Basics of Mechanical Engineering-1

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Week 01

Lecture 4

Scalars and Vectors, Vector Algebra

Welcome to the third lecture in this course. In this lecture, we will be covering the basics of Scalar and Vector. Then we will also be covering Vector Algebra. Friends, we are trying to introduce the concept which later can be used in your engineering career. Before I get into the content of this lecture, in the last two lectures, we have spent time in understanding what are units, type of units and why is units very important.

Any measurement we do without the units has no meaning. And if you are a manufacturing or a fabricating engineer, the units play a very important role. We saw primary units and supplementary derived units. In the next lecture, we went through fundamentals of Newton's law, Inertia, Momentum, Law of Conservation of Momentum. And why are these important?

Because these are going to give you a rough cut idea. An engineer has two face while trying to solve or understand the problem. First, with a rough cut, he has to understand what can be the solution, how big the solution is. Then, he starts working minutely to fine tune his solution. For the rough cut planning, fabrication, understanding, these laws play a very very important role.

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- Addition of vectors, Triangle & Parallelogram Law
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So in this lecture, as I told you, we will try to cover types of physical quantities, then scalar quantity, vector quantity, their representation and its type. How do you add vector? Then Triangle and Parallelogram law. Scalar and Vector product, Vector Algebra and its application, problems on vector algebra, like the previous two lectures, we will have a recap and references. I will give you some problems also for you to fine tune and hone the understanding.

Types of Physical Quantities

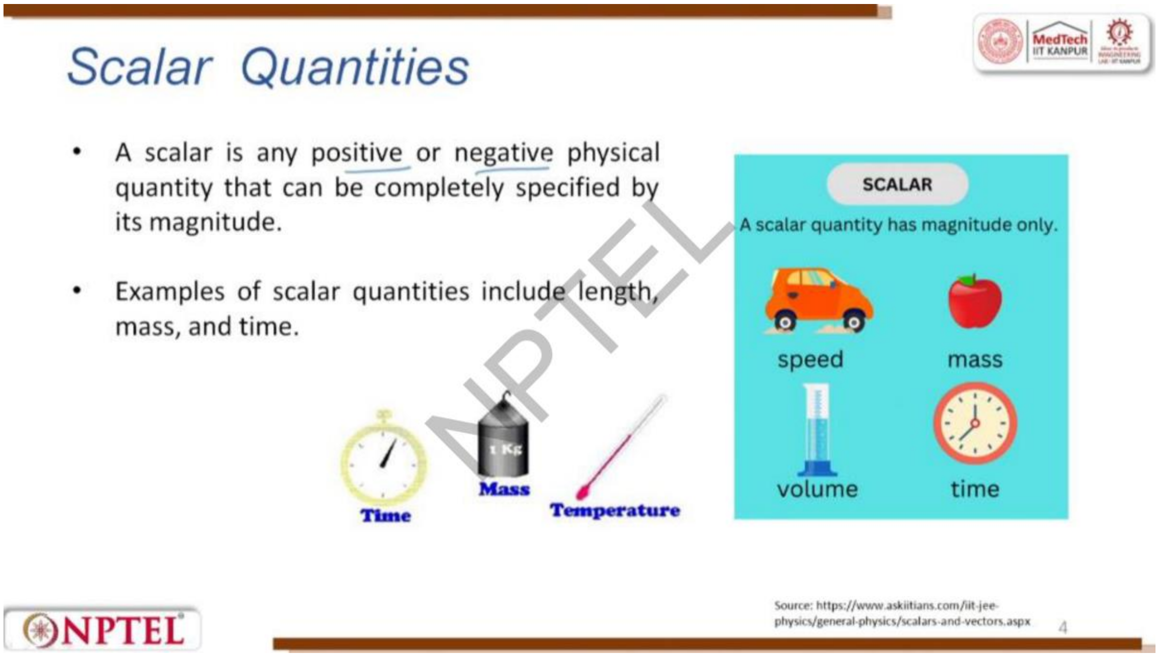
Physical quantities can be classified into the following two categories:

Physical Quantities

Scalar Quantities

Vector Quantities

So type of physical quantities. Physical quantities can be classified broadly into two categories. They are physical quantities. One is Scalar quantities and the other one is called as Vector quantities.



So, let us now look into scalar quantities. Scalar quantities has magnitude only. It says 5 kgs, 5 meters, 22 centimeter, 350 degree Celsius, 5 amps. So it tells you only the scalar quantity and here it only tells you the magnitude, the speed, mass, volume. Volume can be expressed in terms of liter or it can be expressed in terms of millimeter cube or centimeter cube or kilometer cube, whatever it is. Then time in seconds. Scalar is any positive or negative physical quantity that can be completely specified by its magnitude. It can be positive or negative. We have seen examples, but some more example, 15 minutes, 30 seconds.

Next is mass. People have asked questions like sir, year and day are also there. How do you take it? We try to convert it into hours and we always talk in terms of hours. If it is a micro event, which happens in second, millisecond, then we go towards second. So, you should also have a feel for it.

For example, if the event is long, so then we talk in terms of hours. If the event is small, seconds, minutes, again minutes will be converted into seconds or millisecond means you multiply it. So how do you do it? We always convert it 10 to the power 3 positive or 10 to the power minus 3 negative, whatever it is. So you start solving it. Then is mass, then temperature. So all these things are examples of scalar quantities.

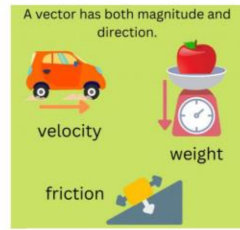
Vector Quantities

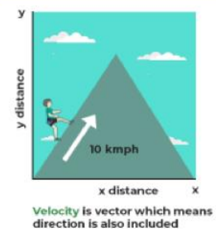
- A vector is any physical quantity that requires both a magnitude and a direction for its complete description.

  or

- Vector quantities are those quantities which have both magnitude as well as direction.

- Examples of vectors are Displacement, velocity, acceleration, force, electric intensity, magnetic intensity etc.





Source: https://sciencenotes.org/scalar-vs-vector-definitions-and-examples/ 5

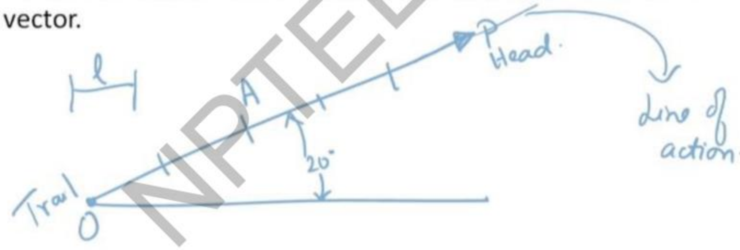
Now let us understand vector quantities. Vector quantity has both magnitude and direction. It says velocity of a car in this direction, weight applied along the gravity, friction which is there on a block, on a slope which is moving, friction is in the downward direction, whatever it is. So, here if you see we have directions also. A vector is any physical quantity that requires both magnitude and a direction for its complete description or the vector quantities are those quantities which have both magnitude as well as direction.

The examples of vectors are displacement which is given always in terms of length scale, Again, velocity will have time scale as well as length scale. Then acceleration, meter per second square, force in terms of Newton, electric intensity, magnetic intensity. So, flux and electrical intensity also you can try to talk in terms of Current per unit area whatever it is, right. So if you see this example 10 kilometers per hour and then in the x axis, we are talking about distance, y axis we are talking about distance.

So he is trying to climb a mountain. So velocity is a vector which means direction is also included.

