, the loads are acting in different directions.

One in, you call it negative direction, another in positive direction. So, there are resisting forces oppositely acting. Suppose at this point, the force is acting. This is the plane at which forces are acting, plane xx. So on the xx plane, there are resisting forces R, right.

And also forces opposite to the forces acting upon the positive axis or right axis, so that is again force R. So here you see a combined figure where load is being applied in two different directions and there are resisting forces. So, this resisting force gives me the value of stress that is resisting force per unit cross section area which is equivalent to my tensile load per unit area. So, this stress tensile stress is $\frac{P}{A}$. Also, strain is again increase in length by original length that is $\frac{dL}{L}$.



In the case of the Compressive load, the forces are acting in the same direction. The things are being compressed. So, here also the force is acting in negative direction. Here also it is in negative direction. The resisting force is also in the same direction here, in this case. Here, the load is acting in this direction.

In figure C, the resisting force is in the opposite direction. But anyway, whatever the resisting force is, here, in case of pull, it becomes push. But still, stress is same, resisting force per unit area, that is push per unit area, that is $\frac{P}{A}$ and strain is also same, that is change in length by original length. However, here the change is decrease in length. This is the difference between compressive and the tensile forces.

Now comes the shear stress. You see here the force is acting in this direction. This is a rivet. This rivet is subjected to a load P in the negative and positive direction. And this is a plane where shear could happen.

So, at a shear plane, the shear stress, this is equal to tau, which is equal to, again, resistance force per unit area. So here, if ϕ is there an angle, obviously, again, the area will turn to $\frac{A}{Cos\varphi}$. And similarly, the changes could happen. So this again, I could put here load per unit length.

Static and Fatigue Loading, Critical Loads

Young's Modulus or Modulus of Elasticity

Hooke's law* *states that when a material is loaded within elastic limit, the stress is directly proportional to strain*

$$\sigma \propto \varepsilon \quad \text{or} \quad \sigma = E.\varepsilon$$

$$E = \frac{\sigma}{\varepsilon} = \frac{P \times l}{A \times \delta l}$$

Table 4.2. Values of *C* for the commonly used materials.

Material	*Modulus of rigidity (C) in GPa i.e. GN/m² or kN/mm²*
Steel	80 to 100
Wrought iron	80 to 90
Cast iron	40 to 50
Copper	30 to 50
Brass	30 to 50
Timber	10

Shear Modulus or Modulus of Rigidity

It has been found experimentally that within the elastic limit, the shear stress is directly proportional to shear strain. Mathematically

$\tau \propto \phi$ or $\tau = C . \phi$ or $\tau / \phi = C$

where τ = Shear stress,

ϕ = Shear strain, and

C = Constant of proportionality, known as shear modulus or modulus of rigidity. It is also denoted by N or G.

The following table shows the values of modulus of rigidity (C) for the materials in every day use:

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Now, let us try to recall the Hooke's laws states that the Young's Modulus says, stress per unit strain for which the relation holds good like this, right. So because stress is proportional to strain, so Stress = Young Modulus × Strain; young modulus is the ratio of stress by strain that is we have already discussed for steel, it is from 80 to 100 for raw tire and from 80 to 90 and so on different materials are given here for timber or wood, it is very low, it is 10.

The young modulus, that means the more the young modulus is, the more strength the material has. So, next is Shear modulus or Modulus of Rigidity. So, it has been found experimentally that within elastic media, the shear stress is directly proportional to shear strain.

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proportional to shear strain. Mathematically,

