Basics of Mechanical Engineering-1

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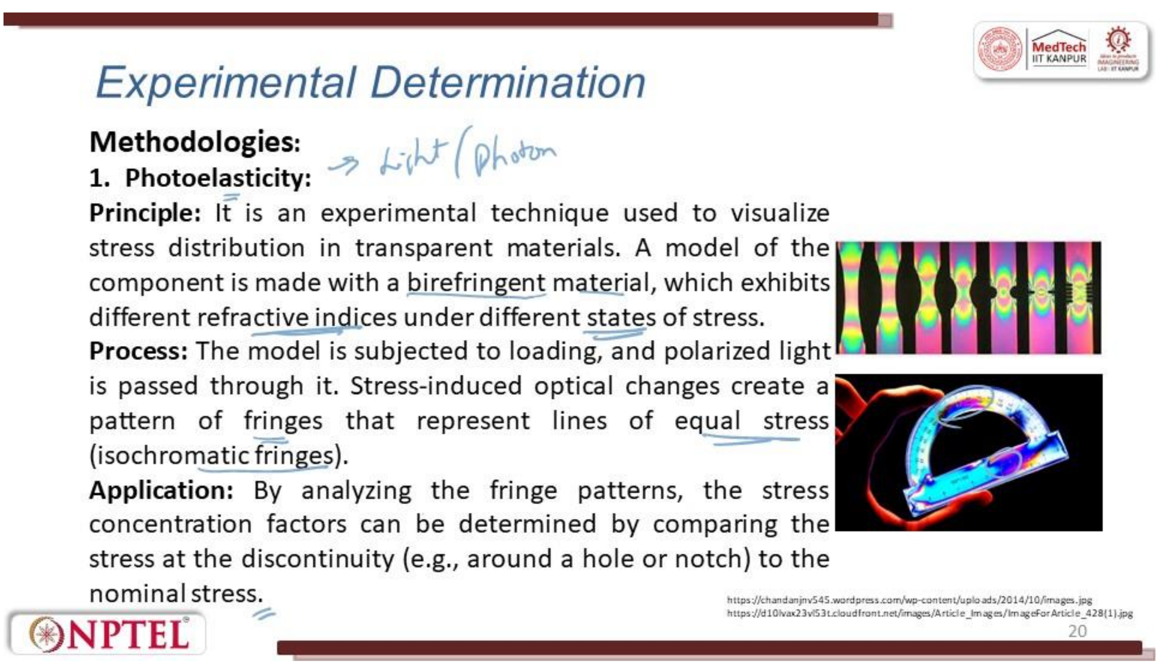
Week 08

Lecture 33

Stress Concentration and Notch Sensitivity (Part 2 of 2)

Welcome to the next lecture on Stress Concentration and Notch Sensitivity.

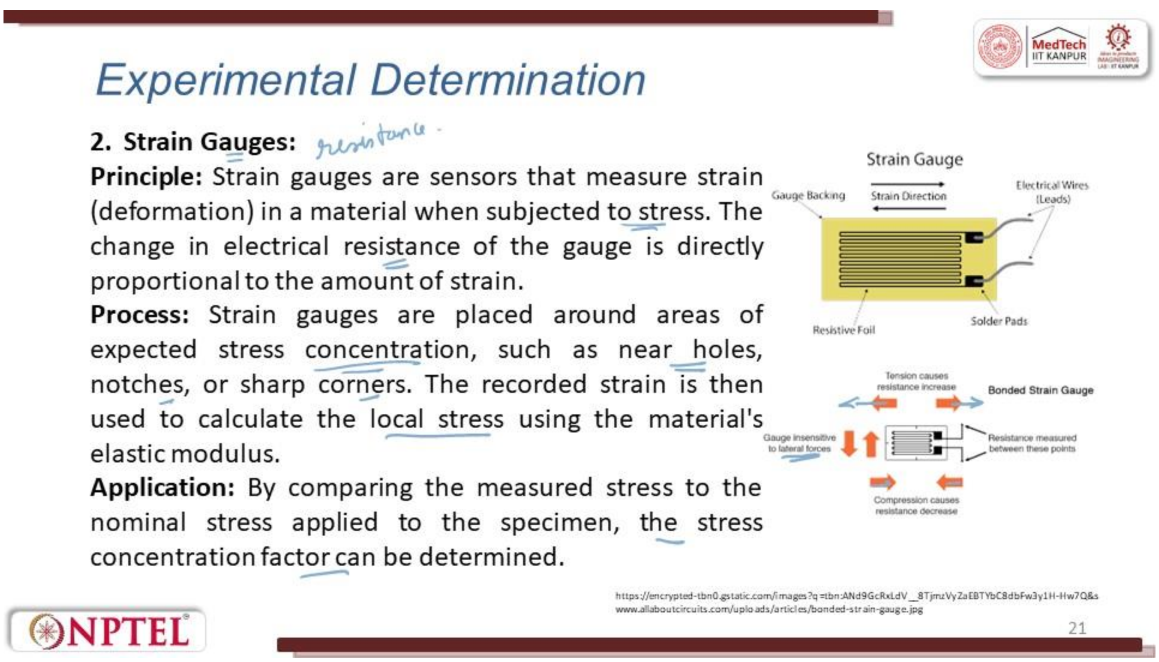


Let us get into the Experimentation Method of determining stress concentration. The Experimental method we will try to use Photoelasticity technique. Photoelasticity technique principles is an experimental technique used to visualize the stress distribution in transparent material. They try to take a transparent material.

When you try to apply load, you will try to see varying color. The light gets diffracted. A model of the component is made with a birefringent material which exhibits different refractive index under different state of stress. So this is a protractor-cum scale. You try to press the plastic one.

