Basics of Mechanical Engineering-1

Prof. J. Ramkumar

Dr. Amandeep Singh

Department of Mechanical Engineering

Indian Institute of Technology, Kanpur

Week 10

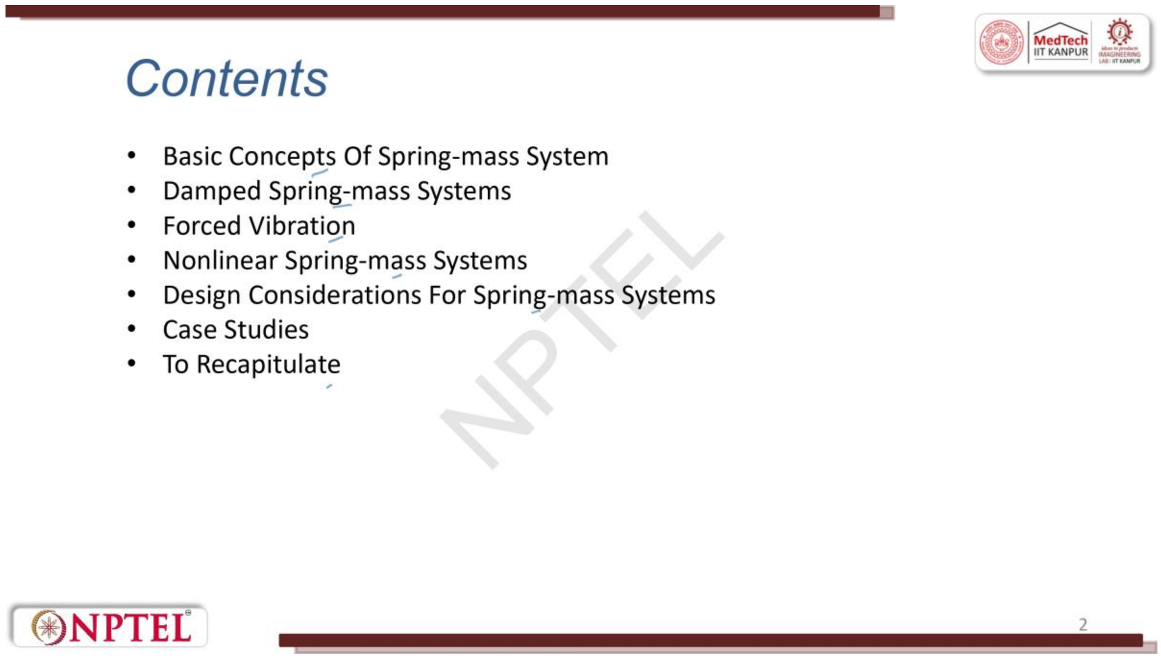
Lecture 40

Spring-Mass Systems (Part 1 of 2)

Friends, welcome to the next lecture on Spring and Mass System. It is a very important topic and spring is one of the most important element in mechanical systems. It absorbs energy and dissipates very slow. As an automobile lover, you would have seen an element. In your shock absorber is spring.

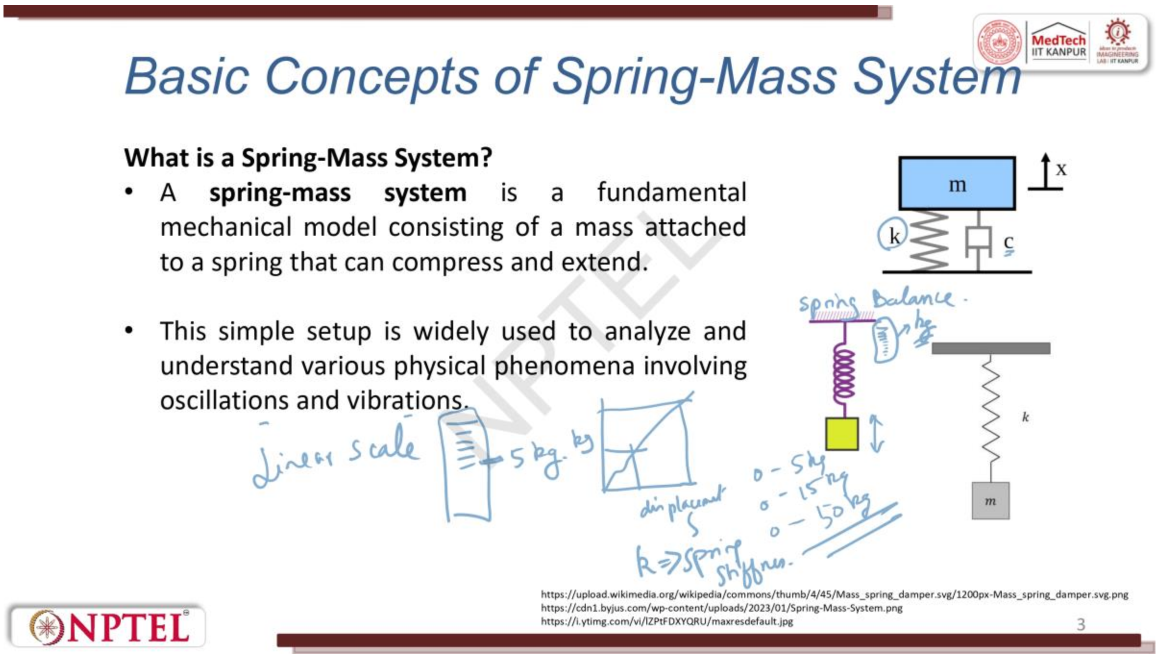
The spring can be of multiple types. For example, it can be a Helical spring. It can be a Leaf spring which is used in buses. Earlier, Leaf spring used to be made out of metal. Today it is made out of elastomers.

In your typical ballpoint pen, again you can see a spring which is put in the front end of the system. So spring is very important element.



In this lecture, we will try to cover the basic concepts. We will try to have Damped Spring-mass Systems, then forced vibration. Nonlinear spring mass system, design consideration for a spring mass system, a case study and then finally you will try to have a recap.

Today when we construct tall structured buildings or a tall statue, we always have to make sure that it withstands the natural disaster. It can be wind, it can be rain, it can be an earthquake. So, in order to balance the weight, again they try to use a Spring-mass system.



