Basics of Mechanical Engineering-1

Prof. J. Ramkumar

Dr. Amandeep Singh

Department of Mechanical Engineering

Indian Institute of Technology, Kanpur

Week 10

Lecture 42

Gears: Basic Concept

Next topic of discussion in the course is going to be Gears. Gears, next to springs, plays a very important role in mechanical systems. Today, in the era of electronics, of course, gear is taking a back step in all the domestic appliances. It is all getting replaced by Microelectromechanical systems. But gear still plays an important role when we talk about high torque, high force transmission.

Wherever you wanted to have a positive drive, whatever I give, it has to be transferred. So, the other way around is gears are like multiplying factors or dividing factors which you can get it done in a mechanical system. Say for example, in a calculator 2 into 3, 6, you multiply. Hydraulic system also you can multiply and reduce. Gears are also in the same way.

You can amplify the speed, amplify the torque and amplify the force whatever it can lift. So gears play a very important role. When I say Gears, there are different types of gears. Spur gear, Helical gear, right. You have Herringbone gear.

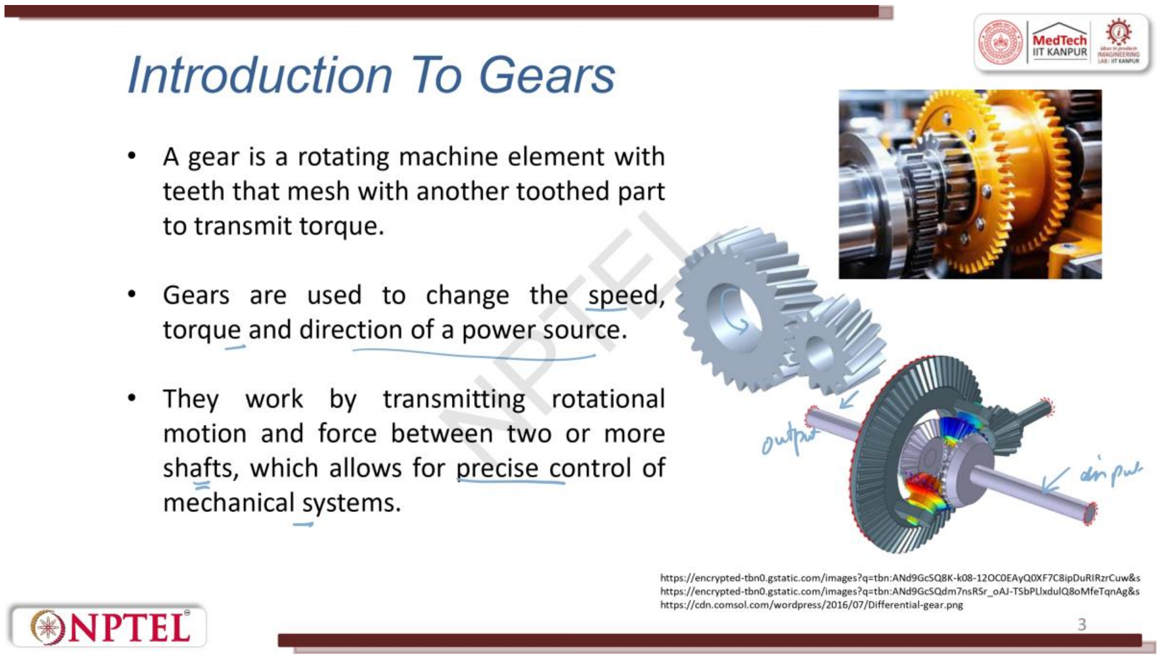
So many different types of gear. Bevel gear, Worm and Worm wheel. So, so many gears are there which will try to give you a multiple output. So understanding gear is very important. From the ancient days to the latest, the advanced dates or to the technology days, gear plays a very important role.

As I gave you an example of opening a door for a temple, opening a door for a cave, opening a door for a kingdom, gears were also used. Bevel gears were used way back 20,000 years ago. So gears is very important.

Contents

- Introduction To Gears
- Gear Terminology
- Conjugate Action in Gears
- Gear Classification
- Gear Kinematics
- Gear Trains
- Derivation - Gear Ratio
- Forces on Gear Teeth
- Stress Analysis in Gears
- To Recapitulate

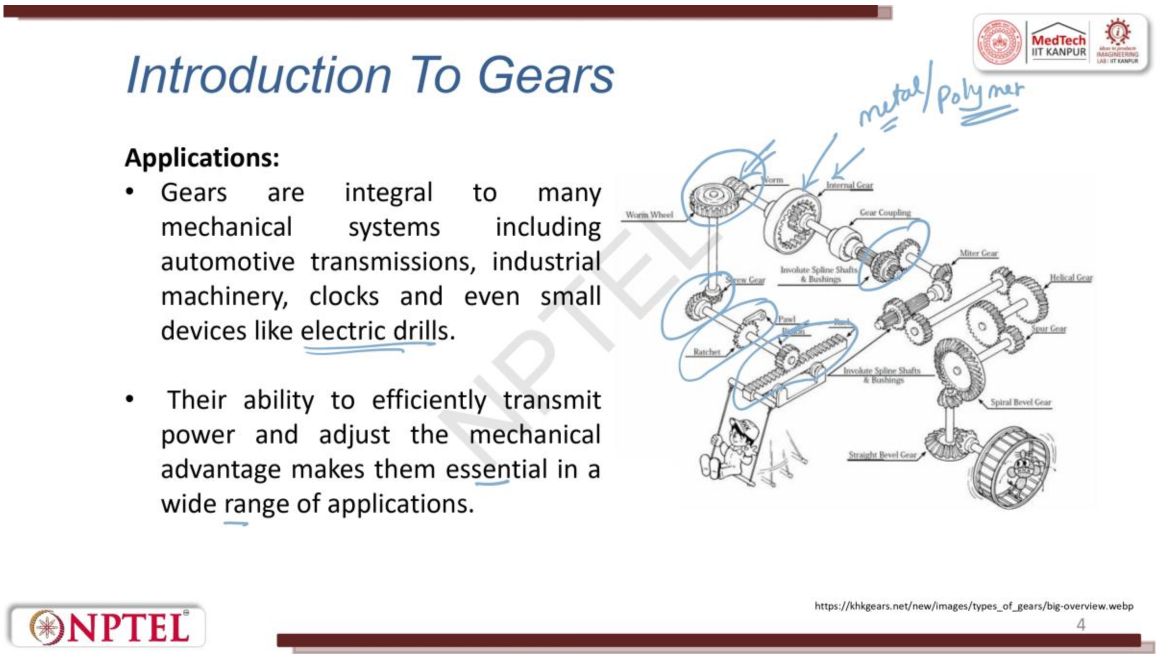
Getting into the content, we will try to see what is gear, few of the Gear Terminologies, Conjugate Actions in Gears, Gear Classification, Gear Kinematics, Gear Train Terminology, Gear Ratio, Force on Gear Teeth, Stress Analysis in Gear and finally we will try to have a recap.



Introduction to Gear. If you look at all the photos here, these are all different types of gears. You can see gears, the gear teeth is along the axis of rotation. It is at an angle inclined to rotation. Here you will see two gears which are trying to mesh. This is an input.

This is an output. So you can try to have multiple factor multiplication or you can try to reduce whatever you want. So all these things are gears. A gear is a rotating machine element with teeths that mesh with another toothed part to transmit torque. The teeth can be inside, it can happen here or it can be on the periphery.

The mating partner should also have gear teeth. Gears are used to change the speed torque and the direction of the power source. So here it is almost like that. They work by transmitting rotation motion and force between two or more shafts which allows for precision control of mechanical systems in Swiss watch where the time used to be very minutely controlled has lot of gears. One of the engineering marv in mechanical engineering domain is watch manufacturing. Very precise watch manufacturing which has happened 150 years ago.



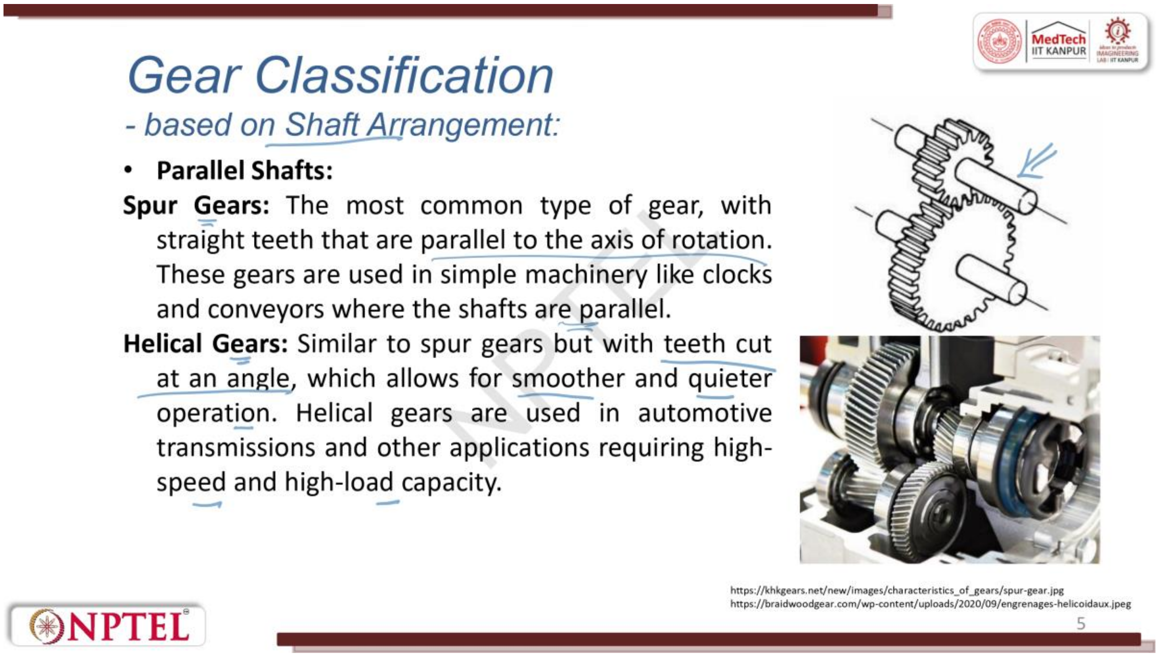
Applications. Gears are integral to many mechanical systems including automobile, automotive transmission, industrial machinery, clocks and even small devices like electric drills. This is what is an example I was trying to talk about, internal gear. So, inside you have a teeth which is made and then you have a pinion which goes around it. So, this will try to rotate around the meshing the teeth, it will go around.

So, this is worm and worm gear. So, wherein which you have a worm and this is the worm wheel. So, here you will have teeths. This will have a similar profile wherein which this and this tries to mate with each other. You can have external gears.

You can have rack and pinion. So wherein which this is a rack which is for straight line motion. Pinion is for rotatory motion. You can have Paul and Ratched mechanism. We saw it in the previous lectures.

Screw gears are also there. Almost all different types of gears are shown here in this figure. Their ability to efficiently transmit power and adjust the mechanical advantage makes them essential in a wide range of applications. These gear material can be made out of metal or polymer. All the toys, they use plastic gears because the loads are not very heavy.

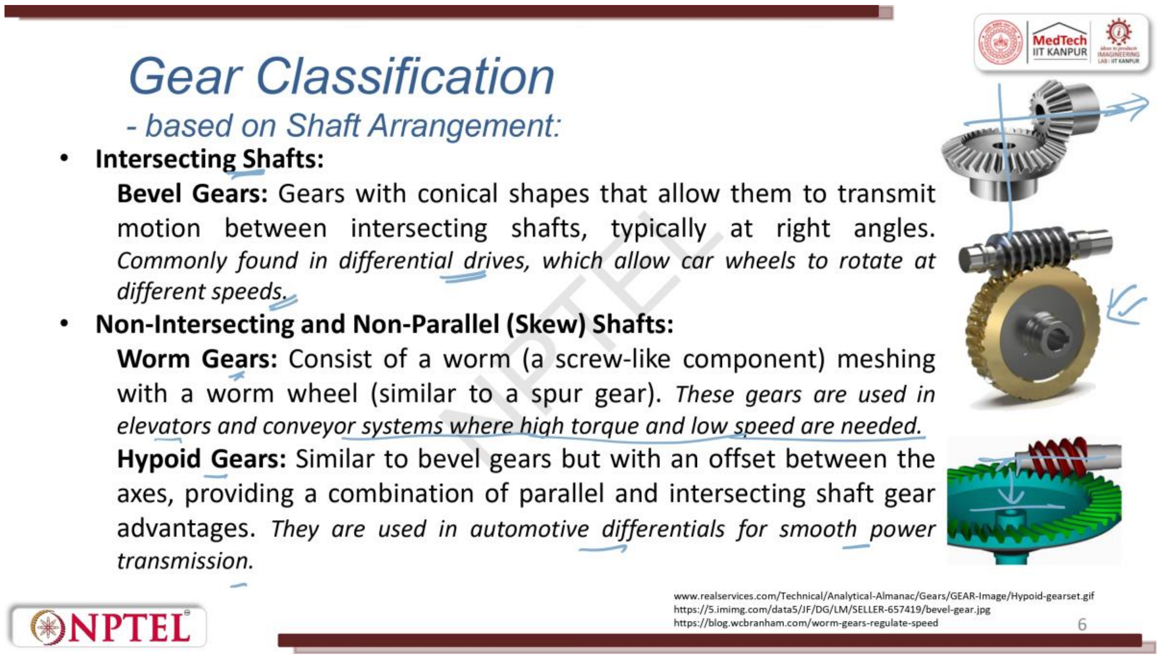
In automobile industry or automotive industry or in machine tool, we always use metal gears. Again, the material for metal can be mild steel, can be stainless steel, can be copper, can be nickel, can be brass. You can use anything of your choice depending upon your requirements.