When the light passes, you try to see the color change. The process. Model is subjected to a loading and polarized light is passed through it. Stress induced optical changes create a pattern of the fringes that represents the line of equal stress, which is called as Isochromatic fringes. This application by analyzing the fringe pattern, the stress concentration factor can be determined by comparing the stress at the disconnects to the nominal stress.

Suppose you want to do it on steel, you will not be able to visualize it on steel. So, what we do is we try to make a model on plastic and we do not put 100 percent of the load. We have a scaled down version of the load, press it and then try to figure out where all are the fringe patterns. So, we try to use a polarized light. The light passes through it. Wherever there is a stress-induced optical change, a pattern of fringe is generated.

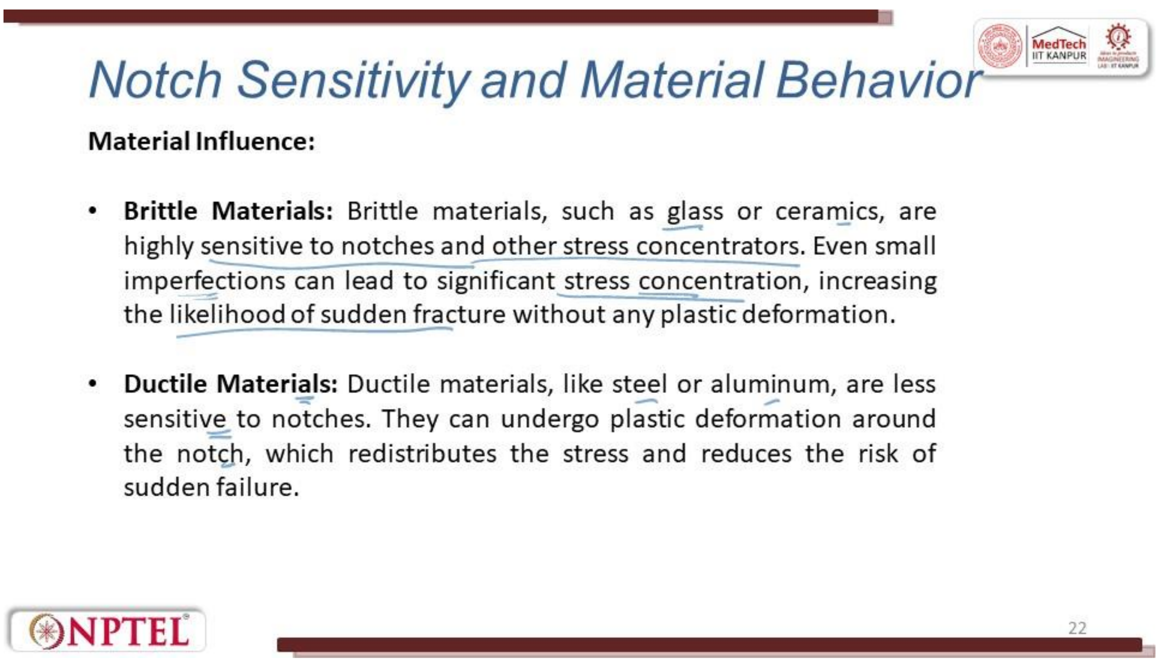


The other way of finding out the stress concentration is by using Strain Gauge. We have already studied about Strain Gauge. Strain gauge tries to talk about the deflection. It can be torsional diffraction, longitudinal diffraction or lateral diffraction, whatever you say.

It is basically to measure the deformation. The principal strain gauge are sensors that measure strain in a material when subjected to stress. The change in the electrical resistance there, photoelasticity, is basically light or photon. Here it is resistance. Change in electrical resistance of the gauge is directly proportioned to the amount of strain.

Strain gauges are placed around areas of expected stress concentration such as near a hole, notch or sharp corners. The recorded strain is then used to calculate the local stress using the material elastic modulus. By comparing the measured stress to the nominal stress applied to the specimen, the stress concentration factors can be determined. If you look at a strain gauge, so when there is a tension happening, there is an expansion of the gauge. The thin wire expands.

When there is a compression, you see here there is expansion reduction. You can also have lateral forces wherein which we see how does it move. So you can also use the lateral to convert into torsional. So the gauge insensitivity to lateral forces is there.



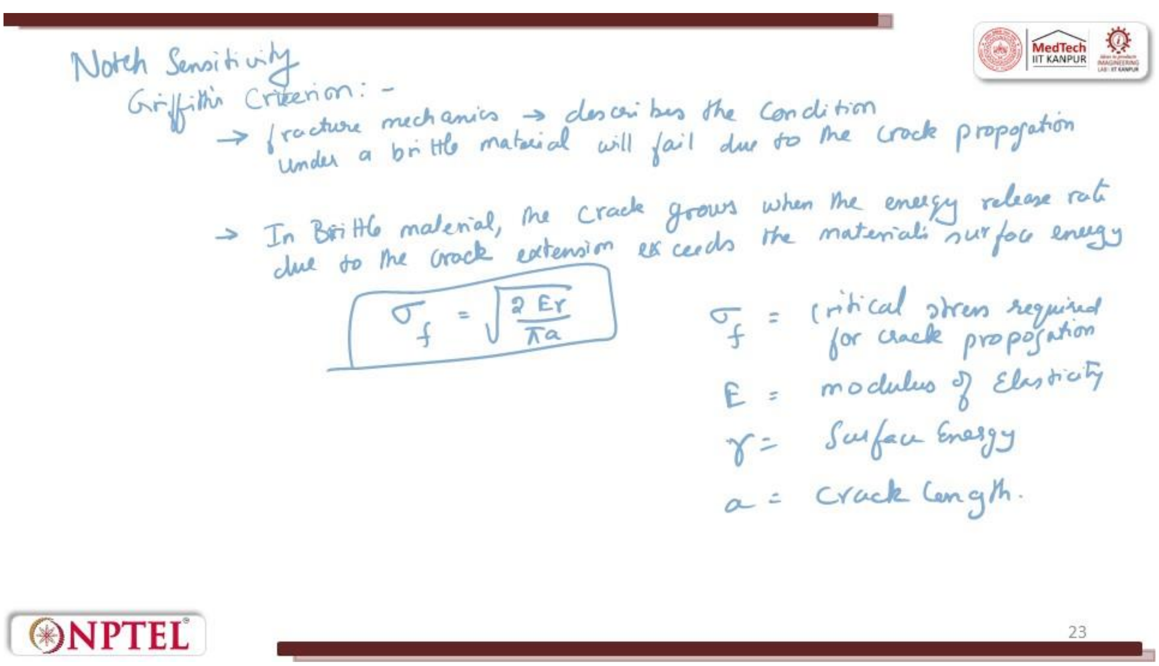
So when we talk about the Notch Sensitivity and Material Behavior, material influence is very high. So Brittle material, Ductile material. Brittle materials such as glass or ceramic are highly sensitive to notch and other stress concentration. See, you have to understand the material property and the beauty of design and plan your product.