What is a Spring-Mass system? A Spring-Mass system is a fundamental mechanical model consisting of a mass attached to a spring that can compress or extend.

When you are trying to travel in a bike, you are sitting. So that's a mass. So when there is a brake applied or when there is an undulation or a pothole, the spring is compressed. And then it is relaxed by this C. So here the mass compresses the spring where k is the stiffness of the spring. You can use the mass also in the downward fashion, where it is used in spring balance.

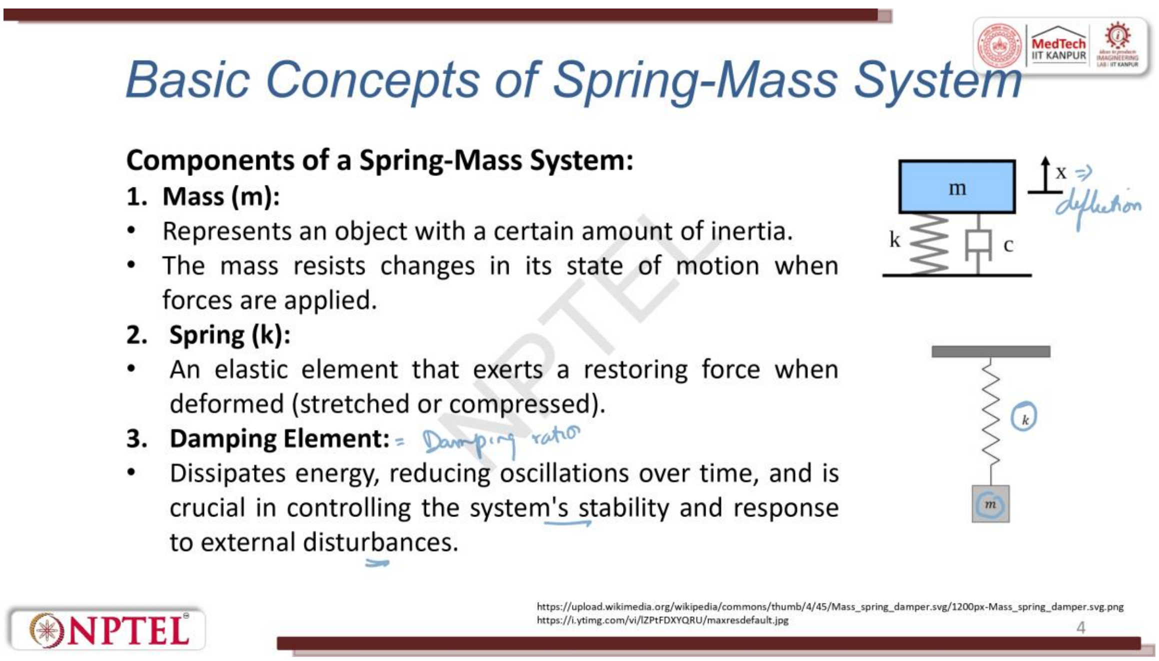
For example, to measure weight, 0 to 5 kgs, 0 to 15 kgs, you can also have 0 to 50 kgs. More the weight, lesser will be the resolution. So you can have a spring attached to a mass. Now here what will happen is you will try to see a graduation. Based on the graduation, you can try to figure out what is the kg's weight.

So how do they do it? They have a linear scale where in which the graduations are there. So, there will be a floating reader. So, it says 5 kg. So, now we would have already calibrated with respect to kg versus displacement.

This displacement in turn is attached to the spring stiffness kg. So you will try to have something like this. So we will always work in the linear portion. So you can even go to Sabji Mandi, vegetable vending and then you can try to buy. When you go to buy clothes in kgs, for example for waste cloth, again you will see a spring balance.

So spring balance, here what happens, the weight is attached and then the spring elongates. So here it is tensile, here it is compression. This simple setup is widely used to analyze and understand various physical phenomena including oscillations and vibrations. You can also put the spring on top of a table and try to see how much does it vibrate. For example, that is how they try to do on a speedy train.

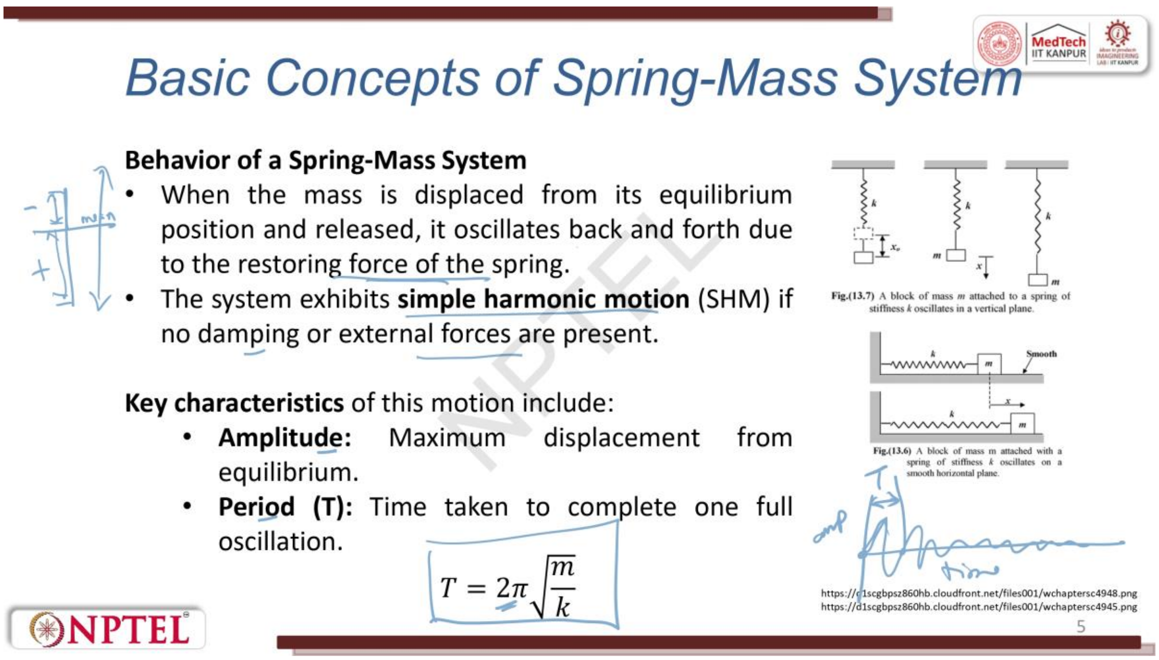
On a high speed train, they used to put a dial gauge and then they used to measure the deflection and say what is the vibration it creates when the train is going at very high speeds and when the brakes are applied.



