Basics of Mechanical Engineering-1

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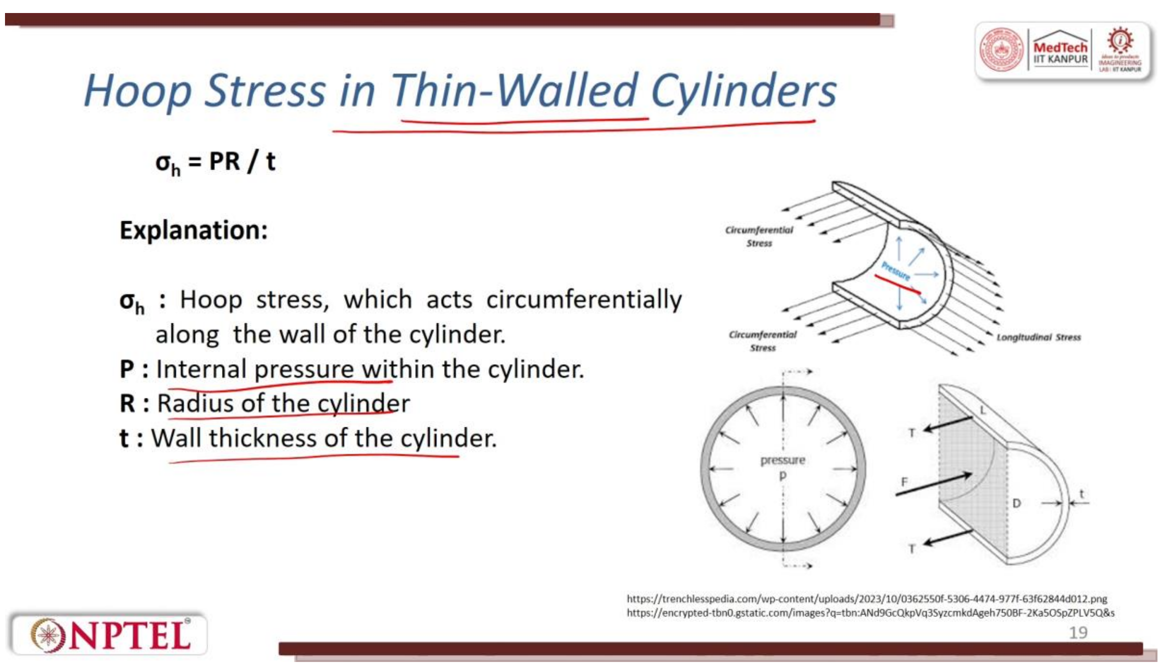
Week 07

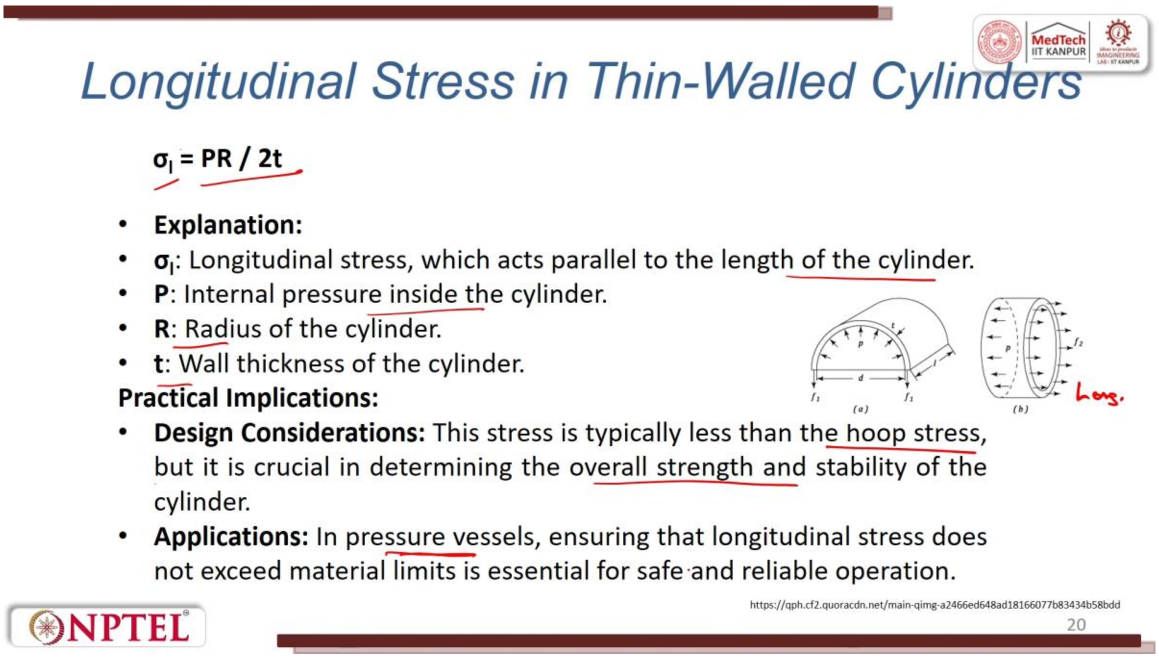
Lecture 30

Tutorial-3 (Part 2 of 2)

Welcome friends to the second part of the tutorial 3 in the course Basics of Mechanical Engineering 1. This course is co-taught by Professor J. Ram Kumar, Department of Design and Mechanical Engineering and Dr. Amandeep Singh Oberoi, Imagining Laboratory. I am Amandeep. We have taken certain numerical statements in the previous lecture, the previous part of this tutorial on Hardness, Toughness, and Static, Fatigue loading and Critical loads. This lecture will be focusing on Stresses in Cylinders as Spheres, Buckling of Columns, and Mohr's circle will try to shed some more light upon.