$\tau \propto \varphi$ or $\tau = C \cdot \varphi$ or $\tau / \varphi = C$

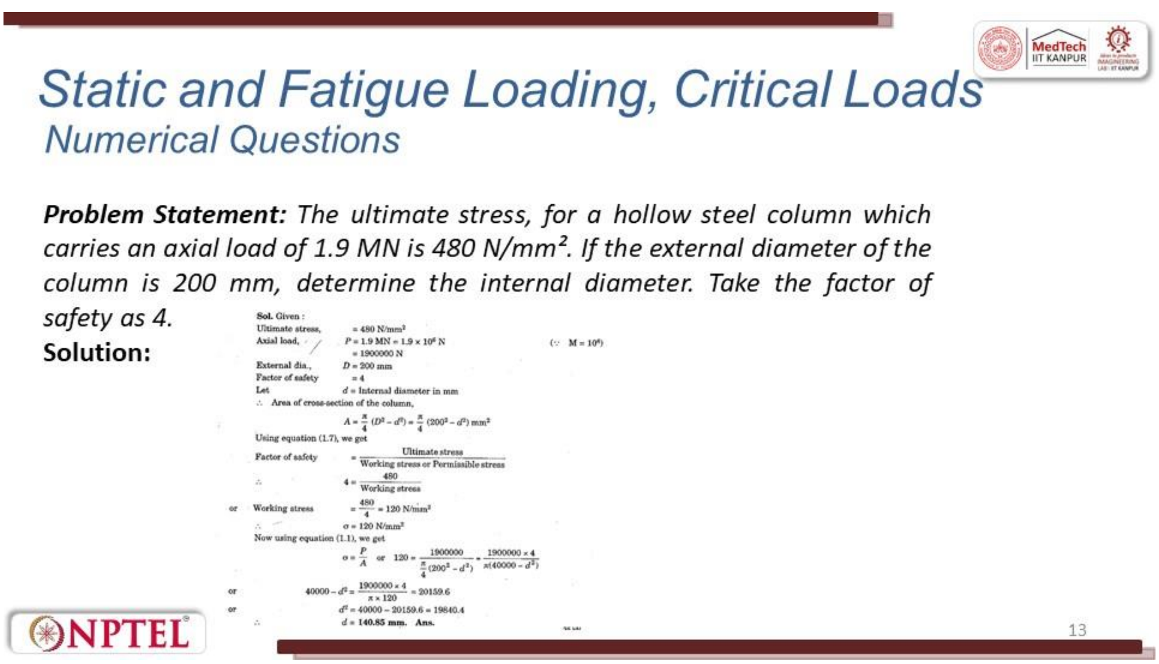
where τ = Shear stress,

φ = Shear strain, and

C = Constant of proportionality, known as shear modulus or modulus of rigidity. It is also denoted by N or G.

The following table shows the values of modulus of rigidity (C) for the materials in every day use.

Now, again for shear modulus, there are tables available. Those tables will be shown you when we will try to show you the laboratory demonstrations.



Let me try to come to a problem statement here. The ultimate stress for a hollow steel column which carries an axial load of 1.9 mega Newton is 480 Newtons per millimeter square. If the external diameter of column is 200 millimeter internal diameter, take factor of safety as 4.

Sol. Given :

Ultimate stress, $= 480 \text{ N/mm}^2$

Axial load, $P = 1.9 \text{ MN} = 1.9 \times 10^6 \text{ N}$ $(\because \text{M} = 10^6)$

$= 1900000 \text{ N}$

External dia., $D = 200 \text{ mm}$

Factor of safety $= 4$

Let d = Internal diameter in mm

$\therefore$ Area of cross-section of the column,

$$A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(200^2 - d^2) \text{ mm}^2$$

Using equation (1.7), we get

$$\text{Factor of safety} = \frac{\text{Ultimate stress}}{\text{Working stress or Permissible stress}}$$

$$\therefore \quad 4 = \frac{480}{\text{Working stress}}$$

or $$\text{Working stress} = \frac{480}{4} = 120 \text{ N/mm}^2$$

$$\therefore \quad \sigma = 120 \text{ N/mm}^2$$

Now using equation (1.1), we get

$$\sigma = \frac{P}{A} \quad \text{or} \quad 120 = \frac{1900000}{\frac{\pi}{4}(200^2 - d^2)} = \frac{1900000 \times 4}{\pi(40000 - d^2)}$$

or $$40000 - d^2 = \frac{1900000 \times 4}{\pi \times 120} = 20159.6$$

or $$d^2 = 40000 - 20159.6 = 19840.4$$

$$\therefore \quad d = \mathbf{140.85 \text{ mm.}} \quad \textbf{Ans.}$$

Static and Fatigue Loading, Critical Loads

Problem Statement: *Determine the changes in length, breadth and thickness of a steel bar which is 4 m long, 30 mm wide and 20 mm thick and is subjected to an axial pull of 30 kN in the direction of its length. Take E = 2 × 10^5N / mm^2 and Poisson's ratio = 0.3.*

Solution:

Sol. Given :

Length of the bar, $L = 4 \text{ m} = 4000 \text{ mm}$

Breadth of the bar, $b = 30 \text{ mm}$

Thickness of the bar, $t = 20 \text{ mm}$

$\therefore$ Area of cross-section, $A = b \times t = 30 \times 20 = 600 \text{ mm}^2$

Axial pull, $P = 30 \text{ kN} = 30000 \text{ N}$

Young's modulus, $E = 2 \times 10^5 \text{ N/mm}^2$

Poisson's ratio, $\mu = 0.3$.

