Basics of Mechanical Engineering-1

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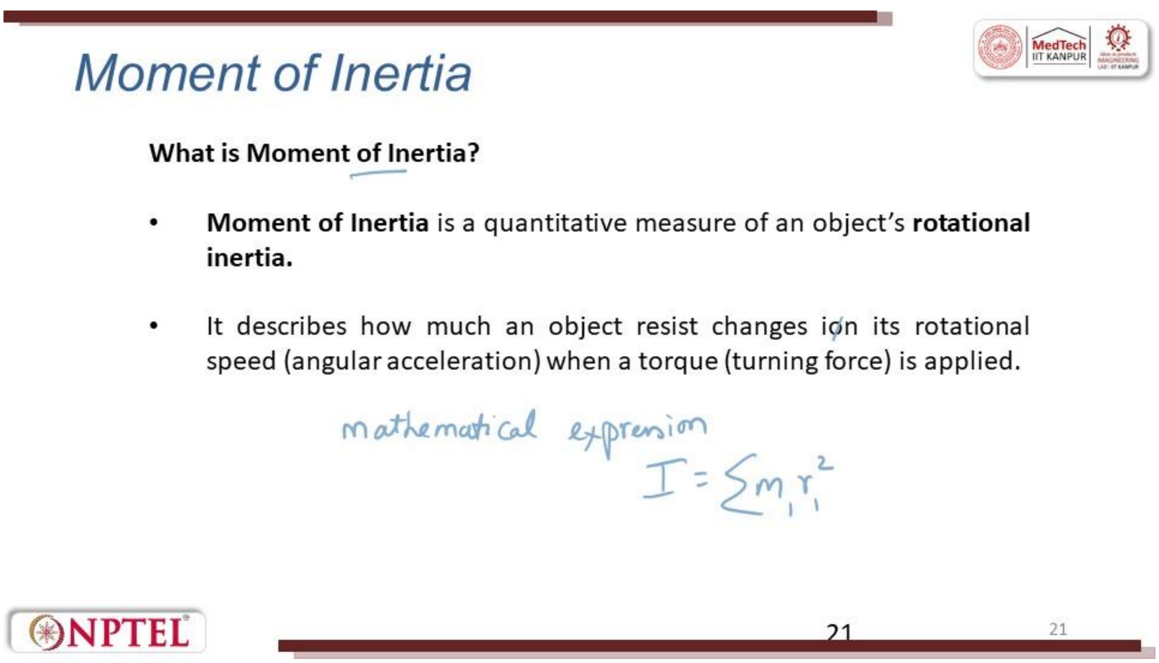
Week 02

Lecture 07

Moment of Inertia and Gravity

Welcome to the next lecture in our course. In this lecture, we will be trying to cover Moment of Inertia and Gravity.