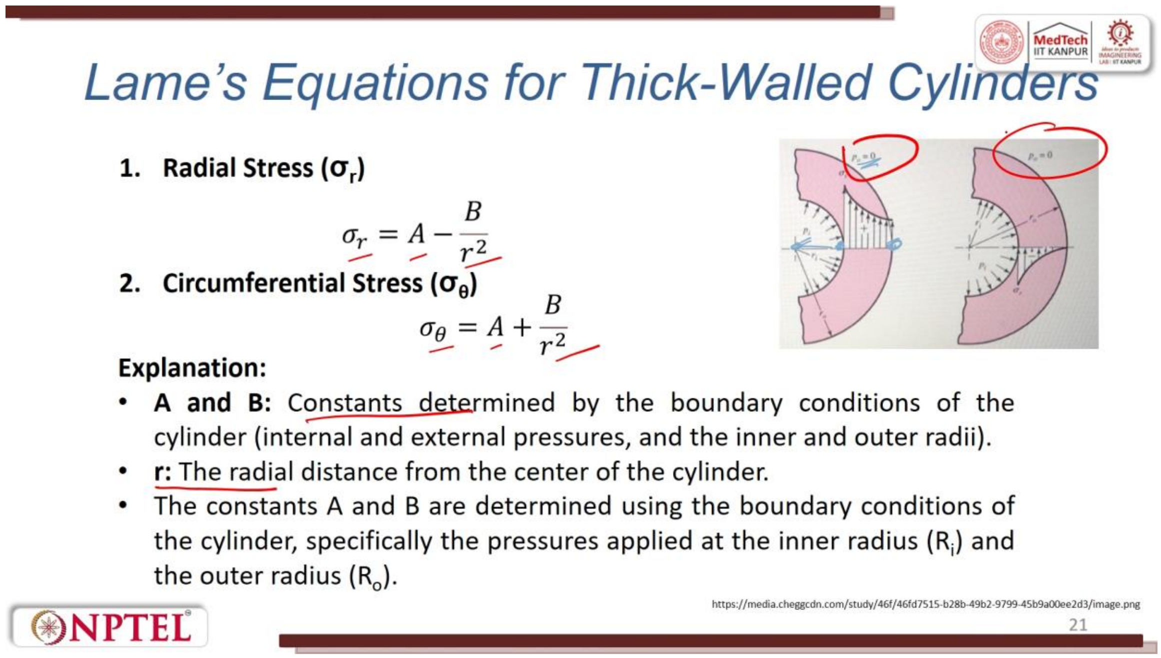
Let us try to recall our concepts on the cylinders. Hoop stress, if you recall, in thin wall cylinders, that is induced with a cylinder having internal pressure within the cylinder and of radius R. So, P is internal pressure, R is radius of the cylinder, t is the wall thickness of the cylinder. We are talking about thin wall cylinder, hoop stretch is PR/t.



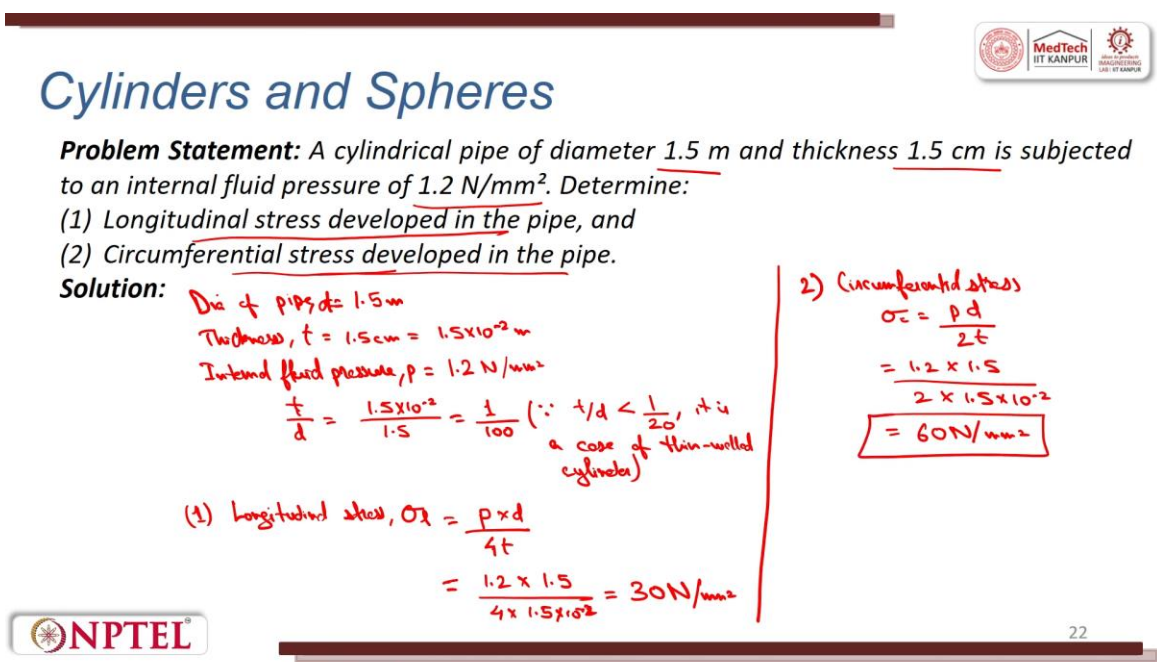
And for a longitudinal stress in thin wall cylinder, you see the longitudinal stress along the radial direction. So, this is longitudinal stress.