So Gear Classification based on shaft arrangement. What is shaft? This is a shaft and on a shaft you press fit and keep a gear or the shaft and gear are integral part of the same part. Both are possible. You can, depending upon the diameter of the gear, so it can be used for multiplication or division. That means to say, for one rotation, it can rotate five times. Both are possible.

So, when you have Parallel Shafts, the gear what we use is Spur gear. The most common type of gear with straight teeths that are parallel to the axis of rotation. This is very easy to manufacture and it also has proper meshing. Thus gears are used in simple machinery like clocks, conveyors where the shafts are parallel. Similar to Spur gear, we will have Helical gear.

The only difference between Spur and Helical is the teeth cut are at an angle. So what is the difference? Why we have to go for this? Here when it runs, it is a smoother and a quieter run will be achieved by helical gear and more contact area is there, so loads can be heavy as compared to that of the Spur gear. Helical gears are used in automotive transmission and other applications requiring high speed and high load capacity.



Intersecting Shafts, Bevel gear. Bevel gear, with conical shape that allows them to transmit motion between intersecting shafts. Typically, at right angles, commonly used in differential drives, which allow car vehicles to rotate at differential speeds. Differential will be used in trucks at the rear end, where you will have wheels, the tyre is attached to this and then you have a gear which tries to convey from the engine to the tyre. So differential drives are possible.

You also have something non-intersecting and non-parallel shafts. The first one what we saw was parallel shafts. Then we saw is intersecting shafts. The third one is non-intersecting and non-parallel shafts. For example, it is worm gear.

This is the worm gear example. So, consists of a worm, a screw like component meshing with a worm wheel. This is a wheel, this is worm. Similar to that of a Spur gear. These gears are used in elevators and conveyor systems where high torque and low speeds are needed.

High torque and low speeds. For example, you want to make a turntable. Turntable also you can try to make it using worm and worm gear. What is turntable? It will be a table.

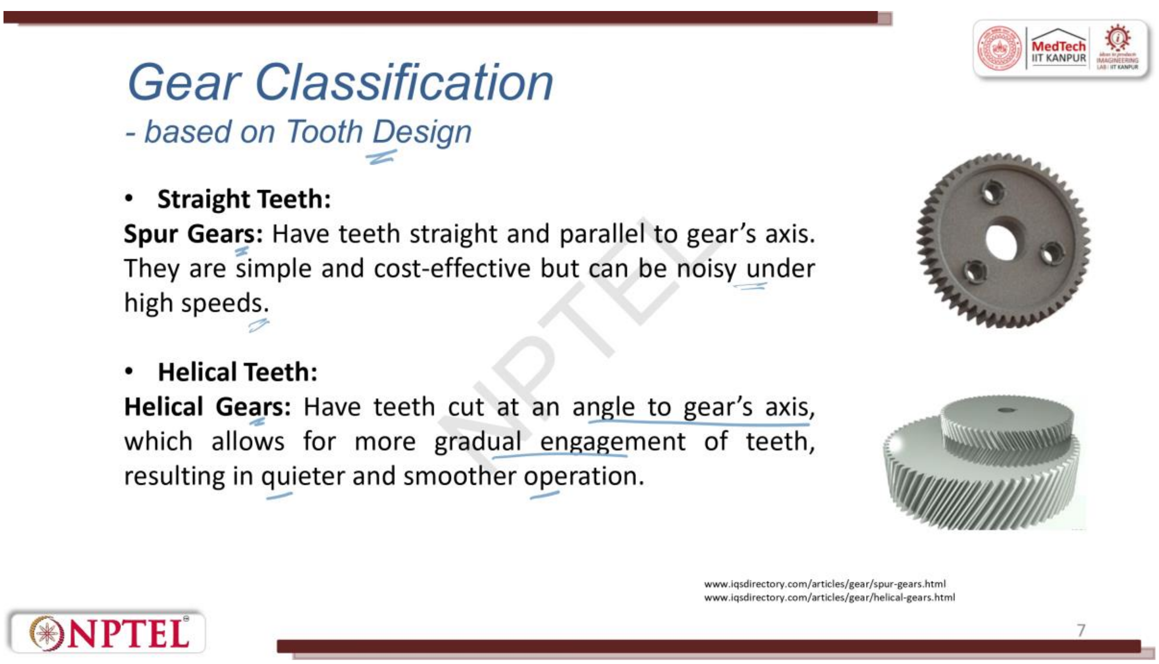
Okay, let me give you an example. When the engine comes from one direction, the engine direction has to be changed. What we do is we move the engine into a turntable.

We rotate the engine in 180 degrees and then move it back. The place where it happens is called as a turntable.

A turntable is always used to rotate using a worm and worm gear. Or you can also try to have a sliding but worm and worm gear is the one which is exhaustively used. Hypoid gear. These gears are similar to bevel gear in the top but with an offset between the axis. So this axis and this axis there is an offset.

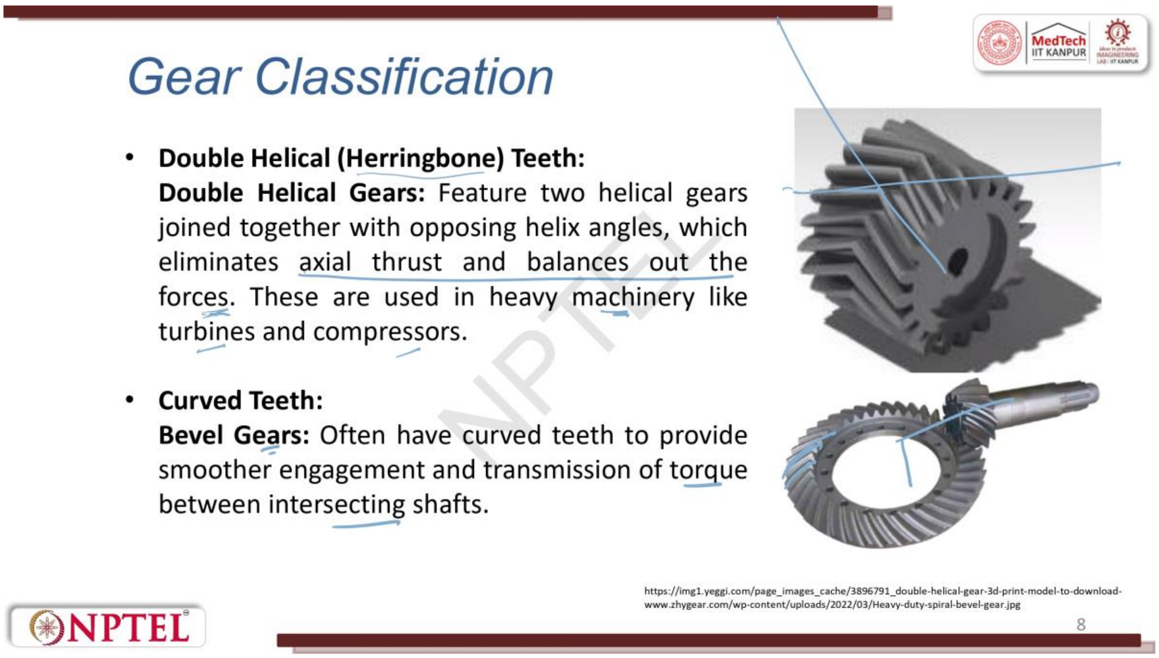
Providing a combination of parallel and intersecting shaft advantage. They are used in automobile and for smooth power transmission. If you see almost all the gears will be used in automobile. Spur gear, Helical gear, Bevel gear, Worm gear and High Paddle gears. Everywhere everything is used.

The only difference is what is the direction, what is the power and what is the speed. Depending upon that it can do. The axis from where the drive is given can be parallel, perpendicular, it can be intersecting, non-intersecting.



So based upon the tooth design classification, Straight teeth. Straight teeth are always found in Spur gear, have teeth straight and parallel to gear axis.

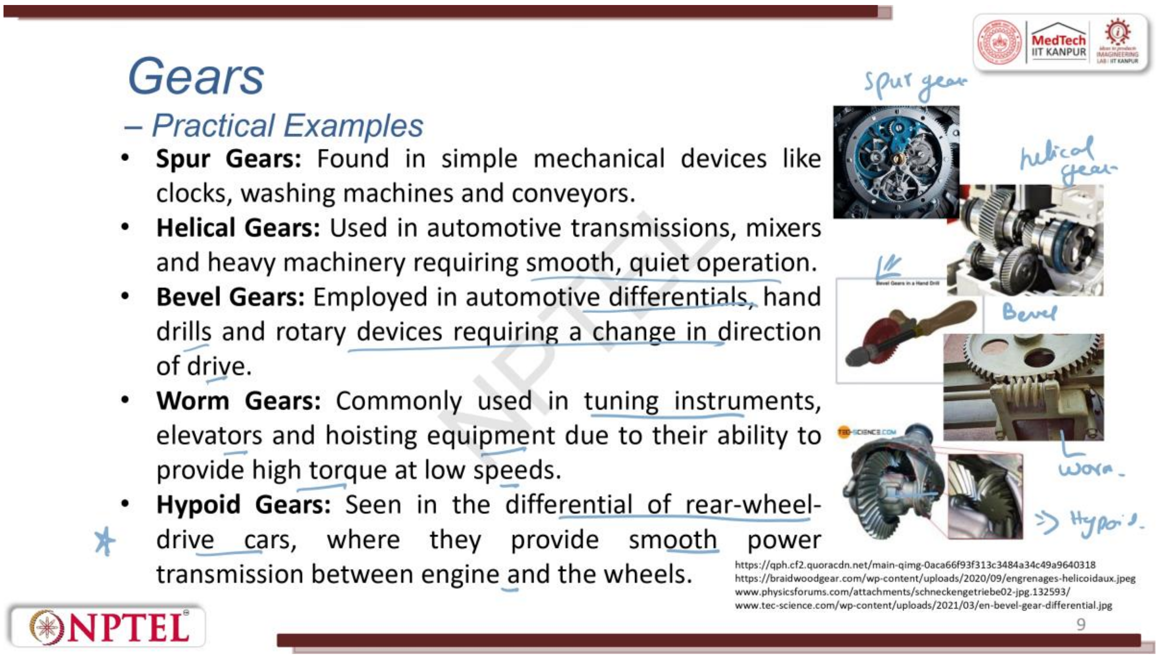
They are simple and cost effective but can be noisy under high speed. In order to avoid the noise, we always go for helical gear. The gear teeth are cut at an angle to the gear axis, which allow for more gradual engagement of teeth, so that it results in quieter and smoother operations.



You have Double Helical gear or teeth. Double Helical teeth is called as Herringbone. Double Helical gear. Features two helical axes. So you can see one and two. This is one and this is two. So two helical gear joins together with opposite helical angles which eliminates axial thrust and balances out the force.

It eliminates axial thrust and balances out the force. These are used in heavy machinery like turbines and compressor. You can also have something called as Curved teeth. The teeth are all curved. So the example is bevel gear.

Often have curved teeth to provide smoother engagement and transmission of torque between the intersecting shafts.



So what are the practical applications? Spur gear found in simple mechanical device like clock, washing machine, conveyors. Helical are used in automobile transmission, mixture and heavy machinery requiring smooth and quiet operation. Bevel gear employed in automobile differentials, hand drilling and rotating device requiring change in direction of drive.

Worm gear, it is used in tuning instruments, elevators and hoisting equipments due to the ability of high torque and low speed. Ellipsoidal gear, seen in differential of rear wheel drive cars where they provide smoother power transmission between the engine and the wheel. So this will be rare hypoid gear seen in differential of rear wheel drive cars. There are front wheel drive cars, rear wheel drive cars. In rear wheel drive cars, hypoid gears are used where they provide smoother power transmission between the engine and the wheels.

