Basics of Mechanical Engineering-1

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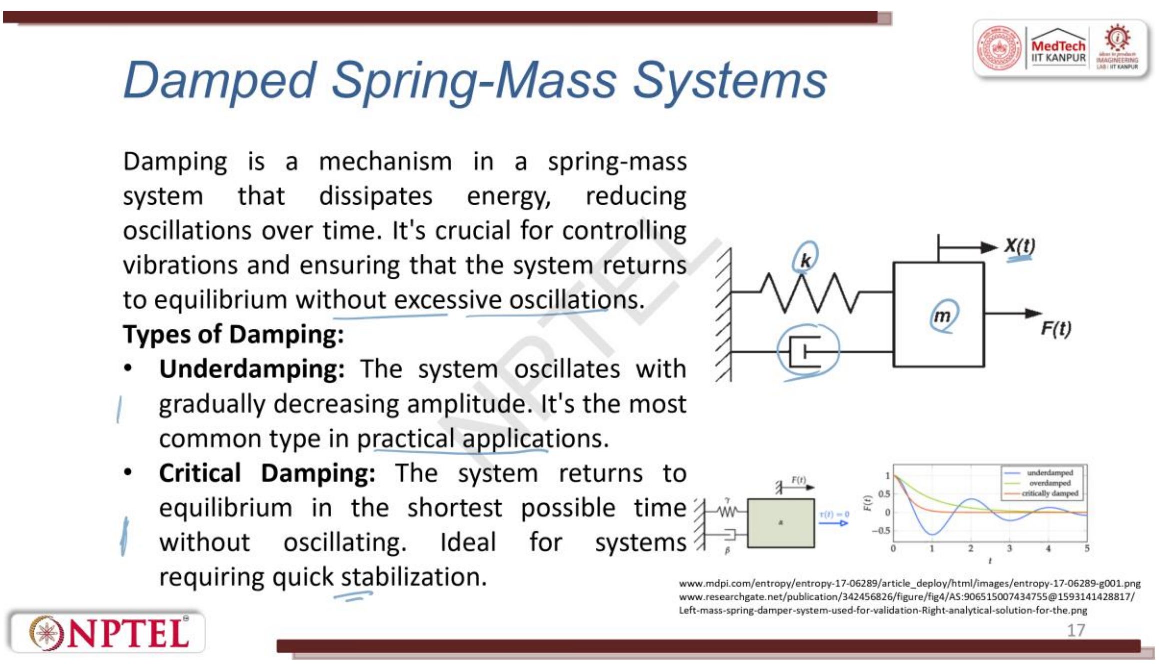
Week 10

Lecture 41

Spring Mass System (Part 2 of 2)

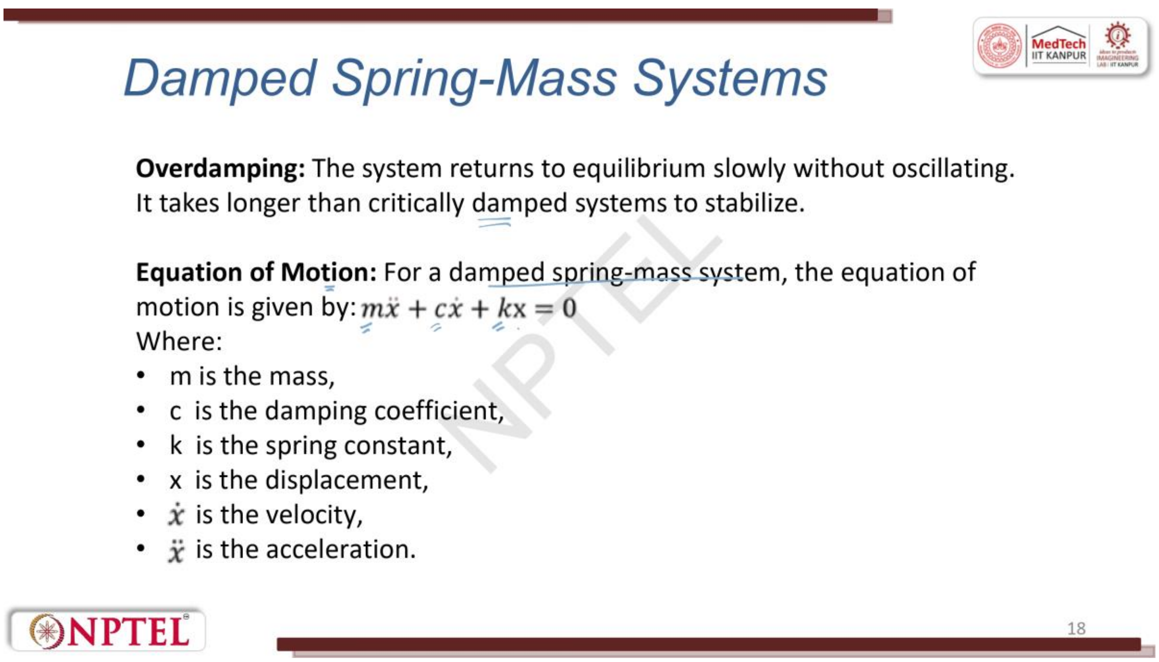
Friends, welcome to the next lecture on Spring and Mass System.



So let us keep understanding more on Damped Spring Mass system. Damping is a mechanism in a spring mass system that dissipates energy reducing oscillation over time. It is crucial for controlling vibration and ensuring that the system returns to equilibrium without excessive oscillation. Two types of damping which we saw under critical damping.

The under damping is the system oscillates with gradual decrease in amplitude. It is the most common type in practical applications. If you have a critical damping, the system returns to equilibrium in the shortest possible time without oscillating, ideal for returning back to quick stabilization. So, this is what we would prefer and generally this is what happens under damping. For Critical Damping, you have to modify the system accordingly.

So, you have a spring which has a stiffness, then m is the mass, you are applying a force, it has a displacement x with respect to time and you have an absorbing energy, a damper.

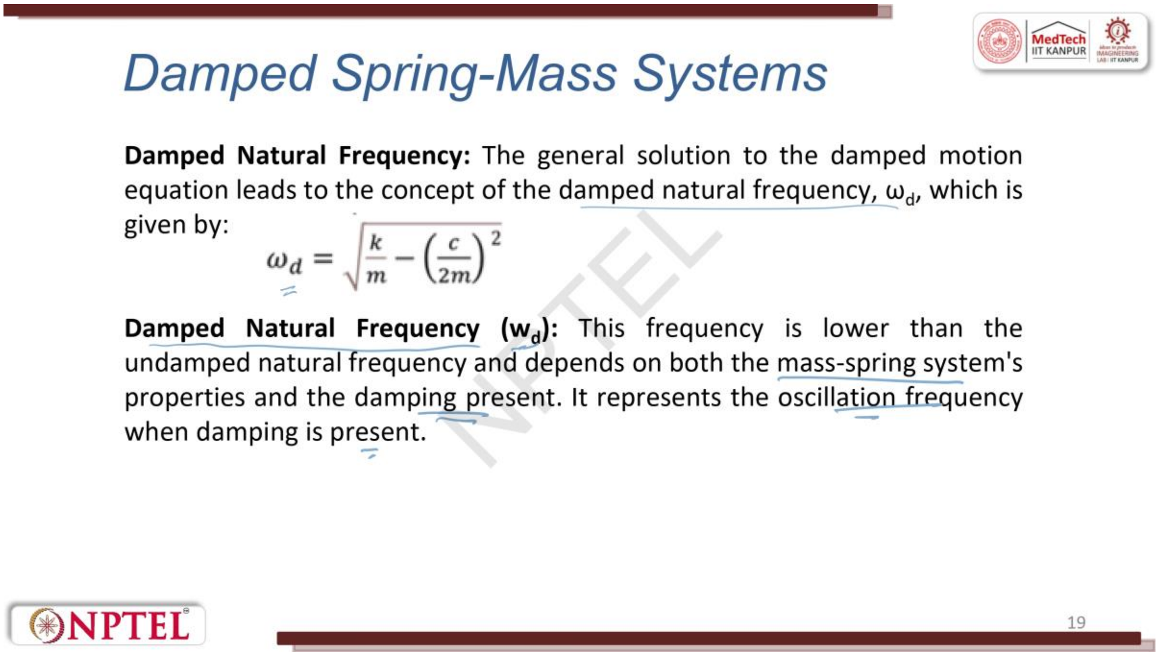


So, Overdamping, the system returns to equilibrium slowly without oscillating, it takes longer time than critical damping system. Now, let us look into the equation of motion. For damping spring mass system, the equation of motion is expressed as

$$m\ddot{x} + c\dot{x} + kx = 0$$

where m is the mass, c is the damping coefficient, k is the spring constant, x is displacement, x dot is velocity, x double dot is acceleration. So, you differentiate.