When we are trying to do on an ampoule which is used for injection, the ampoule is intentionally made to have Very high notch sensitivity such that when the doctor hits, it breaks and he is able to extract the medicine.

You have to choose the proper place and do it. The same way when we try to do the breaking, when you are trying to have a closure and when the closure has to open, we always try to have a brittle material there so that it shears with low force. Again here, the notch sensitivity factor is used and accordingly the design is changed to give the feel for notch sensitivity. Same with respect to a glass which is a tip which you break. You break a tip is also the same.

Even a small imperfection can lead to significant stress concentration. That's why if you have a teapot which is made out of glass, they will try to reinforce at various locations with a soft material to have a spacer between the leg and the glass. While leg movement, the leg has sharp edges that should not damage the glass and when the glass is smooth there will be a shock which will not try to break the glass. So stress concentration increases the likelihood of sudden fracture without any plastic deformation. When we talk about ductile material like steel, aluminium are less sensitive to notch.

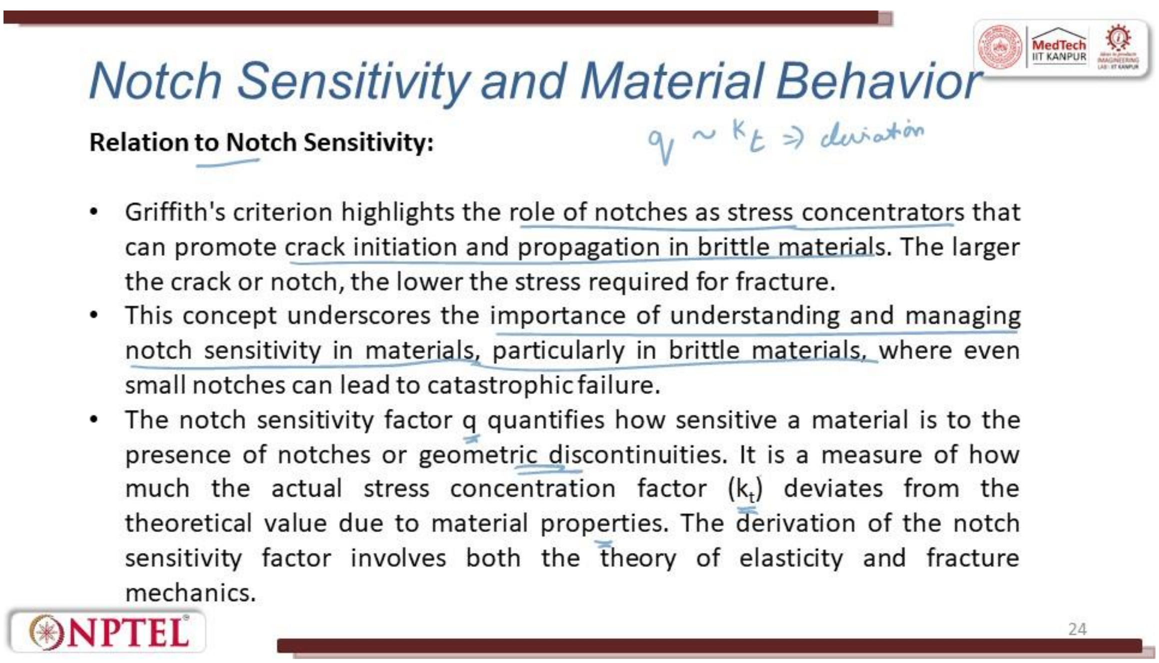
So here in which we try to take care by giving a U notch, we try to give a fillet, all these things are taken. They can undergo plastic deformation around the notch, redistribute the stress, stress flow lines and increase the life of the component.



So now in notch sensitivity, we have Griffith’s criteria so this is a concept from fracture mechanics which describes the condition under a brittle material which will fail due to the crack propagation. So here what Griffiths theory says the brittle material, the crack grows when the energy release rate due to the crack, extension exceeds the material surface energy. So this can be mathematically represented as $\sigma_f = \sqrt{\frac{2E\gamma}{\pi a}}$.

