Basics of Mechanical Engineering-1

Prof. J. Ramkumar

Dr. Amandeep Singh

Department of Mechanical Engineering

Indian Institute of Technology, Kanpur

Week 03

Lecture9

Stress Strain Curve, Elasticity & Poission's Ratio

Welcome to the next lecture of this course.

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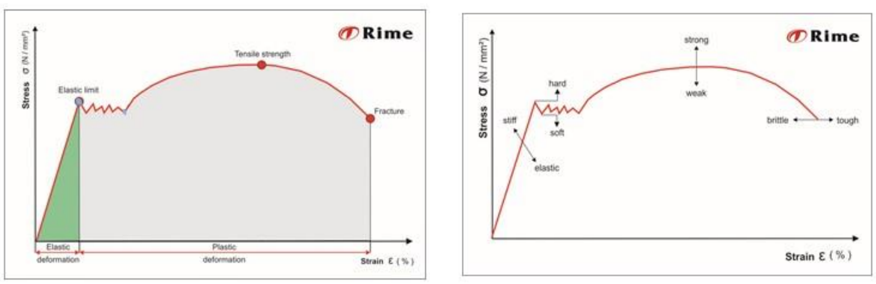
Here in which we will be trying to cover Stress-Strain Curve, Elasticity and Poisson's ratio. In the previous lecture, we were trying to look at Stress-Strain, relationship between Stress and Strain. We were looking into elastic limits. We were looking into plastic limits which was very much required for you to understand if you want to break a material, if you want to deform a material, these properties are very important. Now, in this lecture, we will have little more understanding of the same. So, the content of this lecture is going to be stress-strain curve, elasticity, then we will try to see Hooke's law,

Last class also we saw a relationship stress equal to E into strain where E is the Young's modulus. That is nothing but Hooke's law. Then Proportionality limit and Elastic limit, Plastic Deformation, Ultimate Tensile Strength and Fracture Point. Ultimate tensile strength is the maximum load it can take is ultimate tensile strength wherein which the material has already started deforming. Fracture point is a point where the material yields off.

If you try to take a glass, if you try to throw a stone, it is nothing but an impact load. When the impact load is applied, immediately it fractures. So, fracture point, Poisson's ratio, then real world application and finally we will do a recap.

Stress-Strain Curve



- The stress-strain curve is a graphical representation of the relationship between the stress applied to a material and the strain it experiences.
- This curve provides valuable insights into a material's behavior under load.

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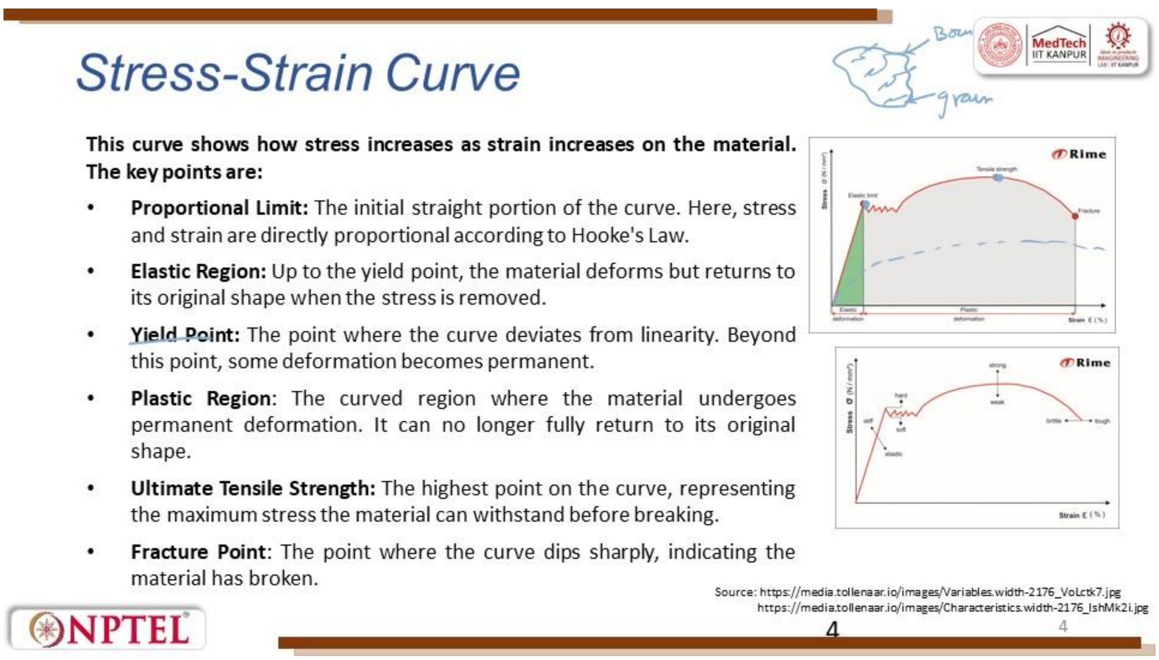
So, this is the Stress-Strain graph for a typical material. So, you have a stress in the y-axis and strain in the x-axis. We have already seen the elastic deformation which is the elastic limit. After the elastic limit starts the plastic. So plastic limit. So elastic limit point and then you have a fall down, fall up. Then you see a yield started increasing. So, this is a yield point.

From here, it starts, the yield point keeps on increasing and it goes to a maximum tensile strength or ultimate tensile strength and then onwards starts the failure. So, this is the plastic deformation and this is the elastic deformation. So here, we try to make it for hard material, this will be there. And for soft, it can go for this small up and down motion, peaks are there. So, this material, when the ultimate tensile strength goes down, it is said to be a weaker material.

When it goes up, it is said to be a stronger material. If it goes in this direction, it is the toughness of the material. And if it goes in this direction, it is going to be a brittle material. So, with this, you can try to talk about the material response to the forces. So, the stress-strain curve is a graphical representation of the relationship between the stress applied to a material and the strain what it experiences.

This curve provides valuable insights into a material behavior under the load. It is as though ECG taken or a pulse taken from a patient. So, we try to infer the health. In the same way by doing so, we try to infer what is the material response which we can get on applying load.



So, these are some of the points in the stress-strain curve. So, the stress-strain curve shows how stress increases as strain increases on the material. The key points are proportionality limit. Proportionality limit is the initial straight portion of the curve here. Stress and strain are directly proportional according to Hooke's law. So, that means to say here within the elastic limit.