So, x differentiate with respect to t velocity, you expect differentiate with respect to one more time t is acceleration. So, the equation of motion for a damped spring mass system is expressed in this form.



So the Damped Natural Frequency is the general solution for the damped motion equation leads to the concept of damped natural frequency which is given as

$$\omega_d = \sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2}$$

This frequency is lower than the undamped natural frequency and depends on both the mass spring system property and the damping present. It represents the oscillation frequency when damping is presented. So, it is a very important concept, Damped Natural Frequency.

Numerical Problem

A spring-mass system has the parameters: Mass, m = 10 kg; Spring constant, k = 250 N/m; Damping coefficient, c=30 Ns/m.
Calculate the damped natural frequency and describe the motion.

Given $m = 10$ kg
$k = 250$ N/m
$C = 30$ Ns/m

Formula = $\omega_d = \sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2}$

$\sqrt{\frac{k}{m}} = \sqrt{\frac{250}{10}} = 5$ rad/s

$\frac{c}{2m} = \frac{30}{2 \times 10} = \frac{30}{20} = 1.5$ rad/sec

$\omega_d = \sqrt{(5)^2 - (1.5)^2} = \sqrt{22.75} = 4.77$ rad/sec

So, let us try to solve a problem to have a better understanding. A spring-mass system has the parameters: Mass, m = 10 kg; Spring constant, k = 250 N/m; Damping coefficient, c=30 Ns/m. Calculate the damped natural frequency and describe the motion.

Solution:

1. **Given:**

   Mass, $m = 10\,\mathrm{kg}$

   Spring constant, $k = 250\,\mathrm{N/m}$

   Damping coefficient, $c = 30\,\mathrm{Ns/m}$

2. **Formula for Damped Natural Frequency (ω_d):**

$$\omega_d = \sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2}$$

3. **Step-by-Step Calculation:**

First, calculate the undamped natural frequency (ω_n):

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{250}{10}} = \sqrt{25} = 5\,\mathrm{rad/s}$$

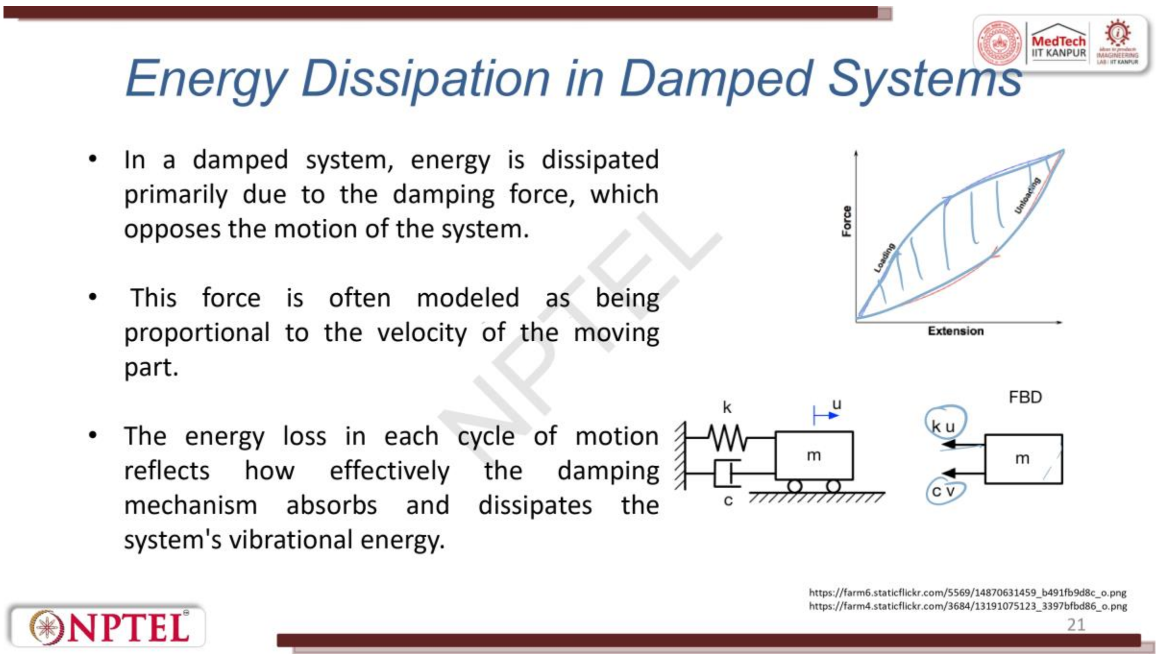
Next, calculate the term $\frac{c}{2m}$:

$$\frac{c}{2m} = \frac{30}{2 \times 10} = \frac{30}{20} = 1.5\,\mathrm{rad/s}$$

Now, calculate the damped natural frequency (ω_d):

$$\omega_d = \sqrt{5^2 - 1.5^2} = \sqrt{25 - 2.25} = \sqrt{22.75} \approx 4.77\,\mathrm{rad/s}$$

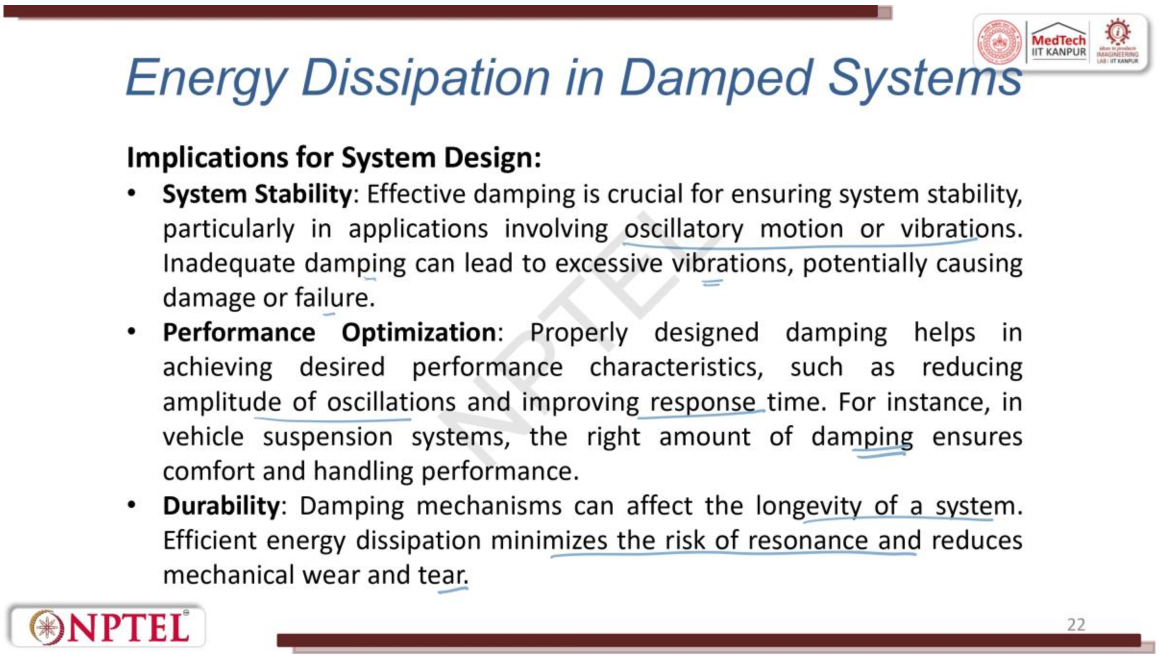
4. **Result:** The damped natural frequency of the system is approximately 4.77 rad/s.