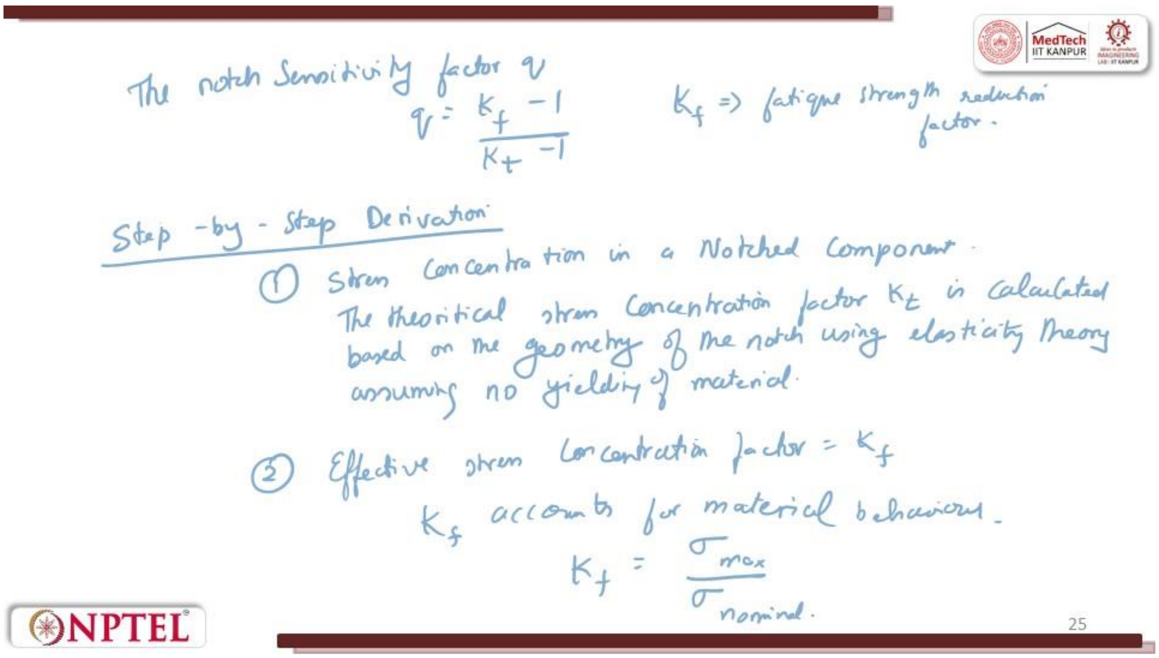
- σ_f is the critical stress required for crack propagation,
- E is the modulus of elasticity,
- γ is the surface energy,
- a is the crack length.

So through this formula, you are able to figure out what is the critical stress required for a crack growth.



So what are the relations to notch sensitivity? Griffiths criterion highlights the role of notch as stress concentrator that can promote crack initiation and propagation in brittle materials. This concept underscores the importance of understanding and managing notch sensitivity in material, particularly in brittle material. The notch sensitivity factor 'q' quantifies how sensitive a material is to the presence of a notch or a geometric discontinuity.

It is a measure of how much actual stress concentration factor Kt deviates from the theoretical value of the given material property. So here what we are trying to talk about is the relationship between q, Kt and what is the deviation. The derivation of the notch sensitivity factor involves both the theory of elasticity and fracture mechanics.



So, the notch sensitivity, which we have already seen, factor q is nothing but $q = \frac{k_f - 1}{k_t - 1}$

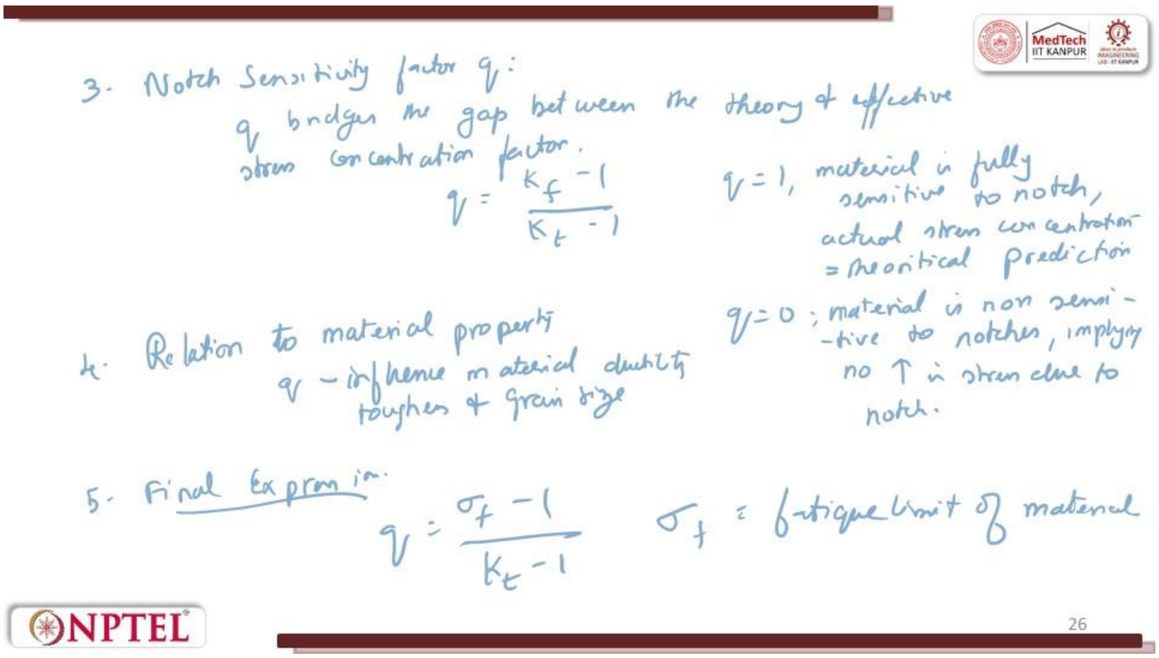
Where,

- k_f is the fatigue strength reduction factor.
- k_t is the theoretical stress concentration factor.