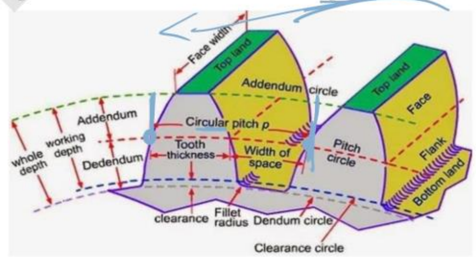
If you look at it, this is a hand drilling machine where in which you use a bevel machine. This is a gear train. It is used in lathe machine for changing the speeds. It is inside a gearbox, Helical gear. Inside a watch, Spur gear.

This is worm. And this is hypoid. Various examples are given. If you see here, you see the axis, you see the inclined angle of the teeth, it will be more clear.

Gear Terminology

The terminology of spur-gear teeth is illustrated in Figure:

- The pitch circle is a theoretical circle upon which all calculations are usually based upon; its diameter is the pitch diameter.
- The pitch circles of a pair of mating gears are tangent to each other. A pinion is the smaller of two mating gears. The larger one is often called the gear.
- The circular pitch **p** is the distance measured on pitch circle, from a point on one tooth to a corresponding point on an adjacent tooth.
- Thus the circular pitch is equal to the sum of tooth thickness and the width of space.



https://mechtics.com/machine/theory-of-machines/gear-terminology-with-defination/

Now let us understand some of the Gear Terminologies. In gear, there are several terminologies. So we will try to cover one after the other. The first one is called as the pitch circle. Pitch circle is the theoretical circle upon which all calculations are usually based on. Its diameter is called as the pitch diameter.

So, this if you see here, pitch circle is written. Pitch circle. You see a dotted line which goes. So the diameter is the pitch diameter for the pitch circle. This is what it is.

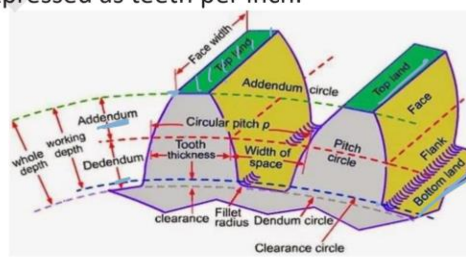
Almost you can see for just for discussion sake it can be half or slightly above half. The pitch circles of a pair of mating gears are tangent to each other. So this one and this one of the mating are tangent to each other. A pinion is the smaller of two mating gears. The larger one is often called as a gear or a bull gear.

So larger gear, smaller gear. The circular pitch p is the distance measured on the pitch circle from the point of one tooth to the corresponding point of the adjacent tooth. Circular pitch. You see here p, circular pitch. So circular pitch is from this point wherever it is starting to the next point of the ending.

It is if you try to take pitch in a sinusoidal way, this is called as a pitch. In a gear it is also the same from this extreme end to this extreme end. So from the end of one teeth to the end of the other teeth. Thus a pitch circle is equal to the sum of tooth thickness and the width of the space. Module.

Gear Terminology

- The **module m** is the ratio of the pitch diameter to the number of teeth. The customary unit of length used is the millimeter. The module is the index of tooth size in SI.
- The **diametral pitch P** is the ratio of the number of teeth on the gear to the pitch diameter. Thus, it is the reciprocal of the module. Since diametral pitch is used only with U.S. units, it is expressed as teeth per inch.
- The **addendum a** is the radial distance between the **top land** and the pitch circle.
- The **dedendum b** is the radial distance from the **bottom land** to the pitch circle.



https://mechtics.com/machine/theory-of-machines/gear-terminology-with-defination/

Module is the ratio of the pitch diameter to the number of teeth. The customary unit of length used is millimeter. The module is the index of a tool size in SI. So, what is module? Module says a ratio of the pitch diameter to the number of teeth.

So, ratio pitch diameter. Pitch diameter p to the number of teeth, let us take it as n, is called as a module. Diametral pitch p is the ratio of the number of teeth on the gear to the pitch diameter. This is the reciprocal of module. Module, inverse of module is diametral pitch p.

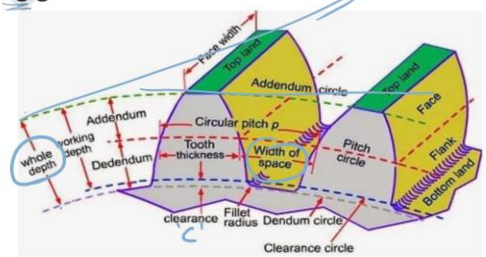
Since diametral pitch is used only with US units, it is expressed as teeth per inch. So module, 1 by module, inverse of module, diametral pitch. Addendum. From the pitch circle diameter to the tip of the gear is called as addendum. From the pitch circle diameter to the root, it is called as addendum.

This is an approximate definition. Let us see. The addendum a is the radial distance between the top land. This is the top land and the pitch circle. Top land and the pitch circle is addendum.

The dedendum is the radial distance between the pit circle or between the land, bottom land. This is called as a bottom. This is called as a top land. This is called as a bottom land. Between the bottom land and the pit circle is called as dedendum.

Gear Terminology

- The **whole depth h_t** is the sum of the addendum and the dedendum.
- The **clearance circle** is a circle that is tangent to the addendum circle of the mating gear. The **clearance c** is the amount by which the dedendum in a given gear exceeds the addendum of its mating gear.
- The **backlash** is the amount by which the width of a tooth space exceeds the thickness of the engaging tooth measured on the pitch circles.



https://mechtics.com/machine/theory-of-machines/gear-terminology-with-defination/

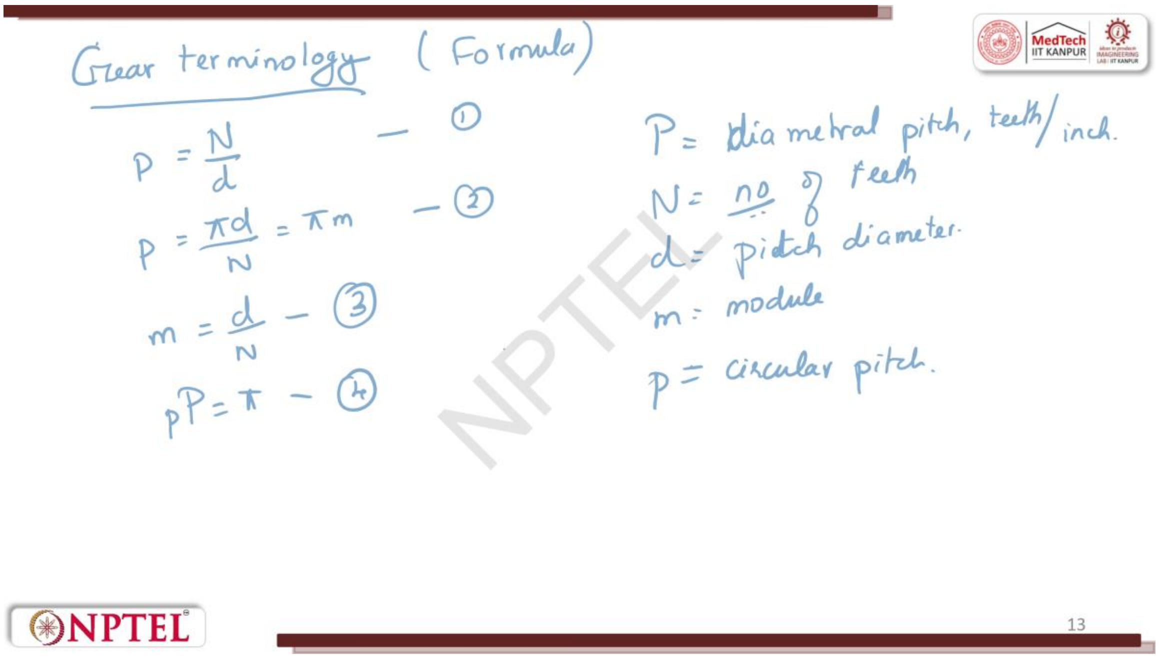
Whole teeth ht. It is the sum of a plus d. It is called as the whole depth ht. Clearance circle is the circle that is tangent to the addendum circle of the mating year. It is tangent to the addendum circle, tangent. So, suppose if it is going like this, it will go like tangent, right.

So, is a circle that is tangent to the addendum circle of the mating gear. The clearance c is the amount by which the addendum in a given gear exceeds the addendum of the mating gear. Clearance c is very important. You see here clearance. So, this is the hole depth, right, a plus d.

This is the working depth. The difference between the two is called as the clearance c. So when two gears meet, there has to be a small clearance. Perfect contact means it is going to be very tough, there will be lot of friction. Backlash is the amount by which the width of the tooth space exceeds the thickness of the engaged gear measured on the pitch circle. So backlash is also a very important phenomena.

So when two teeths meet, there will always be small clearance. This small clearance is called as backlash because what is the effect of backlash? So when you rotate it in the forward direction, you might come back to this position at 0, 0 point. But when I reverse do it, when I want to come back, the angle of rotation need not be the same. If it is not the same, then it is called as backlash.

As and when the wear and tear happens on the teeth, you will always have backlash more. I repeat, width of the space exceeds the thickness of the engaging gear which enters inside.



So, let us see some of the other Gear Terminologies. So, the formulas whatever we have discussed,

$$p = \frac{N}{d} \ (1)$$

$$P = \frac{\pi d}{N} = \pi m \ (2)$$

$$m = \frac{d}{N} \ (3)$$

$$pP = \pi \ (4)$$

where P = diametral pitch, teeth per inch

N = number of teeth

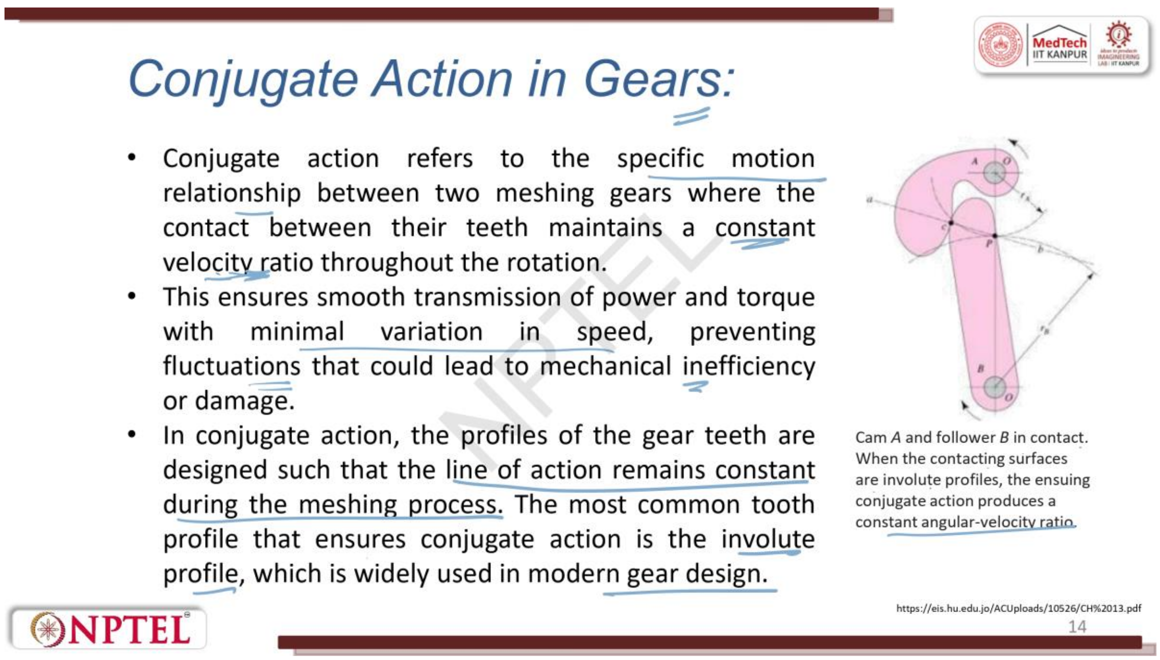
d = pitch diameter, in

m = module, mm

d = pitch diameter, mm

p = circular pitch

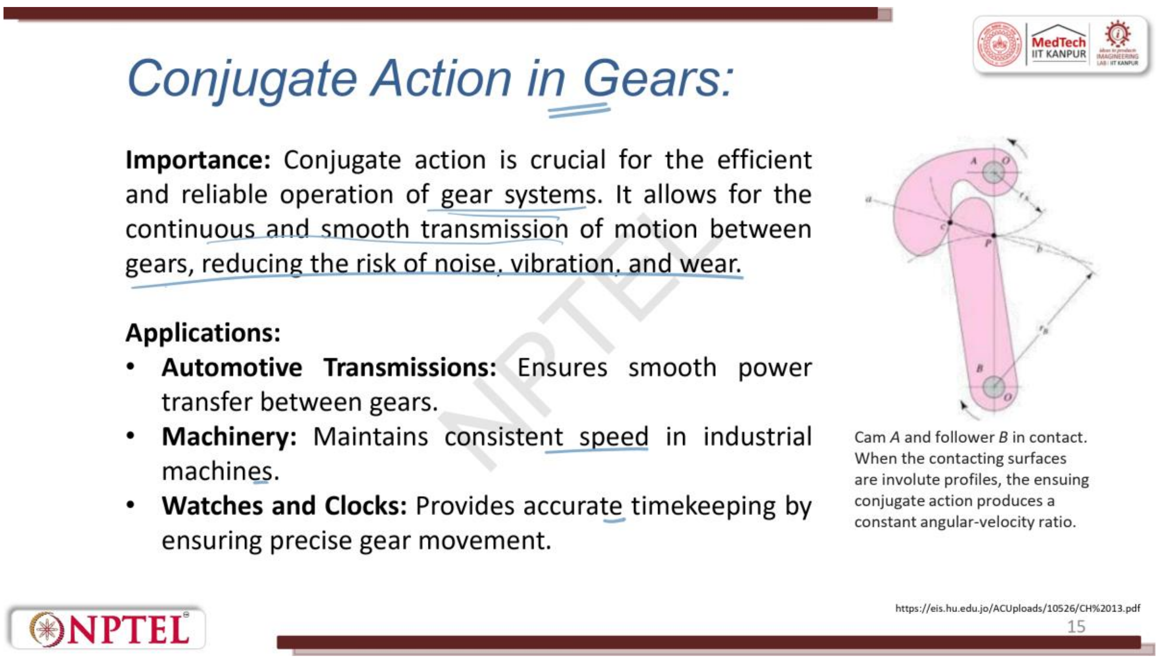
So these are important formulas which we discussed till now.