The components of a Spring-Mass system. It has m, it has k and it also has a damping element. What is m? m represents an object with a certain amount of inertia.

The mass resists change in its state of motion when the force is applied or forces are applied. So here x is the deflection. Spring, an elastic element that exerts a restoring force when deformation happens. So that is called as the Spring or it is called as the Spring Stiffness. Third element is going to be the damping element.

Damping element, it dissipates energy, reduces oscillation or reduce, it dissipates energy by reducing oscillation over time and is crucial in controlling the system's stability and response to external disturbance. Damping is very important because once the spring is taking a load, then the spring has to dissipate the energy such that it can bring the mass back to the original position.



Behavior of Spring-Mass system. When the mass is displaced from its equilibrium position and released, it oscillates back and forth due to the restoring force of the spring. That is the Spring Stiffness.

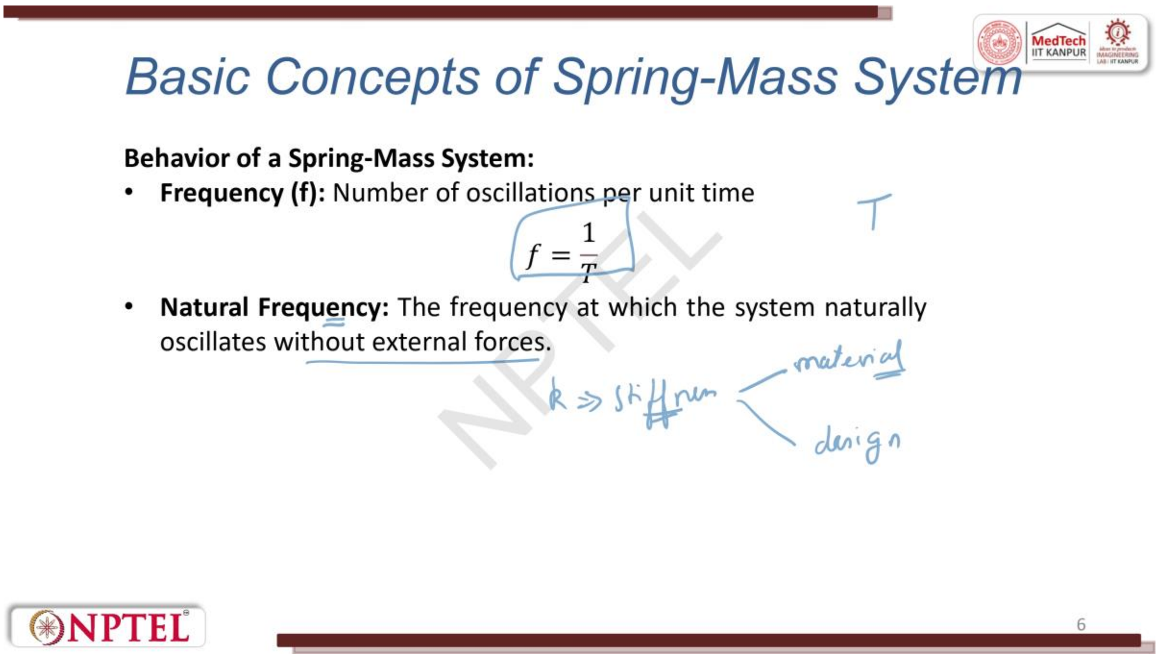
You take a weight, you apply to a spring, you pull the weight and release it. It starts vibrating or it starts oscillating. It oscillates back and forth. Why are we saying back and forth? There will be a mean.

It can go up and down. The displacement can be same or it can be more. So assuming that this is positive and this is negative, this is the mean position. So it will go back and forth. Initially, it will be both sides uniform and over a period of time it slowly dies down.

The system exhibits simple harmonic motion if no damping or external forces are present. It will keep on vibrating. So now there are two things which come into existence. So, what are the two things? What are the key characteristics of this motion?

They include Amplitude and Time. For example, if you try to take Amplitude and Time, so it might go like this. So, the key characteristics of this simple harmonic motion, includes amplitude. The maximum displacement from the mean position or equilibrium. Second thing is what is the time period it takes to complete one cycle.

So one cycle starts from here and it goes up, it comes down and then it goes up. So this becomes your one cycle T. Time taken to complete one full oscillation or cycle. So generally T is represented as $T = 2\pi\sqrt{\frac{m}{k}}$. So, moment you know T, from there we try to derive frequency.