So, here this is sigma longitudinal, $\sigma_l = \frac{PR}{2t}$. The derivations of these were discussed by Professor Ramkumar already. So, longitudinal stress which acts parallel to the length of the cylinder is internal pressure inside the cylinder is PE, R is radius of cylinder, t is wall thickness and practical implications were also discussed that this stress is typically less than the hoop stress but it is crucial in determining the overall strength and stability of cylinder.

Implications were given in the systems such as pressure vessels ensuring that longitudinal stress does not exceed material limits. This is essential for safe and reliable operation.



Also, Lame's equations for thick wall cylinders were discussed in which $\sigma_r = A - \frac{B}{r^2}$ and circumferential stress s$\sigma_\theta = A + \frac{B}{r^2}$. So, where A and B are constants determined by the boundary condition of the cylinders that is internal and external pressures and inner and outer radii. r is the radial distance from the center of the cylinder. So, these were already discussed in the previous lecture. So, these were already discussed in the theory lectures by Professor Ramkumar.



Let me take some examples here. A cylindrical pipe of diameter 1.5 m and thickness 1.5 cm is subjected to an internal fluid pressure of 1.2 N/mm². Determine:

(1) Longitudinal stress developed in the pipe, and

(2) Circumferential stress developed in the pipe.

Solution: Given: Dia. of pipe, d = 1.5 m

Thickness, $t = 1.5 \text{ cm} = 1.5 \times 10^{-2}$

Internal fluid pressure, p = 1.2 N/mm²

$\frac{t}{d} = \frac{1.5 \times 10^{-2}}{1.5} = \frac{1}{100}$, which is less than $\frac{1}{20}$,

hence this is a case of thin cylinder.

i. The longitudinal stress (σ_l)

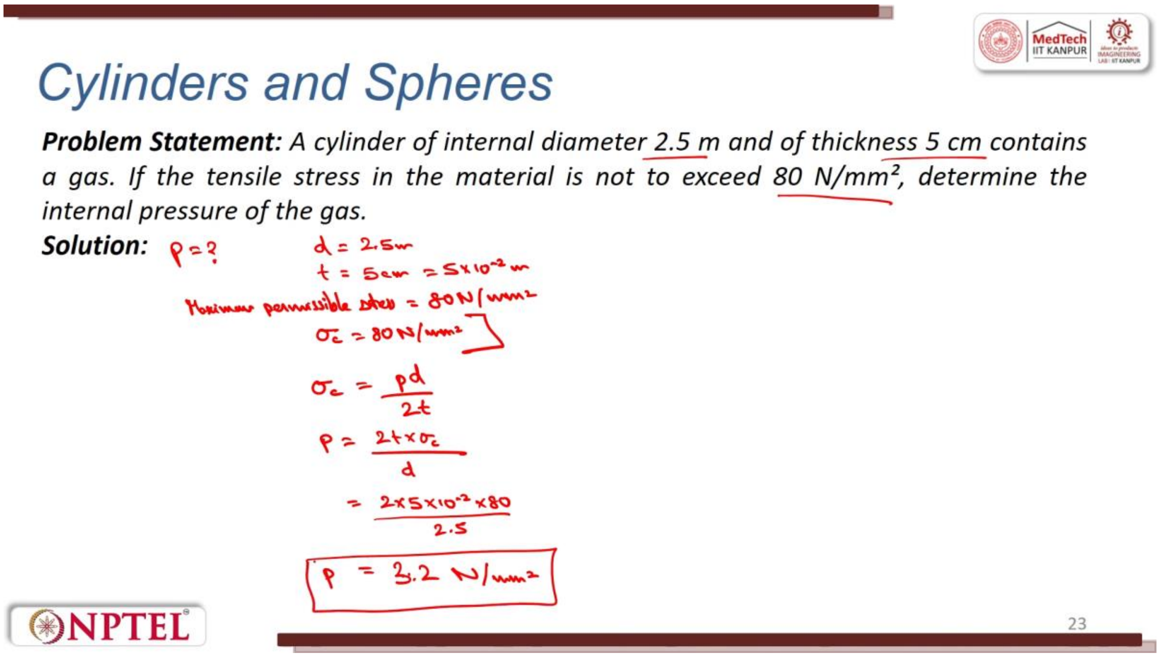
$$\sigma_l = \frac{p \times d}{4t}$$

$$= \frac{1.2 \times 1.5}{4 \times 1.5 \times 10^{-2}} = 30 \text{ N/mm}^2 \text{.} \quad \text{Ans.}$$

ii. The circumference stress (σ_c)

$$\sigma_c = \frac{p \times d}{2t}$$

$$= \frac{1.2 \times 1.5}{2 \times 1.5 \times 10^{-2}} = 60 \text{ N/mm}^2 \text{.} \quad \text{Ans.}$$



Let me see another problem statement. A cylinder of internal diameter 2.5 m and of thickness 5 cm contains a gas. If the tensile stress in the material is not to exceed 80 N/mm², determine the internal pressure of the gas.