Representation of Vector:

- A vector is represented by a straight line with an arrow head.
- Here, the length of the line represents the magnitude and arrow head gives the direction of vector.



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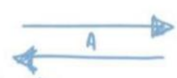
So, then let us understand the representation of vector. A vector is represented by a straight line with an arrow. Here, the length of the line represents the magnitude and the arrowhead gives the direction of vector.

For example, let us try to draw, this is O, this is A, P, this is the trail, this is the head and this is 20 degrees and each distance is L. Here if you see the length of the line represents the magnitude and the arrowhead represents the direction and what is this? This becomes your line of action.

Types of Vectors

Negative Vectors:

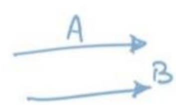
- The negative of a vector is defined as another vector having same magnitude but opposite in direction.



[ie] any vector $\vec{A}$ and its negative vector $[-\vec{A}]$ are shown as above.

Equal Vectors:

- Two or more vectors are said to be equal, if they have same magnitude and direction.



If $\vec{A}$ and $\vec{B}$ are two equal vectors then as shown above:

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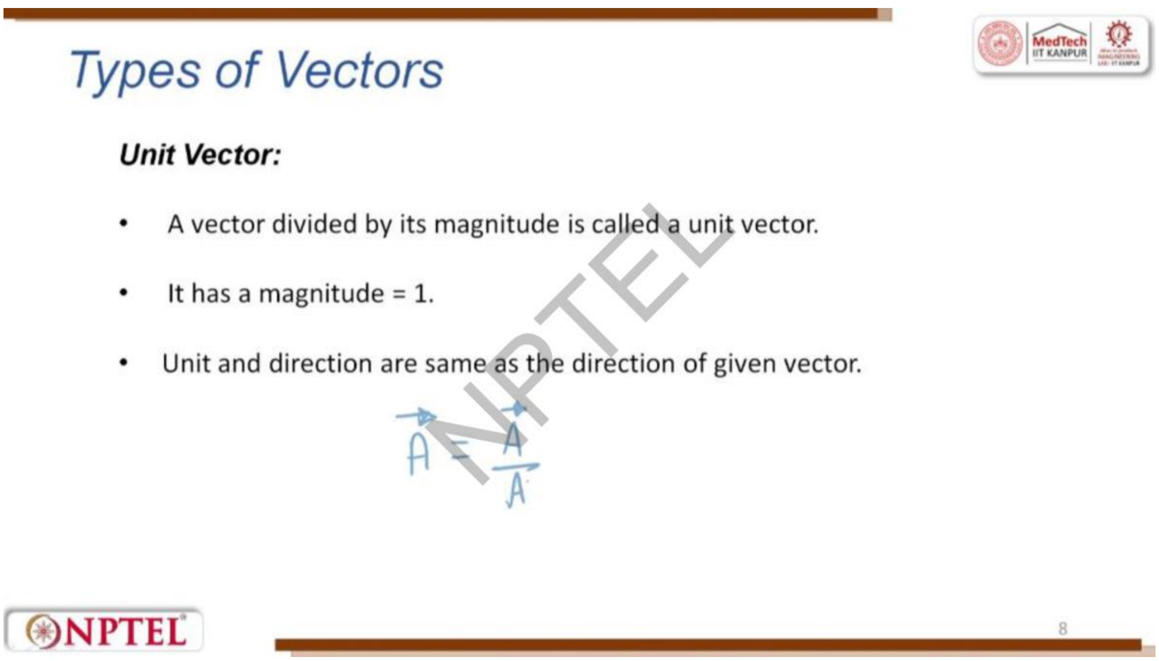
So you have negative vectors, you have equal vectors. So negative vector of a vector is defined as another vector having the same magnitude but opposite in direction. For example, this is this way it goes. So now negative vector will be in this way, okay. So that is any vector which is A, any vector A positive and its negative vector which is minus A, minus A are shown as above, A and minus A. Next is Equal vectors.

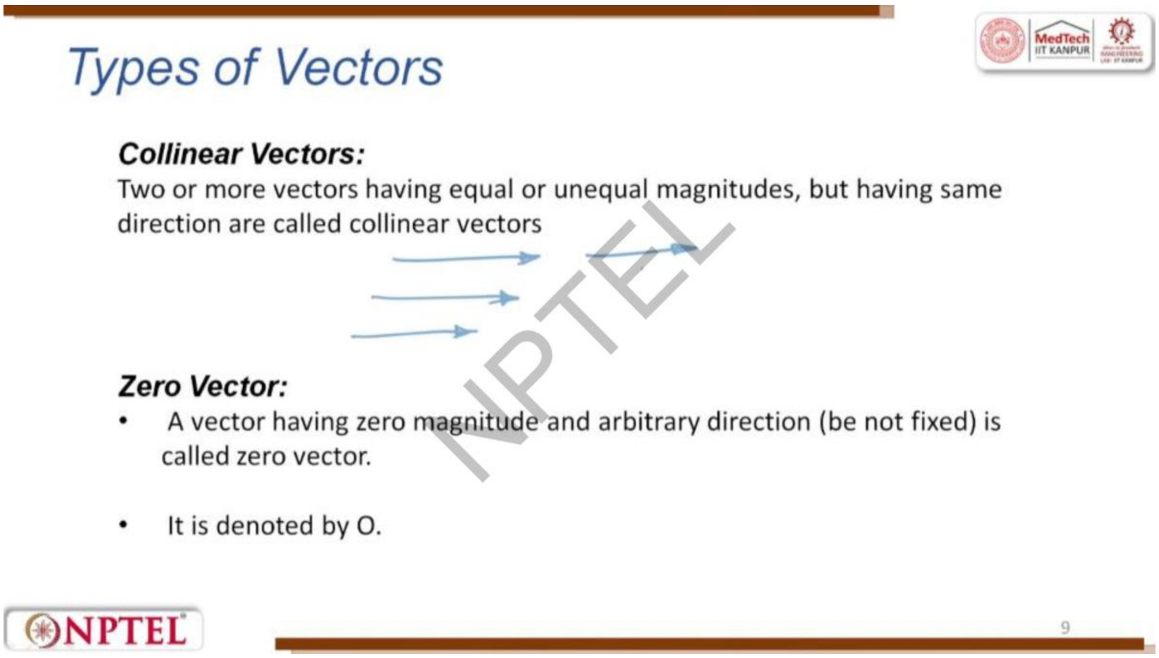
Equal vectors are vectors when two or more vectors are said to be equal if they have same magnitude and direction. For example, if you have this is A and this is B. So here if A and B are two equal vectors.



Then, as shown above, there is also something called as Unit vectors. So, negative vector, equal vector and unit vector. So unit vector means a vector divided by its magnitude is called as a unit vector.