Elastic region up to the yield point. From here starts the yield Irreversible. So up to the yield point, the material deforms but returns to its original shape when the stress is removed. This is elastic. Right. Yield point, the point where the curve deviates from the linearity.

Beyond this point, some small deformation which is permanent is left on the material, yield point. When I want to bend a material, I will try to keep applying force till a point such that it starts bending. That point is called as yielding. So plastic region, once it crosses an elastic limit, it gets into the plastic region. So, in plastic region, the deformation retains the curved region where the material undergoes permanent deformation, it can no longer fully return to its initial shape.

The ultimate tensile strength is the highest point on the curve representing the maximum stress the material can withstand. Till here it does not deform and after here it deforms and when it deforms, it keeps increasing. This will be the highest point after which the material gets into a failure. So how does all these things happen? If you go back to the residual stress, we saw type 3 wherein which you inside a grain there are dislocations, right.

So, type 2 and type 3 residual stress, you will have the dislocations which are inside a grain. All these dislocations will be moving and it will be trying to hit at the grain boundary. So, this is a grain, right. This is the grain boundary. So, it will hit at the grain boundary.

So, once it is trying to hit at the grain boundary, these dislocations will be arrested. Because of this arrest happening, the ultimate tensile strength increases. If you want to reduce the ultimate tensile strength, if you expand the grain, then there is lot of space for the dislocation to move, the yield is expanded. So, once you have touched the ultimate tensile strength, the next is going to be the failure. The failure is the fracture point, the point where the curve dips sharply indicating the material has broken down.

So, in your engineering at your college, you will be trying to do a tensile test. In a tensile test, we will try to get the response. Typically, we try to do a tensile test for a mild steel. We will try to do for ceramic material. We will also try to do a polymer material.

Or you can try to take a ductile material like aluminium. So, in aluminium, what happens? The response will go like this.

Stress-Strain Curve

Stress

- Stress is basically how hard you're pulling on the rope.
- The stronger you pull (more force), the higher the stress.
- It's like the intensity of the squeeze you're putting on the rope.

Strain

- Strain is how much the rope stretches because of your pulling.
- The more you pull, the longer the rope gets compared to its original length.
- It's like how much the rope gives in response to your pulling force.

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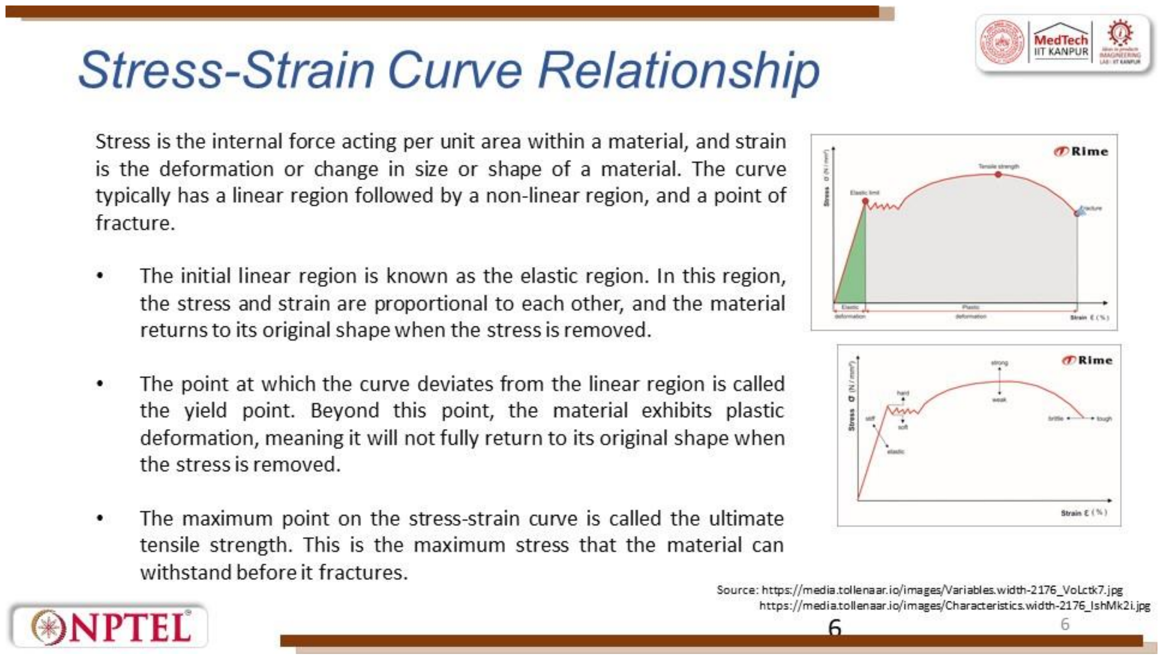
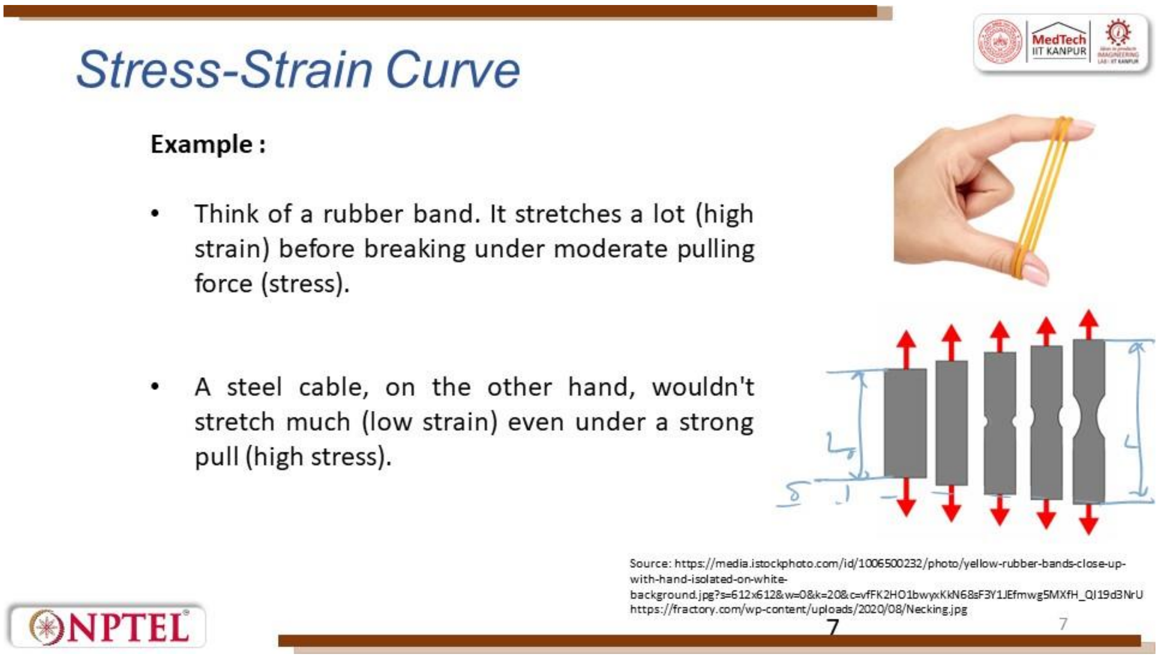
Stress is basically how hard you are pulling on the rope. The stronger you pull, the higher is the stress. It's like the intensity of squeeze you are putting on the rope. Strain is how much rope stretches because of your pull. The more you pull, the longer the rope gets compared to its original length. It's like how much the rope gives in response to your pulling force. So, this is strain and that is stress.