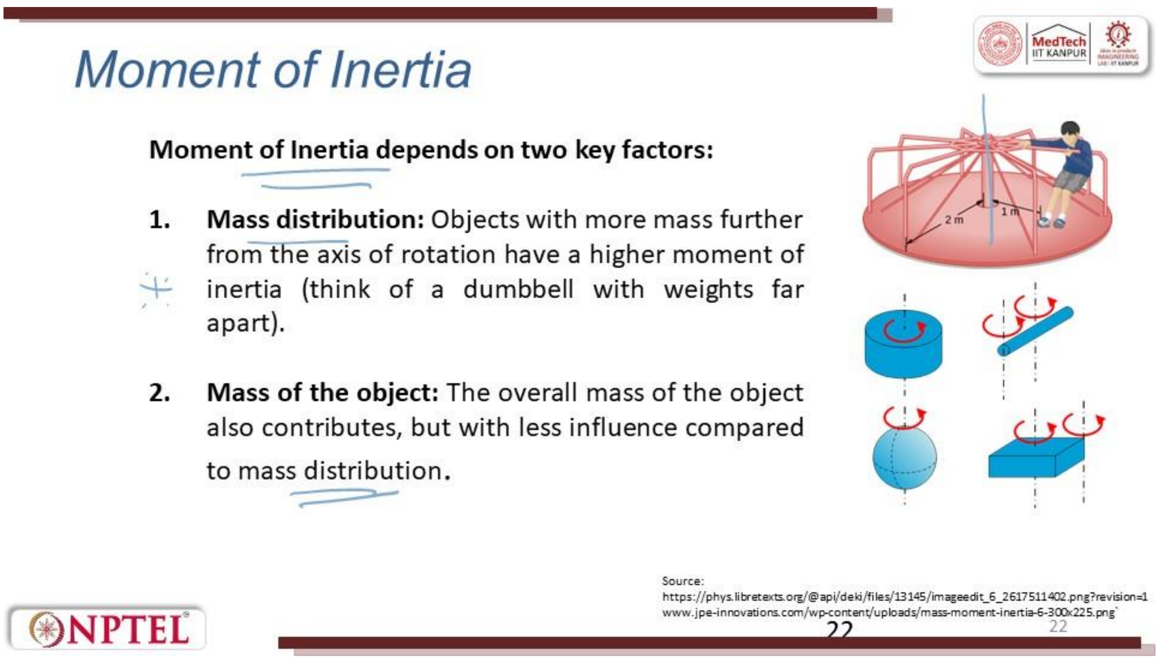


So, the next topic of discussion is going to be Moment of Inertia. Moment of inertia is a quantitative measure of an object's rotational inertia. It describes how much an object resists change in its rotational speed, angular acceleration when a torque turning force is applied.

So, we studied about friction. When we studied about reducing friction, now we are trying to see a moment of inertia. Why this? That was for a linear, this is for rotational, right. For example, a screw is there or a nut is there. What is the torsional force?

When a torque is applied, what happens to the system is called as the Rotational inertia. So here the mathematical expression for moment of inertia I can be represented as

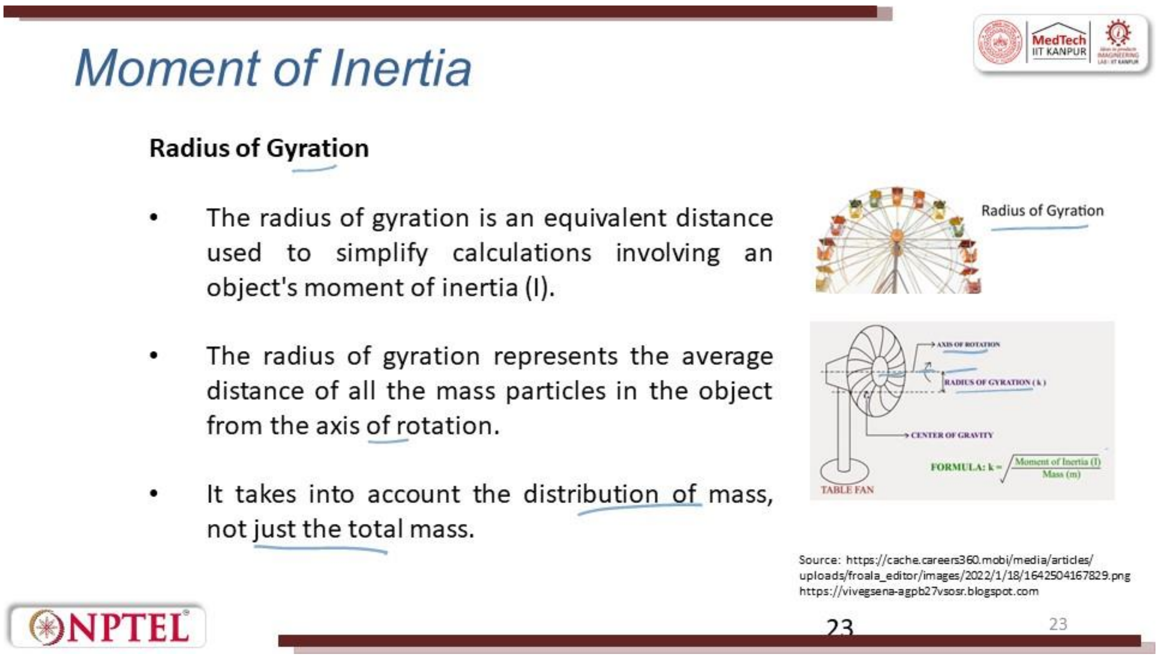
$$I = m_1 r_1^2$$



Moment of inertia depends on two key factors. One is mass distribution, the other one is mass of the object. Objects with more mass further from the axis of rotation have a higher moment of inertia.

So, this is what you have played in your school days. This is a wheel. So, you stand in the wheel. So here you have maximum distribution. Objects with more mass further from the axis of rotation has a higher moment of inertia. Think of a dumbbell with weight far apart as moment of inertia.

Next is mass of the object is also a key factor. The overall mass of the object also contributes, but with less influence as compared to that of mass distribution. So, mass distribution is an important factor. So, these are the two factors where moment of inertia depends.