And its magnitude has to be equal to 1. The unit and the direction are same as the direction of given vector. So it can be written as

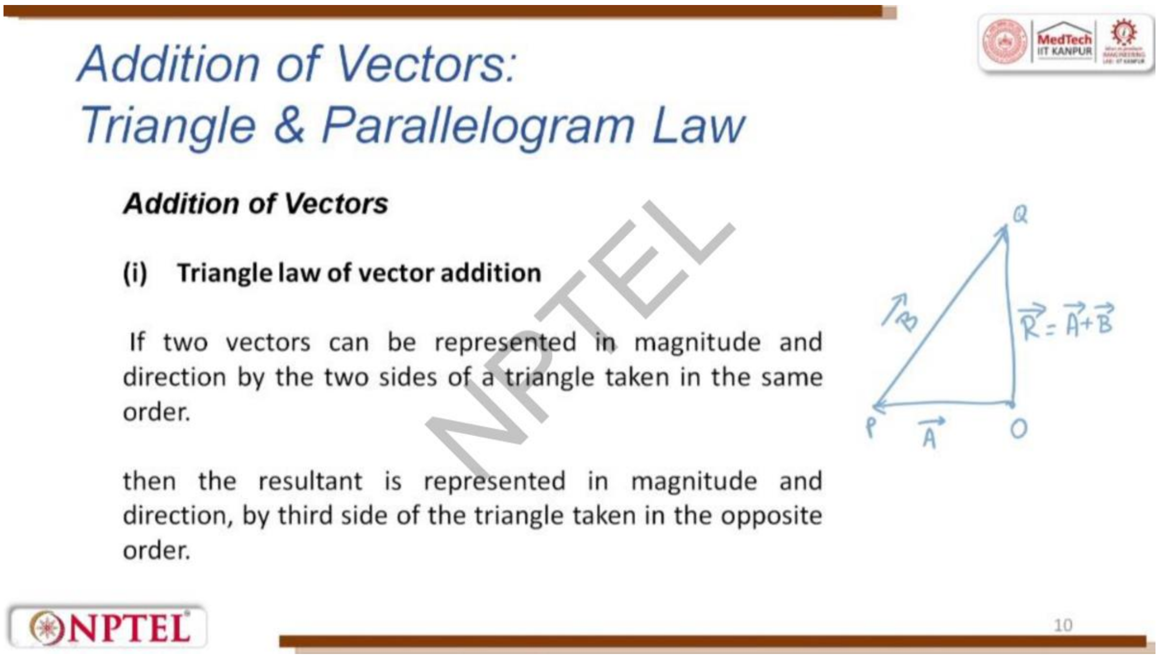
$\rightarrow A = \frac{\rightarrow A}{A}$. So this is called as unit vector.



You also have Collinear vector and Zero vector. Let us look at Collinear vector. When two or more vectors having equal or unequal magnitudes, but having the same direction are called as collinear vectors. For example, it can be this way, this way, So, these are called as collinear vectors. So, if you can see a ball moving, another ball moving, something like that. So, all these balls which are moving, some example can be thought of, right. Zero vector is a vector having magnitude 0 and arbitrary direction 0 but not fixed is called as zero vector.

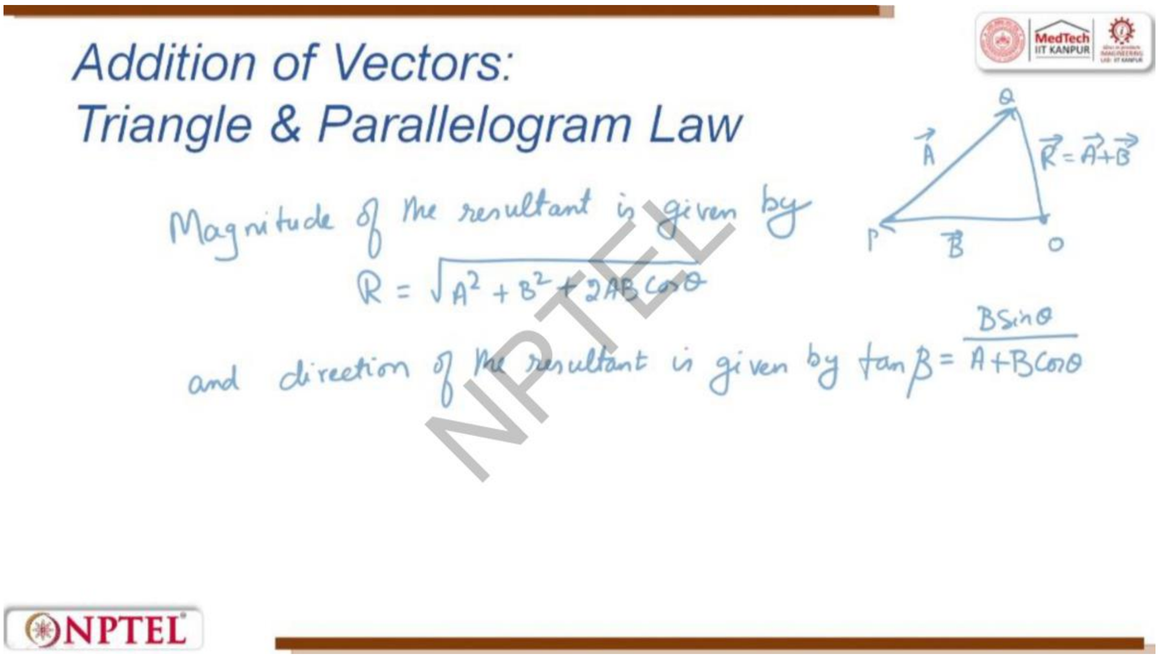
We also have the use of zero vectors. Later in the course, we will try to see where do we use this zero vector. So till now, we saw different types of vector. They are Negative vector, Equal vector, Unit vector, Collinear vector and Zero vector. This is very important.

You might have a question in your mind. Sir, you discussed about negative vector. What is the Positive vector? Whatever is normally written is called as Positive vector, right. A negative is another vector having same magnitude, but same direction becomes positive. And if it is opposite, it becomes negative vector. Now, we have understood scalar quantity, vector quantity types of vector.



With this understanding, we will try to move and understand addition of vectors by Triangle and Parallelogram law. So when I say triangle and parallelogram, first let us try to draw a triangle. Then we have A vector B vector. So now the resultant vector R will be equal to A vector plus B vector. A simple example in engineering when we try to look at more circle, we will try to use some of these vector concepts when we look at merchant circles diagram in metal cutting operation. We will try to look into or we will use these triangle laws for addition of vectors. So in engineering, it has lot of applications.

Now let us understand the fundamentals and keep moving. Many places, when you try to solve in fluid mechanics and in thermodynamics, in aerospace engineering, we will use this addition of vectors triangle law and parallelogram law. So Triangle law of vector addition. If two vectors can be represented in magnitude and direction by the two sides of a triangle taken in the same order, then the resultant is represented in magnitude and direction by the third side of the triangle taken in the opposite direction. So here the arrow goes like this. And the arrow goes like this. Okay. So this is O. This is Q. And this is B.



So the magnitude of the resultant is given by

$$R = \sqrt{A^2 + B^2 + 2ABCos\theta}$$

and direction of the resultant is given by

$$tan\beta = \frac{BSin\theta}{A + BCos\theta}$$

So, now it is good to have figure once again. So, let me draw the figure. This is Q, this becomes A. This becomes B vector, this is O and this is P. So, $\vec{R} = \vec{A} + \vec{B}$. So, we are trying to get the resultant, we are also trying to get the direction of resultant is given by tan beta equal to this.