So, step-by-step derivation First we try to do the stress concentration in a notched component. First we will do that. So the theoretical stress concentration.

Factor Kt is calculated based on the geometry of the notch. Using elasticity theory assuming no yielding of material.

Next what we do is we try to calculate the effective stress concentration factor, which is nothing but K_f. K_f accounts for material behavior and $k_t = \frac{\sigma_{max}}{\sigma_{nominal}}$.

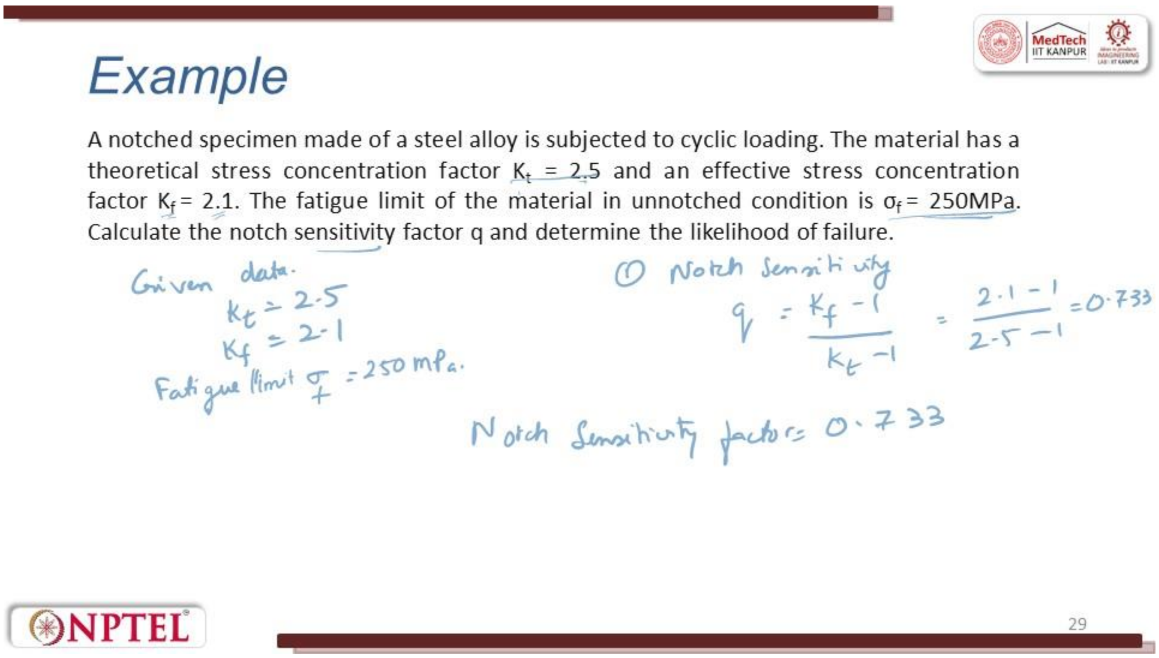


Then we try to calculate the notch sensitivity factor, which is nothing but q. So q generally bridges the gap between the theory and effective stress concentration factor is nothing but $q = \frac{k_f - 1}{k_t - 1}$.

When q = 1, so material is sensitive to notch. So what does it mean? So the actual stress concentration is equal to theoretical prediction. When q = 0 material is non-sensitive to notch implying no increase in stress due to notch So, please keep that in mind.

So, then the fourth step will be relation to material property. So relation to material property, not sensitivity factor q, we will try to see q, how does it influence material ductility? Material ductility, toughness and grain size. So, that is what we see from here.

Then final expression for notch factor, notch sensitivity factor q, final expression will be $q = \frac{\sigma_f - 1}{k_t - 1}$, where σ_f is the fatigue limit of the material. So after this we will see a problem, so you will understand in more details.



So let us look at the problem. So a notch specimen made of a steel alloy is subjected to cyclic load. The material has a theoretical stress concentration factor Kt = 2.5 and the effective stress concentration factor Kf = 2.1. The fatigue limit of the material in unnotched condition is σ_f = 250 megapascals. Calculate the notch sensitivity factor q determining the likelihood of failure.

Solution :