So, the stress is the internal force acting per unit area within a material. Strain is the deformation or change in size or shape of the material. The curve typically has linear region followed by a non-linear region and a point of fracture. The linear region is known as elastic region. In this region, the stress-strain are proportional to each other and the material returns to its original shape when the stress is removed.

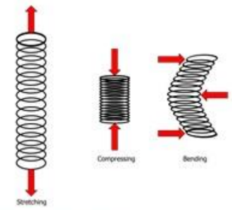
The point at which the curve deviates from the linear region is called as the yield point. Beyond this point, the material exhibits plastic deformation, meaning it will not fully return to its original shape when the stress is removed. The maximum point on the stress-strain curve is called as the ultimate tensile strength. This is the maximum stress that the material can withstand before its failure.

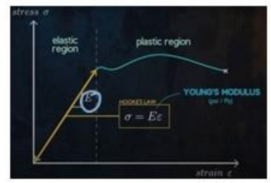


Some of the examples, think of a rubber band, it stretches a lot before breaking under moderate pulling force. Think of a steel cable, on the other hand, wouldn't stretch much even when a strong pull is given. So, here you can see the last class, I explained volume of constancy. This is the constant volume when you try to pull, there is reduction in the area, but the length compensates. So that is why you see here this will be the L naught and this will be the final L. The difference will be noted down here. This is the del.

Elasticity

- Elasticity is the tendency of a material to deform (bend, stretch, or compress) under an applied stress (force per unit area) and then return to its original shape and size once the stress is removed.
- It's like a rubber band that snaps to its original form/shape after you tell it.





- Material stretches or compresses under stress depends on its elastic properties. Different materials have varying degrees of elasticity.
  For instance:

Source: https://mmerevise.co.uk/app/uploads/2022/10/three_springs-2048x1803.png.webp
https://i0.wp.com/engineersblog.net/wp-content/uploads/2023/02/What-is-stress-Strain-and-modulus-of-elasticity-of-steel..jpg?w=613&ssl=1

Elasticity is very important. Elasticity is the tendency of a material to deform. Deformation can happen like bending, stretching or compression. Under an applied stress and then return to its original shape. This is a spring.

You pull the spring. You release it, it will come back to its original position. So, this is tensile, stretching. This is compression. And this is bending.

The material stretches or compresses under stress depending on the elastic property. Different materials have varying degrees of elasticity. So, in this figure, we have introduced a new terminology called E within the elastic limit. The slope what you take talks about the E. E is the Young's modulus of the material.

So, if you look at it, it is like a rubber band that snaps to its original shape, it is always like a spring will also work like a rubber band, right. So, it will all to its original shape original form or shape after you let it go.

Elasticity

- Rubber bands are highly elastic.
  They can deform significantly and return to their original shape.
- Steel is moderately elastic.
  It can deform a small amount elastically but will eventually bend permanently if enough stress is applied.
- Glass is brittle and not very elastic.
  Even a small amount of stress can cause it to fracture (break) without any chance of returning to its original shape.

Source: https://media.istockphoto.com/id/1006500232/photo/yellow-rubber-bands-close-up-with-hand-isolated-on-white-background.jpg?s=612x612&w=0&k=20&c=vfFK2HO1bwyxKkN68sF3Y1JEfmwg5MXfH_QI19d3NrU=

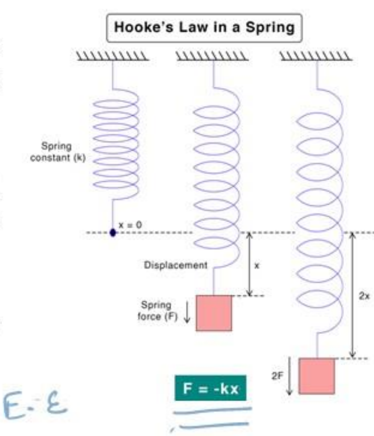
Rubber band are highly elastic, they can deform significantly and return to its original shape. Steel is moderately elastic. It can deform a small amount elastically but will eventually bend permanently if enough stress is applied.

Glass is brittle and not very elastic. Even a small amount of stress can cause it to fracture without any change of returning to its original shape.