Conjugate Action in Gear. Conjugate action refers to the specific motion relationship between two meshing gears where the contact between their teeth maintains a constant velocity ratio throughout its rotation. So this is very important. Conjugate action refers to a specific motion relationship first point between two meshing gears with the contact between the teeth maintained a constant velocity ratio throughout the rotation.

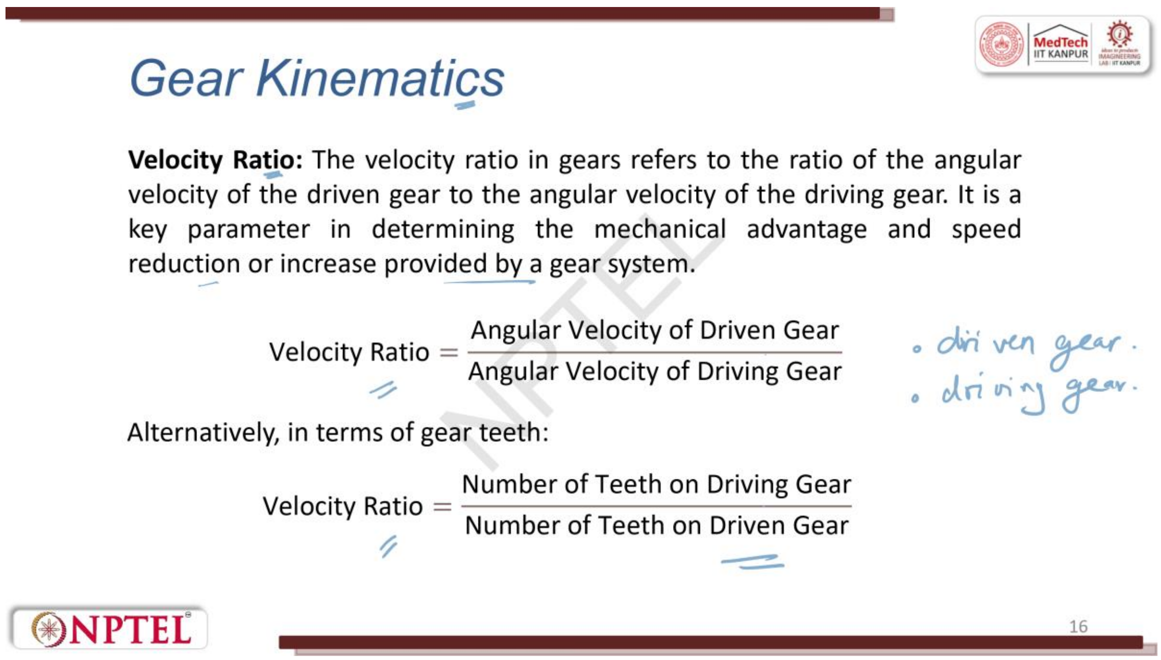
So, we can see here cam A and a follower B in contact. When the contacting surface are involute profiles, the ensured conjugate action provides a constant angular velocity ratio. This ensures smooth transmission of power and torque with minimal variation in speed, preventing fluctuations that lead to mechanical inefficiency, where the constant velocity ratio is maintained. In conjugate action, the profile of the gear teeth are designed such that the line of action remains constant during the meshing process. The most common tooth profile that ensures the conjugate action is the involute profile which is widely used in modern gear design.

Conjugate Action in Gears is very important. You will try to get a constant velocity ratio such that there is no velocity fluctuations so that leading to force fluctuation.



So, the importance is conjugate action is crucial for the efficient and reliable operation in gear systems. It allows for continuous smooth transmission of motion between gears, reducing the risk of noise, vibration and wear. Conjugate action is important.

It finds its application in automobile transmission, ensures smooth power transfer between the gears, machine consistent speed in industry machines, watches and clocks. It provides accurate timekeeping by ensuring precise gear movement. We were talking about velocity ratio.



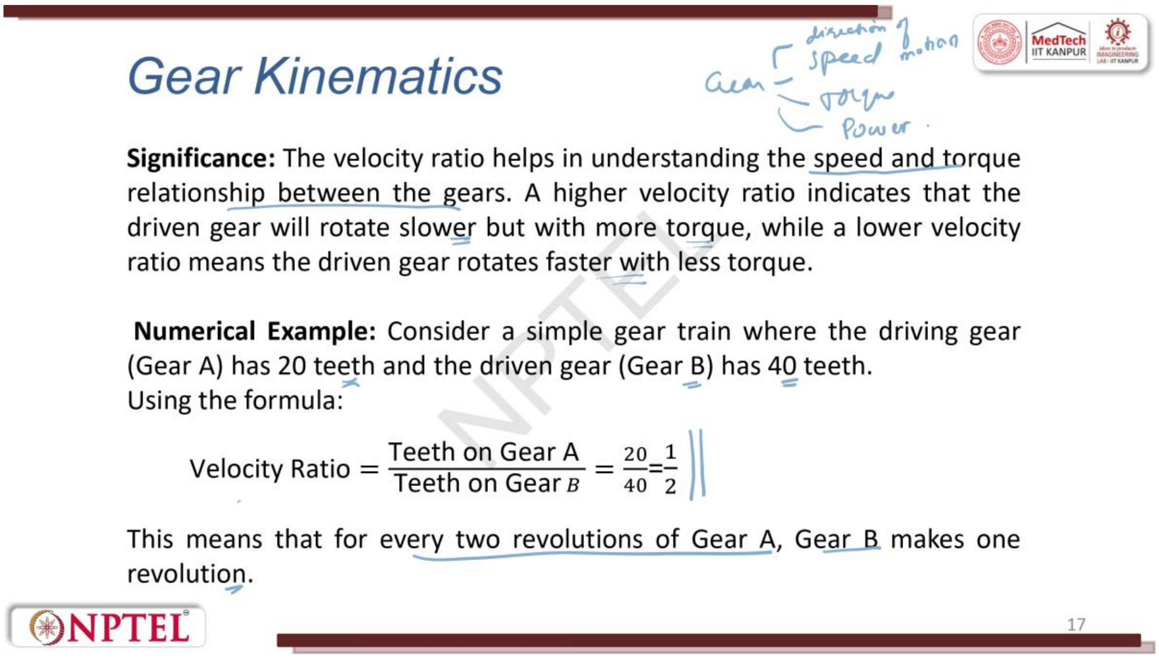
So what is velocity ratio you are supposed to understand? This falls under the topic of Gear Kinematics. So there are two gears. One is called as driven gear. The other one is called as driving gear. The velocity ratio in gear refers to the ratio of the angular velocity of the driven gear, moved gear. Driving gear will be attached to a motor or something.

It is a key parameter in determining the mechanical advantage and the speed reduction increase provided by the gear system. Velocity ratio is written as

$$\text{Velocity Ratio} = \frac{\text{Angular Velocity of Driven Gear}}{\text{Angular Velocity of Driving Gear}}$$

or it can be written as

$$\text{Velocity Ratio} = \frac{\text{Number of Teeth on Driving Gear}}{\text{Number of Teeth on Driven Gear}}$$

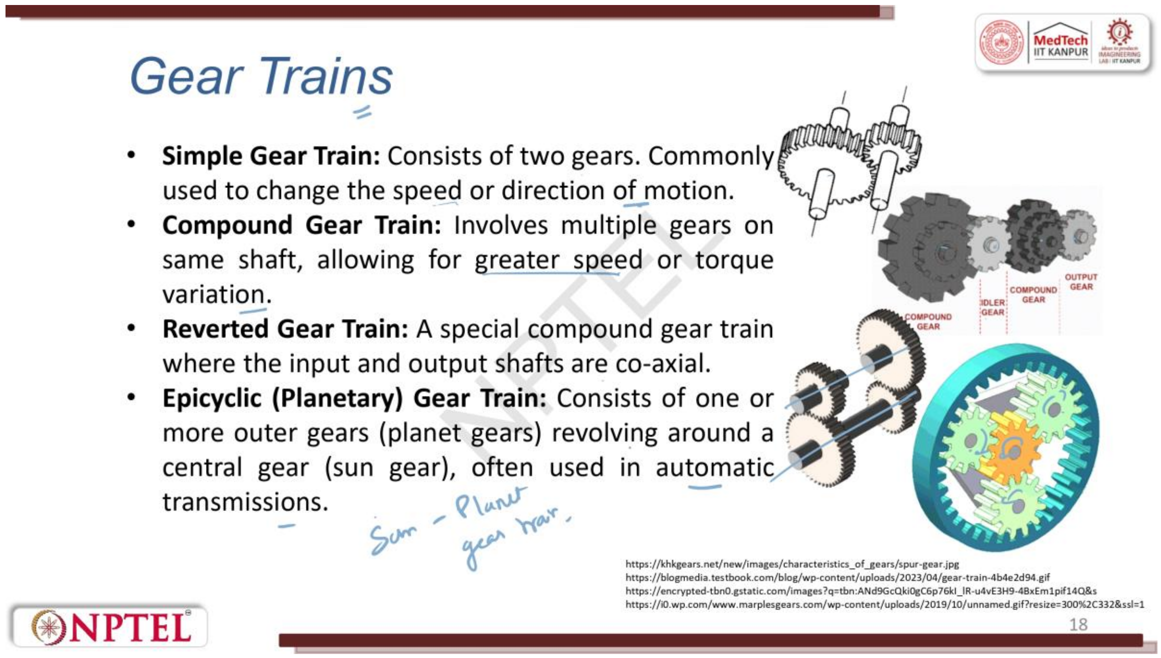


The velocity ratio helps in understanding the speed and the torque relationship between the gears. A higher velocity ratio indicates that the driven gear will rotate slower but with more torque. That's what I said, when you use gear, we try to change the speed, we try to change the torque and we try to change the power. A higher velocity ratio indicates that the driven gear will rotate slower but with more torque, while the lower velocity ratio means the driven gear rotates faster with least torque.

Please understand. Understand the difference. Let's try to solve a simple problem. Consider a simple gear train where the driving gear GA, gear A has 20 teeths and the driven gear, gear B has 40 teeths.

$$\text{Velocity Ratio} = \frac{\text{Teeth on Gear A}}{\text{Teeth on Gear } B} = \frac{20}{40} = \frac{1}{2}$$

This means that for every two rotations of gear A, gear B makes one revolution.



Moving further into gear trains, there are simple gear trains, compound rear gear trains, reverted gear trains, epicyclic gear trains. So simple gear trains are consisting of two gears commonly used to change the speed or the direction of motion. So here you should also make a note, it also changes the direction of motion. So speed or the direction of motion.