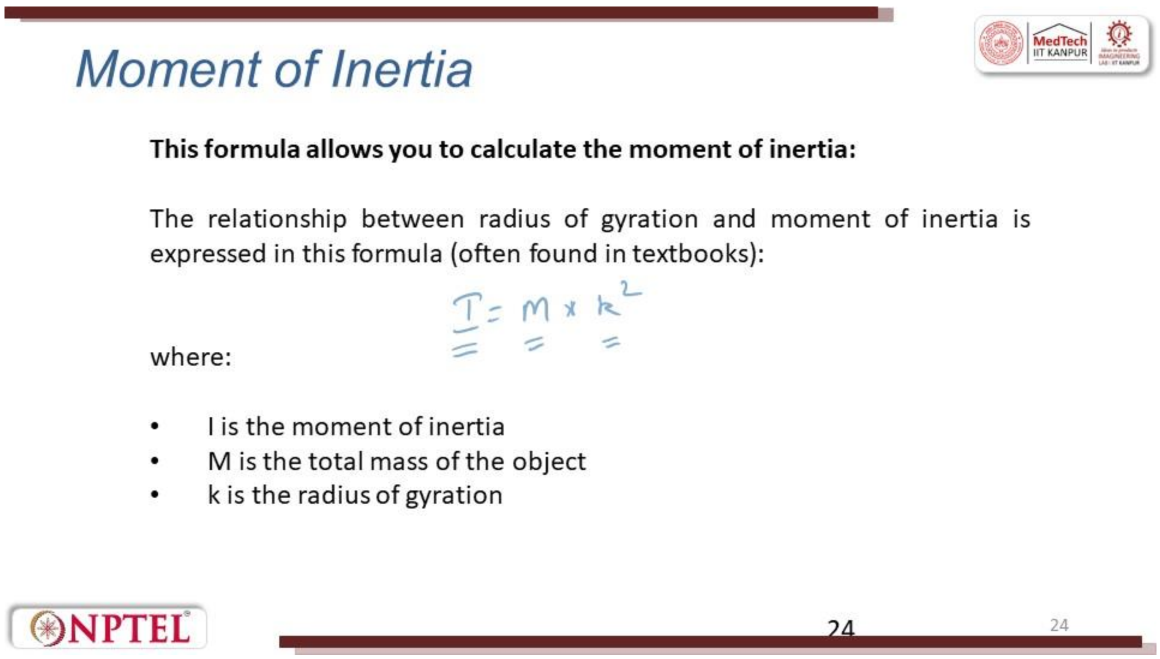


So, Radius of Gyration is an equivalent distance used to simplify calculations involving an object's moment of inertia I. So, this giant wheel is a radius of gyration.

The pedestrian fan what we have, so that is also called as here what this is the axis. So, this center portion is the axis of rotation and the bottom one which is there is called as the center of gravity of the entire pedestal fan. So, the offset whatever is called is called the Radius of Gyration. So, the formula will be

$$K = \sqrt{\frac{I}{M}}$$

The radius of gyration represents the average distance of all the mass particles in the object from the axis of rotation. It takes into account the distribution of mass and not just the total mass. Big difference, distribution of mass and total mass.



So, the relationship between the radius of gyration and the moment of inertia is expressed in the formula

$$I = M \times k^2$$

Where I is the moment of inertia, M is the total mass of the object and k square is the radius of gyration. So, moment of inertia also plays a very very important role. It finds lot of applications.

Moment of Inertia

Applications:

- **Flywheels**: Flywheels use high MOI to smooth out variations in engine power delivery.
- **Machines**: Flywheels regulate speed and maintain consistent power in machines.
- **Pendulums**: MOI affects a pendulum's oscillation period.
- **Stable objects**: MOI is important for designing stable objects.
- **Ice skaters**: Spin faster by pulling arms in, slower with arms out.

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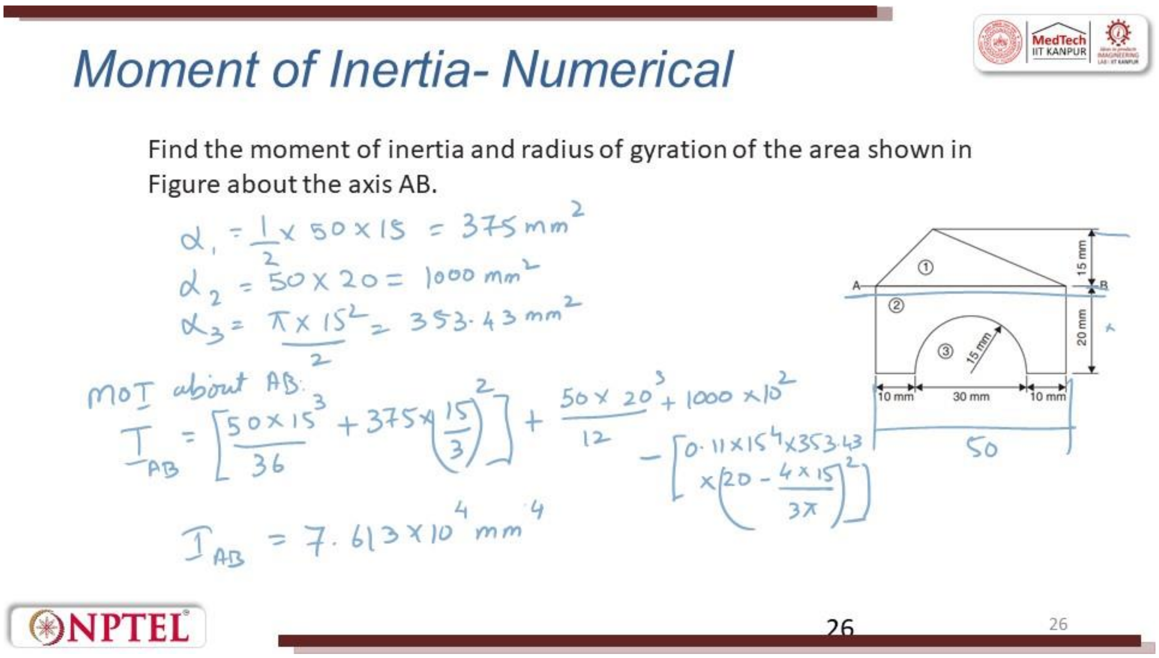
Flywheel use high moment of inertia to smooth out variation in engine power delivery. So, we will have flywhee attached to the engine. So, it is moment of inertia. Machines flywheels regulate speed and maintains consistent power in the machine. For example, when you have a press, printing press or a press to shear some material from the sheet.