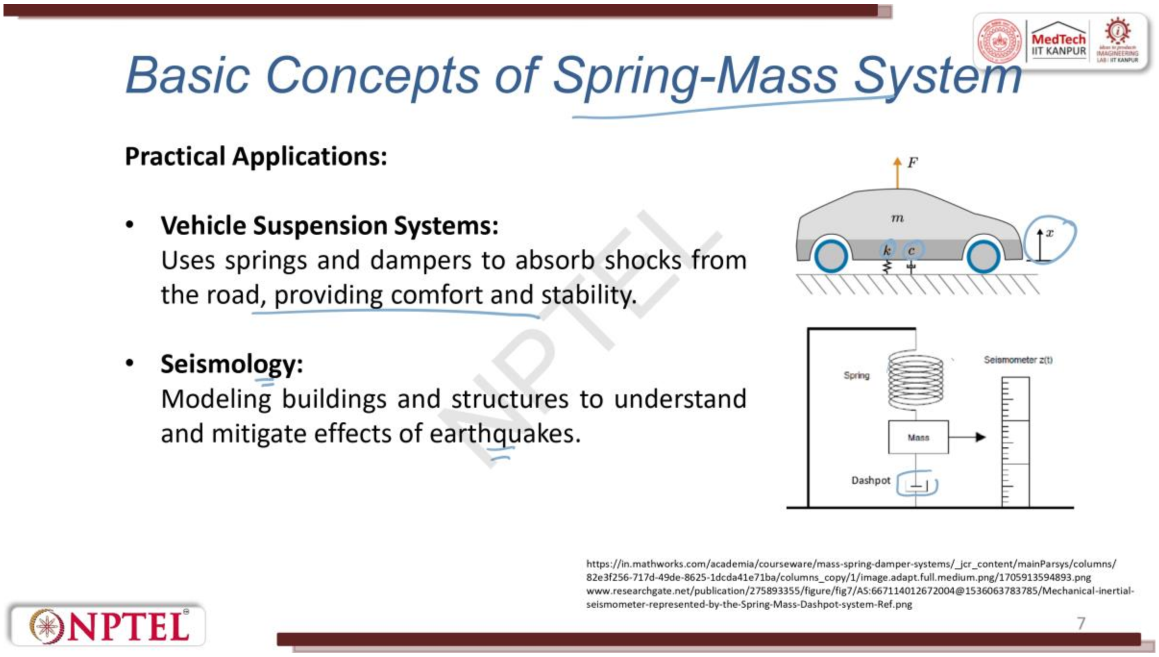


Frequency is the number of oscillations per unit time, f. Every mass or every spring or every object will have its own natural frequency.

The frequency at which the system naturally oscillates without any force is called as Natural frequency. We always try to play around this natural frequency. For example, if there is a natural frequency for a given element, so the vibration, we always make sure it does not go close towards the natural frequency. If it goes close, then there will be a major disaster happening. So frequency of oscillation will be maintained very low compared to the natural frequency of the given object.

So then how will you change the natural frequency? So the frequency can be changed by changing the stiffness. Stiffness can be changed. How will you change the stiffness? The material can be changed or the design can be changed.

So, based on that to some extent the stiffness can be changed. Predominantly we go around changing the material.



So, what are the Practical Applications of Spring-Mass system? So, one is Vehicle Suspension Systems. It is very important.

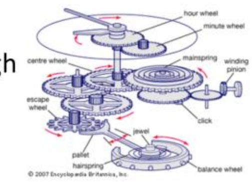
She uses springs and dampers to absorb shocks from the road providing comfort and stability for the passenger. So, here this is the mass of the car. So, here is the spring, here is the damping and then here is the displacement. So, the car will move like this. So, then it is also used in the instrument which is called a Seismology for seismic measurements, modeling buildings and structures to understand and mitigate the effect of earthquake.

So, spring is attached to a mass, this is attached to a dash pod and then you try to see the displacement of the mass with respect to a scale. Seismology is a study of seismic waves and here we are now trying to build buildings. The shape of the building, the weight which is getting distributed on the building, we are trying to design keeping the effect of earthquake or we are trying to mitigate with the earthquake.

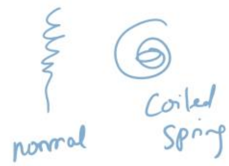
Basic Concepts of Spring-Mass System

Practical Applications:

- **Mechanical Clocks and Watches:**
  Utilize springs and masses to keep accurate time through controlled oscillations.



- **Vibration Isolation Systems:**
  Protect sensitive equipment by absorbing and reducing unwanted vibrations.



- **Engineering and Physics Education:**
  Serves as a foundational model for teaching concepts of dynamics, control systems, and oscillatory motion.

https://cdn.britannica.com/00/27900-004-1CA1D8B5/components-watch.jpg

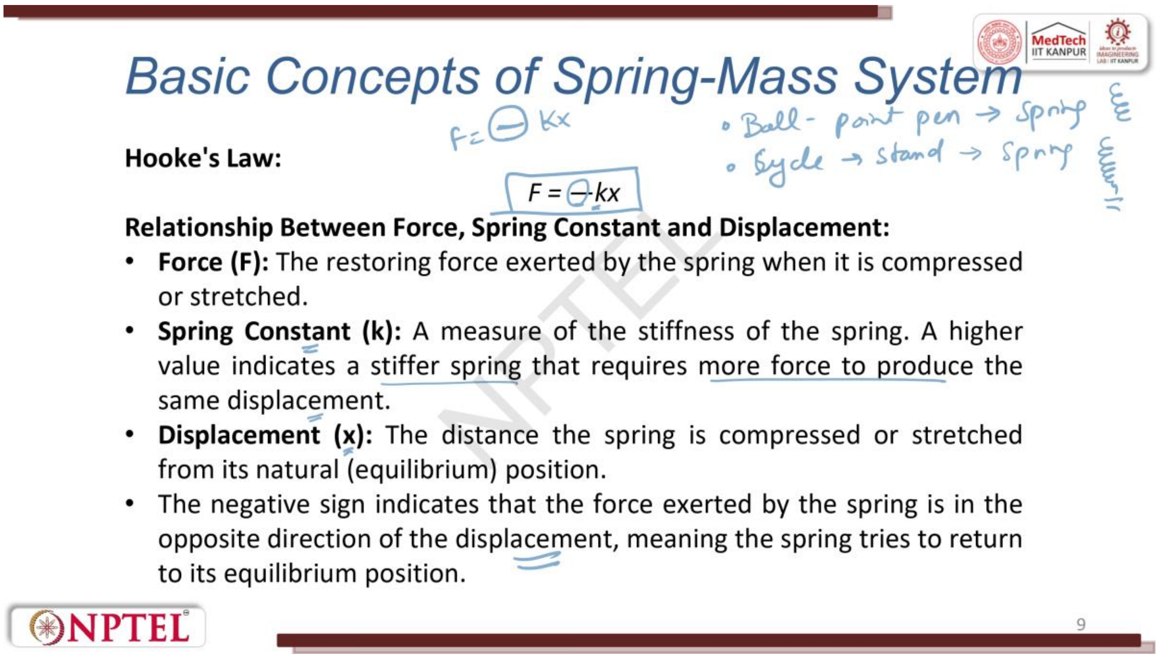
Next is Mechanical Clock and Watches. Utilizes spring and masses to keep accurate time through controlled oscillation.

So here you see there will be lot of gears. There will be a coiled spring. This is a normal spring. You can also have a coiled spring. So in your scooter, this is normal helical spring and this is coiled spring.

In your scooter, kick scooter, when you try to kick, it goes down and then it gets released very fast. They are all coiled springs. So in a watch also we try to use a coiled spring. So the coil spring, when you try to give a key, it starts loading the coil spring and coil spring slowly releases energy. Till it releases complete energy, the watch keeps on working.