Energy Dissipation in Damped System. So, when we try to plot force with respect to extension, so this will be a typical curve it follows during the loading cycle and when you are unloading it will follow this cycle. So, this is called as the Hysteresis loop and the energy inside is the absorption.

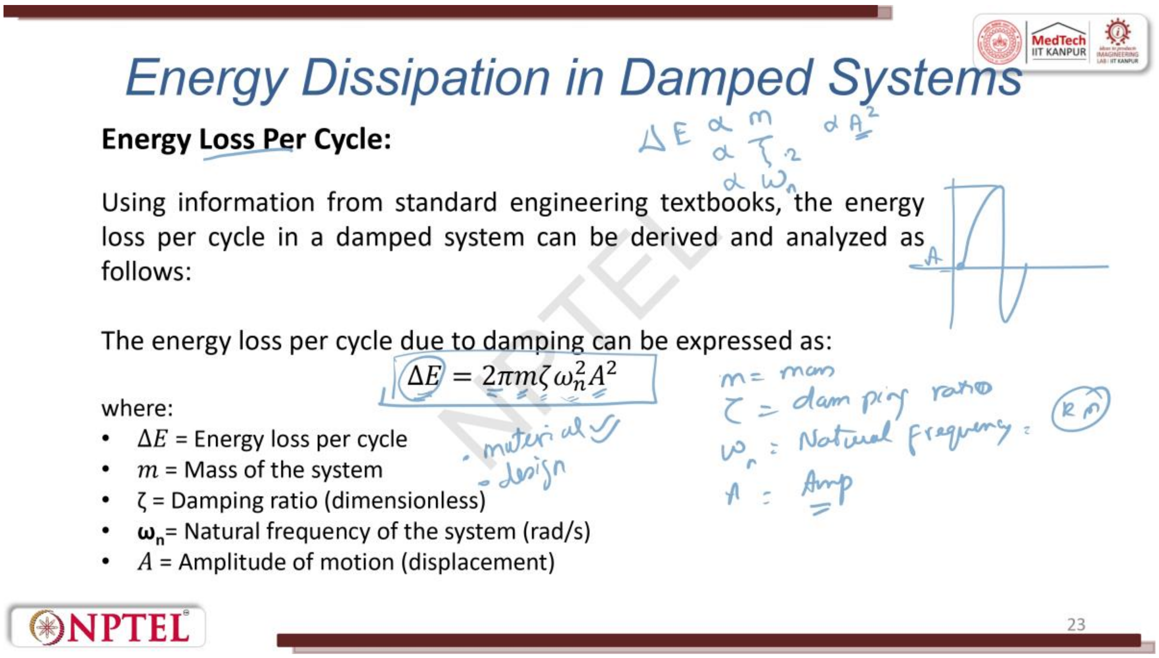
So, in a damping system, the energy is dissipated primarily due to the damping force which possesses the motion of the system. The force is often modeled as being proportional to the velocity of the moving part. The energy loss in each cycle of the motion reflects how effectively the damping mechanism absorbs and dissipates the system vibration energy.

So, this is the schematic diagram. It can be represented like this. You have a spring constant. You have a c damping factor. Then you have a mass which moves on a surface and U is the displacement. So, kU m is written kU and cv.



So the implications for systems design the system stability effective damping is crucial for ensuring system stability particularly in applications involving oscillation motion or vibration inadequate damping can lead to excessive vibration potential cause of failure the performance optimization. Properly designed damping helps in achieving desired performance characteristics, such as reducing the amplitude of oscillation, improving the response time. For instance, in vehicle suspension system, the right amount of damping ensures the comfort and handling performance. Durability.

The damping mechanism can affect the longevity of a system. The effective energy dissipates minimizes the risk of resonance and reduces the mechanical wear and tear. So the durability is also another important factor.



So how do you calculate the Energy Loss Per Cycle? Using information from the standard engineering textbooks, the energy loss per cycle in a damping system can be derived and analyzed as following.

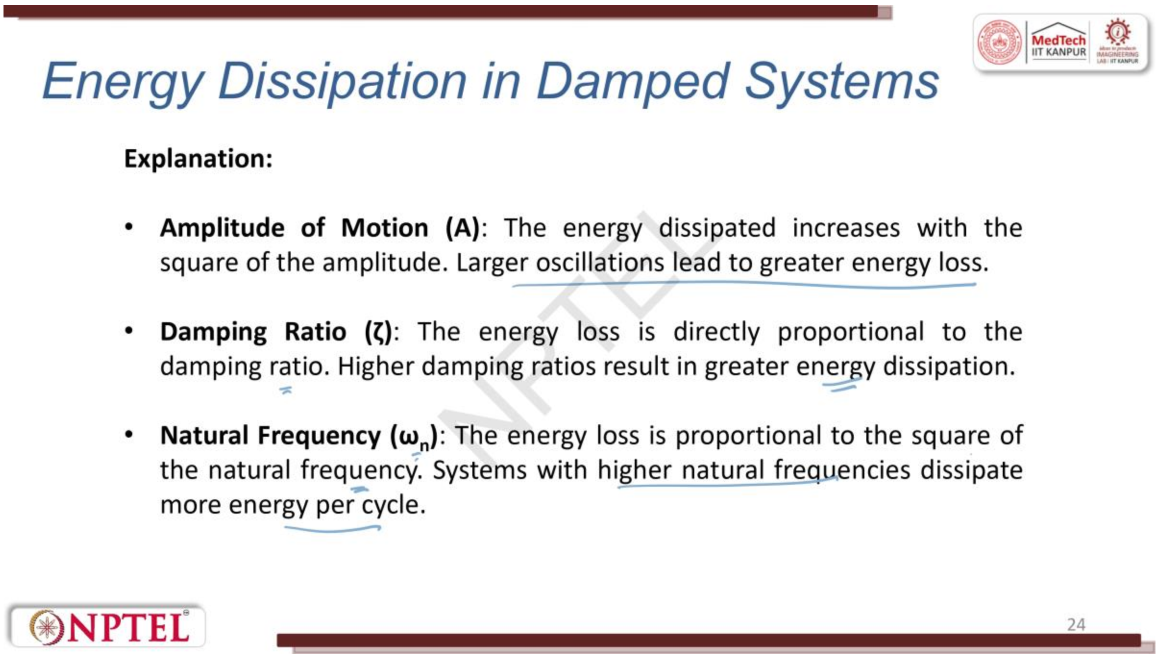
So, this is the energy loss per cycle due to damping equation.

$$\Delta E = 2\pi m \zeta \omega_n^2 A^2$$

where:

- ΔE = Energy loss per cycle
- m = Mass of the system
- ζ = Damping ratio (dimensionless)
- $\boldsymbol{\omega_n}$= Natural frequency of the system (rad/s)
- A = Amplitude of motion (displacement)

So, what all are very important one is amplitude is important A. So, that is what is given here then the next one is Natural Frequency will have k term and m term damping ratio. We have also seen that mass. So the energy loss per cycle due to damping can be expressed as delta E is depends on the mass, depends on the damping ratio, depends on the natural frequency, it also depends on the amplitude. So if you look at it, certain properties are material properties, certain properties are design properties. We always try to play with the material property to try to get the energy loss per cycle high.