Now strain in the direction of load (or longitudinal strain),

$$= \frac{\text{Stress}}{E} = \frac{\text{Load}}{\text{Area} \times E} \qquad \left(\because \text{Stress} = \frac{\text{Load}}{\text{Area}}\right)$$

$$= \frac{P}{A.E.} = \frac{30000}{600 \times 2 \times 10^5} = 0.00025.$$

But longitudinal strain $= \frac{\delta L}{L}$.

$$\therefore \quad \frac{\delta L}{L} = 0.00025.$$

$\therefore$ δL (or change in length) $= 0.00025 \times L$

$= 0.00025 \times 4000 = \mathbf{1.0 \text{ mm.}}$ **Ans.**

Using equation (2.3),

$$\text{Poisson's ratio} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

or $$0.3 = \frac{\text{Lateral strain}}{0.00025}$$

$\therefore$ Lateral strain $= 0.3 \times 0.00025 = 0.000075$.

We know that

$$\text{Lateral strain} = \frac{\delta b}{b} \text{ or } \frac{\delta d}{d}\left(\text{or } \frac{\delta t}{t}\right)$$

$\therefore$ $\delta b = b \times$ Lateral strain

$= 30 \times 0.000075 = \mathbf{0.00225 \text{ mm.}}$ **Ans.**

Similarly, $\delta t = t \times$ Lateral strain

$= 20 \times 0.000075 = \mathbf{0.0015 \text{ mm.}}$ **Ans.**

So, this is another numerical problem in the similar direction where we have lateral strain, where we have the Young Modulus given. Determine the change in length, breadth, thickness of a steel bar. It is length, breadth, thickness.

Sol. Given :

Length of the bar, $L = 4 \text{ m} = 4000 \text{ mm}$

Breadth of the bar, $b = 30 \text{ mm}$

Thickness of the bar, $t = 20 \text{ mm}$

$\therefore$ Area of cross-section, $A = b \times t = 30 \times 20 = 600 \text{ mm}^2$

Axial pull, $P = 30 \text{ kN} = 30000 \text{ N}$

Young's modulus, $E = 2 \times 10^5 \text{ N/mm}^2$

Poisson's ratio, $\mu = 0.3$.

Now strain in the direction of load (or longitudinal strain),

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$$= \frac{P}{A.E.} = \frac{30000}{600 \times 2 \times 10^5} = 0.00025.$$

But longitudinal strain $= \frac{\delta L}{L}$.

$$\therefore \quad \frac{\delta L}{L} = 0.00025.$$

$\therefore$ δL (or change in length) $= 0.00025 \times L$

$= 0.00025 \times 4000 =$ **1.0 mm. Ans.**

Using equation (2.3),

$$\text{Poisson's ratio} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

or

$$0.3 = \frac{\text{Lateral strain}}{0.00025}$$

$\therefore$ Lateral strain $= 0.3 \times 0.00025 = 0.000075$.

We know that

$$\text{Lateral strain} = \frac{\delta b}{b} \text{ or } \frac{\delta d}{d}\left(\text{or } \frac{\delta t}{t}\right)$$

$\therefore$ $\delta b = b \times$ Lateral strain

$= 30 \times 0.000075 =$ **0.00225 mm. Ans.**

Similarly, $\delta t = t \times$ Lateral strain

$= 20 \times 0.000075 =$ **0.0015 mm. Ans.**

Static and Fatigue Loading, Critical Loads

Problem Statement: *A steel rod is 2 m long and 50 mm in diameter. An axial pull of 100 kN is suddenly applied to the rod. Calculate the instantaneous stress induced and also the instantaneous elongation produced in the rod. Take E = 200GN /m²*

Solution:

Sol. Given :

Length, $L = 2 \text{ m} = 2 \times 1000 = 2000 \text{ mm}$

Diameter, $d = 50 \text{ mm}$

$\therefore$ Area, $A = \frac{\pi}{4} \times 50^2 = 625\,\pi \text{ mm}^2$

Suddenly applied load,

$P = 100 \text{ kN} = 100 \times 1000 \text{ N}$

Value of $E = 200 \text{ GN/m}^2 = 200 \times 10^9 \text{ N/m}^2$ $\quad (\because G = \text{Giga} = 10^9)$