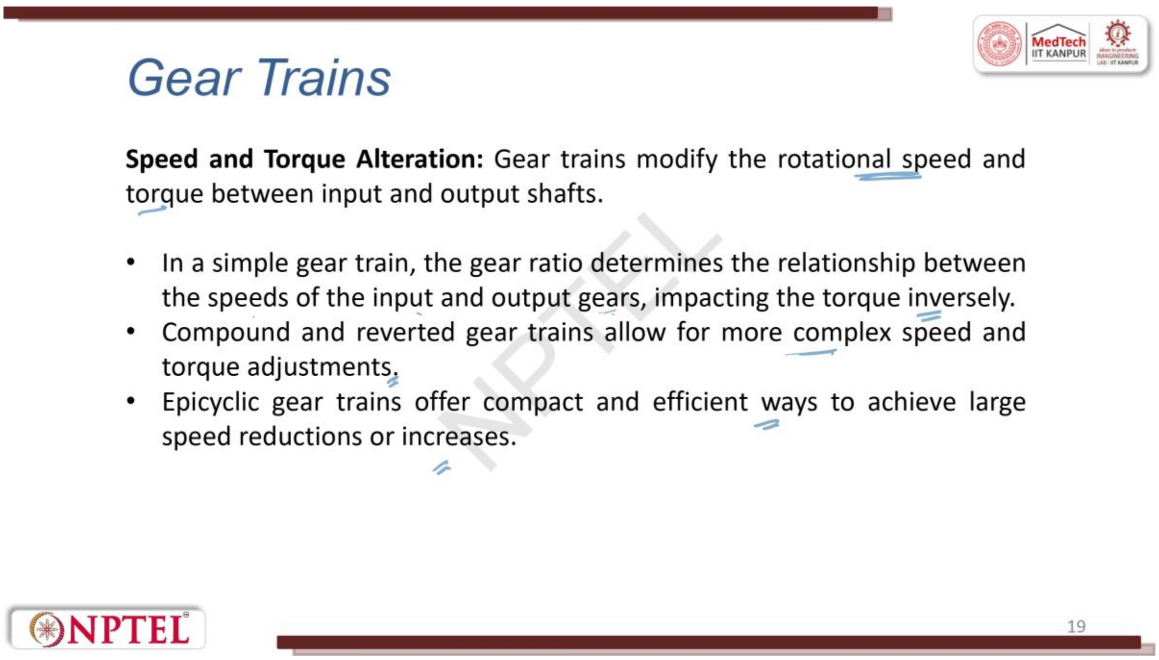
Compound gear involves multiple gears on the same shaft, allowing for faster speed or torque variation. You can have multiple gears. For example, this is called as Compound gears. So you will have an idler in between. You will have a compound gear, you will have an idler, so you will have an output.

You can have multiple things. So here, simple gear train, only two gears. Here, more than two. Involves multiple gears on same shaft, allowing a greater speed or torque variation. When we talk about riveted gear train, a special compound gear train where the input and output shafts are coaxial.

So look at it, they are coaxial. Epicyclic or planetary gear. So here you rotate this. All the other gears rotate. Consisting of one or more outer gears.

These are all called as outer gears. They are called as planetary gears. Revolving around a central gear called the sun gear. It is also called as sun and planet gear train. In the center you rotate all the other gears.

Gears rotate and you can try to get multiple outputs. Often used in automatic transmission.



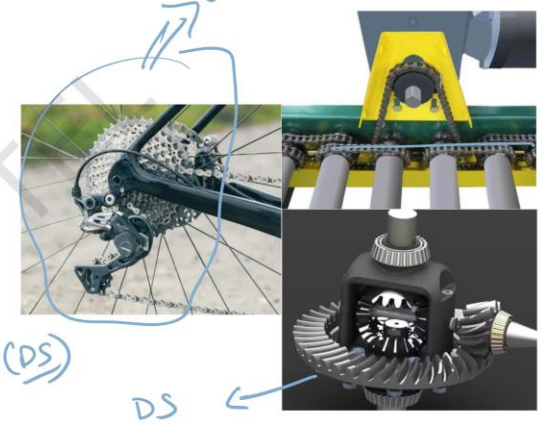
Speed and torque alterations. Gear trains modify the rotational speed and torque with respect to input and output. In a simple gear train, what we saw here, the gear ratio determines the relationship between the speed of input and output gear, impacting the torque inversely.

Compound and riveted gear trains allows for more complex speeds and torque adjustments. Epicyclic train offers compact and efficient way to achieve large speed reduction or increase.

Gear Trains

Practical Applications:

- **Simple Gear Train:** Used in clocks, conveyor systems.
- **Compound Gear Train:** Found in heavy machinery, bicycles.
- **Reverted Gear Train:** Utilized in automotive transmissions.
- **Epicyclic Gear Train:** Key component in automatic transmissions, differential systems. (DS)



www.iqsdirectory.com/articles/conveyors/roller-conveyors/roll-to-roll-chain-driven-roller-conveyor.jpg
www.bikeradar.com/advice/buyers-guides/bike-gears
https://auto.howstuffworks.com/differential2.htm

Various applications. Simple gear train is used in clock and conveyor. Compound found in bicycle. So this is compound in your gear cycle. Where you have multiple gears, right. Multiple teeths. They are all attached. Found in heavy machinery or bicycle.

Reverted gear train utilized in automotive transmission. So automotive transmission you can try to have. This here you can have. Epicyclic gear train key component in automatic transmission and differential system. This is a differential system DS.

Derivation - Gear Ratio

Gear Ratio Formula:

$$\text{Gear Ratio} = \frac{\text{Number of Teeth on Driven Gear}}{\text{Number of Teeth on Driving Gear}}$$

multiplying / dividing => speed

1. **Basic Concept:**

- When two gears are in mesh, the number of teeth on gears determines their rotational speeds. The gear ratio is defined as the ratio of angular velocity of the driving gear to the angular velocity of the driven gear.

2. **Angular Velocity Relationship:** ω

- Let N_1 be the number of teeth on driving gear (Gear 1) and N_2 be the number of teeth on driven gear (Gear 2).
- Let ω_1 and ω_2 be the angular velocities of Gear 1 and Gear 2 respectively.

Since the gears are in mesh, the linear velocity at pitch circle of both gears must be equal: $\omega_1 \times r_1 = \omega_2 \times r_2$

where r_1 and r_2 are the pitch radii of Gear 1 and Gear 2, respectively.

Gear ratio is one of the most important thing which we do.

$$\text{Gear Ratio} = \frac{\text{Number of Teeth on Driven Gear}}{\text{Number of Teeth on Driving Gear}}$$

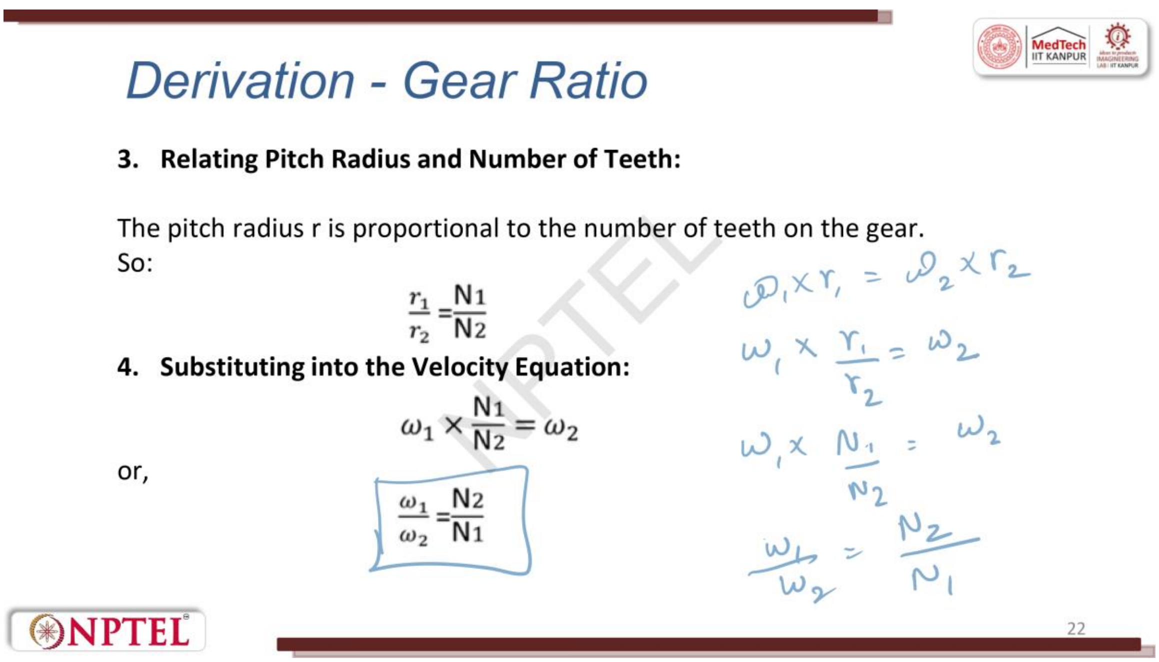
When two gears are in mesh, the number of teeth on the gears determine their rotational speed. Gear ratio will be used for multiplying or dividing the speed.

So it can multiply in terms of increasing the speed or decreasing the speed. The gear ratio is defined as the ratio of the angular velocity of the driving gear to the angular velocity of the driven gear. So angular velocity ratio is always written as omega.

Let ω_1 and ω_2 be the angular velocities of Gear 1 and Gear 2 respectively.

Since the gears are in mesh, the linear velocity at pitch circle of both gears must be equal: $\omega_1 \times r_1 = \omega_2 \times r_2$

where r_1 and r_2 are the pitch radii of Gear 1 and Gear 2, respectively.



Relating the pitch radius and number of teeth, the pitch radius R is proportional to the number of teeth on the gear.

$$\frac{r_1}{r_2} = \frac{N_1}{N_2}$$

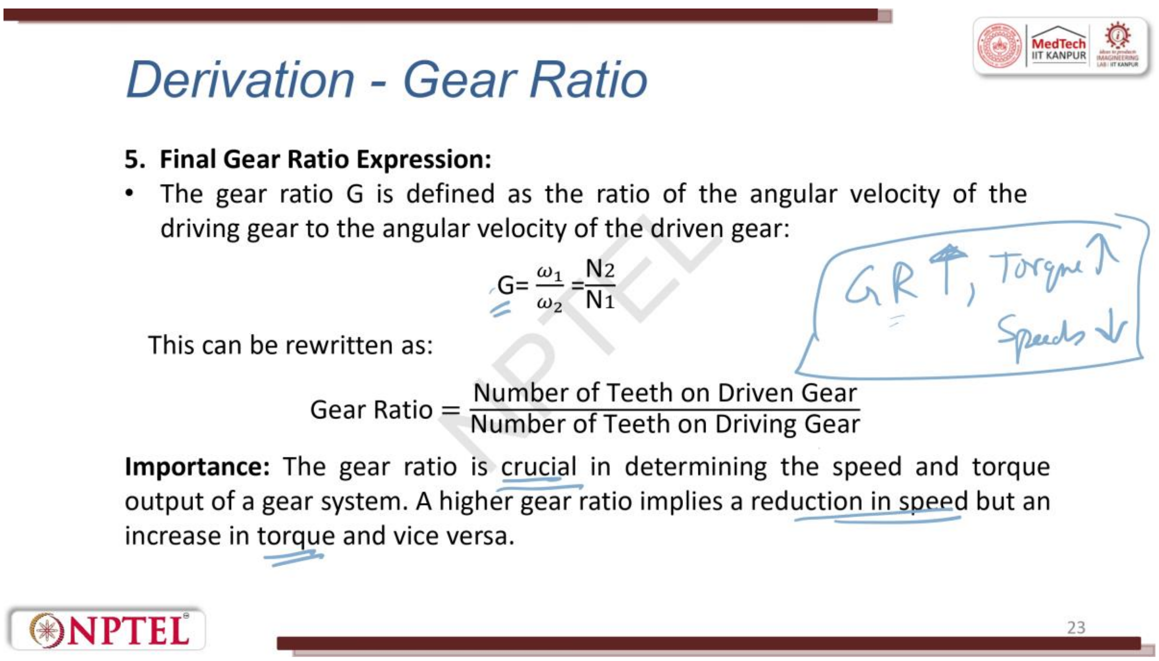
Substituting it to the velocity equation.

$$\omega_1 \times \frac{N1}{N2} = \omega_2$$

Or

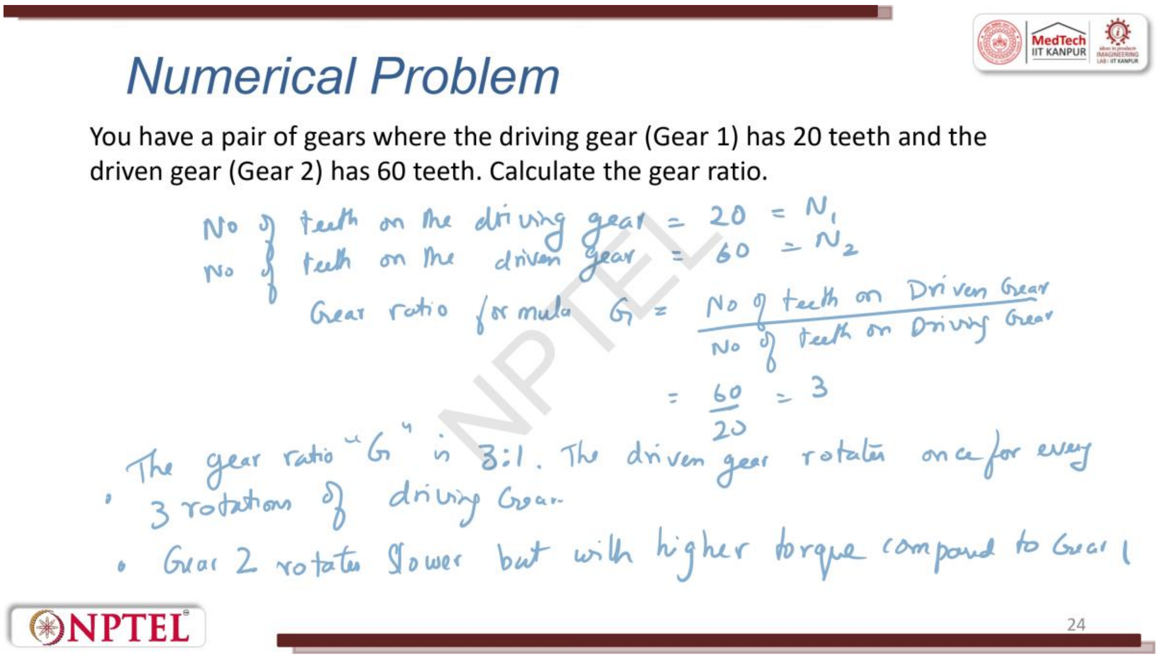
$$\frac{\omega_1}{\omega_2} = \frac{N2}{N1}$$

So what did I get? I get the gear ratio. So gear ratio formula is derived accordingly. So this is the formula.