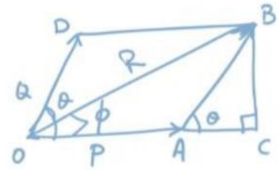
Addition of Vectors:
Triangle & Parallelogram Law

(ii) Parallelogram law of vectors:

It states that if two vectors, acting simultaneously at a point, can be represented both in magnitude and direction by two adjacent sides of a parallelogram,

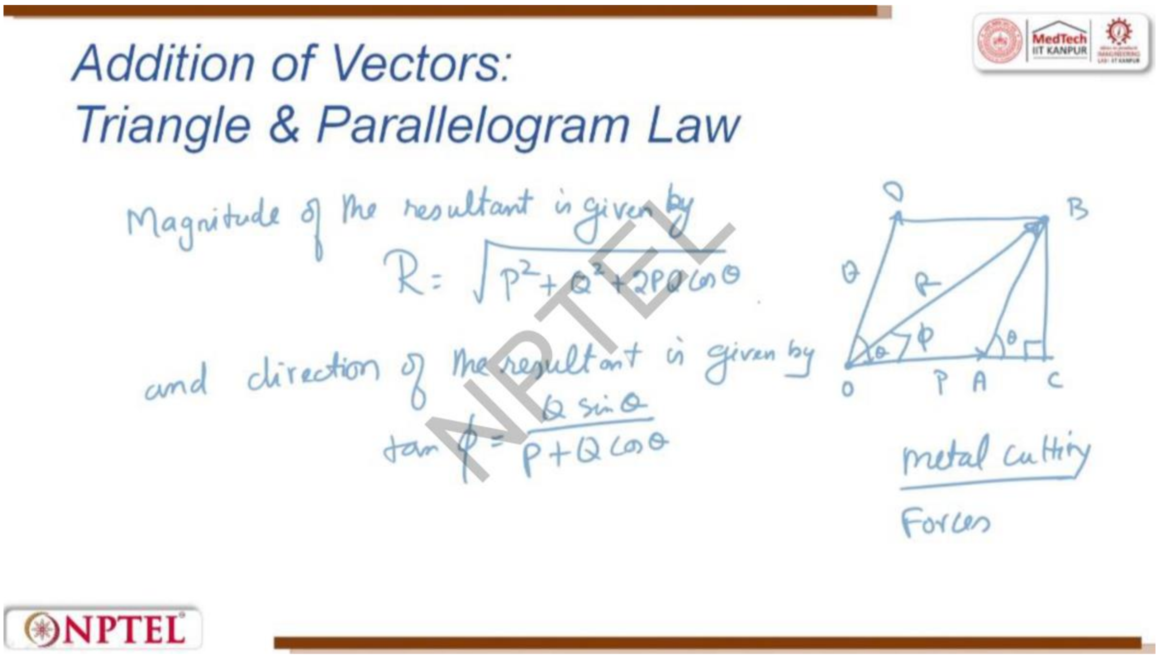
Then

the resultant is represented by the diagonal of the parallelogram passing through that point.



Let us get into the Parallelogram law. So, for that we should first draw a parallelogram. So, this we will represent it as O A Q D B C. This full becomes theta and this becomes phi. So, this is P and this is R, ok. So, if you see OA is P, right. And OD is Q, OB is R, the entire angle between OD and OA is theta. So, then you have AB, we are trying to draw a perpendicular line, BC is perpendicular to the line which is drawn. So, we have a perpendicular. So, AB takes a angle of theta. So the parallelogram law of vector, it states that if two vectors acting simultaneously at a point can be represented both in magnitude and in direction by two adjacent sides of a parallelogram, then the resultant is represented by the diagonal of the parallelogram passing through the point.

So you have OA, OD, OB, AB, right. And then you have OB as the resultant vector, OD you have Q, you have P, right. And then it has an angle which is theta made.



So, now what is the magnitude? Let us write down magnitude of the resultant is given by

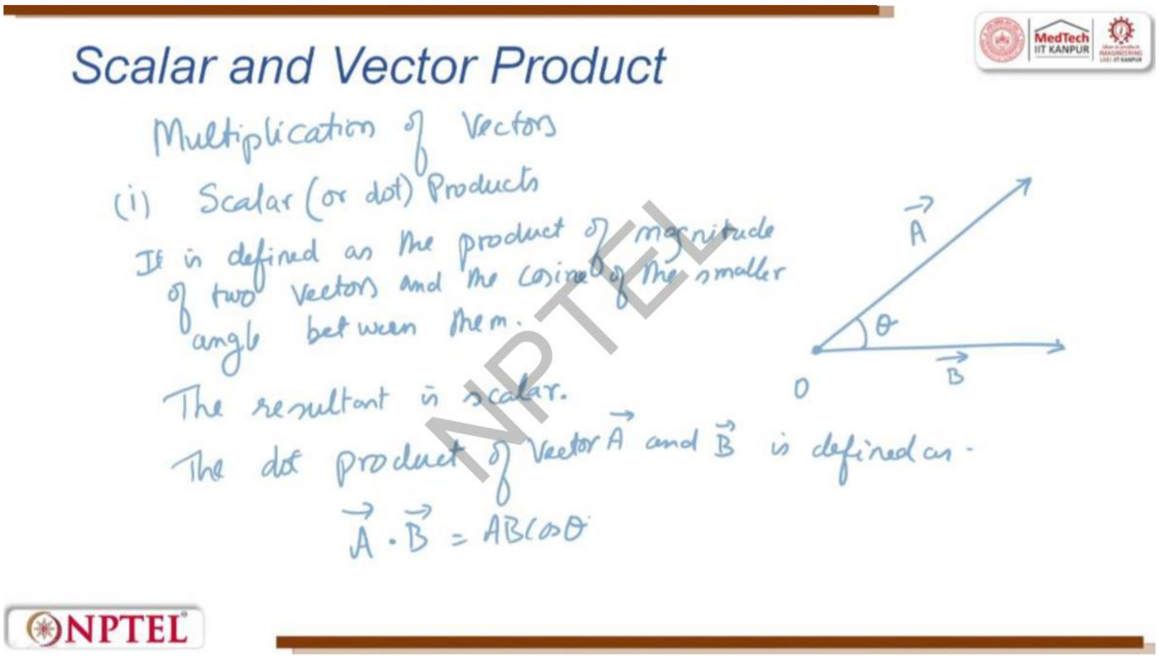
$$R = \sqrt{P^2 + Q^2 + 2PQ\ Cos\theta}$$

Again, I wanted you to understand the figure. So, let us redraw OD, this is represented as Q, this is represented as P. The resultant vector will be this R and this is a point called B. So, now I this becomes your A and now I draw a perpendicular line C. Now this is theta, so this also becomes theta and this becomes phi, okay and direction, if I wanted to do direction of the resultant is given by

$$tan\varphi = \frac{BSin\theta}{P + QCos\theta}$$

As I told you later in metal cutting, if you want to figure out the forces which are required for metal cutting, we will use these parallelogram law and triangle law to solve and find out the forces. Okay, this is one application, Mohr's circle is another application and as I told you in aerodynamics also you have multiple places, these vector addition comes by using this triangle and parallelogram law.

So, the next topic of interest is going to be scalar and vector products. So, again here let us have a drawing for our understanding. This is O, this is theta, this is A vector and this is B vector.