So if you see olden wall clocks, where in which they used to have a key and give motion to the key, they basically load the spring. Then, vibration oscillation systems, it protects sensitive equipments by absorbing and reducing unwanted vibrations. For example, if you buy any of the high resolution or highly sensitive equipments, we always try to put a shock absorber. When you try to install a lathe machine, milling machine or even your washing machine, your grinder, we always have a spacer between your object or your instrument product with respect to ground. So in between what we put is we always try to put a rubber pad or a spring attached thing.

So those are nothing but absorbers which reduces the unwanted vibration. Engineering and Physics Education serves as a foundation model for teaching concepts of dynamics, control systems and oscillation motion. There we use the spring mass system.



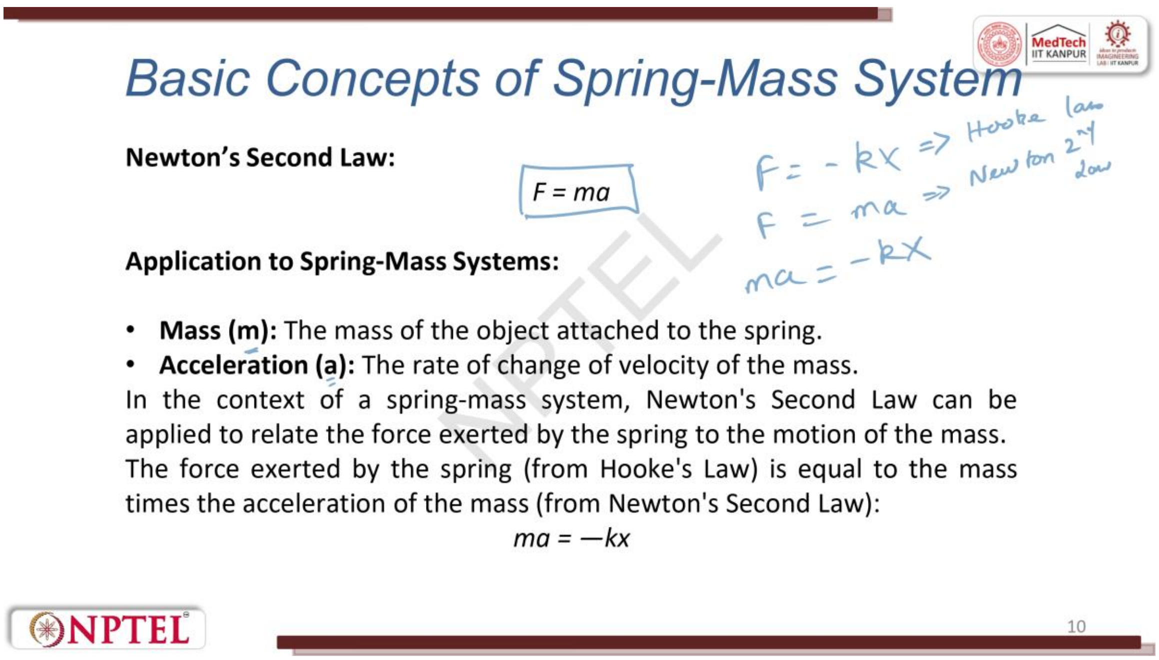
Let us go back to the Hooke's law what we studied when we studied about the uniaxial tensile testing. So we studied $F = -kx$ where F is the restoring force exerted by the spring when it is compressed or stretched.

F is the force. I will just give you an exercise. You can try to remove your ball point pen and then you try to see a spring which is there, then you try to go to your cycle support stand is there. When you park a cycle you place it with a stand; so there you see there is a spring, both the springs are coiled or helical in nature, they are helical here also, it will be helical in nature but the two springs have different stiffness. Why is it so?

It is basically the spring constant is different. A measure of the stiffness of the spring, a higher value indicates stiffer spring that requires more force to produce the same displacement. More the stiffness, more the force required. So when you try ballpoint pen, maximum load may be 2-3 kilos load it will have to take. When we try to use a cycle stand, it has to try to take a load of 15-20 kgs.

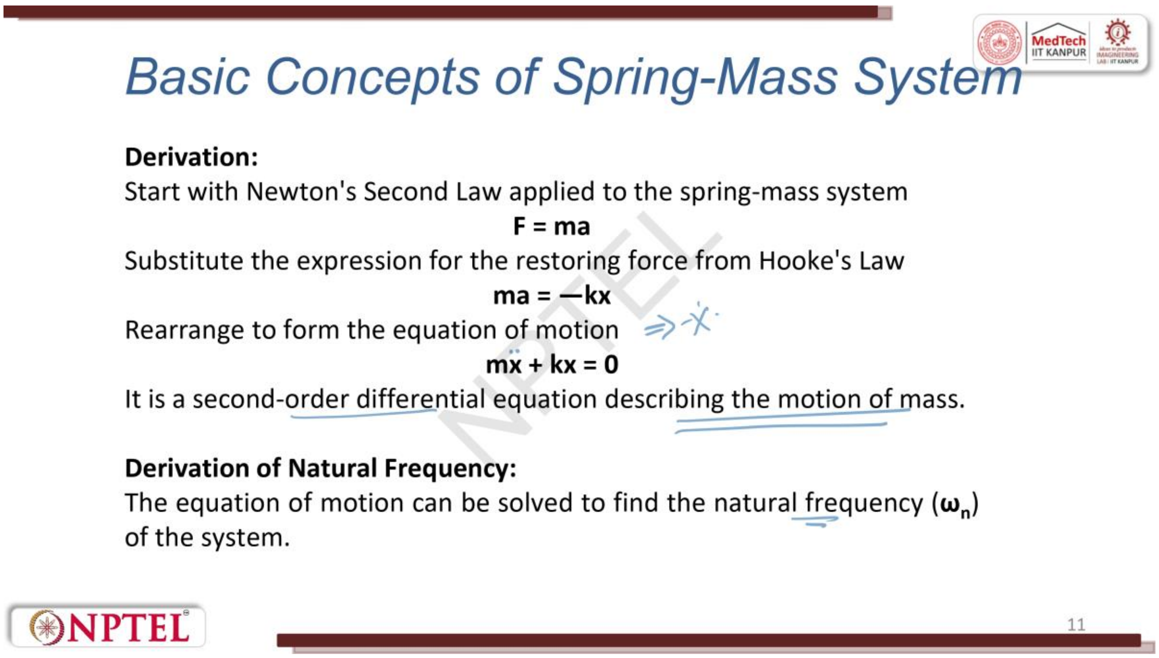
That is what it is. So x is nothing but a displacement. The distance the spring is compressed or stretched from its natural position is called as displacement. The negative sign which is there indicates that the force exerted by the spring is in the opposite direction of the displacement. So, F equal to minus kx, this minus is what we are discussing.