Hooke's Law

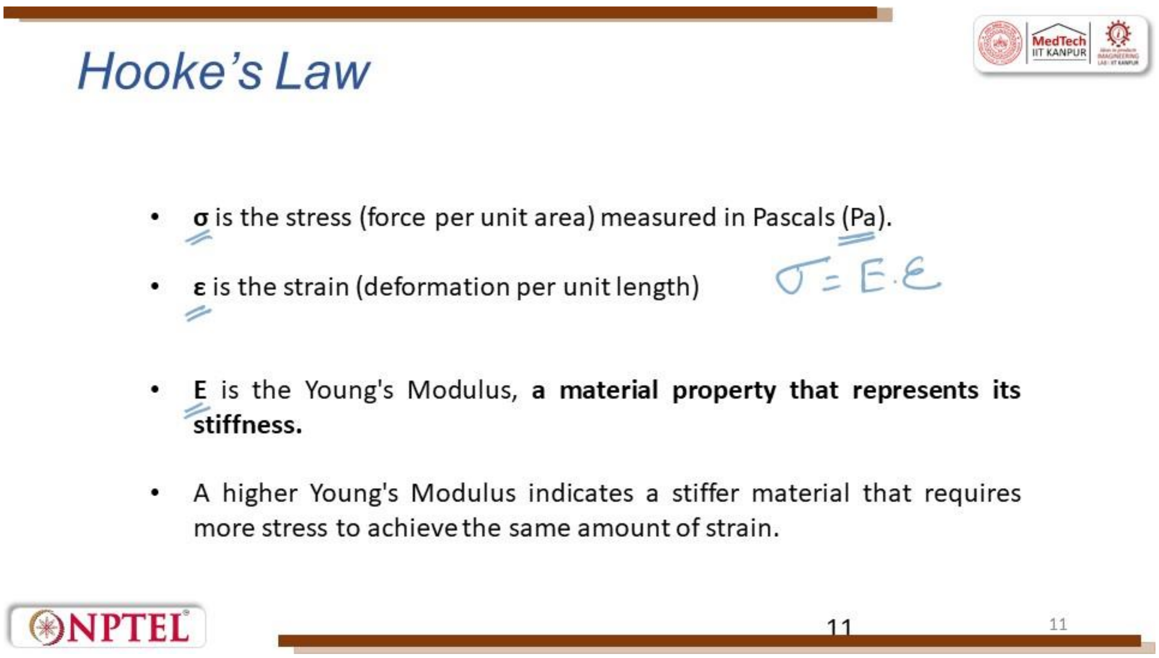
- Hooke's Law describes the relationship between stress and strain within the elastic limit of a material.
- The elastic limit is the maximum stress a material can withstand before it starts to deform permanently.
- Hooke's Law states that stress is directly proportional to strain.

This can be expressed as $\sigma = E \cdot \varepsilon$



Source: https://www.sciencefacts.net/wp content/uploads/2021/07/Hookes Law.jpg

We have already seen Hooke's law. Hooke's law describes the relationship between stress and strain within the elastic limit. This is very important. Stress and strain, if you take in plastic, it is not Hooke's law. The elastic limit is the maximum stress a material can withstand before start to deform permanently. The Hooke's law states that stress is directly proportional to strain. So, the spring constant, when it is expanded, the force there is a displacement. When you increase the force, you can see more displacement f is minus k into x. So here this can be expressed as $\sigma = E.\varepsilon$.



So, the Hooke's law says that σ Young's modulus. So, it is $\sigma = E.\varepsilon$. So, it is in Pascal. So, we always talk in terms of mega Pascals.

Proportional Limit and Elastic Limit

Proportional Limit:

- The proportional limit is the maximum stress up to which stress and strain are directly proportional.
- This means that the material obeys Hooke's Law, and the stress-strain curve is a straight line.
- Beyond the proportional limit, the stress-strain relationship becomes nonlinear, and the material begins to deform plastically.

Proportionality limit is the maximum stress up to which stress and strain are directly proportional. This means that the material obeys Hooke's law and the stress-strain curve is a straight line. Beyond the proportionality limit, the stress-strain relationship becomes non-linear and the material undergoes plasticity.

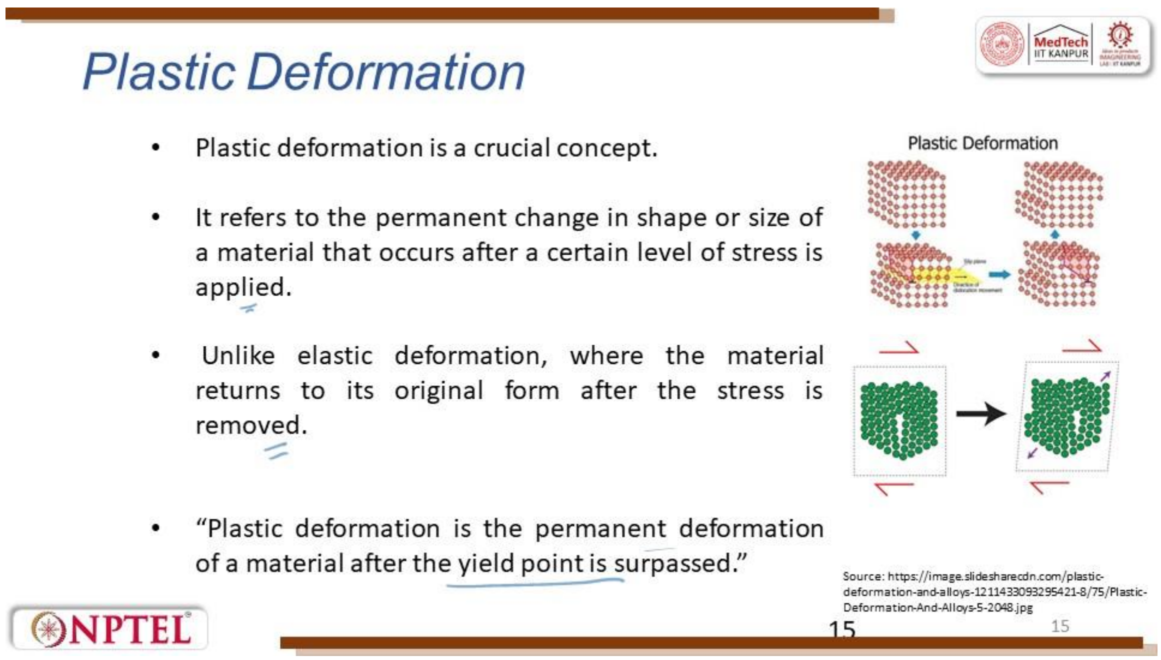
Proportional Limit and Elastic Limit

Elastic Limit:

- The elastic limit is the maximum stress that a material can withstand without permanent deformation.
- If stress exceeds the elastic limit, **some of the deformation will be permanent even after the stress is removed.**
- The elastic limit is often slightly higher than the proportional limit, **but for many engineering materials, the difference is negligible**.

Elastic limit is the maximum stress that a material can withstand without permanent deformation. If stress exceeds the elastic limit, some of the deformation will be permanent even after the stress is removed. The elastic limit is often slightly higher than the proportionality limit.

So, what we are trying to say is you will have this is the proportionality limit and above it you will have an elastic limit. This is PL, this is EL. So, the elastic limit is often slightly higher than the proportionality limit but for many engineering materials, the difference is neglected. So, this is proportionality, stress proportional to strain.

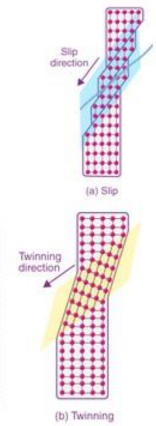


Plastic deformation, it is a crucial concept. It refers to the permanent change in shape or size of a material that occurs after a certain level of stress is applied. So, you can see here, this is in plastic deformation, you see here when you apply load, there is a slipping happening between the stacked atoms, right. So, there is a yield which happens and then the material is moved. So, here if you see, this is a material, there is a shearing. You can see the material is sheared in one direction and the bottom is kept in the other direction.