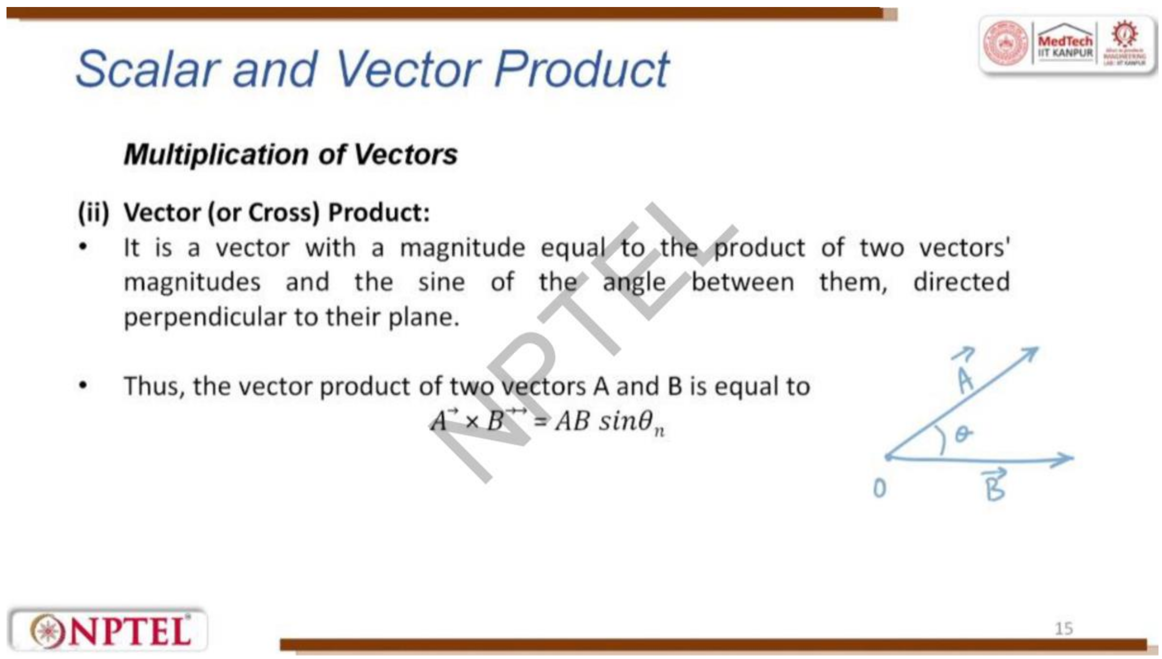


The next topic of interest is going to be scalar and vector products. So we saw addition, now let us look at multiplication of vectors. So, in this first we will try to say scalar or 'Dot' products. So, we will have a diagram here. This is our O, this is θ, this is A vector and this is B vector.

Okay, so scalar it is defined as the product of magnitude of two vectors and the cosine of the smaller angle between them, ok. So, the resultant scalar,

$$\vec{A} \cdot \vec{B} = AB\, Cos\, \theta$$

This is what I said. Scalar or dot product, it is defined as a product of magnitude of two vectors and the cosine of the smaller angle between them. So, $\vec{A} \cdot \vec{B} = AB\, Cos\, \theta$

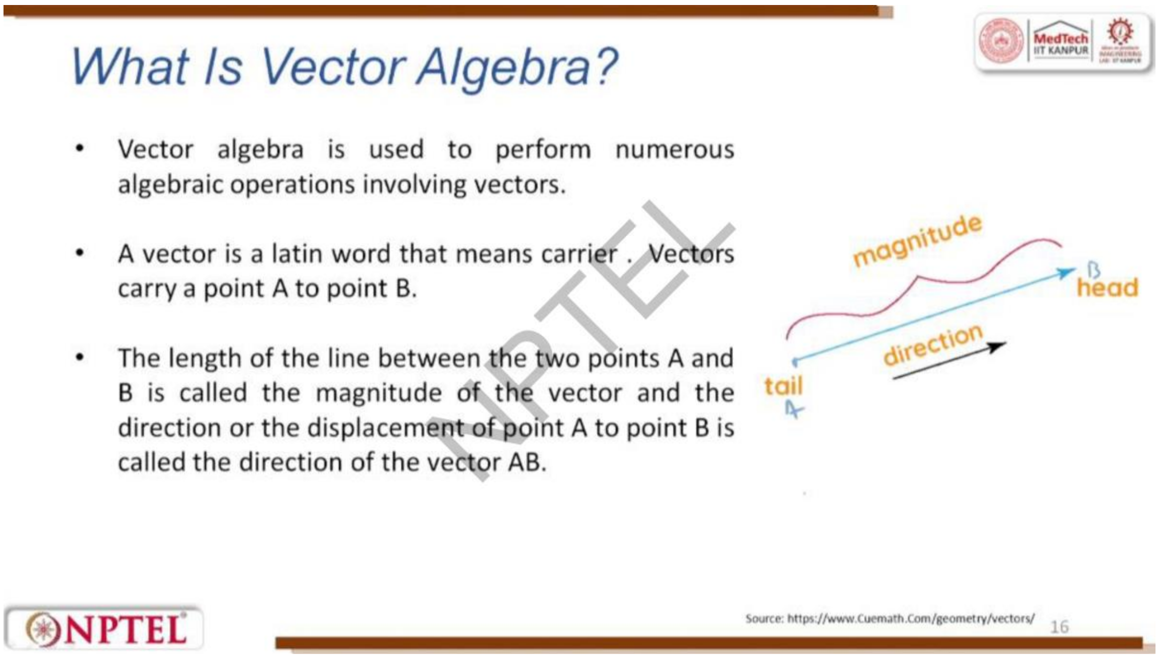


Let us get into the multiplication of vector or cross products. It is a vector with a magnitude equal to the product of two vectors cross product. So we saw scalar or dot product. Now we are seeing vector or 'Cross' product. So it is a vector with a magnitude equal to the product of two vectors and the sine of the angle between them, directed perpendicular to their plane.

So, thus

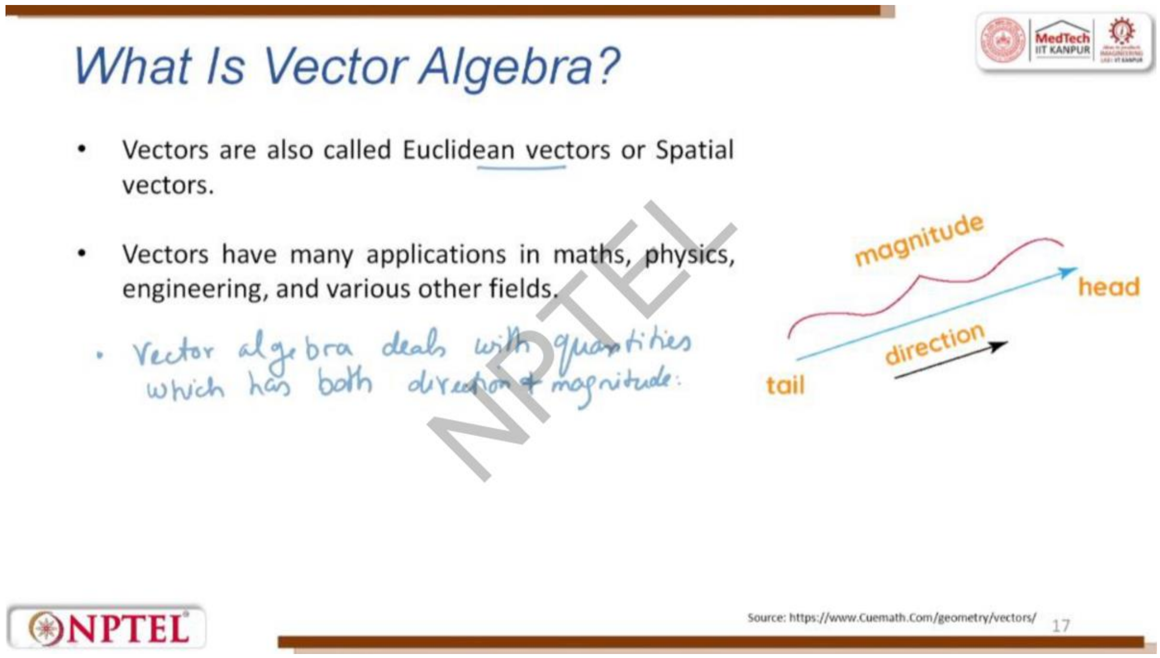
$$\vec{A} \times \vec{B} = AB\, sin\, \theta_n$$

So, this is important. Normal perpendicular to the plane. Theta n normal. So, we saw what is triangle law? What is parallelogram law which is used for addition of vectors? Then we saw multiplication of vectors, scalar product and vector cross product.