1. Given data:

Theoretical stress concentration factor: $K_t = 2.5$

Effective stress concentration factor: $K_f = 2.1$

Fatigue limit of the unnotched material: $\sigma_f = 250MPa$

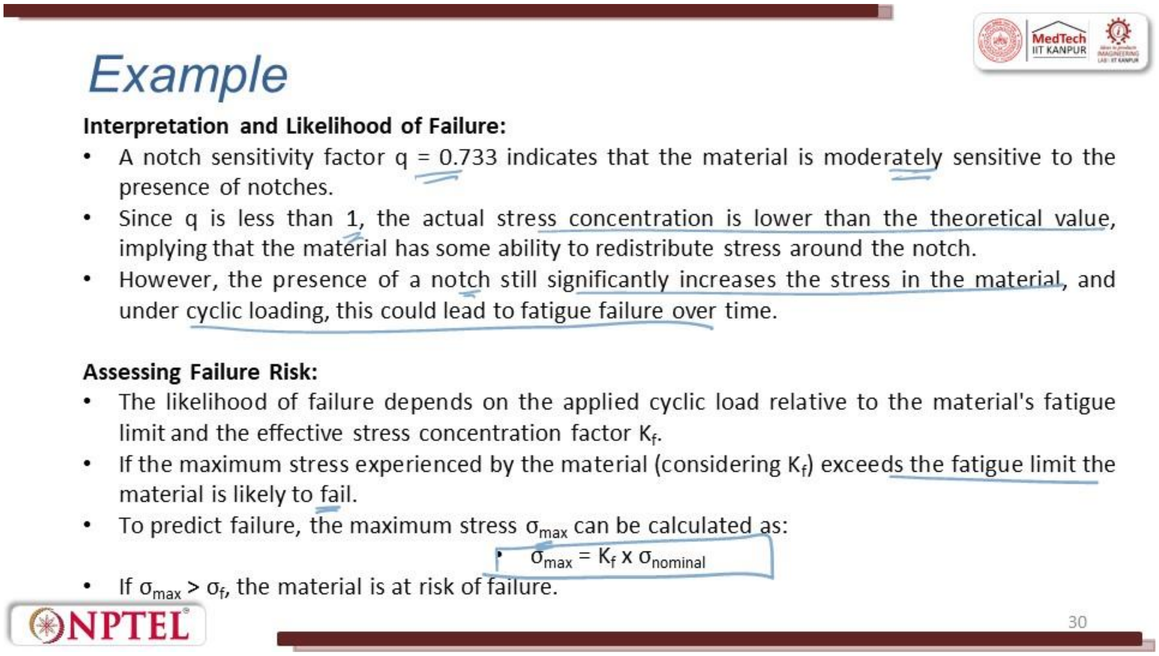
2. Calculate the Notch Sensitivity Factor q:

The Notch Sensitivity Factor q is given by:

$$q = \frac{K_f - 1}{K_t - 1}$$

Substituting the given values:

$$q = \frac{K_f - 1}{K_t - 1} = \frac{2.1 - 1}{2.5 - 1} = \frac{1.1}{1.5} = 0.733$$



So, when we look at this interpretation and likelihood of failure, the notch sensitivity q = 0.733, which indicates the material is moderately sensitive to the notch. Since q is less than 1, the actual stress concentration is lower than the theoretical value.

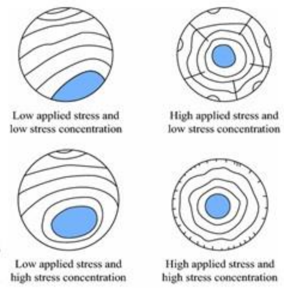
However, the presence of this notch still significantly increases the stress in the material and under cyclic load could lead to early failure. Assessing the risk factor, the likelihood of the failure depends on the applied stress load relative to the material fatigue limits.

If the maximum stress experienced by the material exceeds the fatigue limit of the given material, it likely fails. So to predict the failure, $\sigma_{max} = K_f \times \sigma_{nominal}$. If $\sigma_{max} > \sigma_f$, the material is at a risk to failure. So now we are moving into Fatigue, cyclic loading, right.

Stress Concentration and Fatigue

Impact of Stress Concentration on Fatigue Life

- Stress concentration refers to the localized increase in stress in a material due to geometric discontinuities such as notches, holes, or grooves.
- These discontinuities cause the stress to exceed the nominal value, creating regions where the material is more susceptible to failure, especially under cyclic loading.
- When a material is subjected to repeated or fluctuating loads, fatigue becomes a primary concern. Stress concentrations exacerbate the fatigue process by significantly reducing the number of cycles the material can withstand before failure.
- The presence of notches or other stress concentrators can lead to crack initiation at lower stress levels, shortening the fatigue life of the material.



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The Stress Concentration and Fatigue. Impact of Stress Concentration on Fatigue Life. The stress concentration refers to the localized increase in stress in a material due to geometric discontinuity such as notch, hole or grooves. These discontinuities causes a stress to exceed the nominal value, creates the regions where the material is more susceptible to failure. When the material is subjected to repetitive or fluctuating load, the fatigue becomes a primary concern. Fatigue plus the notch.

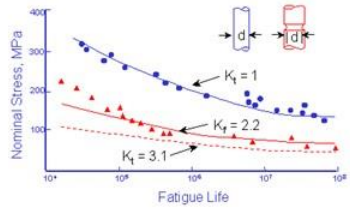
The stress concentration exaggerates the fatigue process by significantly reducing the number of cycles or the material can withstand before failure. The presence of notch or other stress concentration can lead to early crack propagation.

Stress Concentration and Fatigue

Modification of S-N Curves Due to Stress Concentration and Notch Sensitivity

Effect of Stress Concentration: The presence of stress concentrators such as notches modifies the S-N curve by reducing the fatigue strength of the material.

- This results in a lower fatigue limit and a steeper S-N curve, indicating a shorter fatigue life for a given stress level.



Notch Sensitivity: The notch sensitivity factor q affects how much the S-N curve is modified by the presence of a notch.

- A material with high notch sensitivity will show a more pronounced reduction in fatigue life when a notch is present, compared to a material with low notch sensitivity.

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So Modification of SN curve due to Stress Concentration and Notch Sensitivity. This is d and this is the d whatever we have done. So now we have created a notch.

We are trying to see the SN response. Effect of stress concentration. The presence of stress concentration such as a notch modifies a typical SN curve. This results in lowering the fatigue life and steeper SN curves are seen. The notch sensitivity, the notch which is done.

The notch sensitive factor (q affects how much the SN factor is modified by the presence of the notch. So you can do an experiment without a notch, then with a notch and then try to see what is the response, how is the notch sensitivity factor working.

Stress Concentration and Fatigue

Adjusted S-N Curve: The modified S-N curve can be obtained by considering the effective stress concentration factor K_f, which accounts for both the theoretical stress concentration factor K_t and the material's notch sensitivity.

- The actual stress experienced by the material at the notch is multiplied by K_f, leading to a reduced fatigue limit and a shortened fatigue life.

Practical Implications:

Design Considerations: Engineers must account for stress concentrations and notch sensitivity when designing components expected to undergo cyclic loading.