The negative sign indicates that the force exerted by the spring is in the opposite direction of the displacement meaning the spring tries to return to its equilibrium position.



Newton's second law of motion states that F = ma. So, this is an application to the spring mass system, where m is the mass and a is the acceleration. In the context of spring mass system, Newton's second law can be applied to relate the force exerted by the spring to the motion of the mass.

So, earlier we saw F = -kx. Now, we are seeing F = ma. So, now I can equate ma = -kx. That is what is told here. So, the force exerted by the spring Hooke's law. This is Newton's second law and this is the, we are trying equate F and F here.



So, derivation, Start with Newton's Second Law applied to the spring-mass system

F = ma

Substitute the expression for the restoring force from Hooke's Law

ma = —kx

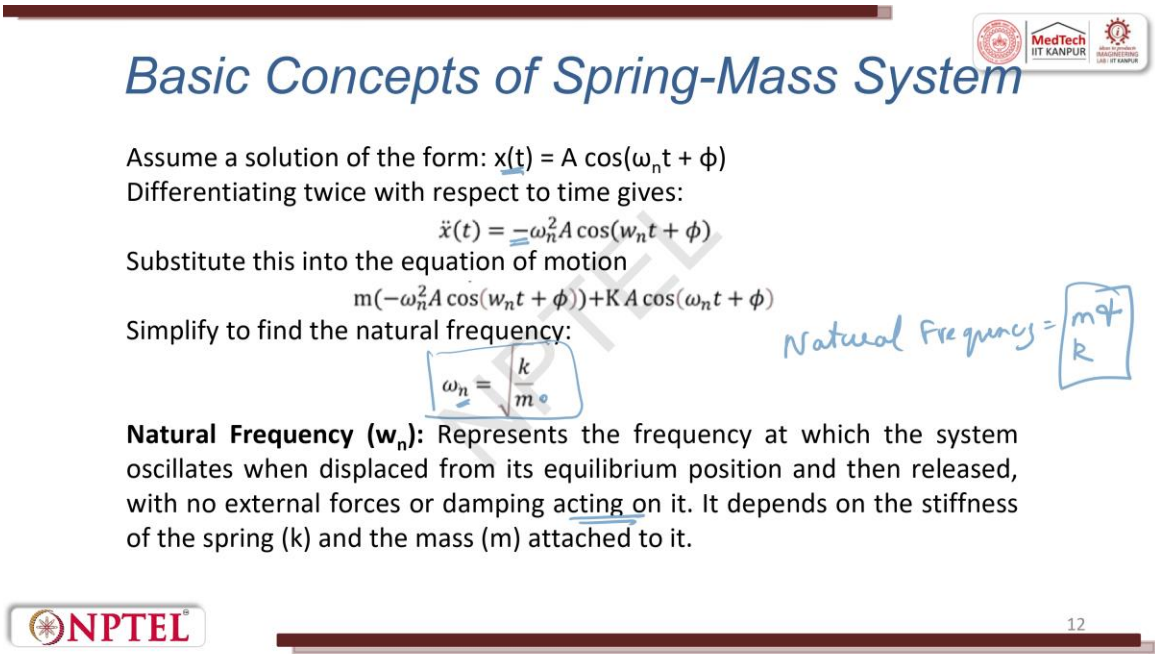
Rearrange to form the equation of motion

mx + kx = 0

It is a second-order differential equation describing the motion of mass.

Derivation of Natural Frequency:

The equation of motion can be solved to find the natural frequency (ω_n) of the system.



Assume a solution of the form: $x(t) = A\cos(\omega_n t + \varphi)$

Differentiating twice with respect to time gives:

$$\ddot{x}(t) = -\omega_n^2 A\cos(w_n t + \phi)$$

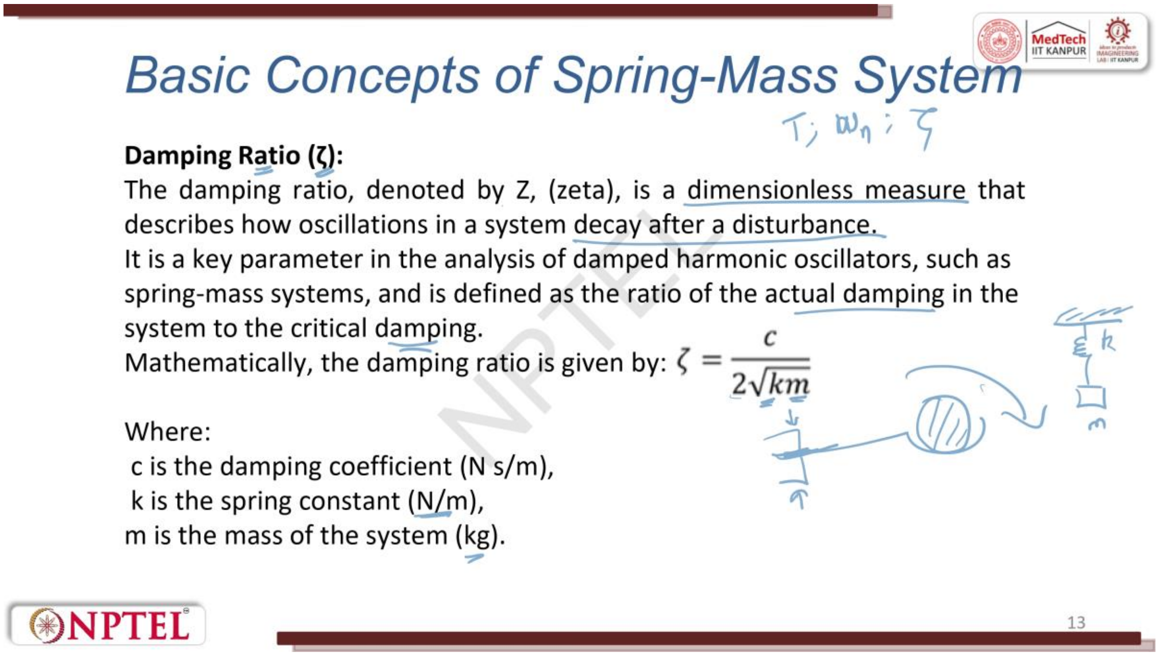
Substitute this into the equation of motion