So, unlike elastic deformation where a material returns to its original form after the stress is removed is plastic deformation is a permanent deformation of material after the yield point is suppressed.

Plastic Deformation

- Plastic deformation occurs beyond the yield point of a material.
- The yield point is the point on the stress-strain curve **where the material transitions from elastic to plastic behavior.**
- Once a material undergoes plastic deformation, **it will not return to its original shape even after the stress is removed.**
- The amount of plastic deformation a material can withstand before failure varies **depending on the material's properties.**



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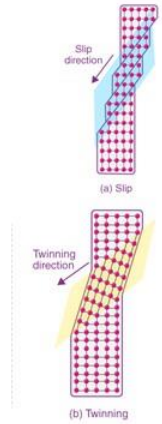
The plastic deformation generally occurs beyond a yield point. This yield point is a point on the stress-strain curve where the material transitions from elastic to plastic behavior. So, from elastic to plastic, once a material undergoes plastic deformation, it will not return to its original shape even after the stress is removed. So here if you see that there is a slip which is happening and there is a twin direction which is there.

The amount of plastic deformation, a material can withstand before failure depends upon the material property.

Plastic Deformation

Examples of Plastic Deformation:

- Bending a metal rod beyond its yield point will cause it to remain permanently bent.
- Squeezing a clay ball will cause it to permanently deform into a new shape.
- The formation of dents in a car bumper during a collision is an example of plastic deformation.



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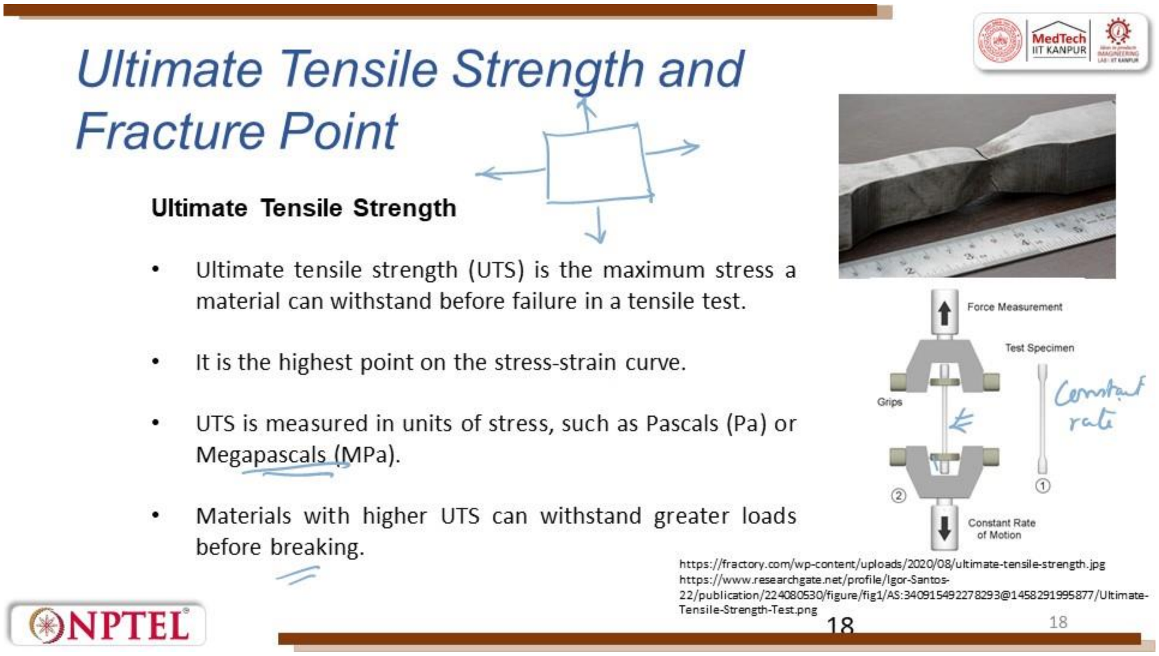
Examples of plastic deformation, bending a metal rod beyond its yield point will cause it to remain permanently bent. Squeezing a clay ball which causes it to permanently deform into a new shape. The formation of dents in a car bumped during a collusion is an example of plastic deformation. So, in plastic deformation, they will try to have as much energy as it can try to store or it can try to take such that the shock of the bump or the accident is not transmitted to a passenger who is driving the car.

So, this is a typical universal testing setup. In this universal testing setup, a standard ASTM sample is held. Depending upon the material, if it is a polymer material, there is a standard, you make it. If it is steel, you make a dog bone sample. So, you make the sample and you load it in the universal tensile testing machine.

You pull it up. So now you see the response. The movement of the top and bottom is done at a constant rate. This is only uniaxial, right, along one axial. Today, you have biaxial.

You put a flat plate and then you pull it. So, you have biaxial tensile testing machine. And you also have triaxial tensile testing machine. Why are we moving from uniaxial to biaxial? Biaxial to triaxial is if you do a triaxial tensile testing machine, you get a much better data point which is close to reality. Universal or uniaxial will give you an approximation.

But there will be an error, biaxial will be much closer. So, you use this and then you try to pull. You can keep a wire here, you can keep a polymer here, you can keep a metal here, you can keep a ceramic here. You try to stretch it. Suppose if the stretching happens very large, if the stretching happens very large, then what you do is you try to do other tests.

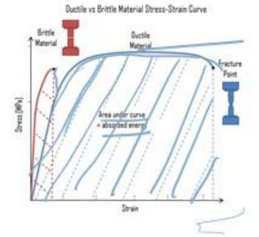


Ultimate Tensile Strength (UTS) is the maximum stress a material can withstand before failure in a tensile test. It is the highest point on the stress-strain curve. It is always reported for a typical metal in Mega Pascal (MPa). Materials with higher UTM can withstand greater loads before failure. After you cross the ultimate tensile strength point, you move towards the failure.

Ultimate Tensile Strength and Fracture Point

Fracture Point

The fracture point is the point on the stress-strain curve where the material breaks.



- Beyond this point, the material cannot sustain the applied stress and fails.
- In some materials, the fracture point may coincide with the ultimate tensile strength, while in others, it may occur at a slightly lower stress level.