Solution: Given: Internal dia. of cylinder, $d = 2.5$ m

Thickness of cylinder, $t = 5$ cm $= 5 \times 10^{-2}$mm

Maximum permissible stress, $= 80$ N/mm²

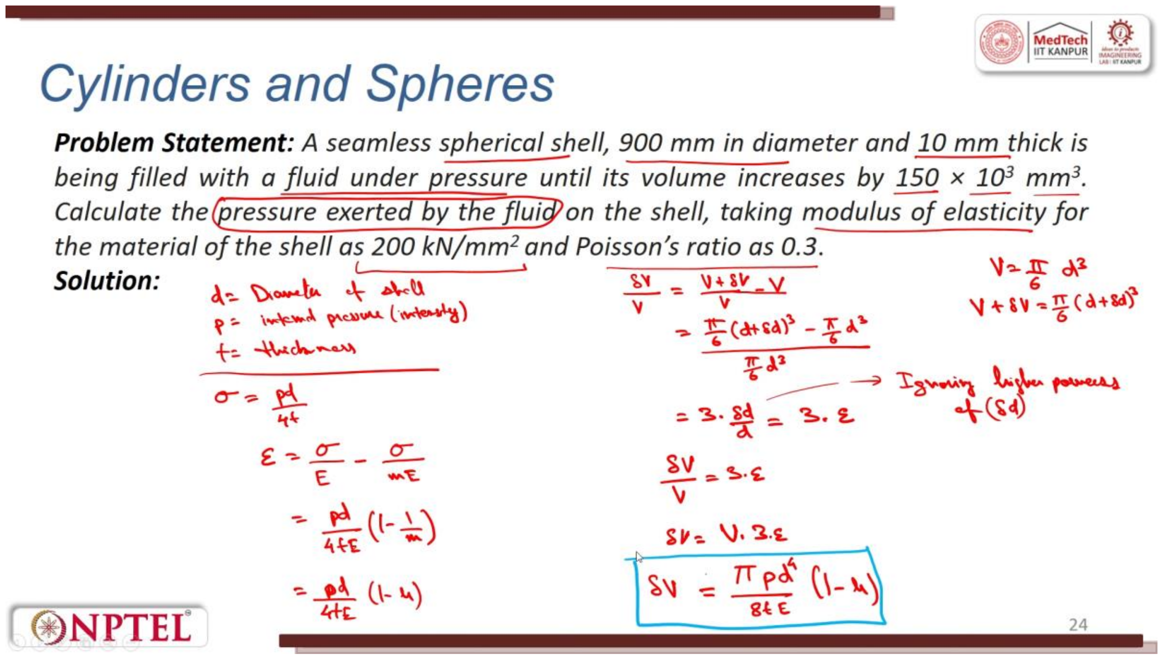
$\sigma_c = 80$ N/mm²

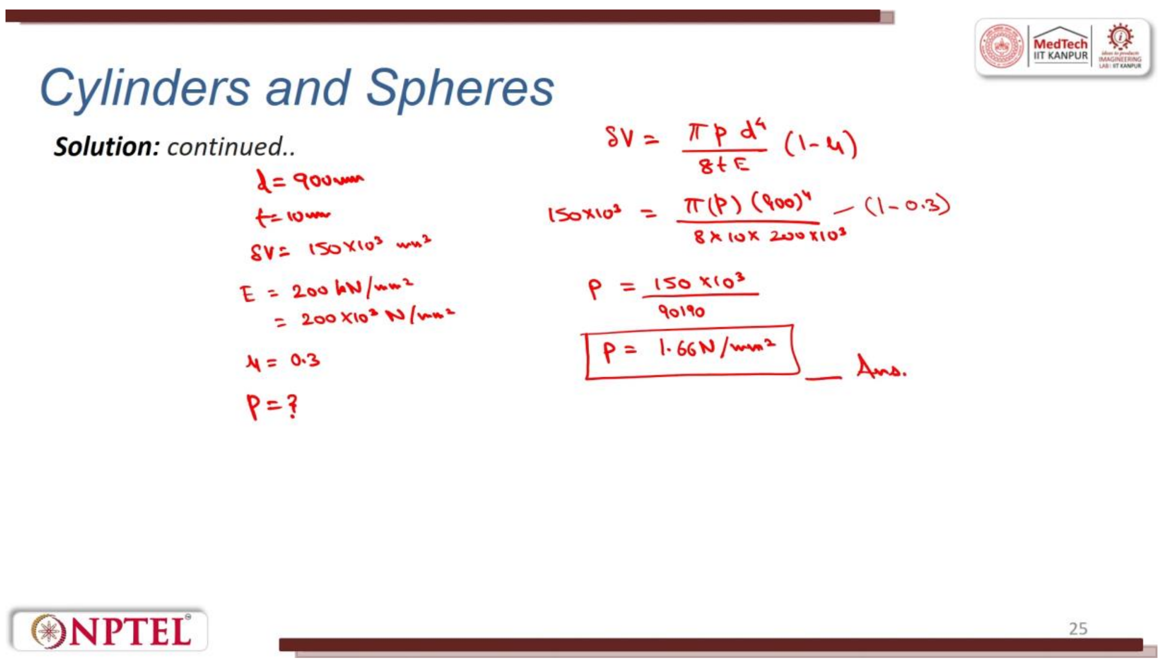
$$\sigma_c = \frac{p \times d}{2t}$$

$$p = \frac{2t \times \sigma_c}{d}$$

$$p = \frac{2 \times 5 \times 10^{-2}}{2.5}$$

$= 3.2$ N/mm² Ans





Here comes now a problem statement on spheres. A seamless spherical shell, 900 mm in diameter and 10 mm thick is being filled with a fluid under pressure until its volume increases by 150 × 103 mm^3. Calculate the pressure exerted by the fluid on the shell, taking modulus of elasticity for the material of the shell as 200 kN/mm^2 and Poisson's ratio as 0.3.

Solution: Given: d = 900mm; t = 10mm; $\delta V = 150 \times 10^3$ mm^3 ; E = 200kN/mm^2

$= 200 \times 10^3$ N/mm^2 ; $\mu = 0.3$;   p = Pressure exerted by the fluid on the shell.

We know that the increase in volume of the spherical shell(δV),

$$150 \times 10^3 = \frac{\pi\, p\, d}{8\, t\, E}(1-\mu) = \frac{\pi\, p\, (900)^4}{8 \times 10 \times 200 \times 10^3} = (1-0.3) = 90190\ p$$

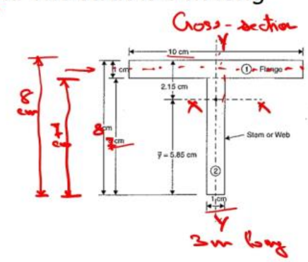
$$p = 150 \times 10^3/90\ 190 = 1.66\ \text{N/mm}^2\ \textbf{Ans.}$$

Columns

Problem Statement: *Calculate the Euler's critical load for a strut of T-section, the flange width being 10 cm, overall depth 8 cm and both flange and stem 1 cm thick. The strut is 3 m long and is built in at both ends. Take E = 2 ×10⁵ N / mm²*

Solution: $l = 3\text{ m} = 3000\text{ mm}$

$E = 2 \times 10^5\ \text{N/mm}^2$



MoI: Y-Y axis has symmetry

C.G. lies on Y-Y axis

$\bar{y}$ = Distance of C.G. of section from bottom end