So, we always have a flywheel, pendulum, moment of inertia affects a pendulum's oscillation period. Moment of inertia is important for designing stable objects. Ice skaters spin faster by pulling arms in and slower with the arm out. That is also moment of inertia.



So let us look into a simple problem where in which the concept of moment of inertia will be clear. Find the moment of inertia and radius of gyration of the area shown in the figure about the axis AB.

So, there is an axis which is A and B. So, you have roof which is 1, then we have a box which is 2 and then in the box we have subtracted a radius which is 3. The distances are given. So, let us try to solve the problem.

$$\alpha_1 = \frac{1}{2} \times 15 = 375\ mm^2$$

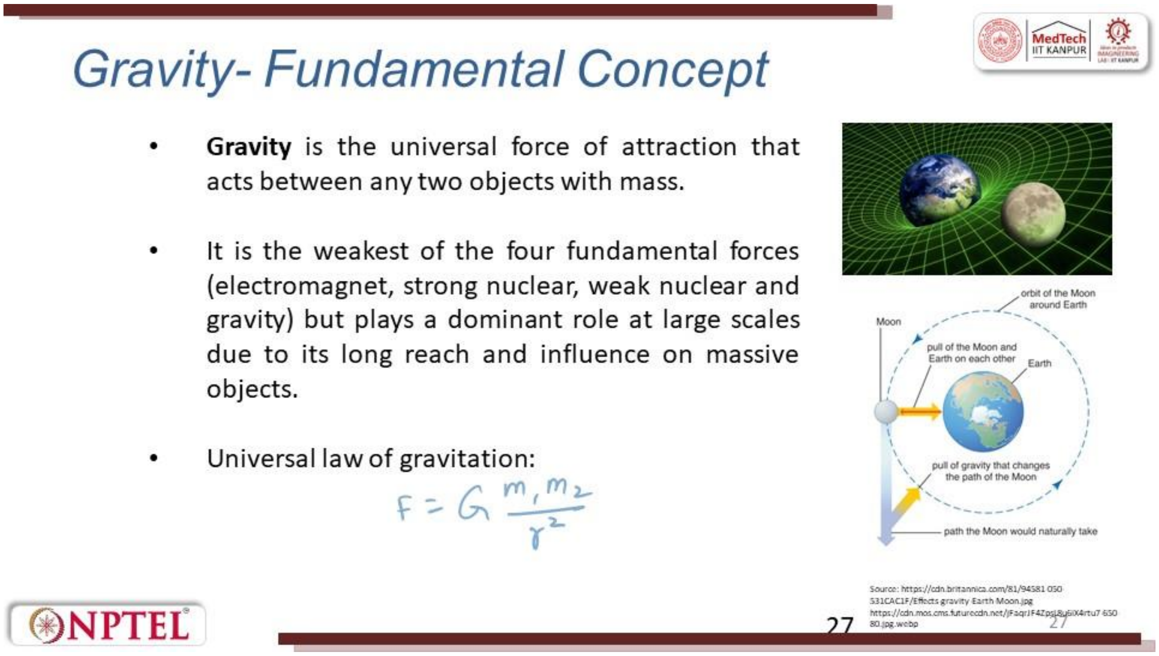
$$\alpha_2 = 50 \times 20 = 1000\ mm^2$$

$$\alpha_3 = \frac{\pi \times 15^2}{2} = 353.43\ mm^2$$

So, now let us try to find out the moment of inertia about the axis MOI about AB.