Source: https://www.nuclear-power.com/wp-content/uploads/2019/11/Stress-strain-curves-Ductile-vs-Brittle-Material-300x273.png

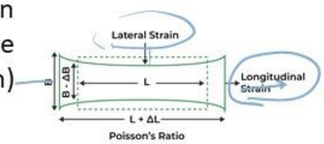
So, the failure will always happen at the fracture point. The fracture point is a point on the stress-strain curve where the material breaks. Beyond this point, the material cannot sustain any applied stress, it fails. In some materials, the fracture point may collide with the ultimate tensile strength, while in others it may occur at slightly lower stress level. So, what are we saying is it can happen along this?

Beyond this point, the material cannot sustain the applied stress and fails. So, if you look into this, so here you see there is an area under the curve. This area under the curve you see here, it says energy absorption. So, when the material is undergoing a shock or impact, the material response, the energy will be absorbed in this area. So that means to say, it will cross elastic and it will get into the plastic region and it will go up to fracture.

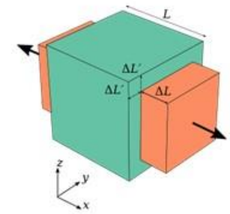
Within this, there is an energy absorbed by the metal. So, if you are planning for impact load, so we will always try to see the maximum energy is absorbed by the material. Cushion falls in this or if you have a rubber, it falls in this.

Poisson's Ratio

"Poisson's ratio is the ratio of the relative contraction in a transverse direction (lateral strain) to the relative elongation in the axial direction (longitudinal strain) when a material is stretched."



- Poisson's ratio provides insight into a material's compressibility.
- A higher Poisson's ratio indicates a greater degree of lateral contraction when the material is stretched.
- Conversely, a lower Poisson's ratio indicates less lateral contraction.



Source: https://media.geeksforgeeks.org/wp-content/uploads/20230725104917/Poisson's-Ratio.png
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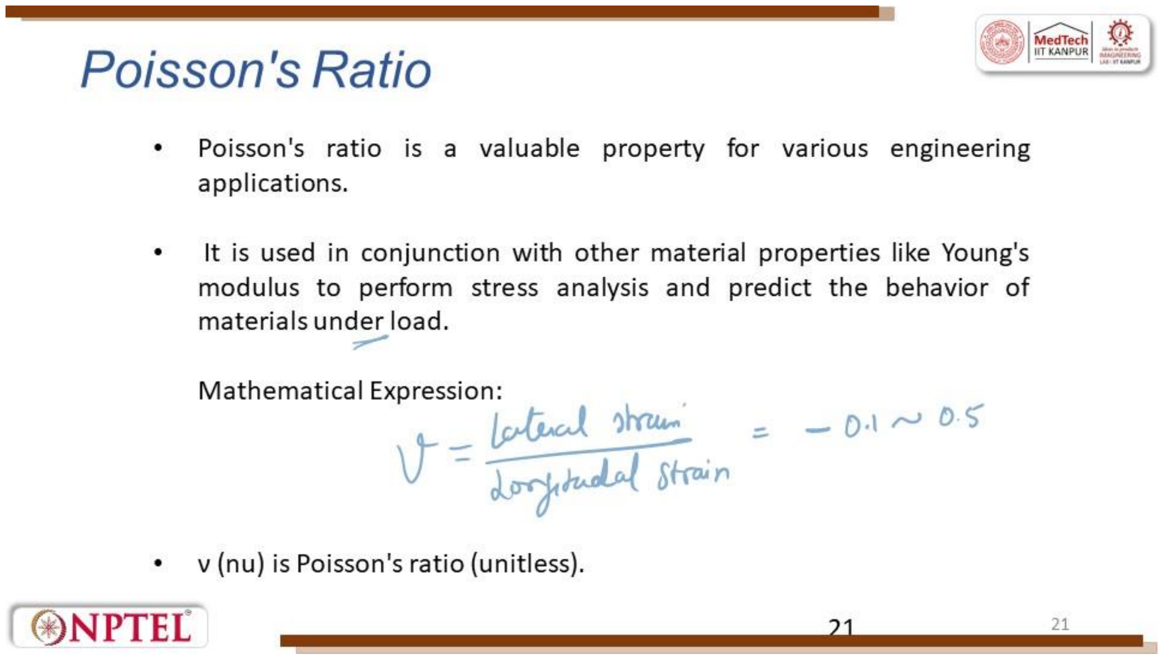
The next concept is Poisson's ratio. Poisson's ratio is a very important terminology. When you do tensile test, we always ask what is the Poisson's ratio. Poisson's ratio is the ratio of the relative contraction in a transverse direction to the relative elongation in the axial direction when a material is stretched. So, when a material is stretched, it is stretched like this. When you stretch like this, it is the ratio of the relative contraction in the transverse direction. Transfers lateral direction to the relative elongation in the axial direction where the material is stretched.

So, it is a ratio between lateral strain divided by longitudinal strain. So, it is always expressed in a terminology called μ. The Poisson's ratio provides insight into the material compressibility. Higher the Poisson's ratio indicates a greater degree of lateral contraction when the material is stretched. Conversely, lower Poisson's ratio indicates less lateral contraction. If it is less lateral contraction, what does it mean? It is going to fail faster.



The Poisson's ratio is a valuable property for various engineering applications. It is used in conjunction with other material property like Young's modulus to perform stress analysis and predict the behavior of the material under load. The Poisson's ratio is expressed as

$$v = \frac{lateral\ Strain}{longitudal\ Strain}$$

It always takes a value in negative for metals. So, it can vary from -0.1, it can go up to 0.5.

Poisson's Ratio

Examples of Poisson's Ratio Values

S. No.	Material	Poisson's ratio
1.	Aluminium	0.330
2.	Brass	0.340
3.	Bronze	0.350
4.	Cast iron	0.270
5.	Concrete	0.200
6.	Copper	0.355
7.	Steel	0.288
8.	Stainless steel	0.305
9.	Wrought iron	0.278

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So here are some of the values for various metals we have displayed. It can take a value you can see from 0.2 to 0.3, 0.27 to 0.33.