→ For Flange, Area, $a_1 = 10 \times 1 = 10\ \text{cm}^2$

y_1 = distance of C.G. of area a_1 from bottom end $= 7 + \frac{1}{2}(1) = 7.5\ \text{cm}$

→ For stem/web, $a_2 = 7 \times 1 = 7\ \text{cm}^2$

$y_2 = \frac{7}{2} = 3.5\ \text{cm}$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{10 \times 7.5 + 7 \times 3.5}{10 + 7} = 5.85\ \text{cm}$$

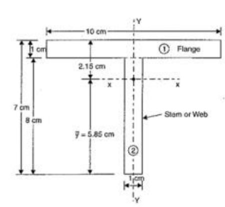
NPTEL

Columns

MoI about X-X and Y-Y axis

$$I_{xx} = \left[\frac{10 \times 1^3}{12} + a_1 \times (2.15 - 0.5)^2\right] + \left[\frac{1 \times 7^3}{12} + a_2 \times (5.85 - 3.5)^2\right]$$

$= 95.298\ \text{cm}^4$



$$I_{YY} = \frac{1 \times 10^3}{12} + \frac{7 \times 1^3}{12} = 83.916\ \text{cm}^4$$

Least value of MoI is for I_{YY}

$I = 83.916\ \text{cm}^4 = 839160\ \text{mm}^4$

$L_e = \frac{l}{2} = \frac{3000}{2} = 1500\ \text{mm}$

P = Euler's critical load $= \frac{\pi^2 E I}{L_e^2}$

$$P = \frac{\pi^2 \times 2 \times 10^5 \times 839160}{1500^2} = 736.19\ \text{kN} \text{ — Ans.}$$

NPTEL

Let me now try to discuss columns. Calculate the Euler's critical load for a strut of T-section, the flange width being 10 cm, overall depth 8 cm and both flange and stem 1 cm thick. The strut is 3 m long and is built in at both ends. Take E = 2 $\times 10^5$ N / mm^6.

Solution: Given: Actual Length l = 3 m = 3000mm

Value of E = 2 $\times 10^5$ N / mm^2

The dimensions of T-section are shown in Fig.19.10

First of all calculate the C.G. of the section The given section is

symmetrical about the Y-Y axis.

Hence the C.G. will lie on y-y axis.

Let y = Distance of C.G. of the section from the bottom end.

For the flange a_1 = Area of the flange = 10 × 1 = 10 cm^2

y_1 = Distance of C.G. of area a_1 from the bottom end.

= 7 + ½ = 7.5 cm.