$$I_{AB} = \left[\frac{50 \times 15^3}{36} + 375 \times \left(\frac{15^2}{3}\right)\right] + \frac{50 \times 20^3}{12} + 1000 \times 10^2 - \left[0.11 \times 15^4 \times 353.43 \times \left(20 - \frac{4 \times 15}{3\pi}\right)^2\right]$$

$$I_{AB} = 7.61 \times 10^4 mm^4$$



The next topic of discussion is going to be gravity. Fundamental concepts. Gravity is the universal force of attraction that acts between any object with mass. It is the weakest of the four fundamental forces. Electromagnetic, strong nuclear, weak nuclear and gravity but plays a dominant role at large scale due to its long range and influence on massive objects. So, the universal law of gravitation is very important.

$$F = G\frac{m_1 m_2}{r^2}$$

Gravity- Fundamental Concept

Gravitational Acceleration (g)

- The constant **acceleration** experienced by an object in a vacuum near Earth's surface when only the force of gravity acts on it.
- This means an object's speed increases by a constant amount each second as it falls.
- Value: Represented by the symbol "g" and has a value of approximately 9.81 m/s2

The acceleration due to gravity is always represented as g. The acceleration due to gravity is a constant. The constant acceleration experienced by an object in a vacuum near the earth's surface when only the force of gravity acts on it. This means an object's speed increases by a constant amount each second as it falls. The value for g is acceleration 9.8 m/s^2.

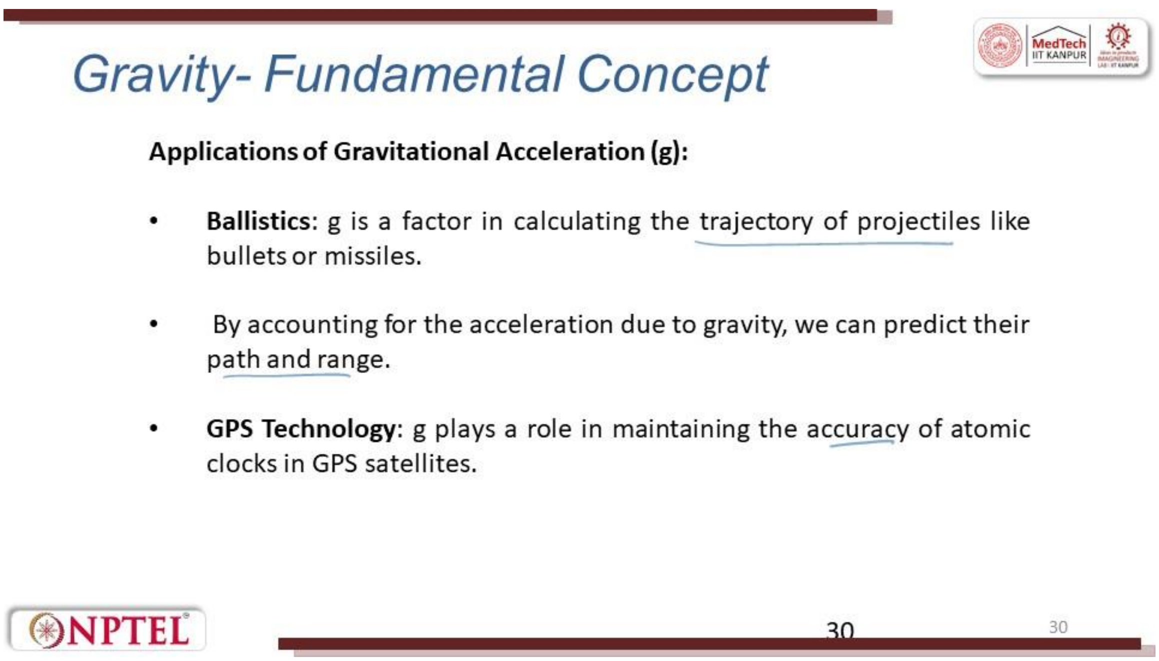
Gravity- Fundamental Concept

Applications of Gravitational Acceleration (g):

- **Spacecraft Design**: g is crucial for calculating the launch velocity needed for rockets and spacecraft to overcome Earth's gravity and escape its atmosphere.
- **Satellite Orbits**: The value of g is used to determine the orbital paths and speeds of satellites around Earth.
- By precisely calculating the balance between Earth's gravity and the satellite's inertia, we can ensure stable and predictable orbits.

So, some of the applications of gravity. Spacecraft design, g is crucial for calculating the launch velocity needed for rockets and spacecrafts to overcome Earth's gravity and escape its atmosphere. So, gravity is very important. Then the satellite orbit, the value of g is used to determine the orbital path and the speed of satellite around the earth. By precisely calculating the balance between the earth gravity and the satellite inertia, we can ensure stable and predictable orbit.