$= \frac{200 \times 10^9}{10^6} \text{ N/mm}^2 \quad (\because 1 \text{ m} = 1000 \text{ mm} \quad \therefore \text{m}^2 = 10^6 \text{ mm}^2)$

$= 200 \times 10^3 \text{ N/mm}^2$

Using equation (4.5) for suddenly applied load,

$\sigma = 2 \times \frac{P}{A} = 2 \times \frac{100 \times 1000}{625\,\pi} \text{ N/mm}^2 = \mathbf{101.86 \text{ N/mm}^2}$. **Ans.**

Let dL = Elongation

Then $dL = \frac{P}{E} \times L = \frac{101.86}{200 \times 10^3} \times 2000 = \mathbf{1.0186 \text{ mm}}$. **Ans.**

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There is another problem very simple similar to the one that we have been doing in the previous tutorial. A steel rod is 2 meters long and 50 millimeters in diameter, an axial pull of 100 kilo newton is suddenly applied to the rod, calculate the instantaneous stress induced and also the instantaneous elongation produced in the rod d is given here.

Sol. Given :

Length, $L = 2 \text{ m} = 2 \times 1000 = 2000 \text{ mm}$

Diameter, $d = 50 \text{ mm}$

$\therefore$ Area, $A = \frac{\pi}{4} \times 50^2 = 625\,\pi \text{ mm}^2$

Suddenly applied load,

$P = 100 \text{ kN} = 100 \times 1000 \text{ N}$

Value of $E = 200 \text{ GN/m}^2 = 200 \times 10^9 \text{ N/m}^2$ $\quad (\because G = \text{Giga} = 10^9)$

$= \frac{200 \times 10^9}{10^6} \text{ N/mm}^2 \quad (\because 1 \text{ m} = 1000 \text{ mm} \quad \therefore \text{m}^2 = 10^6 \text{ mm}^2)$

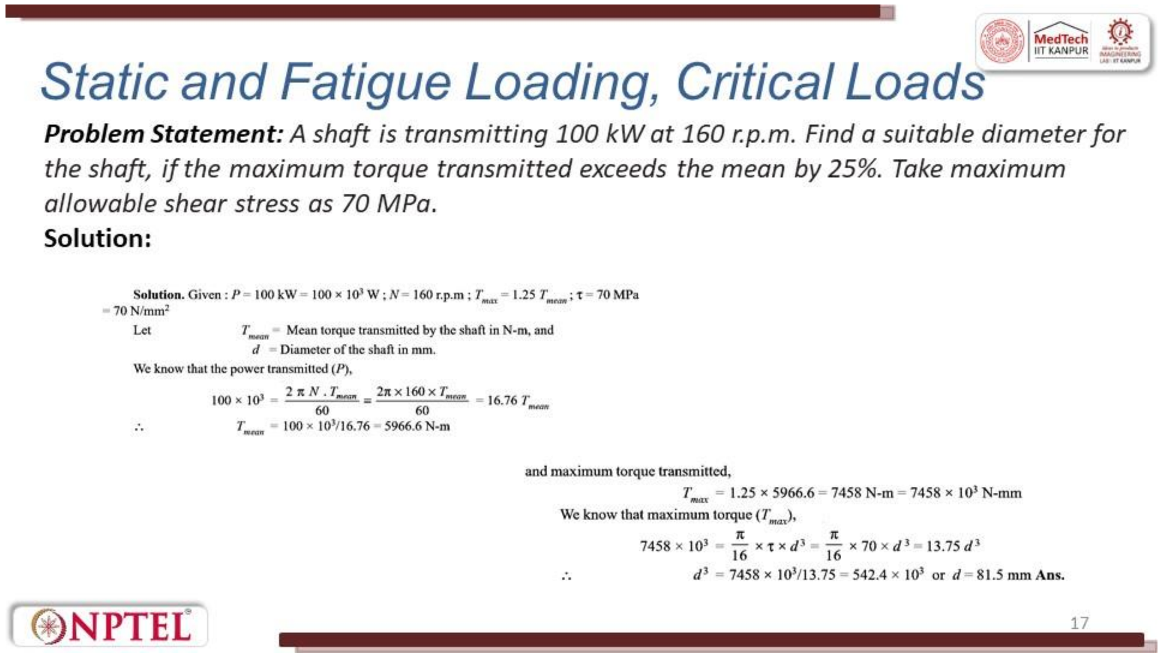
$= 200 \times 10^3 \text{ N/mm}^2$

Using equation (4.5) for suddenly applied load,

$\sigma = 2 \times \frac{P}{A} = 2 \times \frac{100 \times 1000}{625\,\pi} \text{ N/mm}^2 = \mathbf{101.86 \text{ N/mm}^2}$. **Ans.**