Final gear ratio expression G is, expressed as omega 1 by omega 2 is equal to N2 by N1. Gear ratio G is defined as the ratio of the angular velocity of the driving gear to the angular velocity of the driven gear. So it can be rewritten as number of teeth on driven gear divided by number of teeth on driving gear. The gear ratio is a very crucial factor in determining the speed and torque output of the gear system. Higher the gear ratio implies reduction in speed but an increase in torque. If GR is high, then torque is high.

Speeds are low. This is very important. Because when the gear train ratio is high, torques are very high. I can lift heavy things. Speed will be low. So heavy things when I lift, the speed will be low.



So let us try to solve a simple numerical problem to understand the concept whatever we have seen till now. You have a pair of gears where the driving gear G1 has 20 teeth and the driven gear G2 has 60 teeth. Calculate the gear ratio.

Solution:

Step 1: Identify the Given Data

Number of teeth on the driving gear N1= 20

Number of teeth on the driven gear $N_2 = 60$

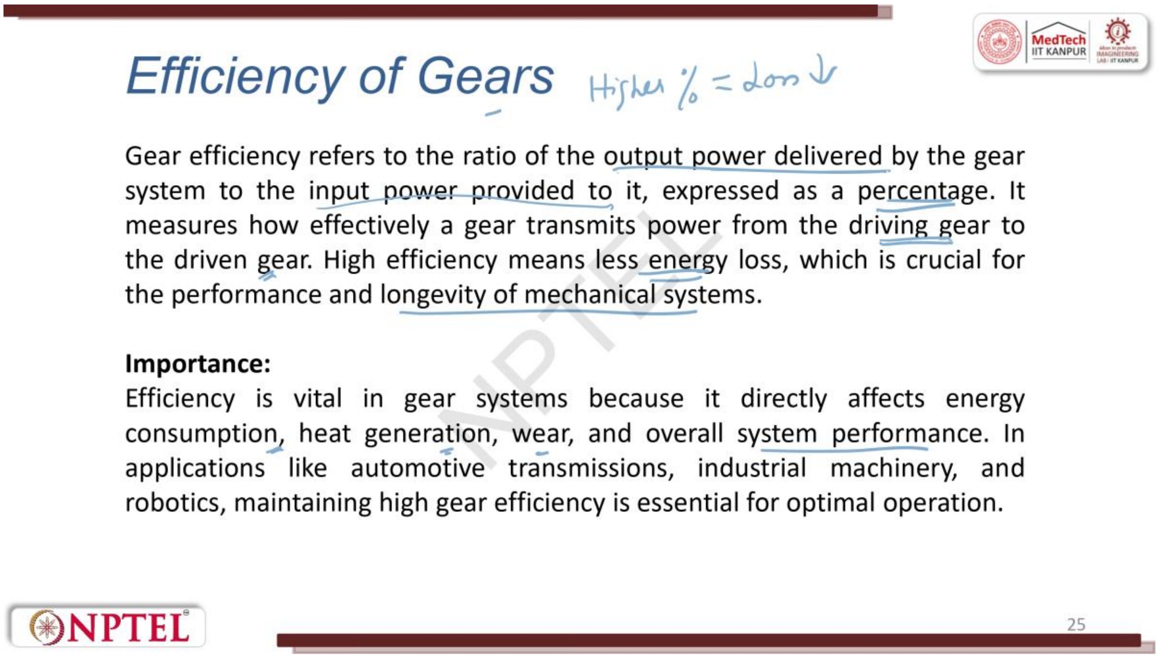
Step 2: Apply the Gear Ratio Formula The gear ratio G is given by:

$$\text{Gear Ratio} = \frac{\text{Number of Teeth on Driven Gear } (N_2))}{\text{Number of Teeth on Driving Gear}(N1)}$$

Substituting the values: $G = \frac{60}{20}$

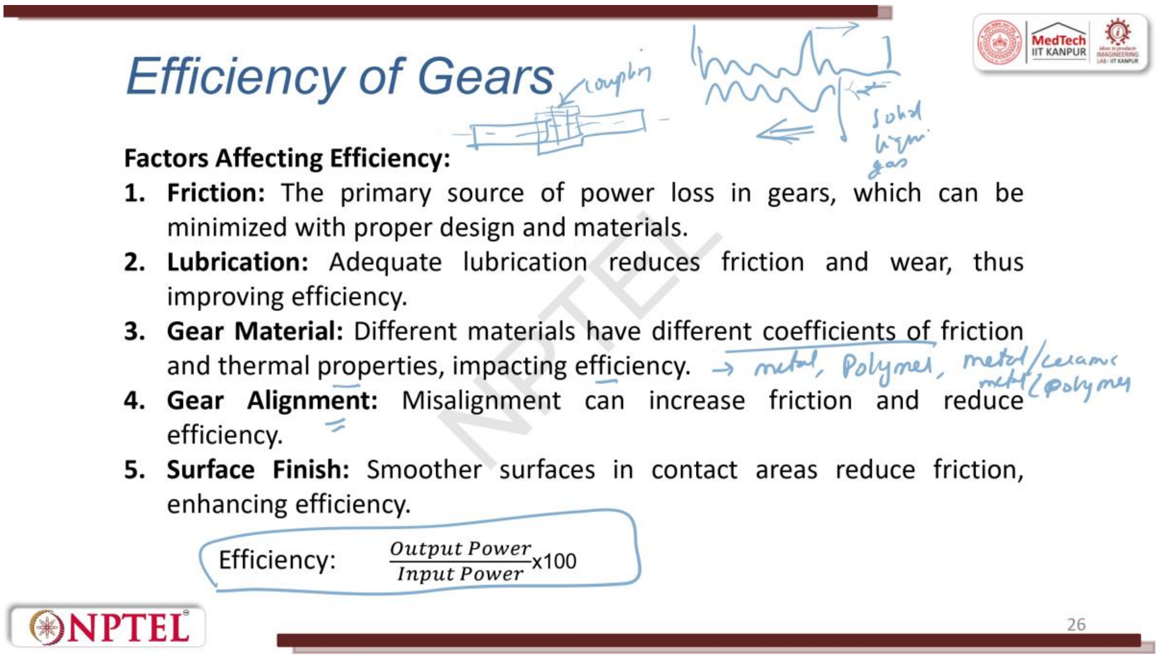
Step 3: Simplify the Ratio, G = 3

- The gear ratio is 3:1, meaning the driven gear (Gear 2) rotates once for every three rotations of the driving gear (Gear 1).
- This indicates that Gear 2 rotates slower but with higher torque compared to Gear 1.



So, Efficiency of the Gear. Gear efficiency refers to the ratio of the output power delivered by the gear system to the input power provided to it, which is expressed generally in terms of percentage. It measures how efficiently gear transmits power from the driving gear to the driven gear. High efficiency means less energy loss. So, higher the percentage, loss will be lesser, which is crucial for the performance and the longevity of a mechanical system.

Output by input, output power delivered by input power provided. Efficiency is vital in gear systems because it directly affects the energy consumption, heat generation, wear and the overall systems performance. This is very much used in automotive transmission, industrial machinery, robotics. Maintaining high gear efficiency is essential for optimal operations.



So the Factors Affecting the Efficiency are going to be Friction, Lubrication, Gear Material, Gear Alignment and Surface Finish.

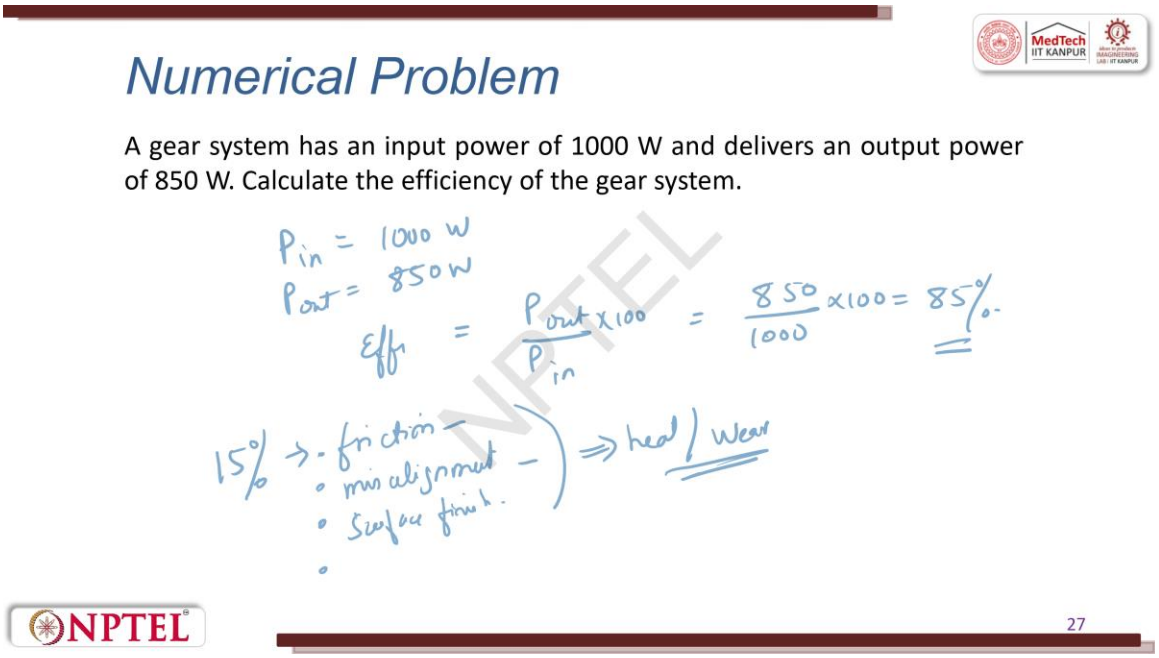
Friction when there are undulations on the surface and there is a matching undulation on the surface, this is moving, so there is going to be a huge friction, the primary source of power loss in gear which can be minimized with proper design and material is friction lubrication in between these two you can try to have a solid liquid or a gas lubrication. Adequate Lubrication reduces friction and wear, thus improving the efficiency. The Gear Material can be chosen depending upon the friction and the load of transmission. So different coefficient of friction, thermal properties and impacting efficiency.

So you can try here metal polymer. You can also try on a metal ceramic coating or metal polymer coating. So this is only to reduce the friction. But the other load will be taken by the metal part of it. Gear Alignment.

So gear misalignment can increase the friction and thereby reducing the efficiency. So there is a shaft. There is one more shaft. So, between these two shafts, the alignment should be proper. So, that is why today we use something called as flexible coupling.

This takes care of the gear misalignment problem. Surface finish has a direct influence on the friction. Smoother surface in contact area reduces the friction.

Efficiency: $\frac{Output\ Power}{Input\ Power}$x100



Let us try to solve another simple problem to understand the power and efficiency.

A gear system has an input power of 1000 W and delivers an output power of 850 W. Calculate the efficiency of the gear system.

Given that: Input Power P_{in} = 1000 W Output Power P_{out} = 850 W

Apply the Efficiency Formula

Efficiency= $\frac{P_{out}}{P_{in}}$x100

Substituting the values:

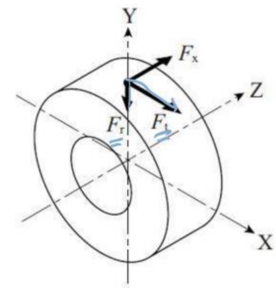
Efficiency =$\frac{850\ W}{1000\ W}$x100 Efficiency=85%

The gear system operates with an efficiency of 85%, meaning that 15% of the input power is lost, likely due to friction, heat or other factors.

This is a reasonable efficiency for many practical applications but efforts to reduce losses could improve performance further.