For the stem or web, a_2 = Area of stem = 7 × .1 = 7cm^2.

$$y_2 = \frac{7}{2} = 3.5 \text{ cm.}$$

$$\bar{y} = \frac{a1 \times y1 + a2 \times y2}{a1 + a2} = \frac{10 \times 7.5 + 7 \times 3.5}{10 + 7}$$

$$= \frac{75 + 24.5}{17} = \frac{99.5}{17} = 5.85 \text{ cm.}$$

Moment of inertia about x-x axis and y-axis.

$$I_{xx} = \left[\frac{10 \times 1^3}{12} + a1 \times (2.15 - 0.5)^2\right] + \left[\frac{1 \times 7}{12} + aa \times (5.85 - 3.5)^2\right]$$

$$= 95.298 \text{ cm}^4$$

$$I_{yy} = \frac{1 \times 10^3}{12} + \frac{7 \times 1^3}{12} = 83.33 + 0.583 = 83.916 \text{ cm}^4$$

The least value of moment of inertia is about y-axis.

∴ $I = I_{yy} = 83.916\ cm^4 = 839160\ mm^4$.

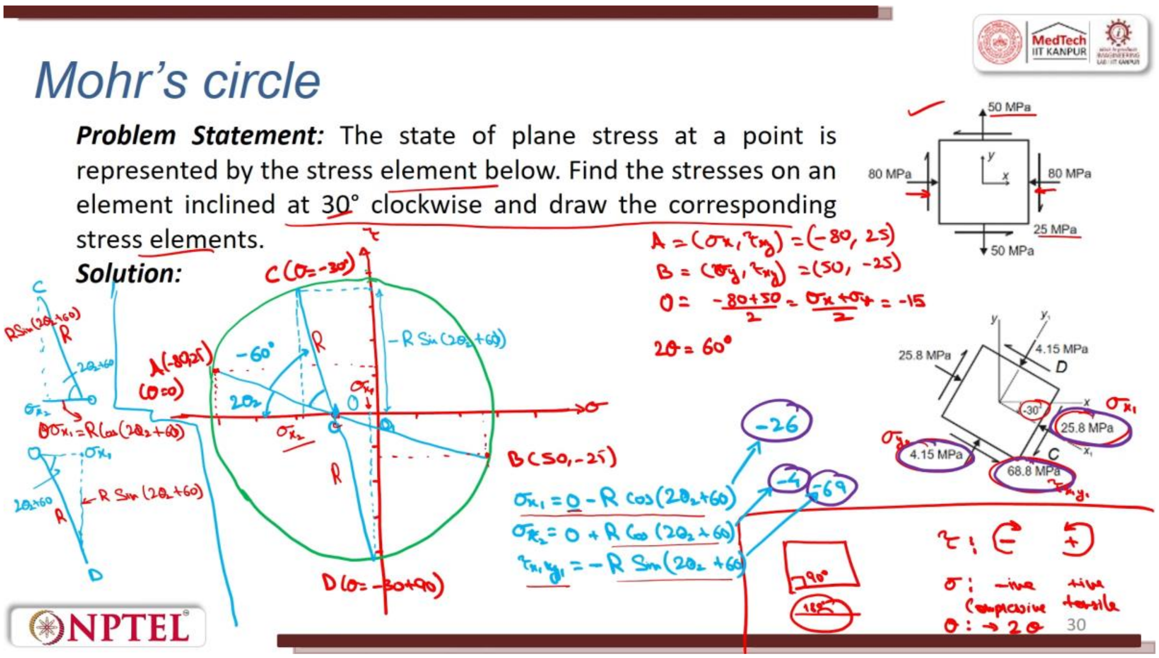
∴ $L_e = \frac{l}{2} = \frac{3000}{2} = 1500\ mm$

P = Euler's critical load.

$$P = \frac{\pi^2 EI}{Le^2}$$

$$= \frac{\pi^2 \times 2 \times 10^5 \times 839160}{1500^3}$$

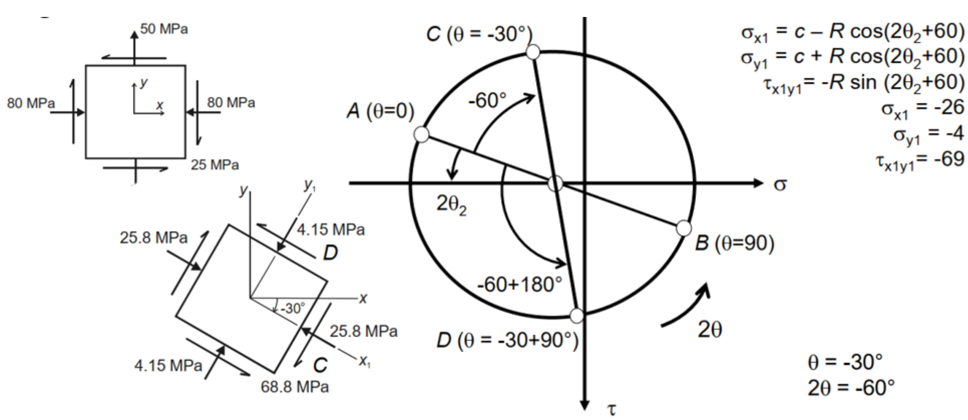
= 736.19 kN. **Ans**.