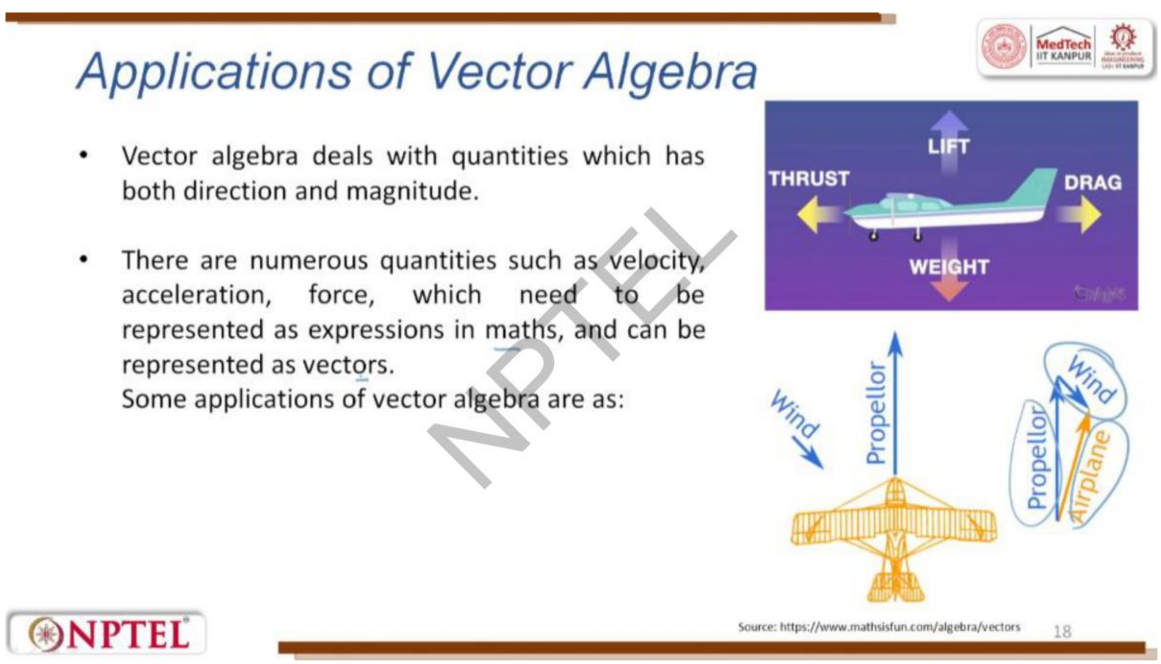


Now let us try to understand little bit of vector algebra. Vector algebra is used here to perform numerous algebraic operations involving vectors. Vector need not be in one direction. It can have multiple directions. So if you have multiple directions, addition of these vector, subtraction, multiplication, all these things come.

So vector algebra is very important to further process the data. A vector is a Latin word that means 'carrier'. Vector carry a point A point B. The length of the line between point A and B is called as the magnitude of the vector and the direction or the displacement of point A to point B is called as the direction of AB.



Vectors are also called as 'Euclidean vectors' or 'Spatial vectors.' Sometimes they call it as Euclidean vectors. Vectors have many applications in math, physics, engineering and various other fields. Vector algebra deals with quantities which has both direction and magnitude.



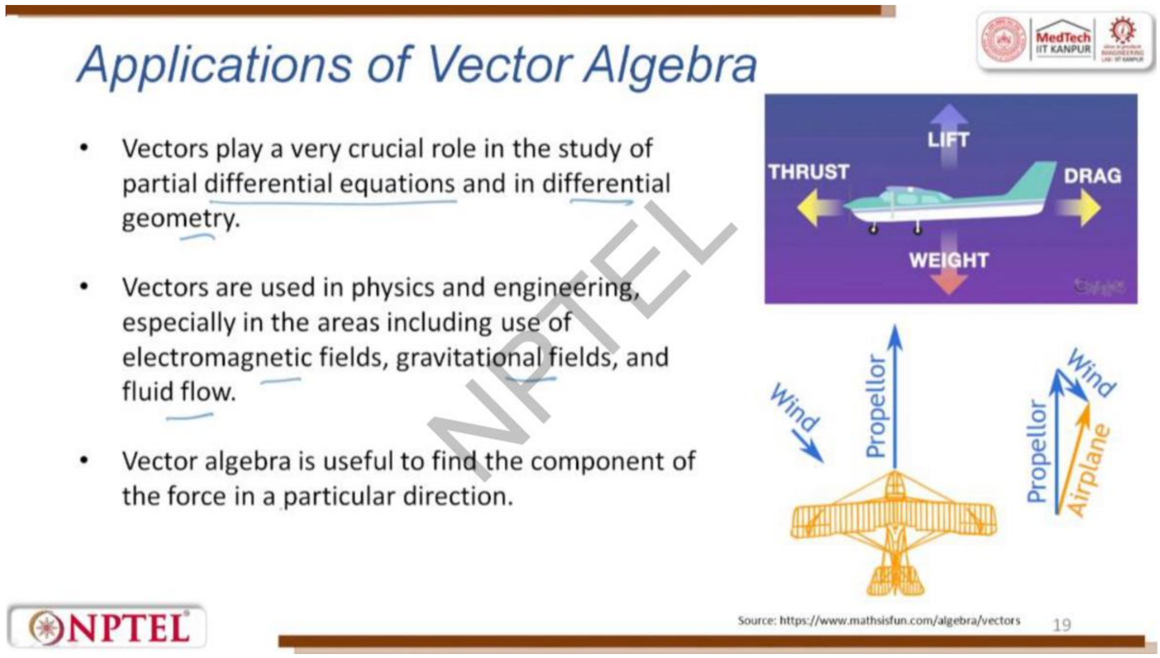
Let us see some of the applications. I have been time and again saying that in aerospace they do it. This is a typical flight which goes.

The forward direction we call it as thrust or the pull. Then in the backward is a drag. So when it tries to go in the air, it is called as lift and the gravity weight comes down. So this is direction whichever you can draw for a plane. Typically for a plane when it is balancing, you will see.

So thrust in the forward direction, drag in the reverse direction, lift in the upward and weight in the downward. So now when the aeroplane is trying to propel or I can put it glider, when it is trying to propel, you can see the propeller is moving in one direction. Wind is coming in the angle direction. Now the plane will move in this direction. So now you see how vector algebra is used for a real time application.