Forces on Gear Teeth

- During the operation of gears, forces are transmitted from one gear to another through the contact of their teeth.
- These forces are crucial because they determine how effectively the gears transmit torque and rotational motion.
- The forces on gear teeth must be carefully analyzed to ensure that the gears can withstand the loads without failure, excessive wear, or deformation.

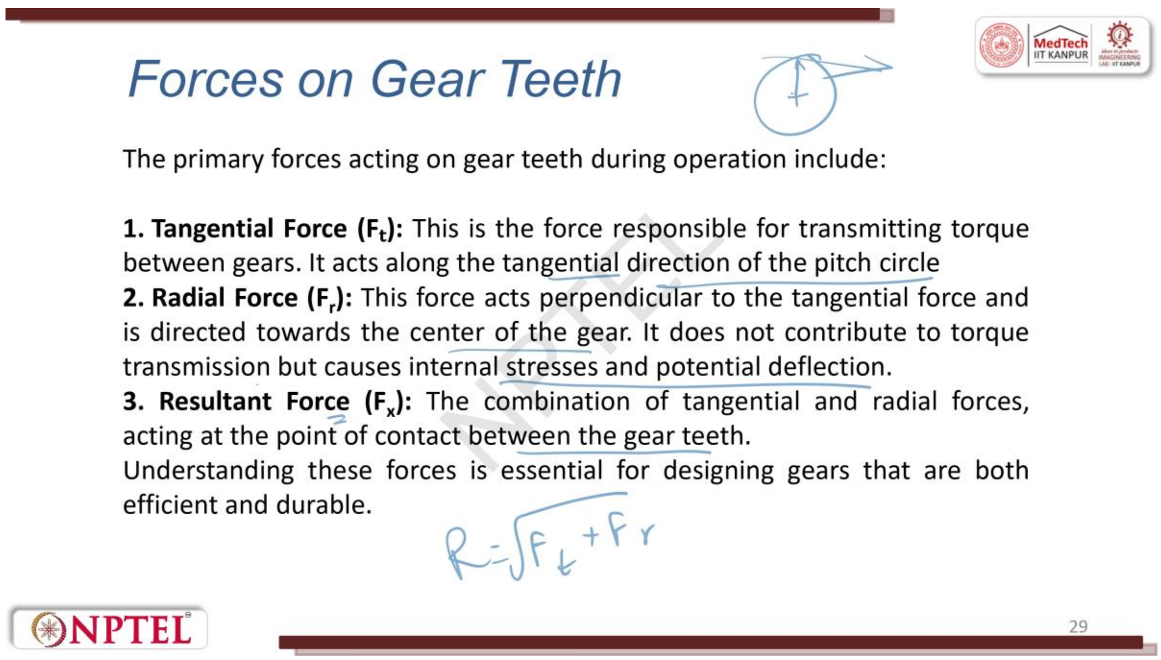


https://khkgears.net/new/images/Gear-Forces/Fig.-12.1-Direction-of-Forces-Acting-on-a-Gear.jpg

So what are the forces which are generally involved in a gear? So Forces on a Gear Teeth. During the operation of a gear, forces are transmitted from one gear to the another through the contact of the teeth. These forces are crucial.

So you can see here, there is Fx, there is Fx along the x direction, there is Fr along the radial direction, there is Ft which is the torque direction. So, these forces are crucial because they determine how efficiently the gear transmits torque and rotation motion.

The forces on the gear may be carefully analyzed to ensure that the gears can withstand the load without having excessive wear or deformation. So, the forces are very important to be calculated.



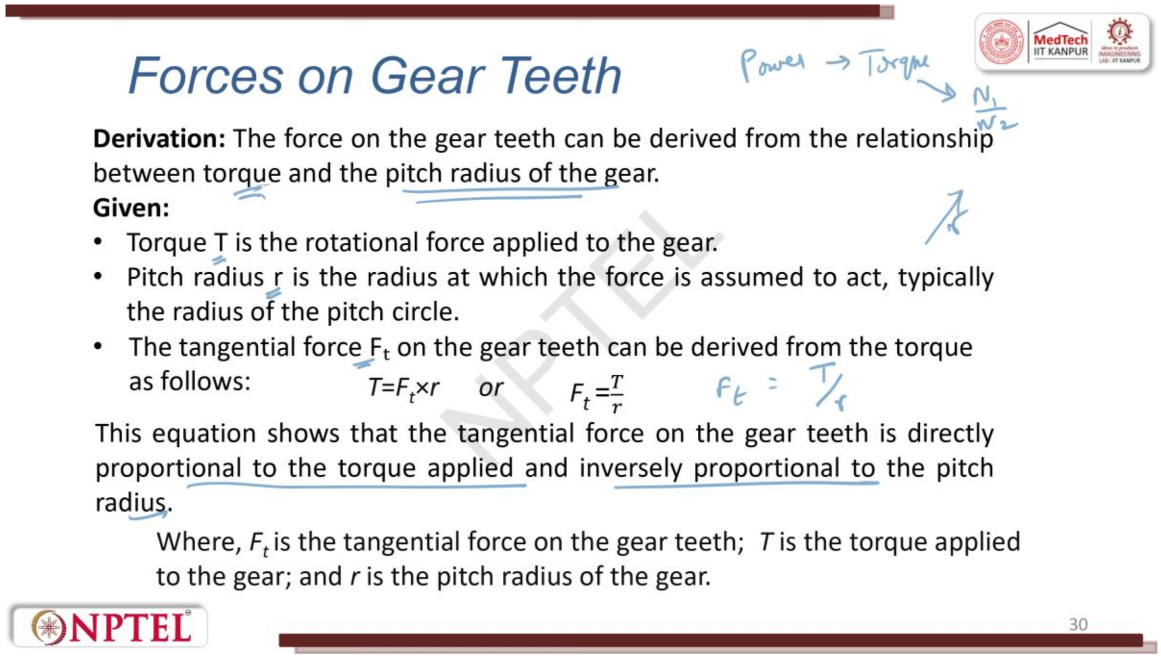
So, when we talk about forces, there are three. Two forces, then the resultant of these two forces becomes the third force. Tangential force.

This is a force responsible for transmitting torque between the gears. It acts along a tangential direction of the pitch circle, tangential force. Radial force is the downward force or it is a force from the radius. So the Radial force, the force acts perpendicular to the tangential force and is directed towards the center of the gear. It does not contribute to the torque transmission but causes internal stresses and potential deflection.

Bending of the gear happens. Radial force is also very important. So for transmission, it will be tangential. For radial, it will be the internal stresses and the potential deflection. A combination of these two,

$$R = \sqrt{F_t + F_r}$$

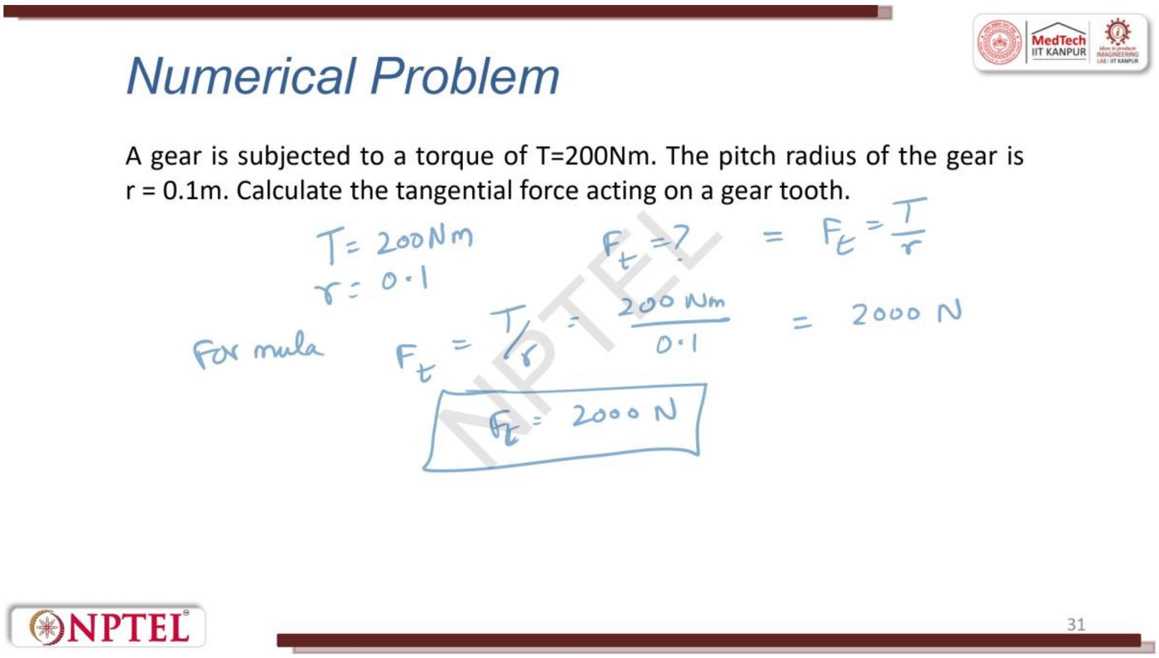
So resultant force is a combination of tangential force and radial force acting on a point of contact between the gear teeths. Understanding all these forces are very important.



So a small simple derivation we will try to do because here we are more interested in power and power that is always found from the torque. The torque in turn can be found from the ratio N1/N2 or N2/N1, whatever it is.

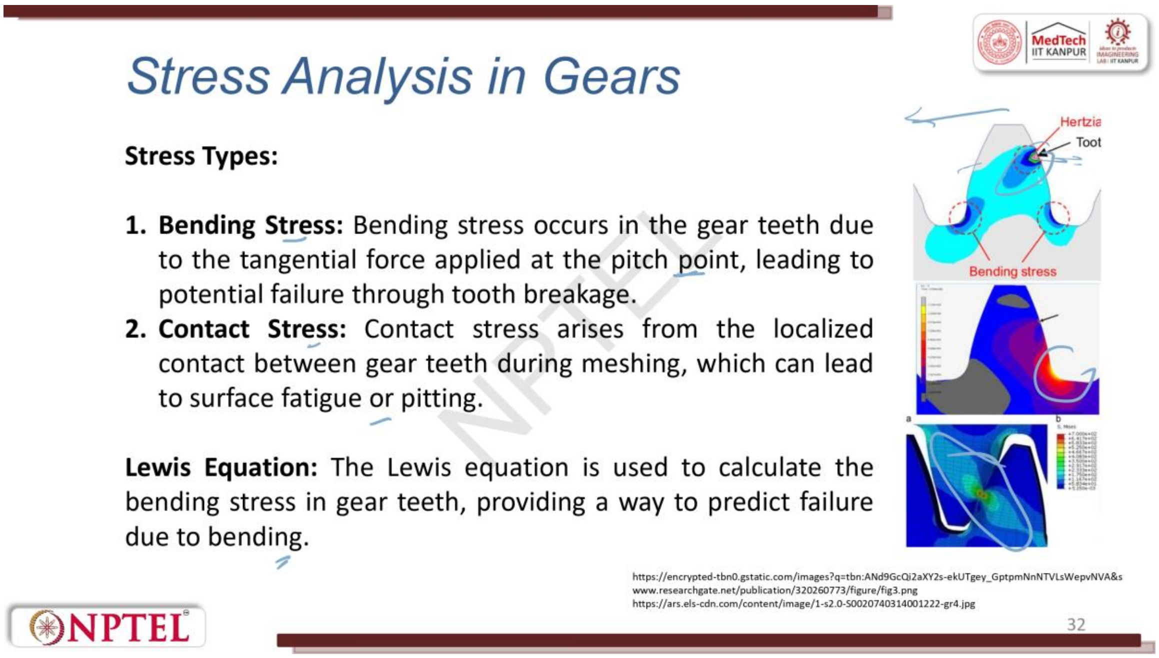
Derivation: The force on a gear teeth can be derived from the relationship between torque and the pitch radius of the gear. The torque T is the rotational force applied on the gear. The pitch radius R is the radius at which the force is assumed to act, typically the radius of the pitch circle. We saw the radius of a pitch circle, radius R. So the tangential for Ft is nothing but torque. Or if you want to find out the tangential force, it is going to be torque divided by radius.

This will try to give me my tangential force. This equation clearly shows me that the tangential force on the gear train is directly proportioned to the torque applied and inversely proportioned to the pitch radius. If the pitch radius is large, then the tangential force is going to be low. Where Ft is the tangential force on the gear train, T is the torque applied to the gear and R is the pitch radius.



Based on the understanding, we will try to solve a simple problem. The gear train has a torque which is 200 Newton meter. The pitch radius R is equal to 0.1. What is Ft? Ft is nothing but T by r, right. So, the torque is given, radius is given.