So, I would request you all to see the recent success of our ISRO and understand and appreciate how is the spacecraft design made and how is the satellite orbit design made.



When we are trying to talk about ballistic application, g is a factor in calculation of trajectory of projectile like bullet or missile. By accounting for acceleration due to gravity, we can predict the path and the range. So now in India we talk about air to air, air to water, water to water. Then we try to talk land to land. All these things are missiles. When it moves in a trajectory, gravity plays a very important role.

The GPS technology, g plays a role in maintaining the accuracy of atomic clock in GPS satellites. There also gravity plays a very important role.

Gravity- Fundamental Concept

Gravitational Potential Energy (U):

- A type of stored energy an object possesses due to its position within a gravitational field.
- Imagine gravity pulling the object down, like a stretched spring. The higher the object is positioned.
- The more potential energy it has, similar to a tighter spring storing more energy.

$U = mgh.$

- U: Gravitational Potential Energy (Joules)
- m: Object's mass (kilograms)
- g: Gravitational acceleration (≈ 9.81 m/s²)
- h: Object's height above a reference point (meters)

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In gravity, we have gravitational potential energy (U), a type of stored energy an object possesses due to its position within a gravitational field. Imagine gravity pulling the object down like a stretched spring. The higher the object is positioned.

The more potential energy it has, similar to the tighter spring storing more energy. Energy, the $U = mgh$. U is the gravitational potential energy in joules, m is in mass, gravitation g and h is the object height above the reference point.

Gravity- Fundamental Concept

Applications of Gravitational Potential Energy (U):

Energy Generation:

- **Hydroelectric Dams**: Use the potential energy of stored water to generate electricity.
- **Roller Coasters**: Gravitational potential energy at the top is converted into kinetic energy for thrilling rides.

Energy Storage:

- **Pumped Storage Hydro**: Acts like a giant battery, storing excess energy by pumping water uphill during low-demand periods. When electricity demand is high, the water is released to generate electricity.

So, the application of gravitational potential energy U is equal to energy generation where in which we have hydroelectric dams. Use the potential energy of storing water to generate electricity, Roller coaster.

The gravitational potential energy at the top is converted into kinetic energy for thrilling rides. So that is also gravitational potential energy U. The energy storage pumped storage hydro acts like a giant battery storing excess energy by pumping water uphill during low demand period. When electricity demand is high, the water is released to generate electricity. So, you can see gravitational potential energy plays a very important role in several applications.

Gravity- Numerical

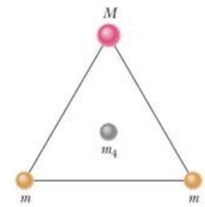
As seen in Figure two spheres of mass m and a third sphere of mass M form an equilateral triangle, and a fourth sphere of mass m4 is at the center of the triangle. The net gravitational force on that central sphere from the three other sphere is zero.

(a) what is M in terms of m?

(b) If we double the value of m4, what then is the magnitude of the net gravitational force on the central sphere?

(a) The distance between any of the spheres at the corners & the sphere at the center is

$$\gamma = \frac{1}{2}\cos 30^\circ = \frac{1}{\sqrt{3}}$$



Source: www.toppr.com/ask/question/as-seen-in-fig-1336-two-spheres-of-mass-m-and-a-third-sphere-of/

As seen in the figure, two spheres of mass m and m and a third sphere of mass m form an equilateral triangle. The fourth sphere of mass m4 is at the center of the triangle. The net gravitational force on that central sphere from the three other spheres is zero. What is m in terms of m? That we should see. Then if we double the value of m4, what then is the magnitude of the net gravitational force on the central sphere?

So, let us try to solve this problem. The first part of the problem is we are trying to solve the distance between any of the spheres at the corners and the sphere at the center is

$$r = \frac{1}{2}Cos30° = \frac{1}{\sqrt{3}}$$

Gravity- Numerical

where l is the length of one side of the equilateral triangle.
The net ↓ contribution caused by the two bottom most spheres (each of mass m) to the total force on m_4 has magnitude.

$$2F_y = 2\frac{Gm_4 m}{r^2}\sin 30^\circ = 3\frac{Gm_4 m}{l^2}$$

This must equal the magnitude of the pull from M, so

$$2F_y = \frac{Gm_4 m}{(l/\sqrt{3})^2} = 3\frac{Gm_4 m}{l^2} \Rightarrow m = M$$

(b) m_4 cancels, then the amt of mass in the centre of sphere is not relevant to the problem.

Net force = 0

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Where L is the length of one side of the equilateral downward contribution caused by the two bottom most spheres each of mass m to the total force on m4 has magnitude

$$2F_y = 2G\frac{m_4 m}{r^2}Sin30° = 3G\frac{m_4 m}{l^2}$$

$$m = M$$

So, this is for problem a and for problem B is since m4 cancels in the last step then the amount of mass in the center of sphere is not relevant to the problem. So, $Net\ Force = 0$, ok. So, moving Gravity in orbital mechanics.