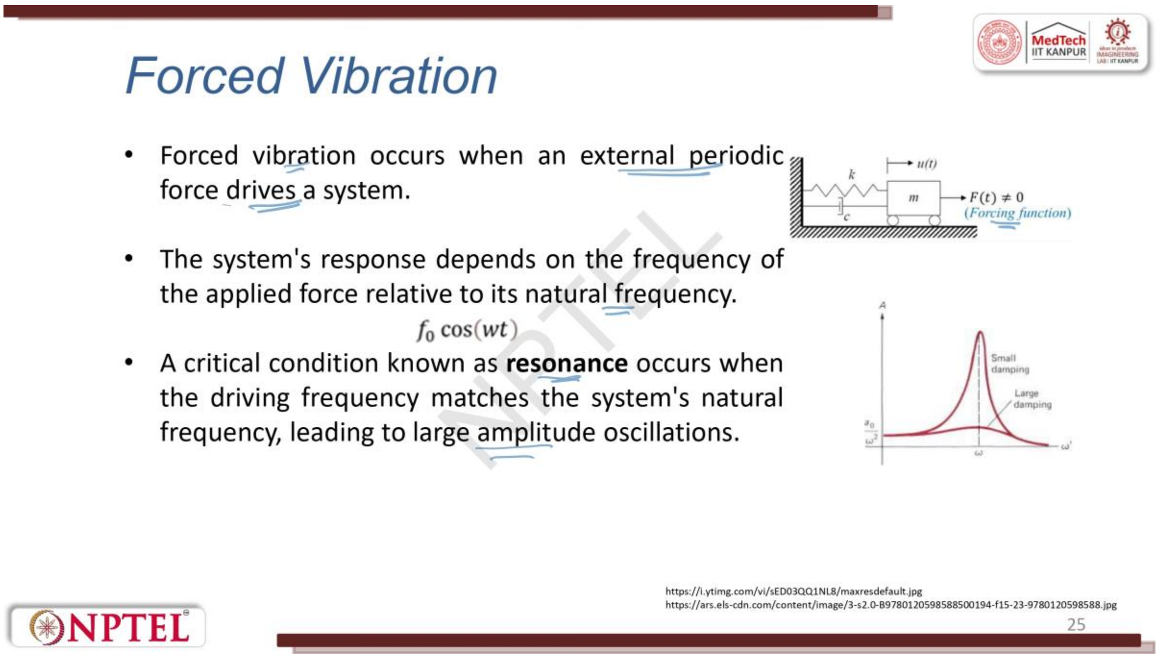


The Amplitude of Motion is A. The energy dissipated increases with the square of the amplitude. Larger oscillation leads to greater loss. Then damping factor, the energy loss is directly proportioned to the damping ratio. Higher the damping ratio results in greater energy dissipation. But if you see here, if you keep on increasing the damping ratio, the vehicle also becomes heavy.

So natural frequency (**ω_n**), the energy loss is proportioned to the square of the natural frequency. Why are we saying square? Right? So the energy loss is proportioned to the square of the natural frequency. The system with higher natural frequencies dissipate more energy per cycle.

So what are we trying to express? We are trying to say E is directly proportioned to mass, is directly proportioned to damping factor zeta, it is directly proportioned to square of the natural frequency. And it is also proportion to square of the amplitude. So, that is what we are trying to say. If the amplitude is high, then the large oscillations leads to greater energy loss.

Higher damping ratio results in greater energy dissipation. The system with higher natural frequency dissipates more energy per cycle. So, these are very important. Now, once we have understood the energy dissipation in damping systems.



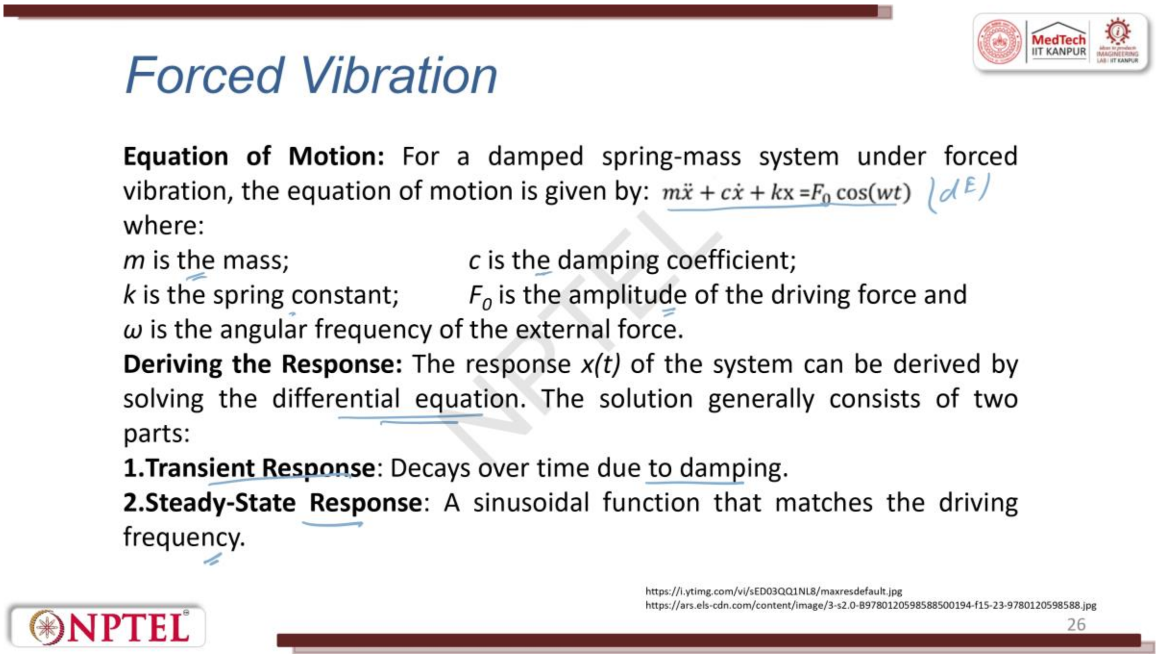
Now let us try to understand little bit more on Forced Vibration. Forced vibration occurs when an external periodic force drives a system.

It is called as Forced Vibration. The system's response depends on the frequency of applied force relative to its natural frequency.

$$f_0 \cos(wt)$$

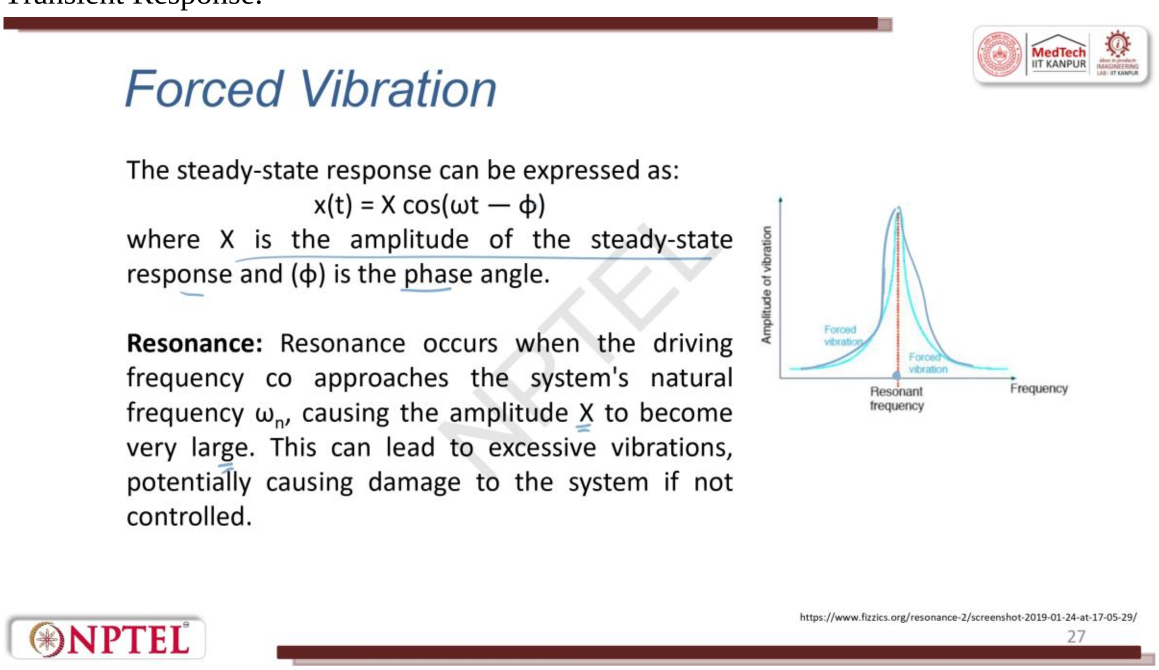
So here we are representing the amplitude with respect to omega. A critical condition known as resonance occurs when the driving frequency matches with the system frequency leading to a large amplitude oscillation.