- By understanding and applying the correct stress concentration factors and modifying the S-N curve accordingly, they can better predict fatigue life and design safer, more reliable components.

Example Applications: The impact of stress concentration on fatigue life is critical in components such as gears, crankshafts, and aircraft structures, where failure due to fatigue can lead to catastrophic outcomes.

Adjusting SN curve is very important when we are trying to do a notch. Modifying SN curve can be obtained by considering the effective stress concentration factor (Kf) which accounts for both the theoretical stress concentration factor (Kt) and the material notch sensitivity. If you take Kf, this will try to adjust SN.

So, what are the practical implications for the stress concentration and fatigue? During design, the engineer must take into account for stress concentration, notch factor sensitivity when designing components expect to undergo cyclic load. By understanding and applying the correct stress concentration factors, the SN curve is modified. So modified SN curve value gives you the correct response. Earlier you remember in bearing, we were trying to look at SN curve, stress versus number of cycles.

There used to be a different curve response. Now you add stress concentration to it and see the modified one. So lot of places this is very much used. The impact of stress concentration on fatigue life is critical in components such as gears, shafts, crankshafts or aircraft structures where failure due to fatigue can be catastrophic.

Example

A steel component with a circular notch has a nominal stress of $\sigma_{nominal}$=150 MPa under cyclic loading. The theoretical stress concentration factor is K_t=3, and the material has a fatigue limit of σ_f=250 Mpa in an unnotched condition. The notch sensitivity factor q is 0.8. Estimate the fatigue life of the component using the modified S-N curve.

Given

Nominal Stress $\sigma_{nom} = 150$ MPa.

$K_t = 3$

Fatigue limit in unnotched $= \sigma_f = 250$ MPa.

Notch Sensitivity $= 0.8$

Calculate the effective stress concentration factor ; $K_f = 1 + q \times (K_t - 1)$

$K_f = 1 + 0.8 \times (3 - 1) = 2.6$

Example

3. Determine the actual max stress $\sigma_{max} = K_f \times \sigma_{non}$

$\sigma_{max} = K_f \times \sigma_{nom} = 2.6 \times 150$ MPa $= 390$ MPa.

So max stress $\sigma_{max} = 390$ MPa.

4. Compare σ_{max} with fatigue limit

$\sigma_{f\,not} = \frac{\sigma_f}{K_f} = \frac{250}{2.6} \simeq 96.15$ MPa.

Since $\sigma_{max} = 300$ MPa which is higher than notch fatigue limit of 96.15 MPa, the component is expected to have a reduced life.

Let us now try to solve a simple problem. A steel component with a circular notch has a nominal stress of sigma nominal equal to 150 MPa under a cyclic load. The theoretical stress concentration factor Kt is equal to 3. The material has a fatigue failure sigma f is this. The notch sensitivity factor q is also given. Now you are trying to predict what is the fatigue life of the component using modified SN curve.

1. Given Data:

Nominal stress $\sigma_{nominal}$ = 150 Mpa

Theoretical stress concentration factor: $K_t = 3$

fatigue limit in unnotched condition σ_f = 250 Mpa

notch sensitivity factor: $q = 0.8$

2. Calculate the Effective Stress Concentration Factor: K_f

The Effective Stress Concentration Factor: K_f is given by:

$K_f = 1 + q \times (K_t - 1)$

Substituting the values:

$K_f = 1 + 0.8 \times (3 + 1)$

$K_f = 1 + 0.8 \times 2 = 1 + 1.6$

$K_f = 2.6$

3. Determine the Actual Maximum Stress σ_{max} :

The maximum stress at the notch is calculated using:

$\sigma_{max} = K_f \times \sigma_{nominal}$

Substituting the values:

$\sigma_{max} = 2.6 \times 150\text{MPa}$

= 390 Mpa

So, the maximum stress σ_{max} is 390 Mpa.

Design Strategies to Minimize Stress Concentration

1. **Adding Fillets:** Smooth transitions at corners reduce peak stress by distributing the load more evenly.
2. **Changing Geometries:** Tapering thickness or using rounded holes prevents abrupt stress increases, leading to a more uniform stress distribution.
3. **Material Selection:** Choosing ductile materials lowers the risk of failure at stress concentration points by absorbing and redistributing stress more effectively.

Practical Examples:

- **Gears:** Fillets at the base of gear teeth reduce stress concentration, extending gear life.
- **Shafts:** Adding fillets to keyways reduces stress, increasing durability.
- **Aircraft Structures:** Rounded edges and increased thickness around openings prevent crack propagation, enhancing safety.

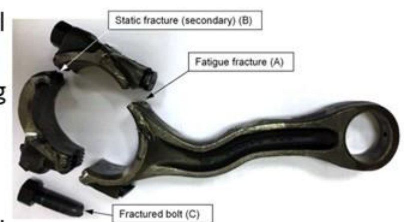
So what are the Design Strategies to Minimize the Stress Concentration? We add fillets, we change geometry, we choose a proper material.

In practice, gears, shafts, aircraft structures, they all use the design strategies to minimize the stress concentration.

Case Studies in Stress Concentration

Case Study 1: Analysis of a Failed Mechanical Component

Component: A connecting rod in an internal combustion engine.
Issue: The connecting rod failed prematurely during operation.



Analysis:

- The failure occurred at the fillet between the rod and the cap, a common location for stress concentration.