Poisson's Ratio

- **Examples of Poisson's Ratio Values**
- **Steel**: 0.288 (moderately compressible) - Steel is a common construction material with a moderate Poisson's ratio of 0.288. This means that when a steel beam is stretched under tension, it will also contract slightly in width. This slight change in dimension is important to consider when designing structures and machines using steel.
- **Rubber**: 0.5 (highly compressible) - Rubber is known for its elasticity and high Poisson's ratio of 0.5. When a rubber band is stretched, it will significantly contract in width relative to its elongation in length. This property makes rubber ideal for applications where sealing and form-fitting are required.
- **Cork**: 0.1 (slightly compressible) - Cork is a natural material with a low Poisson's ratio of 0.1. This indicates that cork is relatively incompressible and maintains its shape well under stress. This property makes cork valuable for applications such as gaskets and vibration dampeners.

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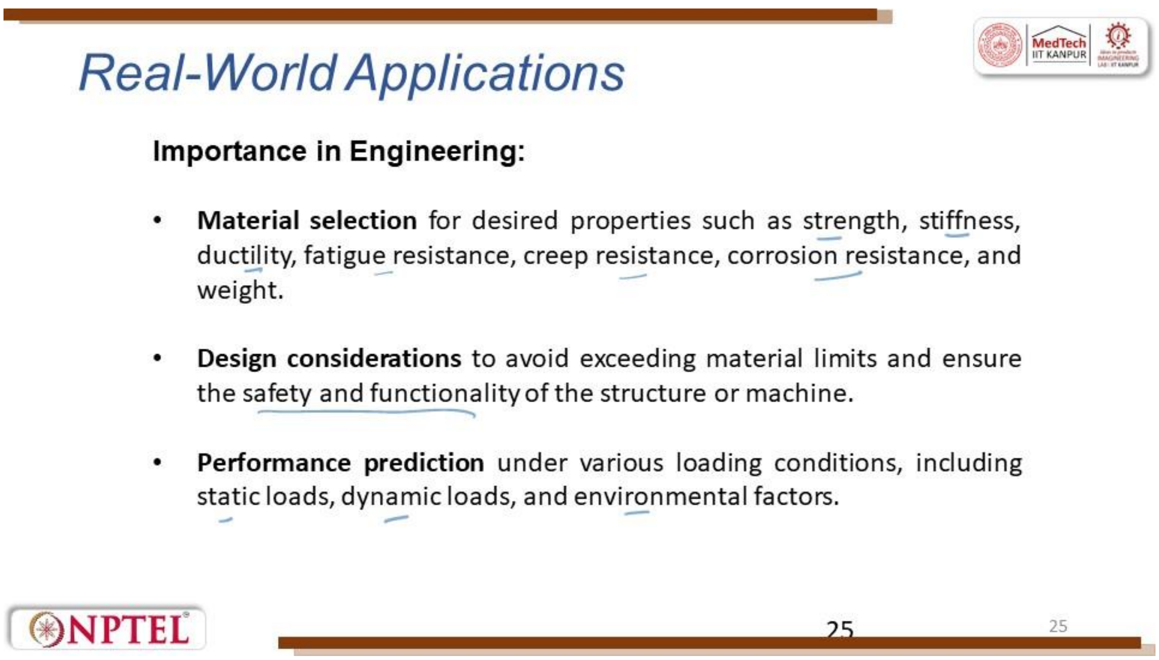
So, examples of Poisson's ratio values, steel it is 0.228, Steel is used commonly for construction material with a moderate Poisson's ratio of 0.8. This means that the steel beam is stretched under tension. It will also contract slightly in the width. This slight change in dimension is important to consider when designing structures and machines using steel. This is very important.

Next is rubber. Rubber, it will try to have 0.5. Rubber is known for its elasticity and high Poisson's ratio of 0.5. When rubber band is stretched, it will significantly contract in width relative to its elongation in length. This property makes rubber ideal for application when sealing and form fitting are required.

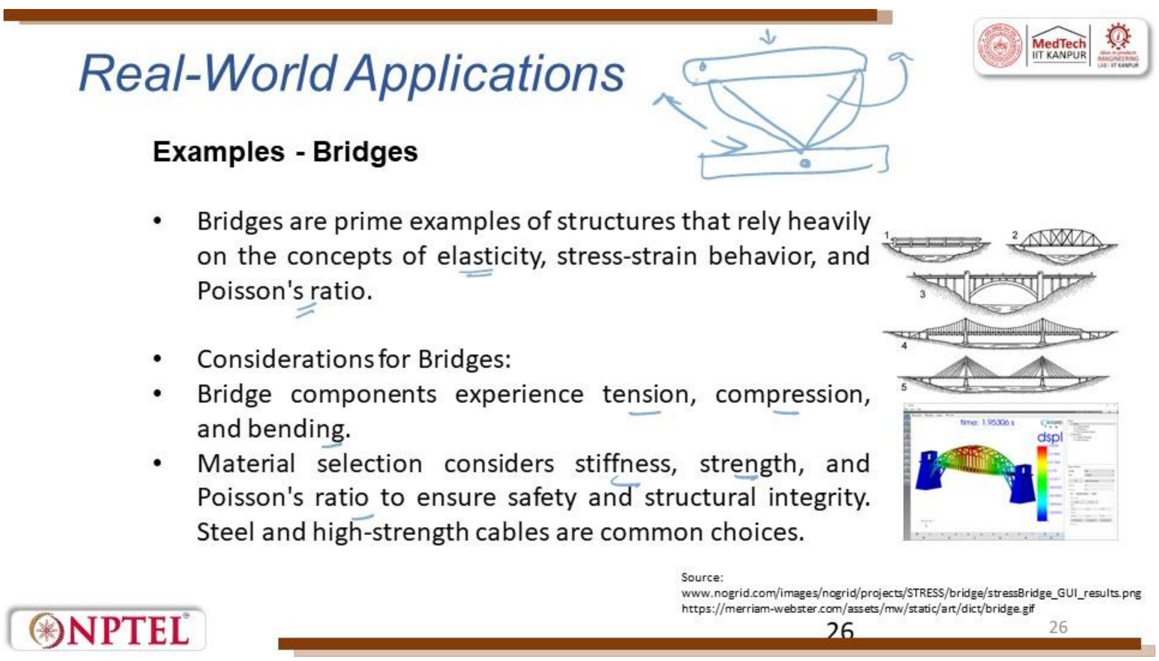
Let us take cork. Cork is a natural material which has a Poisson's ratio of 0.1. This indicates that the cork is relatively incompressible and maintains its shape well under stress. This property makes cork valuable for applications such as gasket and vibration dampers. So how do we choose different materials for application is got out from the Poisson's ratio.

Suppose if there is not much of tensile property loading or compression, then we will always try to use Poisson's ratio for measurement as an advantage.