So, this is resonance. So, if you look at the figure, there is $F(t) \neq 0$ a forcing function. So, we can represent it like this. This is called as Forced Vibration. So, here if you see the plot $\frac{a_0}{\omega^2}$ we will see a small damping and a large damping.



So, equation of motion for a damped spring mass system under forced vibration, the equation of motion is going to be $m\ddot{x} + c\dot{x} + kx = F_0 \cos(wt)$

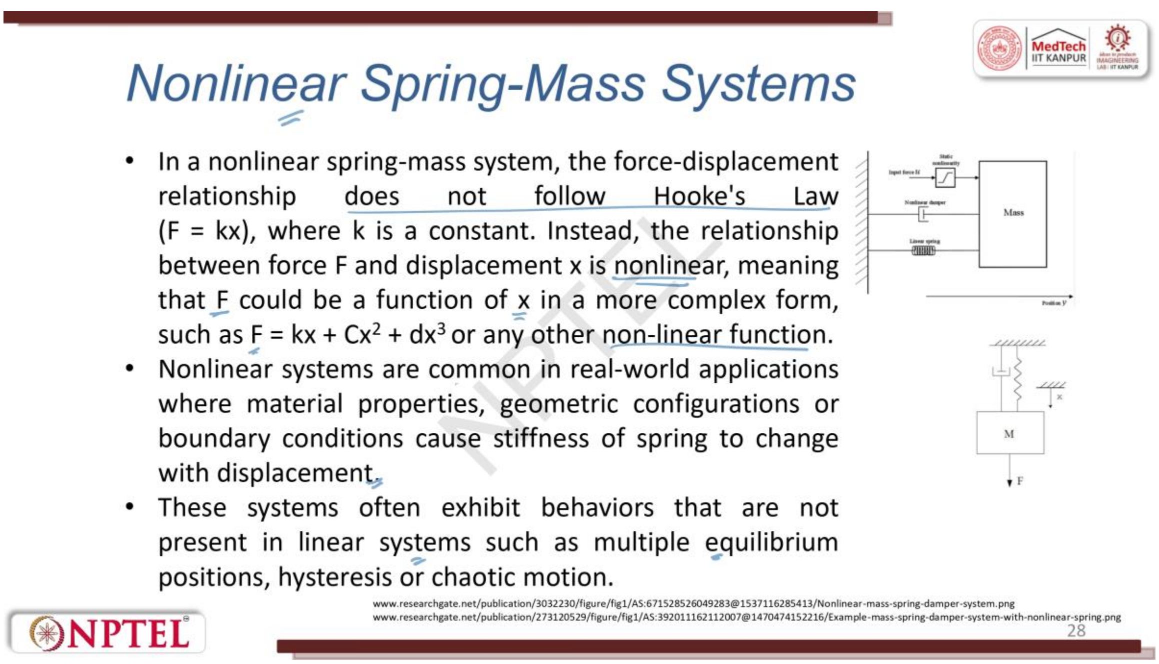
The m is the mass, c is the damping coefficient, k is the spring constant, f0 is the amplitude of the driving force and omega is the angular frequency of the external force. Deriving the response, the response x of t for the system can be derived by solving the differential equation. The solution generally occurs in two parts. First part we call it as Transient Response.



The second part we call it as Steady State Response. We try to solve the differential equation DE. This is why we study differential equation exhaustively in engineering. DE will try to give two parts, transient response which tries to talk about decay over time due to damping and the other one is steady state response, a sinusoidal function that matches the driving frequency. So we will try to get both these values, transient response and then steady state response, both we get to solve in this forced vibration.

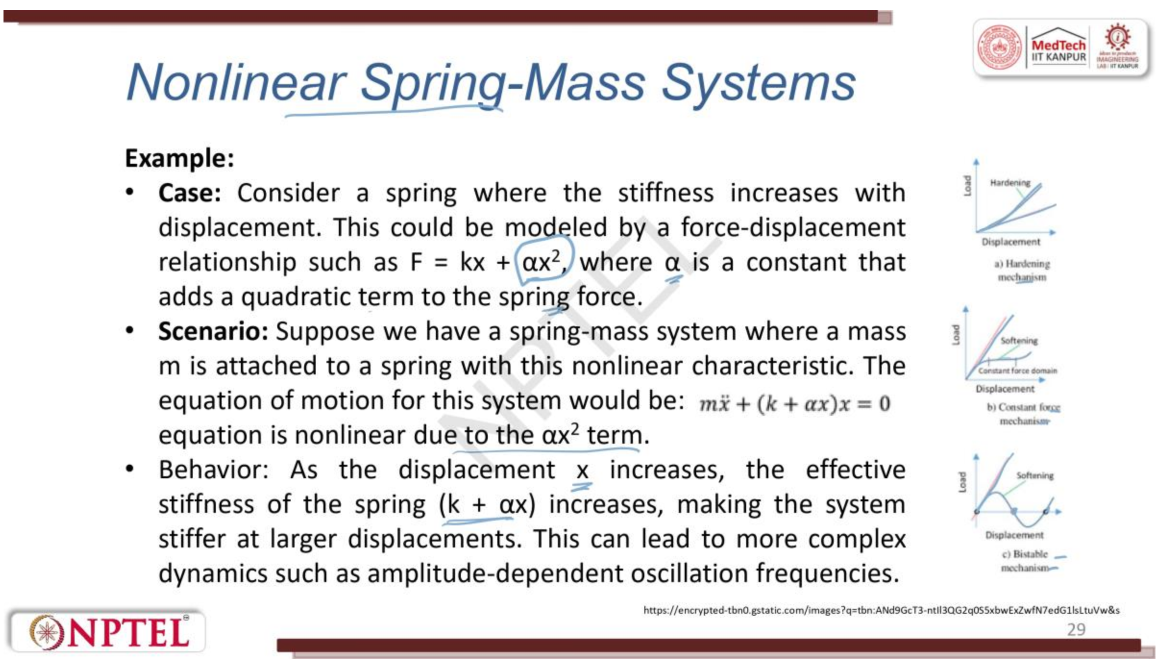
A steady state response can be expressed as $x(t) = X \cos(\omega t - \varphi)$. X is the amplitude of the steady state response and phi is the phase angle. So, you have amplitude of vibration, forced vibration you see here. This is how the response is. This is the resonant frequency.

We try to shift it or we try to reduce it. Both we can try to do. So this is the amplitude of forced vibration response. The resonance occurs when the driving frequency approaches the system's natural frequency omega n, causing the amplitude x to become very large. This leads to excessive vibration and premature failure. When we talk about forced vibration, we were trying to solve the forced vibration for a linear system.



Now, let us try to understand Nonlinear Spring Mass system. In a nonlinear spring mass system, the force displacement relationship does not follow Hooke's law. We always generalize it so that we understand the system in a simplified manner. But in reality, it is never linear and it also never follows Hooke's law where k is the constant and instead the relationship between the force F and the displacement is nonlinear, meaning F is a function of x in a more complex form, such as $F = kx + Cx^2 + dx^3$.