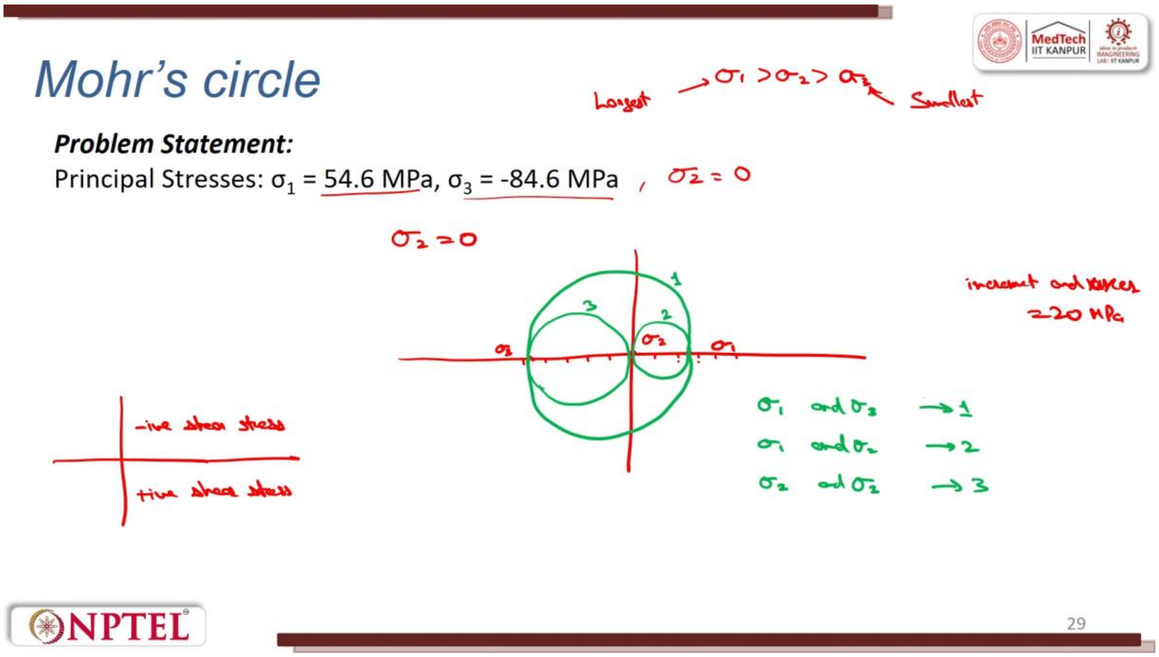


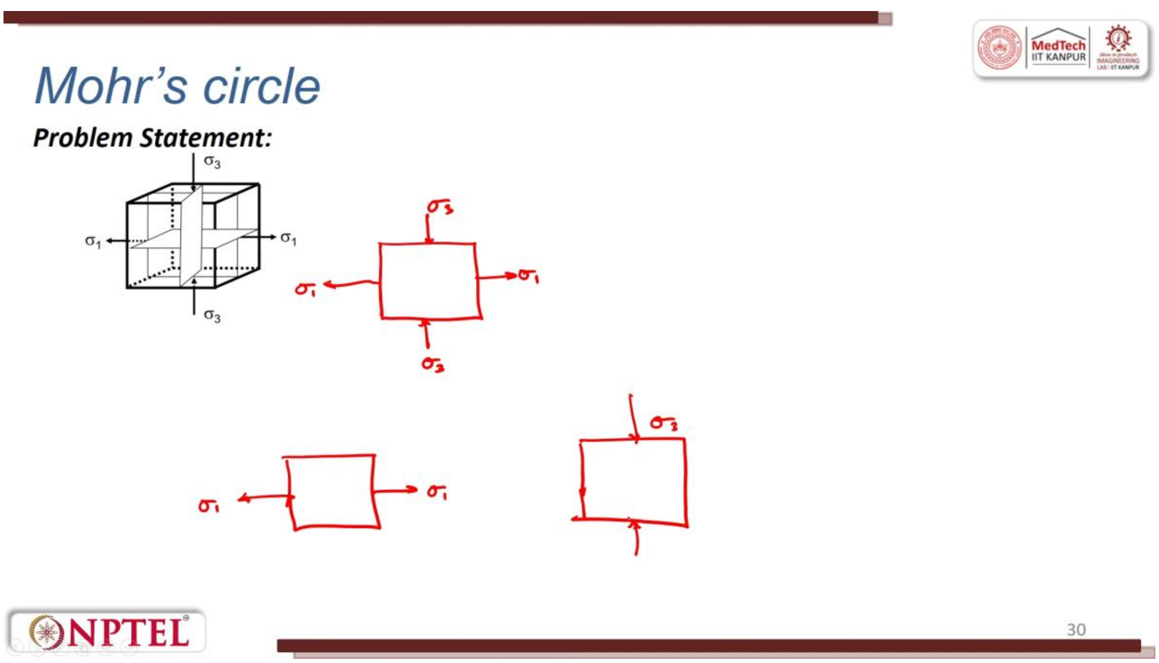
Now, we will try to come to the Mohr circle directly. Mohr circle, if you recall the concept, there were certain rules that shear stress tau is negative when we move in clockwise direction and it is positive when we move in anticlockwise direction. Similarly, sigma normal stress is negative when we talk about compressive load and it is positive when we talk about tensile load, right. And value of theta is always 2 times.