Let dL = Elongation

Then $dL = \frac{P}{E} \times L = \frac{101.86}{200 \times 10^3} \times 2000 = \mathbf{1.0186 \text{ mm}}$. **Ans.**



Next numerical is talking about a shaft that is transmitting 100 kilowatts at 160 rpm. Now we are talking about shaft, that means a torque would come. Find a suitable diameter of the shaft. If maximum torque transmitted exceeds the mean by 25%, take minimum allowable shear stress as 70 MPa.

Solution. Given : $P = 100$ kW $= 100 \times 10^3$ W ; $N = 160$ r.p.m ; $T_{max} = 1.25\ T_{mean}$; $\tau = 70$ MPa $= 70$ N/mm^2

Let T_{mean} = Mean torque transmitted by the shaft in N-m, and

d = Diameter of the shaft in mm.

We know that the power transmitted (P),

$$100 \times 10^3 = \frac{2\,\pi\,N\,.\,T_{mean}}{60} = \frac{2\pi \times 160 \times T_{mean}}{60} = 16.76\ T_{mean}$$

$$\therefore \quad T_{mean} = 100 \times 10^3/16.76 = 5966.6 \text{ N-m}$$

and maximum torque transmitted,

$$T_{max} = 1.25 \times 5966.6 = 7458 \text{ N-m} = 7458 \times 10^3 \text{ N-mm}$$

We know that maximum torque (T_{max}),

$$7458 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 70 \times d^3 = 13.75\ d^3$$

$$\therefore \quad d^3 = 7458 \times 10^3/13.75 = 542.4 \times 10^3 \quad \text{or} \quad d = 81.5 \text{ mm}$$ **Ans.**

Static and Fatigue Loading, Critical Loads

Problem Statement: *A shaft is transmitting 97.5 kW at 180 r.p.m. If the allowable shear stress in the material is 60 MPa, find the suitable diameter for the shaft. The shaft is not to twist more that 1° in a length of 3 metres. Take C = 80 GPa.*

Solution:

Solution. Given : $P = 97.5$ kW $= 97.5 \times 10^3$ W ; $N = 180$ r.p.m. ; $\tau = 60$ MPa $= 60$ N/mm^2 ; $\theta = 1° = \pi / 180 = 0.0174$ rad ; $l = 3$ m $= 3000$ mm ; $C = 80$ GPa $= 80 \times 10^9$ N/m^2 $= 80 \times 10^3$ N/mm^2

Let T = Torque transmitted by the shaft in N-m, and

d = Diameter of the shaft in mm.

We know that the power transmitted by the shaft (P),

$$97.5 \times 10^3 = \frac{2\pi N.T}{60} = \frac{2\pi \times 180 \times T}{60} = 18.852\,T$$

$\therefore$ $T = 97.5 \times 10^3/18.852 = 5172$ N-m $= 5172 \times 10^3$ N-mm

Now let us find the diameter of the shaft based on the strength and stiffness.

1. ***Considering strength of the shaft***

We know that the torque transmitted (T),

$$5172 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78\,d^3$$

$\therefore$ $d^3 = 5172 \times 10^3/11.78 = 439 \times 10^3$ or $d = 76$ mm ...(*i*)

2. ***Considering stiffness of the shaft***

Polar moment of inertia of the shaft,

$$J = \frac{\pi}{32} \times d^4 = 0.0982\,d^4$$

We know that $\frac{T}{J} = \frac{C.\theta}{l}$

$$\frac{5172 \times 10^3}{0.0982\,d^4} = \frac{80 \times 10^3 \times 0.0174}{3000} \text{ or } \frac{52.7 \times 10^6}{d^4} = 0.464$$

$\therefore$ $d^4 = 52.7 \times 10^6/0.464 = 113.6 \times 10^6$ or $d = 103$ mm ...(*ii*)

Taking larger of the two values, we shall provide $d = 103$ say 105 mm **Ans.**

Another problem statement is there that I will also leave for you to do. It is given shaft is transmitting 97.5 kilowatts at 180 rpm. If the allowable shear stress in the material is 60 megapascal, find the suitable diameter of the shaft. The shaft is not to twist more than one degree in length of 3 meters take C here as 80 gigapascal.