Any other nonlinear function, this is one type of nonlinear function. You can have any other type of nonlinear functions. The nonlinear systems are common in real world application where material property, geometry configuration or the boundary conditions cause stiffness of a spring to change with displacement, nonlinear is reality. These systems often exhibits behavior that are not present in the linear system such as multiple equilibrium position or hysteresis or chaotic motions.

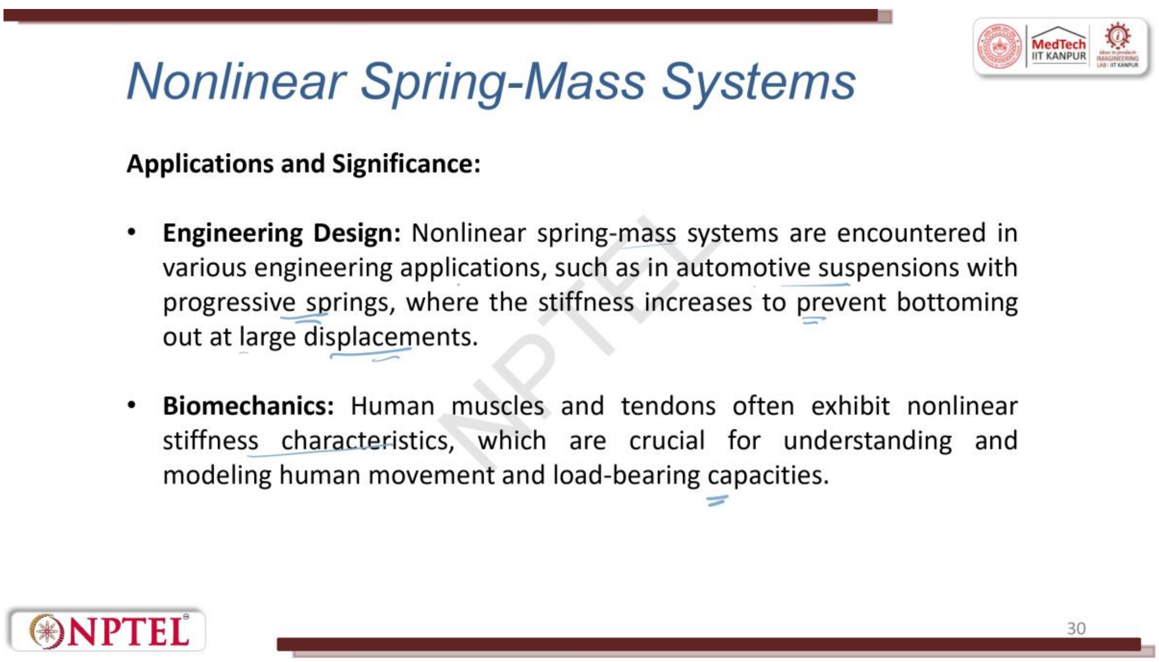


Example of nonlinear spring mass systems. Case: Consider a spring with a stiffness increases with displacement. This could be modeled by a force displacement relationship such as F equal to kx plus alpha x square, where alpha is a constant that adds to quadratic term to the spring force. Look at it. This term scenario suppose we have a spring mass system where the mass m is attached to a spring with this nonlinear characteristics, the equation of motion of the system would be m double dot plus k plus alpha x into x equal to 0 the equation is nonlinear due to is alpha x square term. As the displacement x increases, the effective stiffness of the spring k plus alpha x increases, making the system stiffer at large displacements.

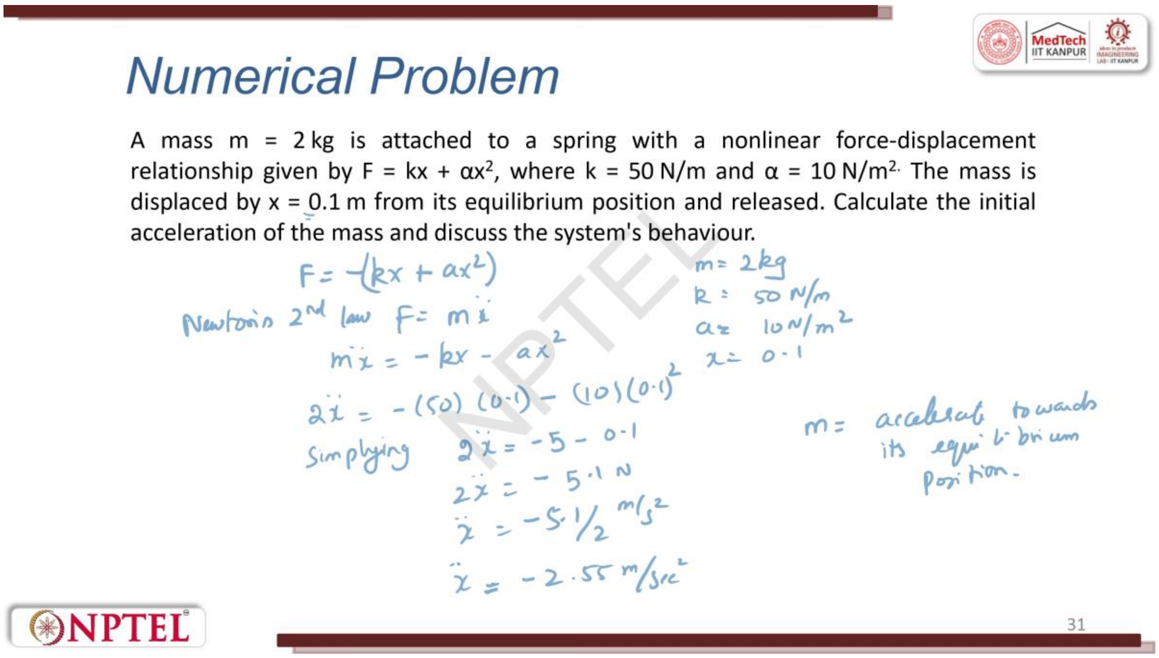
This can lead to more complex dynamics such as amplitude dependent oscillation frequency. So, you can see here this is the load versus displacement, this is what is a spring constant, this happens due to a phenomena called as hardening and then this is the linear form, this is something called as softening. So, here you have constant force domain. This is hardening mechanism. This is constant force domain.

This is bistable domain. So, you have this is the thing and you have a softening one. So, you have one system and other system. It is bistable domain. Mechanism is used.

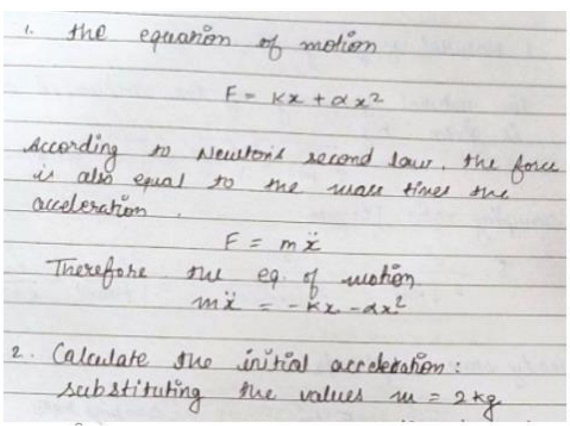
So, now you see you can start playing more and more with the nonlinear spring mass system to get better oscillation at a lower frequency, higher frequency, band pass frequency you can try to get everything through mechanical systems.