Gravity in Orbital Mechanics

Kepler's Laws: These three laws describe the elliptical motion of satellites around a central body.

- **Elliptical Orbits**: Paths are ellipses, not circles.
- **Equal Area Law**: Satellite sweeps equal area in equal time intervals.
- **Orbital Period vs. Distance**: Farther orbits take longer.

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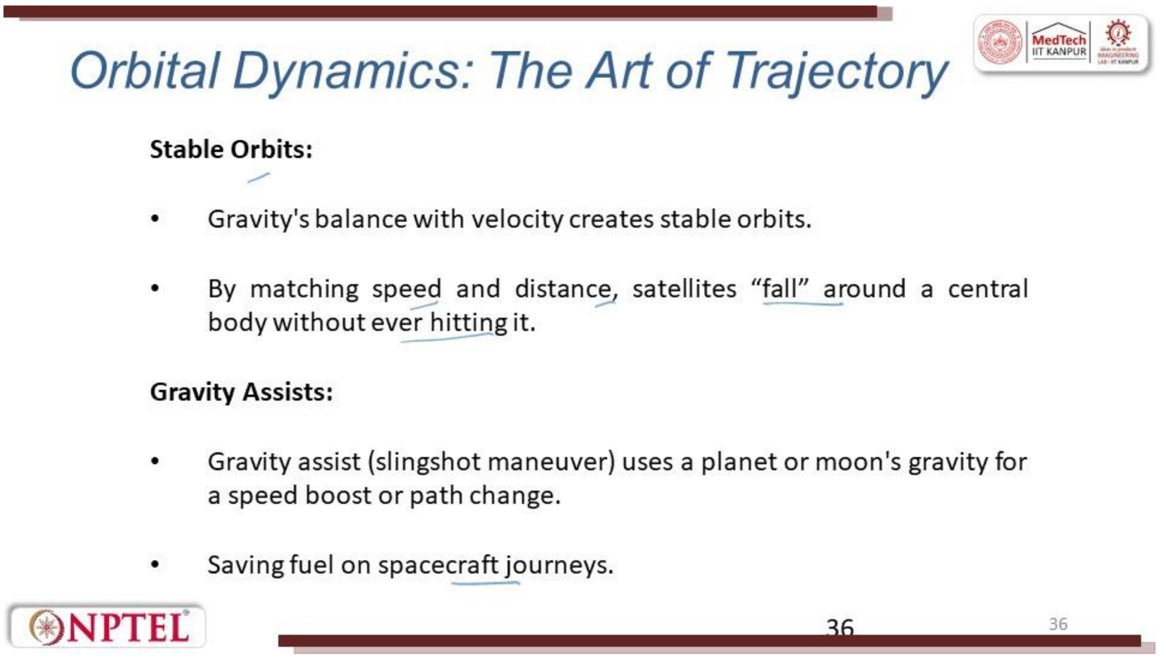
So, we have Kepler's law. These three laws describe the elliptical motion of satellites around a central body. Elliptical orbit, path or ellipse not circle. Equal area law.

Satellite sweeps equal area in equal time intervals. Orbital period versus distance interval. Further orbits take longer. So, these are the three laws described in Kepler's law.



Moving further in orbital dynamics, the art of trajectory, we have stable orbits, we have gravity assist. Stable orbits are gravity balanced with velocity creates stable orbits. By matching speed and distance, the satellites fall around a central body without ever hitting it. So stable orbit, you play with this. The gravity balance with velocity which creates a stable orbit. So, moment you increase or decrease the velocity, you can try to pull it.

By matching speed and distance, satellites fall around a central body without ever hitting it. The gravity assist uses a planet or moon's gravity for a speed boost or path change, here in which it saves fuel on the spacecraft journey. So here you see how the gravity plays an important role in orbital mechanics.

Orbital Mechanics & Orbital Dynamics

Real-World Example:

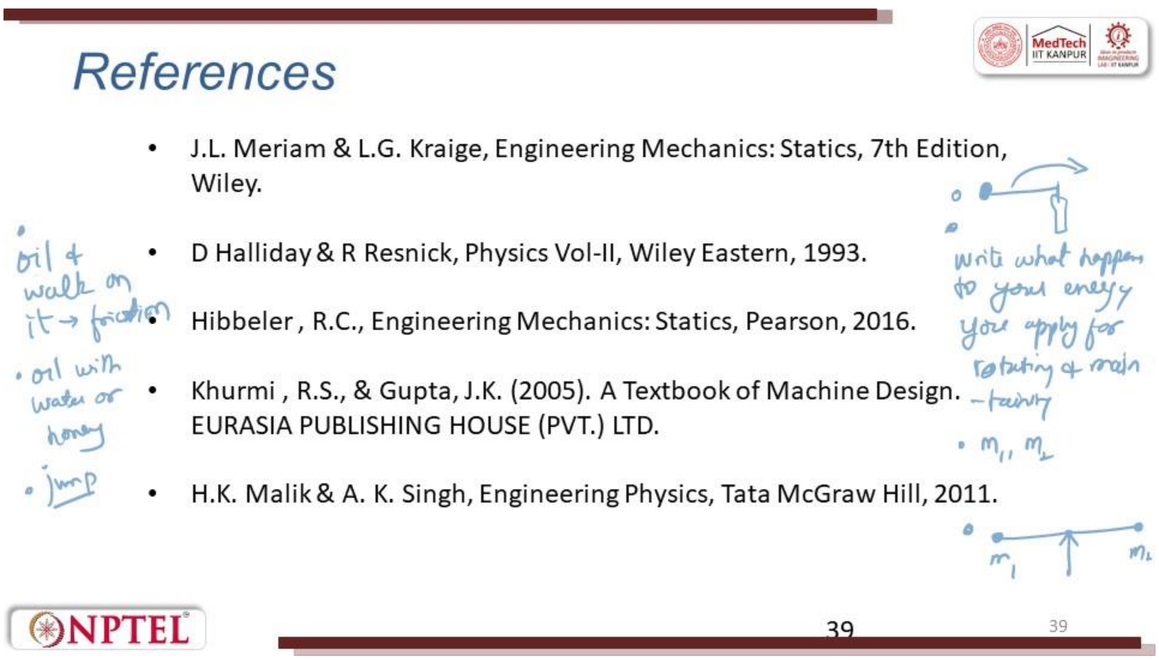
- The Voyager 1 spacecraft famously utilized multiple gravity assists from Jupiter and Saturn to propel itself farther into interstellar space, a testament to the power of orbital mechanics.
- By harnessing the principles of gravity and orbital mechanics, we can design efficient and innovative space missions, furthering our exploration of the cosmos.

Real life examples are Voyager 1 spacecraft, famously utilized, multiple gravity assists from Jupiter and Saturn to propel itself further into interstellar space, a testament to the power of orbital mechanics. By harnessing the principle of gravity and orbital mechanics, we can design efficient and innovative space missions, furthering our exploration of the cosmos. So, gravity and orbital mechanics play a very important role in orbital mechanics.

To Recapitulate

- Why are the topics of friction, lubrication, moment of inertia, and gravity important in mechanical engineering?
- What is the difference between static friction and kinetic friction?
- What methods of lubrication are commonly used, and in what scenarios are oil, grease, and solid lubricants preferred?
- What is the moment of inertia, and why is it significant in rotational motion?
- What is the universal law of gravitation, and how does it describe the force between two masses?
- How does gravitational acceleration affect free-falling objects, and what is its value on Earth?

To recapitulate what we studied in this lecture, why are the topics of friction lubrication moment of inertia and gravity important for mechanical engineers? What is the difference between static friction and dynamic friction? What methods of lubricants are commonly used? What is moment of inertia and why is it significant in rotational motion what is the universal law of friction? And how does it describe the force between two masses? How does gravitation acceleration affect free fall objects? And what is the value on earth?



Before I conclude, here are the references. I would like to post you some problems for you to consider. The first problem is you try to tie a mass with a string. And you try to hold the string by your hands. Start rotating it. And as and when the speed goes larger and larger and larger, try to write down what happens to your energy you apply for rotating and maintaining.

This will be the maintaining. So, this exercise will give you lot of understanding when the object is rotating and with a mass. Now what do you do is you try to change the mass m1 to m2 and then start doing it. The next one will be tried to balance m1 with a string and in the opposite side you have m2 and start repeating the same exercise. Now you have a detailed analysis of force, energy what you spend and see how does it rotate.

So, this is one part of the exercise. The second part of the exercise is going to be try to pour oil on a small floor and walk So here the first case is you try to take oil and walk on it. Note down what happens to the friction without the oil and with oil. Now next what you do is you change oil with water or honey and try to do the same exercise.

When you do the same exercise, you will start understanding and appreciating what is the influence of friction, what is the influence of viscosity, what is a hydrodynamic lubrication, what is partial lubrication, all these things you will be able to understand. And the next one is you try to jump on the film whatever you created on it and then walk.

So that means to say you pour oil or you pour water on the tile and then you jump on it and then walk. So here what are you trying to do is you are trying to disturb the lubrication film and then you start walking. So please try to do this exercise so that you start enjoying the lecture whatever we have discussed.

Thank you very much.