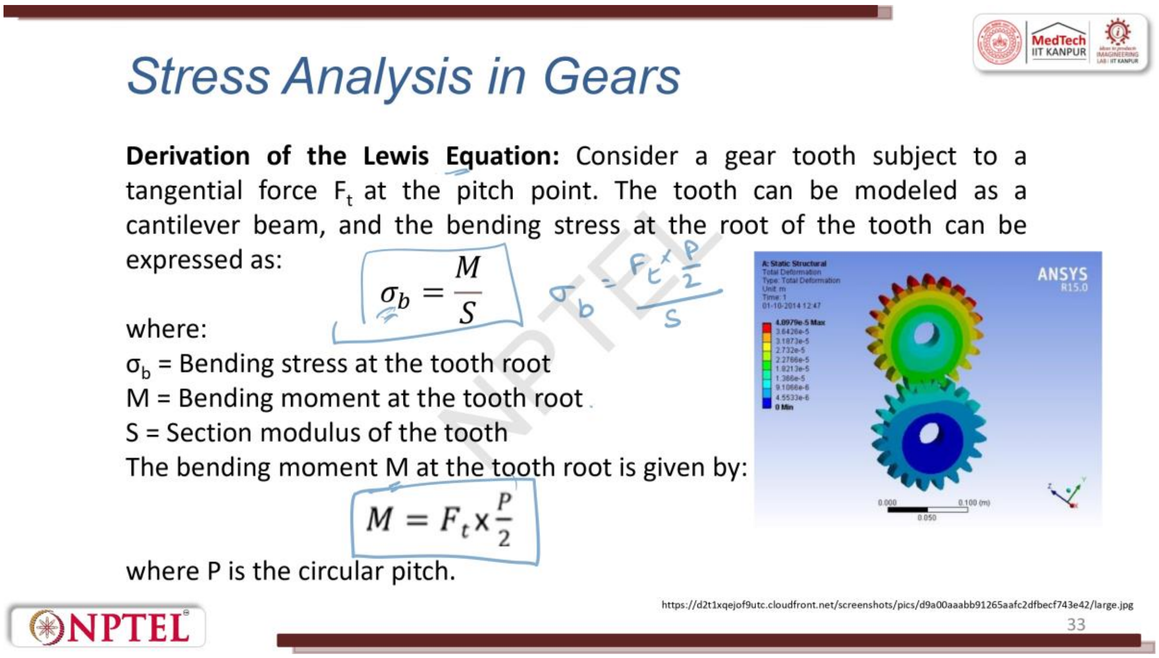
So, from the formula, Ft equal to T by r, we get 200 newton meter divided by r, which is 0.1, which is equal to 2000 newton. So, the tangential force Ft is equal to 2000 Newton.



So, the stresses, now we saw after seeing the force, force per unit area is stress, Stress Analysis. There are two types of stress analysis; generally we do to understand the bulk bar figure. We try to do a Bending stress and then we also try to calculate the Contact stress.

Bending stress occurs in the gear teeth due to the tangential force applied at the pitch point leading to a potential failure through tooth break. So bending stress occurs in the gear teeth due to the tangential force applied on the pitch. Tangential force applied on the pitch point leading to the potential failure through a tooth breakage. Contact stress arises from the localized contact between the gear teeth during meshing. So during meshing which can lead to surface fatigue or pitting.

Lewis equation is used to calculate the bending stress in gear teeth, providing a way to predict failure due to bending.



So what does this Lewis equation say? Consider a gear tooth subjected to a tangential force Ft at a pitch point. Pitch point is on the pitch circle a point. The tooth can be modeled as a cantilever beam and the bending stress at the root of the tooth can be expressed as $\sigma_b = \frac{M}{S}$.

What is M? M is the bending moment at the tooth point. S is the sectional modulus of the tooth. So with this we can try to figure out what is the bending stress in a gear. The bending moment at the tooth root is given as

$$\sigma_b = \frac{F_t \times \frac{P}{2}}{S}$$

Stress Analysis in Gears

The section modulus S for a gear tooth can be expressed in terms of the face width F_m and the Lewis form factor Y:

$$S = F_m \times Y$$

Substituting these into the bending stress formula gives:

$$\sigma_b = \frac{F_t \times \frac{P}{2}}{F_m \times Y} = \frac{F_t \times P}{2 \times F_m \times Y}$$

To simplify, we multiply both sides by 2:

$$\sigma_b = \frac{F_t \times P}{F_m \times Y}$$

The sectional modulus of a gear can be found out by $S = F_m \times Y$. What is Fm? It is the face width and a Lewis form factor Y. So now if we put everything into the equation for finding out the bending stress, it is $\sigma_b = \frac{Ft \times \frac{P}{2}}{Fm \times Y} = \frac{F_t \times P}{2 \times Fm \times Y}$. So, to simplify, we multiply both sides by 2. So, we finally get $\sigma_b = \frac{F_t \times P}{Fm \times Y}$.

Stress Analysis in Gears

Where:

σ_b = Bending stress

F_t = Tangential force

P = Pitch

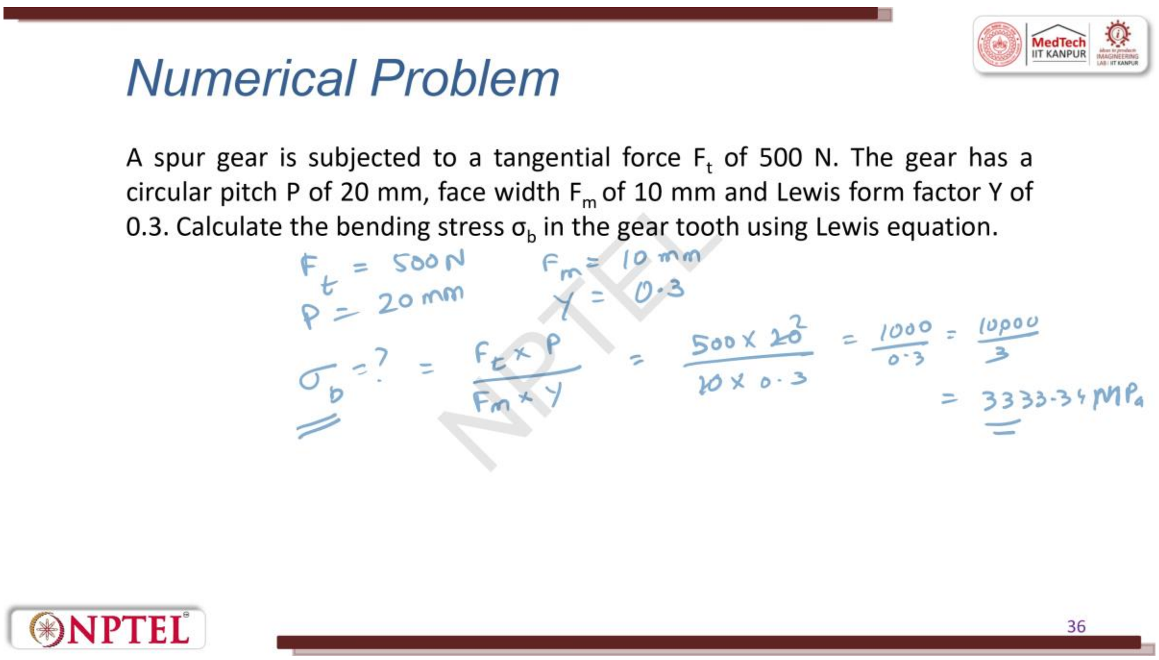
F_m = Face width

Y = Lewis form factor, which depends on the gear tooth geometry

Application:

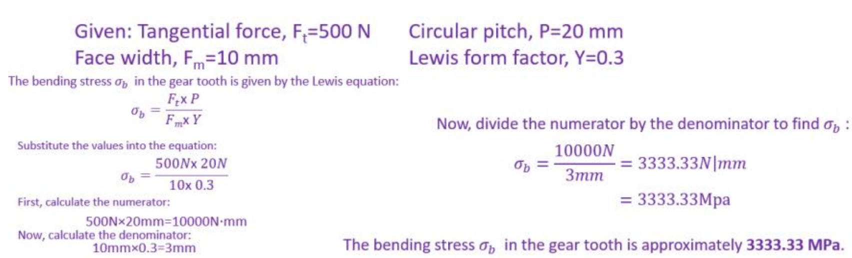
- The Lewis equation is crucial in gear design to ensure that the gears can handle the applied loads without bending failure.
- It helps in selecting appropriate materials and gear dimensions.

This is how we get the bending stress for a gear where sigma b is the bending stress, P is the pitch, Y is the Lewis form factor, tangential for Ft and face width is Fm. So, where do we use this? Lewis equation is crucial in gear design to ensure that the gears can handle applied load without bending failure. It is used predominantly in selecting the proper material and the gear dimensions.



Let us do a simple problem to understand the concept what we studied. A spur gear is subjected to a tangential force F_t of 500 N. The gear has a circular pitch P of 20 mm, face width F_m of 10 mm and Lewis form factor Y of 0.3. Calculate the bending stress σ_b in the gear tooth using Lewis equation.

Solution:



Given: Tangential force, F_t=500 N   Circular pitch, P=20 mm

Face width, F_m=10 mm   Lewis form factor, Y=0.3

The bending stress σ_b in the gear tooth is given by the Lewis equation:

$$\sigma_b = \frac{F_t x\, P}{F_m x\, Y}$$

Substitute the values into the equation:

$$\sigma_b = \frac{500N x\, 20N}{10x\, 0.3}$$

First, calculate the numerator:

500N×20mm=10000N·mm

Now, calculate the denominator:

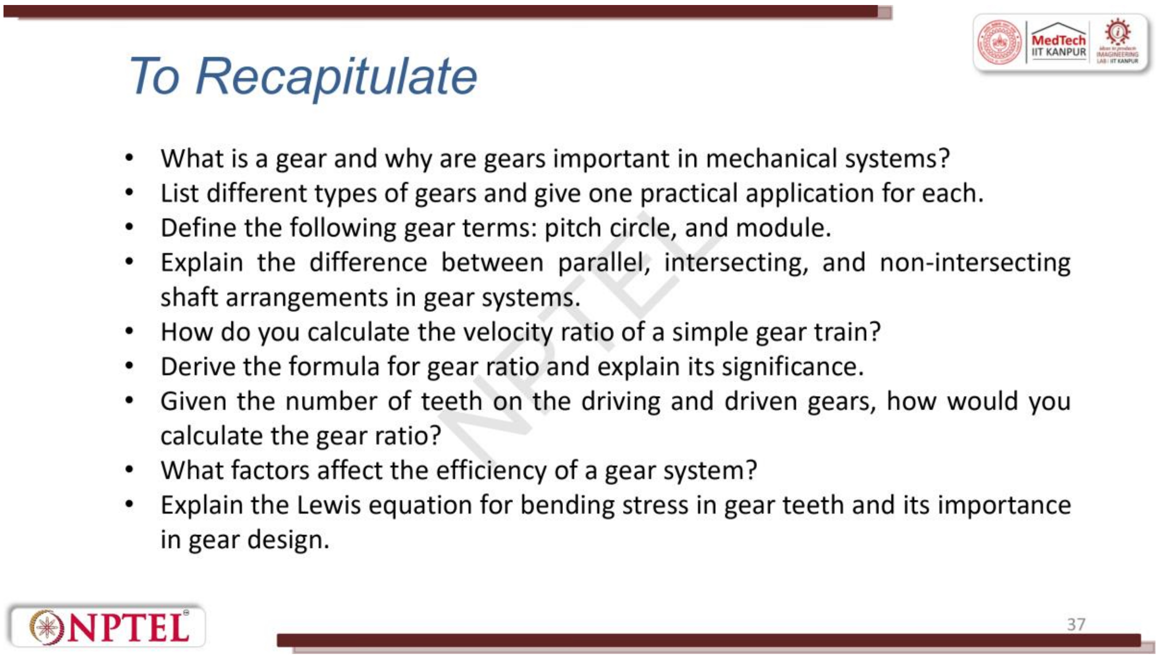
10mm×0.3=3mm

Now, divide the numerator by the denominator to find σ_b :

$$\sigma_b = \frac{10000N}{3mm} = 3333.33N|mm$$

$$= 3333.33\text{Mpa}$$

The bending stress σ_b in the gear tooth is approximately **3333.33 MPa**.



To recap what all we have covered in this lecture, we have first seen what is gear, why is gear important, different types of gear, different types of teeth in a gear, then what is pitch circle, module.

What are the difference between parallel, intersecting and non-intersecting shaft arrangements in a gear? How do you calculate the velocity ratio of a simple gear train? Derive the formula for gear train and explain its significance. Number of teeth on the driving and driven gear. How would you calculate the gear ratio?

What is a efficient gear system? When do we use Lewis equation while designing a gear?

References

1. Shigley, J.E., Mitchell, L.D. and Saunders, H., 1985. Mechanical engineering design.
2. Bhandari, V.B., 2010. Design of machine elements. Tata McGraw-Hill Education.
3. Khurmi, R.S. and Gupta, J.K., 2005. A textbook of machine design. S. Chand publishing.
4. Rattan, S.S., 2014. Theory of machines. Tata McGraw-Hill Education.

These are the reference book we have gone through.

Thank you very much.