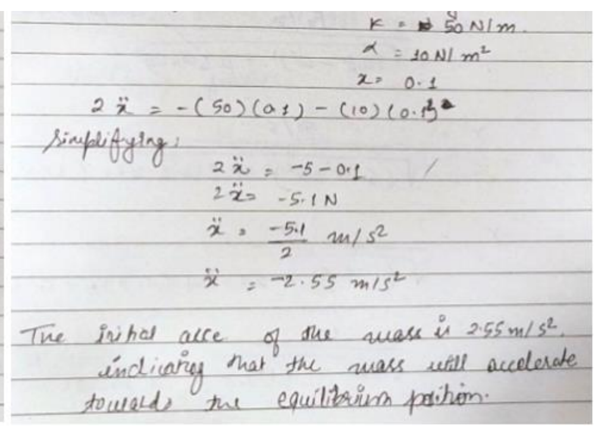


So, the application and significance engineering design nonlinear spring mass systems are encountered in various engineering applications such as automobile suspension with progressive springs where the spring increases to prevent the bottoming out at the large displacement. In biomechanics, when you are trying to mimic a muscle or when you are trying to make a biped robo, human muscles and tendons often exhibit nonlinear stiffness characteristics, which are crucial for understanding and modeling of human movement and load-bearing capacity.



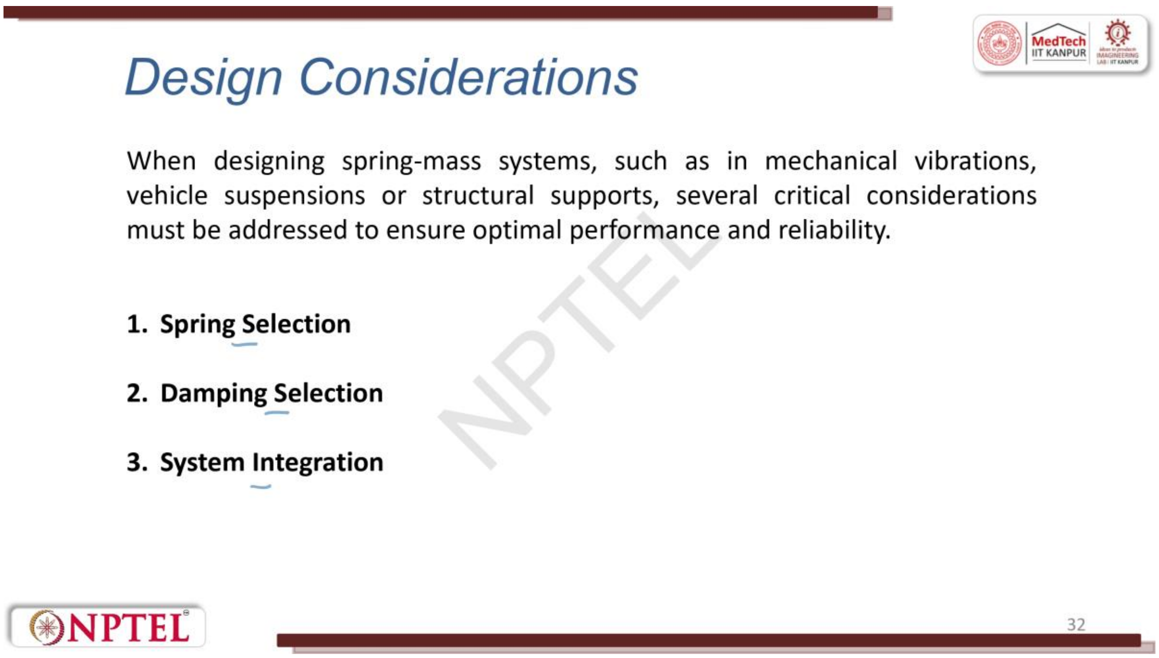
Let us work on a simple problem to understand the concept whatever we have studied. A mass m = 2 kg is attached to a spring with a nonlinear force-displacement relationship given by $F = kx + \alpha x^2$, where k = 50 N/m and α = 10 N/m^2. The mass is displaced by x = 0.1 m from its equilibrium position and released. Calculate the initial acceleration of the mass and discuss the system's behaviour.



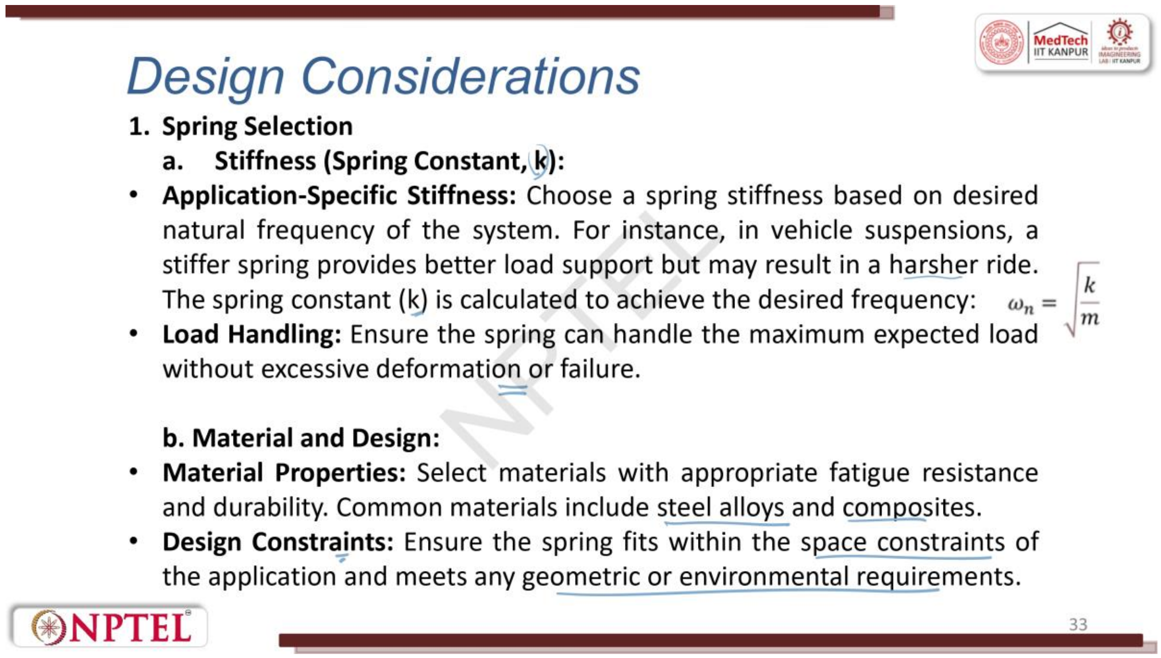


k equal to 50 Newton meter or a equal to 10 Newton meter square, x equal to 0.1. So, now 2x dot is equal to minus 50 to 0.1 minus 10 into 0.1 the whole square. So, simplifying it, what we get is 2x double dot equal to minus 5 minus 0.1. So, 2 double dot is equal to minus 0.1 So, x double dot is equal to minus 5.1 divided by 2 meter per second square.

So, x double dot is finally equal to minus 2.55 meter per second square. So, the initial acceleration of the mass is 2.55 meter per second square where the mass will accelerate towards the equilibrium position. So, here the mass will accelerate towards its equilibrium position.



So, what are all the design considerations we should have while trying to look into spring mass system? When designing a spring mass system such as a mechanical vibration, vehicle suspension or structural support, several critical conditions must be addressed. Spring selection, Damping selection and System integration.