$$m(-\omega_n^2 A\cos(w_n t + \phi)) + K\,A\cos(\omega_n t + \phi)$$

Simplify to find the natural frequency:

$$\omega_n = \sqrt{\frac{k}{m}}$$

Natural Frequency (w_n): Represents the frequency at which the system oscillates when displaced from its equilibrium position and then released, with no external forces or damping acting on it. It depends on the stiffness of the spring (k) and the mass (m) attached to it.



So what is Damping Ratio? I said the energy gets dissipated and it almost the vibration attains zero, so now when you talk about that so there is something called as damping which happens and when you look into the system we also said in the beginning, damping element, right? So, this damping element, we will have this damping ratio. So, damping ratio zeta. Damping ratio denoted by z or zeta is a dimensionless measurement. When I say ratio, it will be dimensionless measurement that describes how oscillation in the system decay after a disturbance.

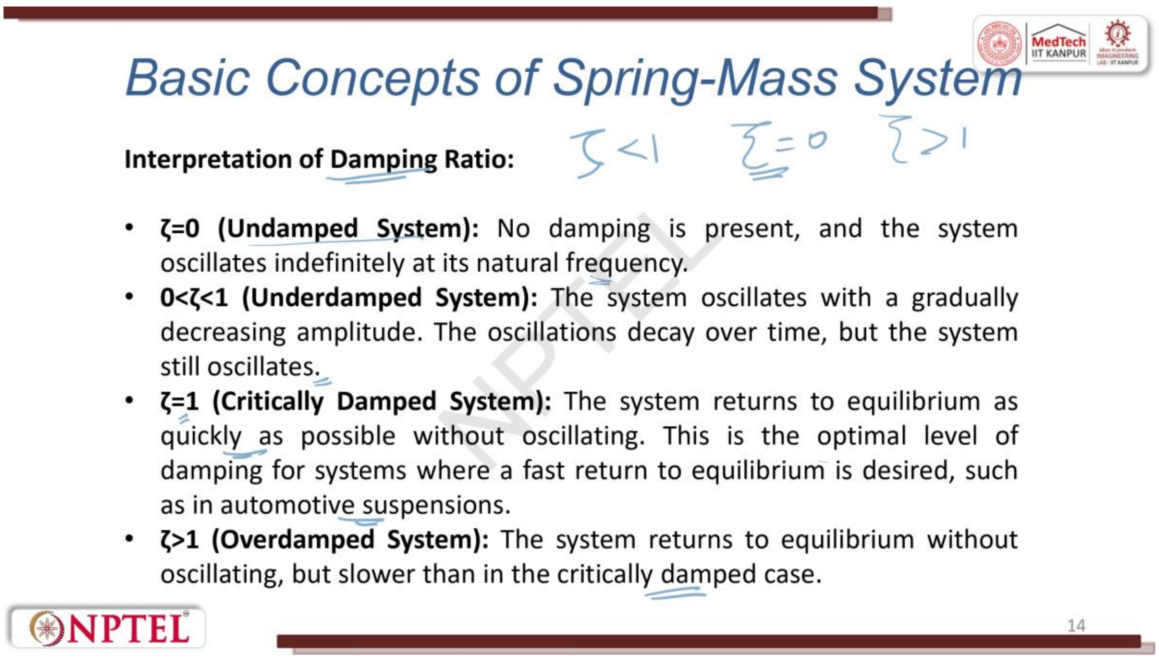
You have a spoon, you have a maybe an orange here and then you try to close this or hold it in your mouth. So, here there is a force which is getting applied. So moment you apply a load like this and then you pull it, it starts vibrating. This vibration does not continue for a longer time. It vibrates and then dies.

So that's what is the decay after a disturbance. It is a key parameter in the analysis of damping harmonic oscillators such as spring mass system and is defined as a ratio of the actual damping in the system to the critical damping. So, here zeta can be represented as

$$\zeta = \frac{c}{2\sqrt{km}}$$

You see spring stiffness and mass keeps on coming in multiple formulas when we talk about spring mass system in terms of time, yes it came. In terms of natural frequency omega, it came.

When you talk about zeta, the damping ratio, again it comes. So please be careful about the formulas. So what is c? c is the damping coefficient which is represented in units like Newton S per meter. The spring constant is Newton per meter and the mass is always in kgs.



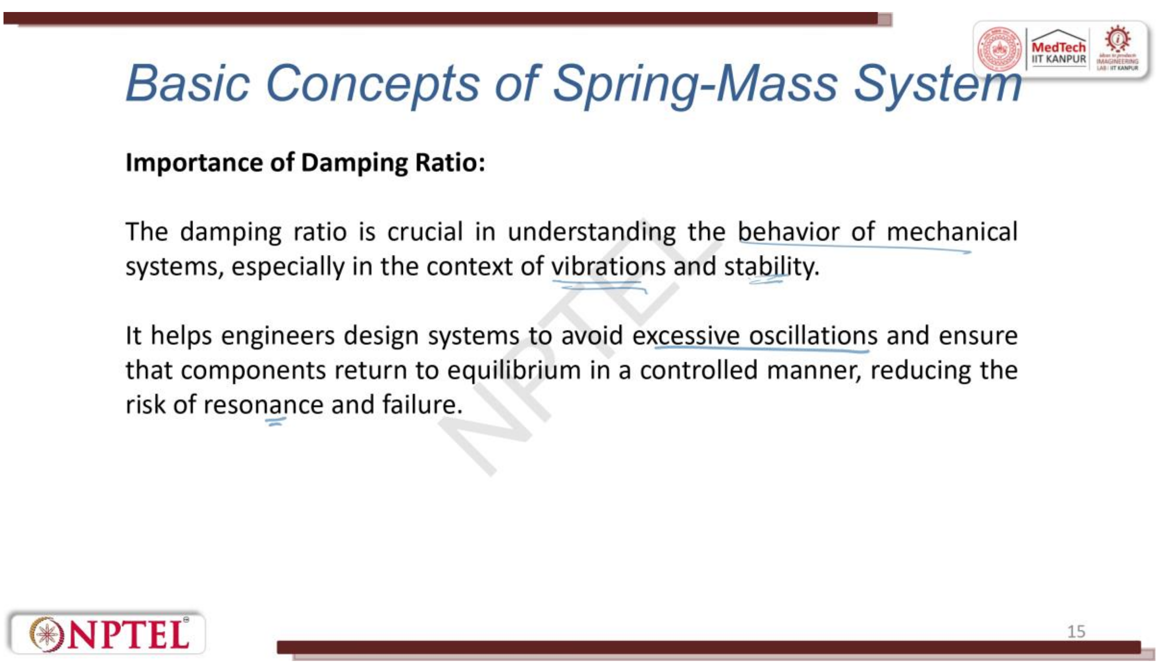
So, Interpretation of the Damping Ratio. So, what are all possible? Zeta can be less than 1, zeta can be equal to 0, zeta can be greater than 1. So, when **$\zeta=0$**, the system is called as Undamped system. No damping is present and the system oscillates indefinitely at its natural frequency.