Solution. Given : $P = 97.5$ kW $= 97.5 \times 10^3$ W ; $N = 180$ r.p.m. ; $\tau = 60$ MPa $= 60$ N/mm^2 ; $\theta = 1° = \pi / 180 = 0.0174$ rad ; $l = 3$ m $= 3000$ mm ; $C = 80$ GPa $= 80 \times 10^9$ N/m^2 $= 80 \times 10^3$ N/mm^2

Let T = Torque transmitted by the shaft in N-m, and

d = Diameter of the shaft in mm.

We know that the power transmitted by the shaft (P),

$$97.5 \times 10^3 = \frac{2\pi N.T}{60} = \frac{2\pi \times 180 \times T}{60} = 18.852\,T$$

$\therefore$ $T = 97.5 \times 10^3/18.852 = 5172$ N-m $= 5172 \times 10^3$ N-mm

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1. ***Considering strength of the shaft***

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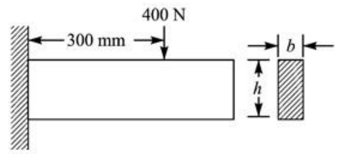
$\therefore$ $d^4 = 52.7 \times 10^6/0.464 = 113.6 \times 10^6$ or $d = 103$ mm ...(*ii*)

Taking larger of the two values, we shall provide $d = 103$ say 105 mm **Ans.**

Static and Fatigue Loading, Critical Loads

Problem Statement: *A beam of uniform rectangular cross-section is fixed at one end and carries an electric motor weighing 400 N at a distance of 300 mm from the fixed end. The maximum bending stress in the beam is 40 MPa. Find the width and depth of the beam, if depth is twice that of width.*



Solution:

Solution. Given: $W = 400$ N ; $L = 300$ mm ; $\sigma_b = 40$ MPa $= 40$ N/mm^2 ; $h = 2b$

The beam is shown in Fig. 5.7.

Let b = Width of the beam in mm, and

h = Depth of the beam in mm.

$\therefore$ Section modulus,

$$Z = \frac{b\,.\,h^2}{6} = \frac{b\,(2b)^2}{6} = \frac{2\,b^3}{3}\ \text{mm}^3$$

Maximum bending moment (at the fixed end),

$$M = W.L = 400 \times 300 = 120 \times 10^3 \text{ N-mm}$$

We know that bending stress (σ_b),

$$40 = \frac{M}{Z} = \frac{120 \times 10^3 \times 3}{2\,b^3} = \frac{180 \times 10^3}{b^3}$$

$\therefore$ $b^3 = 180 \times 10^3/40 = 4.5 \times 10^3$ or $b = 16.5$ mm **Ans.**

and $h = 2b = 2 \times 16.5 = 33$ mm **Ans.**

NPTEL

One last problem statement that will take today is about this figure that is given here, it is given a beam of uniform rectangular cross section is fixed at one end and carries an electric motor weighing 400 newton, this electric motor here it is fixed here in at this end not at a distance of 300 millimeter from this fixed end, this distance is given 300 millimeters.

Solution. Given: $W = 400$ N ; $L = 300$ mm ; $\sigma_b = 40$ MPa $= 40$ N/mm^2 ; $h = 2b$

The beam is shown in Fig. 5.7.

Let b = Width of the beam in mm, and

h = Depth of the beam in mm.

$\therefore$ Section modulus,

$$Z = \frac{b\,.\,h^2}{6} = \frac{b\,(2b)^2}{6} = \frac{2\,b^3}{3}\ \text{mm}^3$$

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$\therefore$ $b^3 = 180 \times 10^3/40 = 4.5 \times 10^3$ or $b = 16.5$ mm **Ans.**

and $h = 2b = 2 \times 16.5 = 33$ mm **Ans.**

In relation to the topics of hardness, toughness or static and fatigue loading and critical loads. Also, bending movement numerical is also discussed. I will further talk about some problem statements in the stresses in cylinders, something of buckling of columns and Mohr circle in the next part of tutorial.

Thank you.