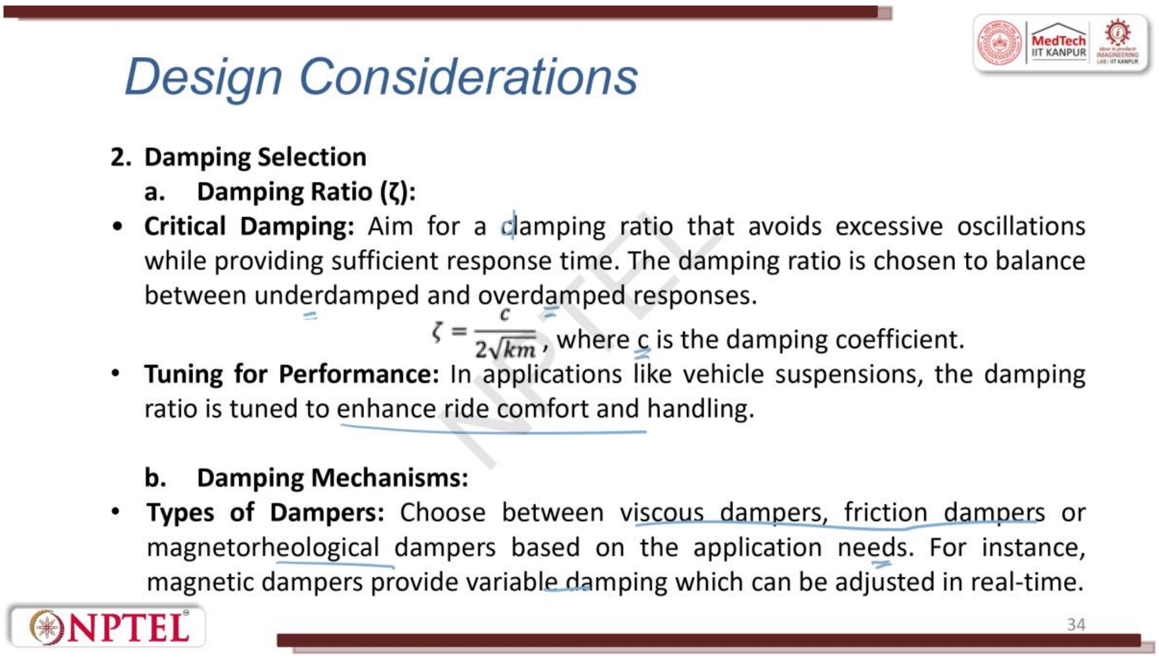
So, under Spring Selection, you have Stiffness which is a spring constant K. The application and specific stiffness. Choose a spring stiffness based on desired natural frequency of the system. For instance, in vehicle suspension, a stiffer spring provides better load support but may result in a harsher drive or a ride.

The spring constant k is calculated by achieving the desired frequency

$$\omega_n = \sqrt{\frac{k}{m}}$$

Load handling ensures the spring can handle the maximum accepted load without excessive deformation or failure. So, k plays a very important role. Next is Material and Design. Material properties such as select materials with appropriate fatigue resistance and durability. Polymer, elastomer can be thought of or in metals you can think of steel alloys and composites.

Design consideration ensures the spring fits within the space constraints and application and meets any geometric and environmental requirement or design constraints.



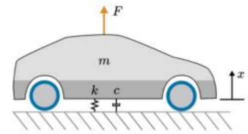
Damping selection, Damping Ratio, critical damping aims for a damping ratio that avoids excessive oscillation while providing sufficient response time. The damping ratio is chosen to balance between under damping and over damping where

$$\zeta = \frac{c}{2\sqrt{km}}$$

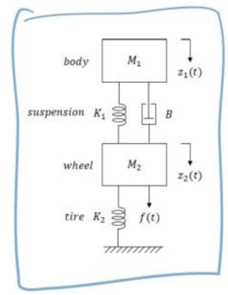
where c is the damping coefficient. Tuning for performance is also one of the most important in design consideration.

Case Studies

1. Vehicle Suspension Systems:



- **Application:** In automotive design, suspension systems are engineered to provide a balance between ride comfort and handling.
- **Design Considerations:** Engineers select springs and dampers to achieve an optimal damping ratio and natural frequency that suits different driving conditions and vehicle loads. For example, high-performance sports cars use stiffer springs and tuned dampers to enhance handling, while luxury vehicles use softer springs and dampers to improve ride comfort.



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In application like vehicle suspension, the damping ratio is tuned to enhance the ride comfort and handling.

So damping mechanisms are also part of damping selection. Types of Dampers. Choose between viscous damper, friction damper and magnetorheological damper based upon the application.

Viscous dampers, friction dampers are predominantly spring system, spring wherein which there is a contact between the two surface is friction damper. Viscous damper is you have an oil which tries to damp and then magnetorheological is you apply a magnetic field and try to stiffen the oil.

For instance, magnetic dampers provide variable damping can be adjusted in real time depending upon the magnetic field you apply.

Design Considerations

3. System Integration

a. Frequency Response:

- **Avoid Resonance:** Ensure the natural frequency of the system does not match the frequency of external excitations to avoid resonance. This requires careful selection of spring constants and damping parameters.

b. Environmental Factors:

- **Temperature and Corrosion:** Consider environmental conditions that may affect the performance of springs and dampers, such as temperature extremes and exposure to corrosive substances.

System Integration. System Integration depends on two things. One is frequency response which is avoid resonance. Ensure the natural frequency of the system does not match with the system of external excitation to avoid resonance.

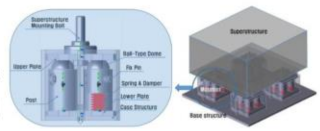
Environmental factors such as temperature and corrosion also plays a very important role while considering it for design. The temperature and corrosion considers environmental conditions which may affect the performance of a spring or a damper.

Let us take a case study which is Vehicle Suspension System. So, application in automotive design suspension systems are engineered to provide balance between comfort and handling. So, this is a schematic which is expressed and this is the block diagram which is expressed a body of mass m suspension, then you have a damping, then you have a wheel, then you have a tyre, then you have a function t which is expressed.

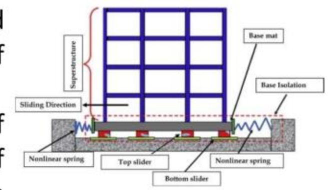
So, engineers select springs and dampers to achieve an optimum damping ratio and the natural frequency that suits different driving conditions and vehicle loads. For example, higher performance sports cars use stiffer springs and tuned dampers to enhance handling, while luxury car use soft springs and damp to improve comfort riding.

Case Studies

2. Building Seismic Isolation:



- **Application:** In earthquake-prone areas, buildings are designed with base isolators to absorb and dissipate seismic energy.
- **Design Considerations:** Base isolators use spring-mass systems with carefully chosen stiffness and damping characteristics to minimize the impact of ground motion on the building.
- The design ensures that the natural frequency of the isolators is lower than the frequencies of expected seismic activities, preventing resonance and protecting the structure from damage.



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When we talk about buildings, Seismic Isolation, you can see this is the superstructure of a building and here they try to keep at the base some dampers. It can be pad or it can be even rollers, which roll and then dissipate energy. In earthquake prone areas, buildings are designed with a base isolator to absorb and dissipate seismic energy.

Design consideration, base isolators use spring damp system which carefully choose the stiffness and damping characteristics to minimize the impact of ground motion to the building. The design ensures the natural frequency of the oscillator is lower than the frequency of the expected seismic activity preventing resonance and protecting the structure from getting damaged.

To Recapitulate

- What is the spring constant and how does it influence the behavior of a spring-mass system?
- Explain Hooke's Law and how it applies to a spring-mass system.
- What is the natural frequency of a spring-mass system and how is it calculated?
- What is damping in a spring-mass system and why is it important?
- Define the damping ratio and explain its role in determining the behavior of a damped system.
- What is the equation of motion for a damped spring-mass system?
- How would you calculate the steady-state amplitude of a spring-mass system under harmonic excitation?
- What challenges arise in analyzing nonlinear spring-mass systems?

Friends, in this lecture, we saw what is spring constant, how does it influence the behavior of a spring mass system. Hooke's law, how is it applied to spring mass system. What is natural frequency of a system, how is that calculated.

What is damping in a spring mass system and why is it so important. Define the damping ratio and explain its importance. Role in determining the behavior of the damping system. Then equation of motion for a damped spring mass system was discussed. Then finally we tried to calculate a steady state amplitude of a spring mass system under harmonic. Nonlinear spring mass system was also discussed very briefly.

Thank you very much.