Very important, please note, undamped system. When the system is underdamped, the system oscillates with a gradually decreasing amplitude. The oscillation decays over time, but the system still oscillates. When you say **$\zeta=1$**, it is critically damped systems. The systems returns to equilibrium as quick as possible without oscillating.

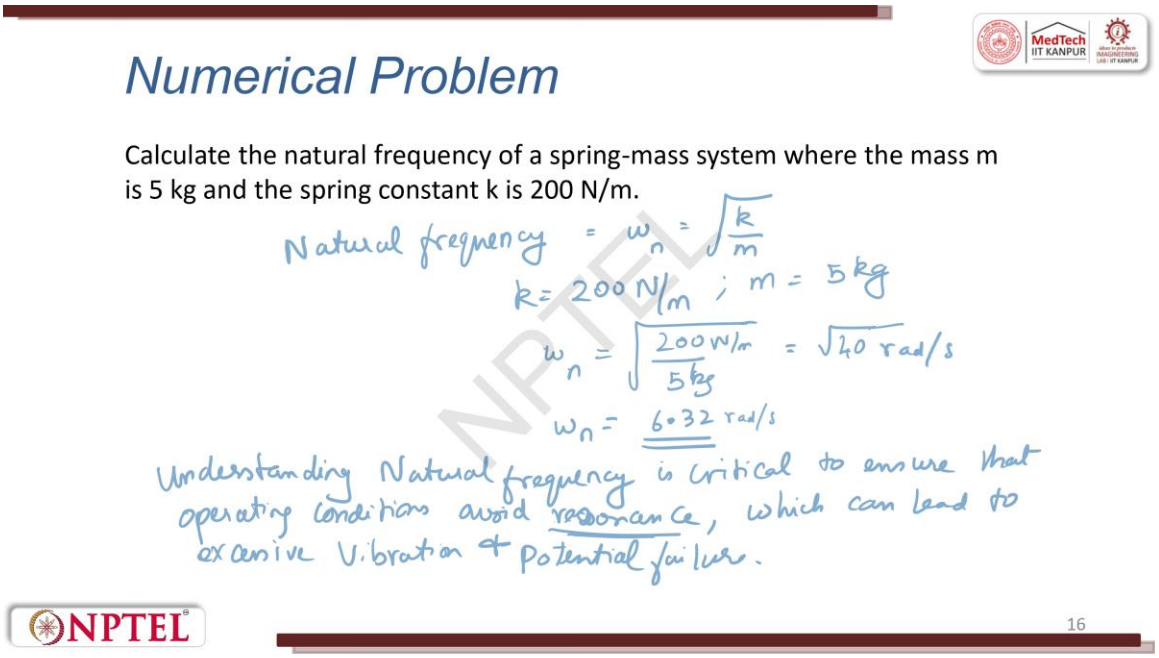
For example, you need to have this in cars. When you hit a pothole, if there is an oscillation which is happening, it has to be arrested very fast. Maybe in one oscillation or in the second oscillation it should die. So that is called as a Critical Damped Oscillation. The system returns to the equilibrium as quick as possible without oscillating.

This is the optimum level of dampening for systems where a fast return to equilibrium is desired such as in automobile suspensions. When it is greater than 1, it is called as over damped. The system returns to equilibrium without oscillating but slower than the critical damping case. So the difference you should note down. In critical damped system, the system returns to equilibrium as quick as possible without oscillating.

Here in over damping, the system returns to equilibrium without oscillating but slower than the critical damped case. So, in both the case, when you go back to zeta, you see a spring stiffness and mass. So, the mass whatever you apply in the spring mass system. So, you had a spring, then you had a mass. So, this has to be damped. So, this is spring k, this is mass m and this is mounted.



So, Importance of Damping Ratio, the damping ratio is crucial in understanding the behavior of a mechanical system, especially in the context of vibration and stability. If some object keeps on vibrating, then it is going to bring lot of uncomfortable situation for the object or the system. Stability is very important. It helps engineers design systems to avoid excessive oscillation and ensures that components return to equilibrium in a controlled manner, reducing the risk of resonance and failure.



Let us try to work on a very simple problem to understand the concept whatever we have gone through. Calculate the natural frequency of a spring mass system where mass is 5 kg and the spring constant k is 200 Newton meter.

Given: Mass, m = 5 kg; Spring constant, k = 200 N/m

Natural Frequency Formula: $\omega_n = \sqrt{\frac{k}{m}}$

Substitute the Given Values: $\omega_n = \sqrt{\frac{200\ \text{N/m}}{5\ \text{kg}}}$

4.Calculate the Natural Frequency: $\omega_n = \sqrt{40rad^2}/s^2$

ω_n =6.32rad/s^2

Result: The natural frequency of the spring-mass system is **6.32 rad/s^2**.

Significance: Understanding the natural frequency is critical in designing mechanical systems to ensure that operating conditions avoid resonance, which can lead to excessive vibrations and potential failure.

Thank you very much.