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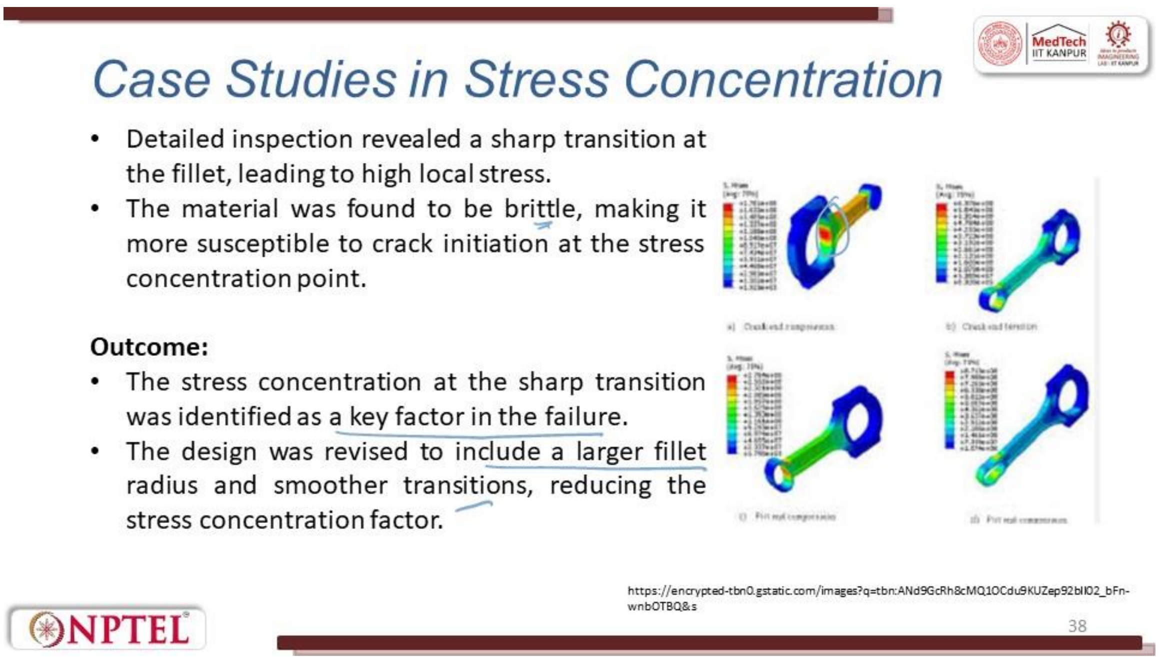
Let us look at simple case studies. Case study in stress concentration, here in which case study 1 analyzes the failed mechanical component, a connecting rod in an internal combustion engine.

The connecting rod failed prematurely during operation. On analysis it was figured out the fillet which was given was less by giving proper rounding out between the rod and the cap the performance of the component could be increased.

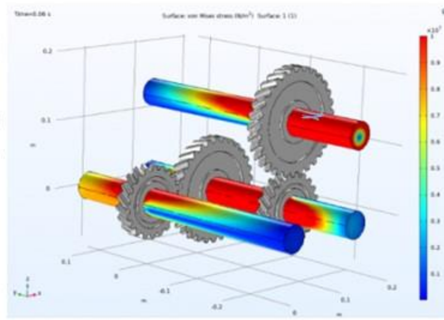


A detailed inspection revealed that the sharp transition at the fillet leads to the local stress. The material was found to be brittle and that also changed the outcome. Stress concentration at that sharp transition was identified as a key factor of the failure and the design was revised by inducing or including a larger fillet radius and a smooth transition reducing the stress concentration factor.

Case Studies in Stress Concentration

Case Study 2: Success Story in Redesign to Minimize Stress Concentration

Component: A shaft used in a high-speed gearbox.
Issue: The original design featured sharp keyways, leading to frequent failures due to stress concentration.



Redesign:

- The keyways were redesigned with larger fillet radii and smoother transitions.

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Let us take another example of a gear, a success story in redesigning to minimize the stress concentration. A shaft used in a high speed gearbox. Issue is the original design featured sharp keyways. Keyways will be here, sharp keyways leading to frequent failure due to stress concentration. The keyway was redesigned and the stress concentration levels were brought down and the performance was enhanced.

Case Studies in Stress Concentration

- Surface finishing processes were improved to eliminate micro-cracks that could serve as initiation points for failure.

Outcome:

- The redesigned shaft experienced significantly lower stress concentrations.
- The service life of the shaft was extended by more than 50%, and the frequency of failures was drastically reduced.

Surface finishing process was introduced to eliminate sharp edges. The outcome, the redesigned shaft experienced significant lower down of the stress concentration. The surface life of the component could be extended by 50%.

To Recapitulate

- What is stress concentration and why is it important in mechanical design?
- Explain the concept of notch sensitivity and how it influences material behavior near discontinuities.
- What are common causes of stress concentration in mechanical components?
- Explain how stress concentration impacts the fatigue life of materials.
- Describe the role of the stress concentration factor (K_t) in evaluating the strength of a component.
- What is Griffith's criterion and how does it relate to notch sensitivity?
- Why is it essential to consider notch sensitivity in the design of components subjected to cyclic loading?
- What is the significance of notch sensitivity factor (q)? How is it calculated?
- Derive the expression for the notch sensitivity factor (q) using fracture mechanics principles.

To recap what we saw in this lecture is we understood what is stress concentration, why is it important. The concept of notch sensitivity and its influence on the material behavior was discussed.

The common cause for stress concentration in mechanical components were seen. The stress concentration impact on the fatigue life of material was also discussed. Describing the role of stress concentration factor Kt was carried out.

The Griffiths criterion and how does it relate to notch sensitivity was covered. The essential to consider notch sensitivity in design components were seen, then the notch sensitivity factor q was calculated and finally we saw case studies wherein which using the stress concentration redesigning the part for enhancing the performance was discussed.

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These are the references which are used in this lecture preparation. So thank you very much.