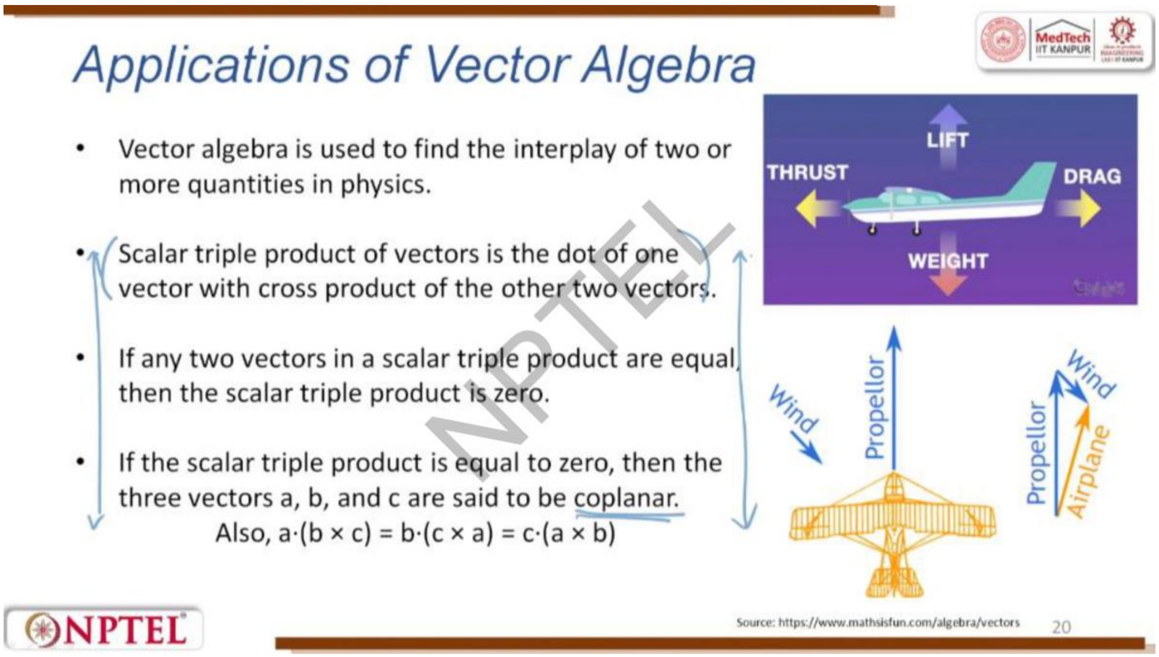
The same way you can also draw for a screwdriver where you are trying to fasten two wooden boards. When two wooden boards have to be fastened together, we use screw and we have a screwdriver. Now the screwdriver what it gives is a torsion. A torque right, so now the angle will be like this and you will try to do all the other balancing of forces. So you see wind is coming in this direction, I have represented wind here, propeller is going in this direction, now the plane will move in the resultant one.

So the vector algebra deals with quantities which has both direction and magnitude. There are numerous quantities such as velocity, acceleration, force which needs to be represented, which needs to be expressed in maths and can be expressed as vectors. So here is one of the application of Vector Algebra.



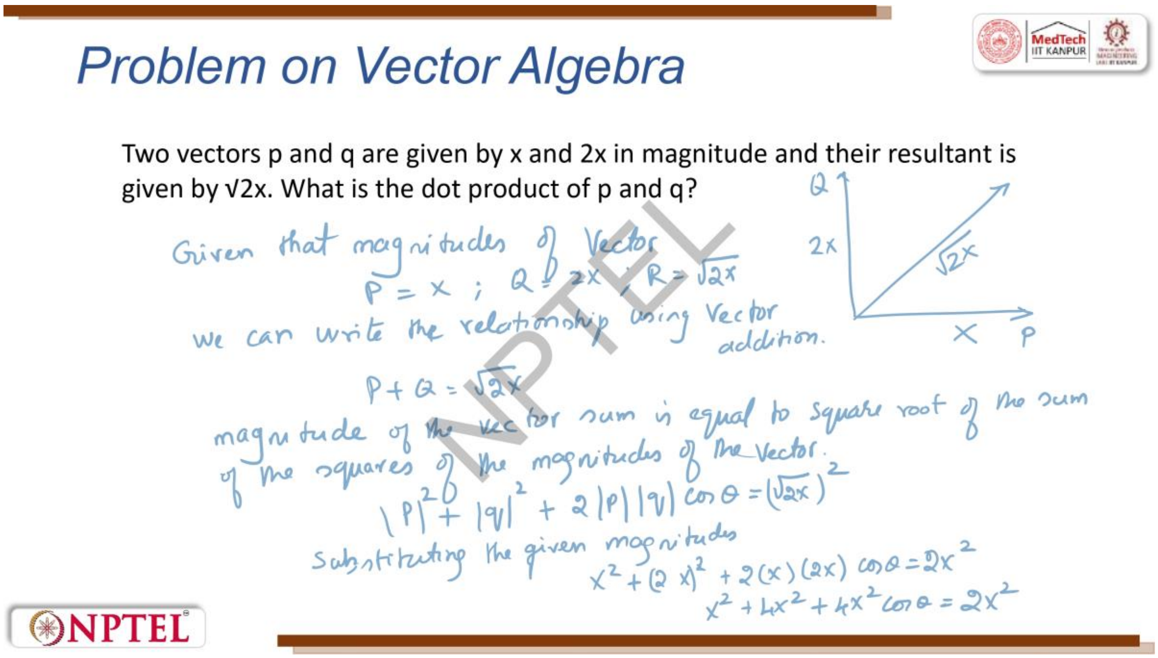
The vectors play a very crucial role in the study of partial differential equations and in differential geometry. Differential equations, whatever we study and then differential geometry also we can see the application.

Vectors are used in physics and engineering, especially in the area of electromagnetic field, gravitational field and in mechanical and chemical and aero, we try to see the fluid flow. Everywhere we use exhaustively these vector. Vector algebra is useful to find the components of forces in a particular direction.



The vector algebra is used to find the interplay of two or more quantities in physics. Scalar triple product of vectors is the dot of one vector with cross product of other two vectors.

This is very important. We will use it later, but you please make a note of it. If any two vectors in a scalar triple product are equal, then the scalar triple product is 0. If the scalar triple product is equal to 0, then the three vectors A, B and C are said to be coplanar. They are going to exist in one plane. So also a . (b x c) = b . (c x a) = c . (a x b). These are very important which you will try to use these concepts while solving problems in the future.

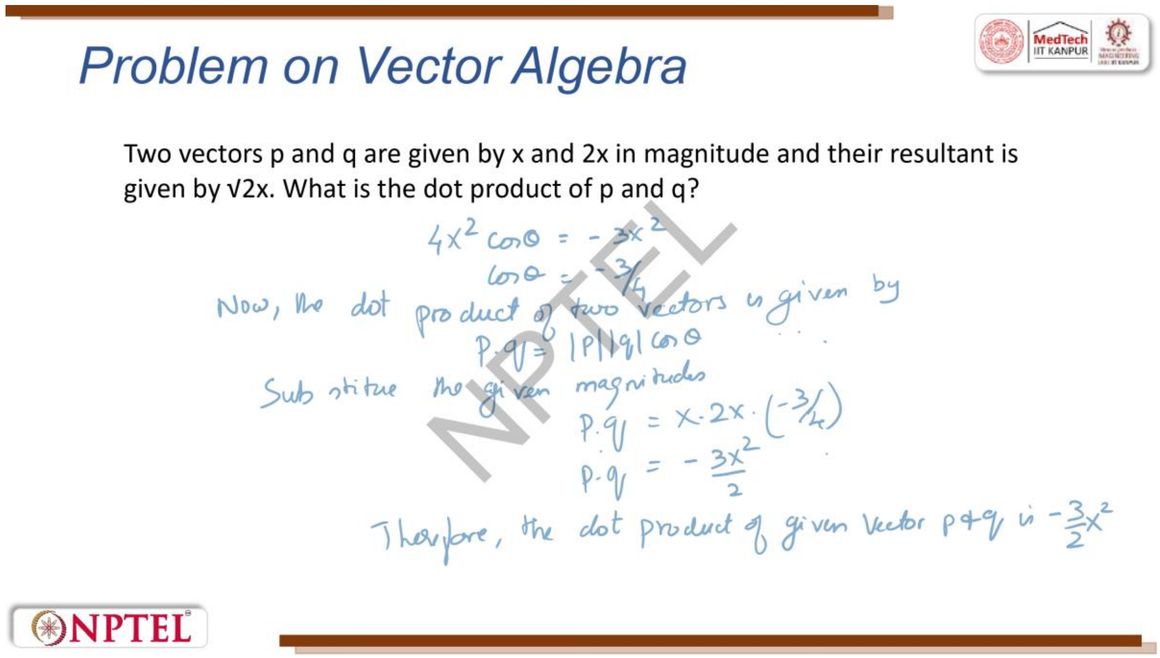


Problems on vector algebra. We have two problems. So let us try to solve the problem. Two vectors P and Q are given by x and 2x in magnitude. And their resultant is given by 2x. So two vectors. Let us take x. 2x or we can say 2x. pq their resultant which is 2 to x. What is the dot product of p and q?