Problem Statement: The state of plane stress at a point is represented by the stress element below. Find the stresses on an element inclined at 30° clockwise and draw the corresponding stress elements.

Solution:







Let me take some more examples here to explain it further. So, here it is given principal stress is σ_1 = 54.6 MPa, σ_2 = -84.6 MPa and element in the plane stress that is σ_z = 0. So, the other principal stresses are also 0.

Solution:

Since the element is in plane stress (σ_z = 0), the other principal stress is zero.

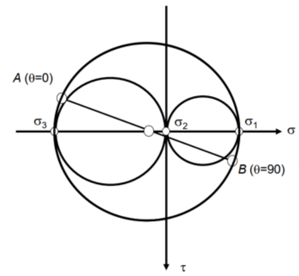
σ_1 = 54.6 MPa σ_2 = 0 MPa σ_3 = -84.6 Mpa

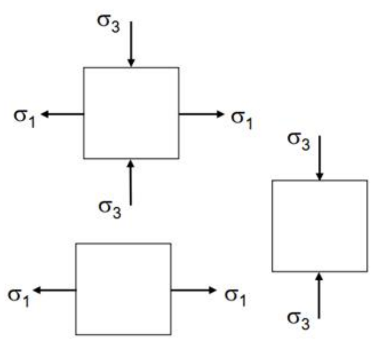
This means three Mohr's circles can be drawn, each based on two principal stresses:

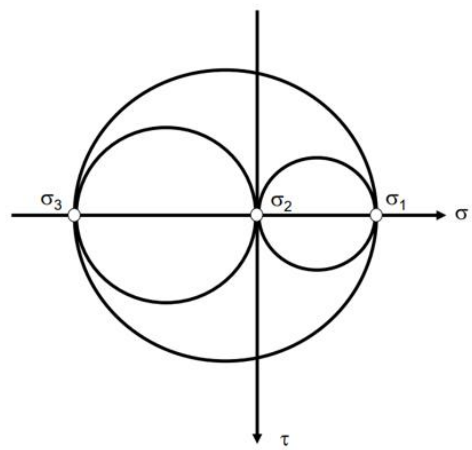
σ_1 and σ_3

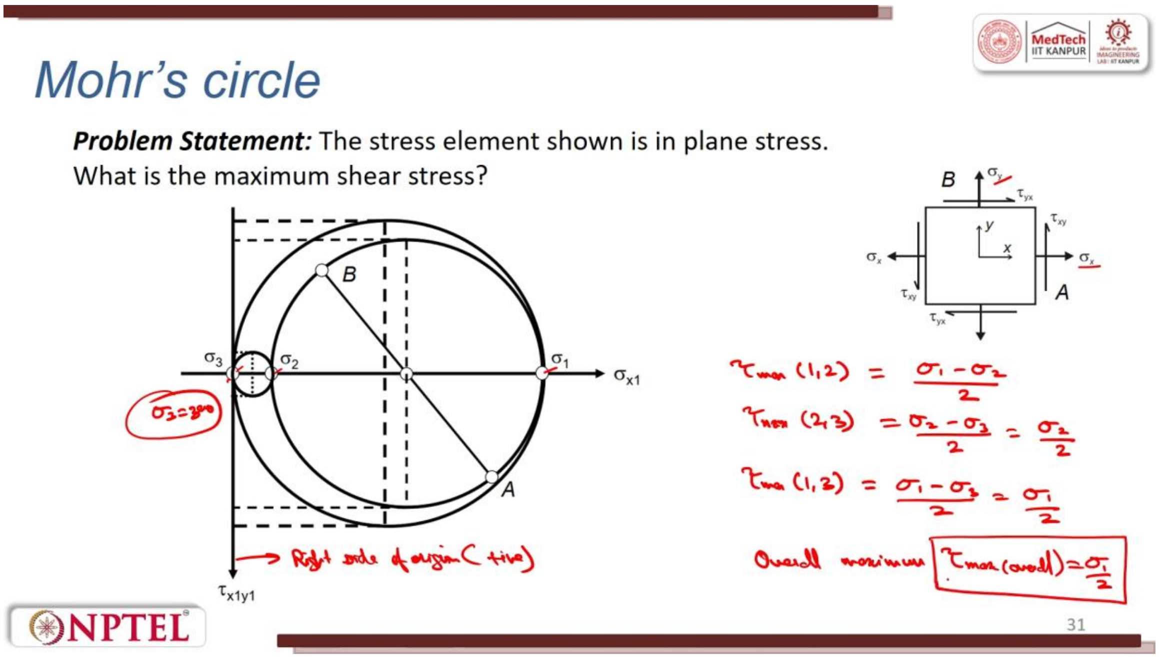
σ_1 and σ_2

σ_2 and σ_3









Now, the last problem has a Mohr circle already given.

Solution:

$$\tau_{\max(1,2)} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_{\max(2,3)} = \frac{\sigma_2 - \sigma_3}{2} = \frac{\sigma_2}{2}$$

overall maximum $$\boxed{\tau_{\max(1,3)} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_1}{2}}$$

References:

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With this, I am closing this tutorial and we will meet in the next week where we will discuss the laboratory demonstrations on the solid mechanics theory and tutorials that we have discussed in the previous weeks.

Thank you.