So, let us try to solve this problem. Even that magnitudes of vector, p = X, q = 2x, and R = $\sqrt{2x}$. So, this is given. So, what we can write is, we can write the relationship using vector addition. So, that is p + q = $\sqrt{2x}$. So, the magnitude of the vector sum is equal to square root of the sum of the squares.

I am writing in full detail so that when you go through the slides, you will understand things much better. Magnitudes of the vector. So that is nothing but $p^2 + q^2 + 2pqcos\theta = (\sqrt{2x})^2$. So now substituting the given magnitudes, what do we get? We get $x^2 + (2x)^2 + 2(x)\,(2x)\,Cos\theta = 2x^2$

So, what do we get? $x^2 + 4x^2 + 4x^2\,Cos\theta = 2x^2$. So, what is the dot product of p and q? We will get that $x^2 + 4x^2 + 4x^2\,Cos\theta = 2x^2$.



So, now we are continuing with the same problem. So, that will be $4x^2\ cos\theta = -3x^2$

$$Cos\theta = \frac{-3}{4}$$

Now, the dot product of two vectors is given by P dot Q equal to p modulus q modulus cos theta. Now, substituting the given magnitudes what we get is $p\ .q\ =\ x.2x\ .\frac{-3}{4}$. So, $p\ .q =\ \frac{-3x^2}{2}$. Therefore, the dot product of given vector p and q is $\frac{-3}{2}x^2$.

To Recapitulate

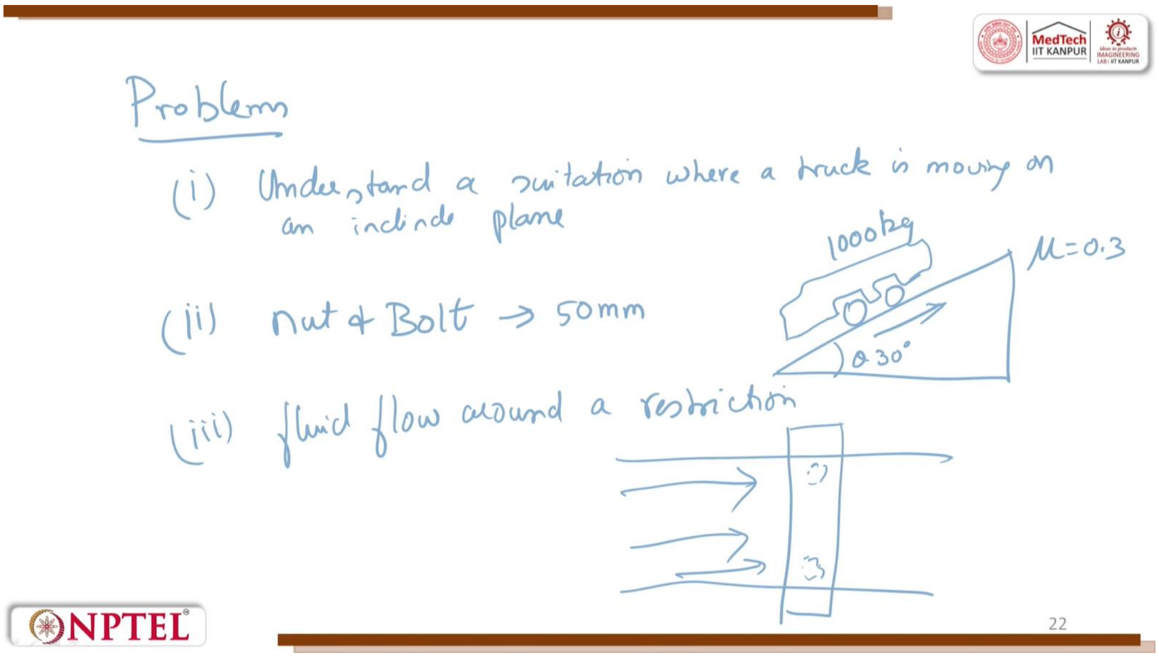
- Define Scalar quantities with examples.
- What are Vector quantities and how are they represented?
- What are various types of vector quantities?
- State and describe Triangle & Parallelogram Law of addition of vectors.
- What do you understand by Dot product and Cross product of vectors?
- What is Vector algebra? State its applications.

So, in this lecture, we saw the scalar quantities with examples, vector quantities and their representation. Various types of vector quantities, then triangle and parallelogram law of addition of vectors. We understood the dot product and the cross product of vectors. Finally, we saw vector algebra and its application in aerospace.

References

- Hibbeler, R.C., Engineering Mechanics: Statics, Pearson, 2016.
- Halliday D. & Resnick R., Physics Vol-II, Wiley Eastern, 1993.
- Malik H.K. & Singh A.K., Engineering Physics, Tata McGraw Hill, 2011.

These are the references which we have used in making this lecture.



So, in this lecture, I would like to give you real life problems for your understanding so you will try to solve and if you have difficulty in the tutorials, we can discuss it in length, first problem is you try to understand a situation where a truck is moving on an inclined plane.

A truck is moving in an inclined plane. The truck is going to go up. The slope is given as 30 degrees. So now please try to draw all the forces which are involved with the directions and try to see how can you solve this problem by using vector addition or dot product. The only thing is the weight. Assume a weight of 1000 kgs. That is the weight of the truck which is moving on an inclined plane at 30 degrees.

So if you want to take it to a real life, there is a bridge. The truck is climbing on the bridge. So now try to figure out what are the forces and how do you find out what is the effort which is required for the engine. And next comes the friction component. You can assume the friction mu equal to 0.3.

That is a road, tyre and road friction. The next problem is you will have to do all the forces like what I showed in glider. You saw that I had a lift direction, weight direction. So you have to do all those things. Next is let us try to take a nut and a bolt situation.

Where there is a nut, you have a bolt, you have a spanner and the situation is, the spanner is getting tightened. So, now you will have to draw the diagram, identify the forces and try to solve the problem, find out what will be the torque required. And for calculation sake, you can try to take the nut dimension as 50 millimeter. The third thing is I am trying to make a fluid to flow around a restriction. And what is the situation?

The situation is there is a river which is flowing and then there is a bridge which is constructed on top. So the bridge will have pillars. So these pillars are there. Try to take a situation of a fluid flow which is there. So this is the stream of fluid which is flowing and there is a obstruction which is there on a bridge, a hole, a pillar is made and the bridge is made on the top.

So in this situation, try to figure out what are all the fluid is flowing, what are all the forces which come and how do you try to solve because when the fluid is flowing, it will try to go around the pillar. So these are three situations which all are real-time applications. Try to figure out with the understanding, whatever you have in the subject and wherever you want, you assume a magnitude for yourself. So this also will try to give you a feel that what is the understanding you have on magnitude, right. Fluid flow and a vehicle which is going up, a torque which is applied on a nut.

So you have to find out what is a torque and what are the forces, vector addition, vector multiplication, whatever it is you want to do, please do it and try to solve these three problems for your understanding. Thank you very much.