The real-world applications, the importance in engineering is material selection for desired properties such as strength, stiffness, ductility, fatigue resistance, creep resistance, corrosion resistance and weight. For all these properties are looked for choosing a material for a real time application. Design consideration to avoid exceeding material limits and ensure safety and functionality of the structure, we always try to do material testing.

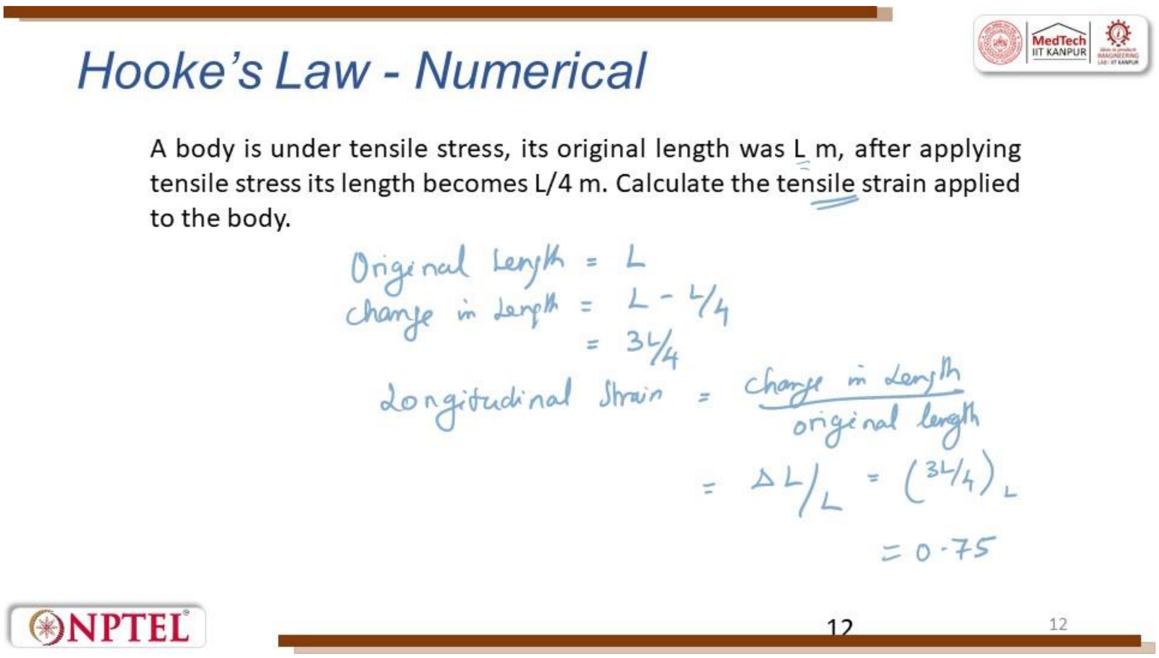
Performance prediction under various loading conditions include static load, dynamic load and environmental load. Static load is you have a building which is constantly standing. Dynamic load is if the building is always disturbed or if there is a wind which blows on and off or if there is a seismic wave happening, then it is a dynamic loading. Environmental factors are corrosion. So, for all these things, material properties are very important.



Let us take a bridge as an example. Bridges are prime example of structures that rely heavily on the concept of elasticity, the stress-strain behavior and the Poisson's ratio. Consideration for the bridge. Bridge components experience tension, compression and bending. What is tension?

When you have a structure like this and it is mounted, so something like this. So, you have a bolt here. So, this is tensile and suppose if this is swinging, then comes the bending and the load when a truck goes on top of it is compression. The material selection considers stiffness, strength and poissons ratio to ensure the safety and the structural integrity of the bridge. Steel and high strength cables are commonly used.

You can see here these are all steel structures. You can also have high strength cables which are used. Cables are nothing but it is a single wire which is twisted. So, you will have a bundle of wires. We form a cable.



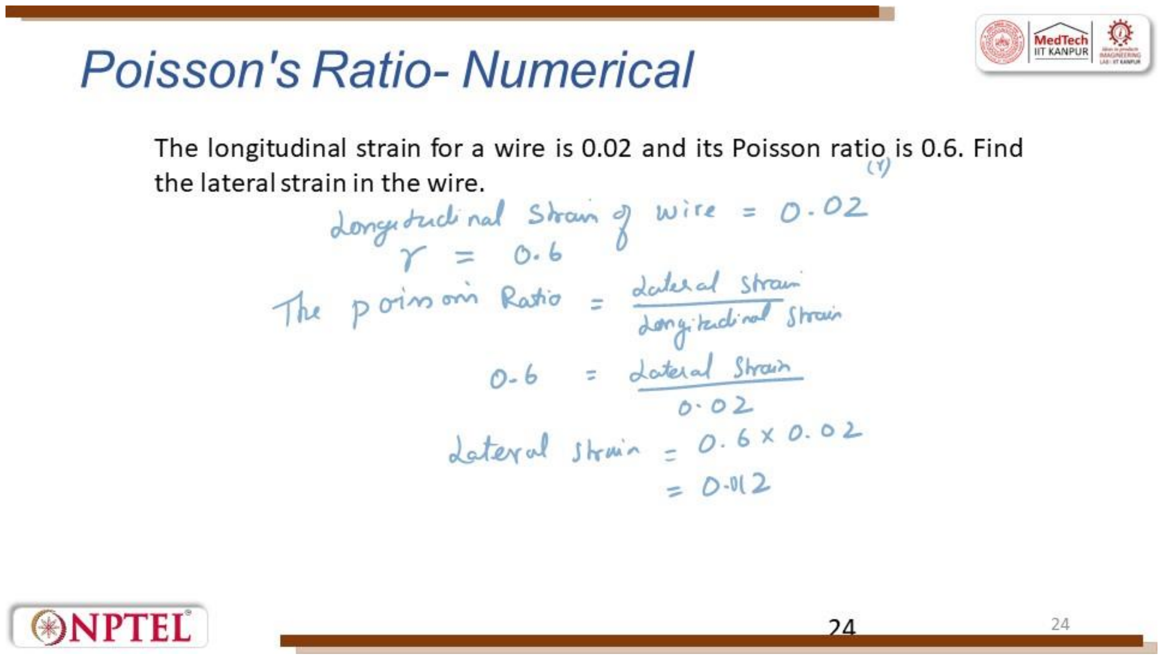
Let us try to solve a problem in Hooke's law. A body is under tensile stress. Its original length was L m. After applying tensile strength, its length becomes L/4 m. Calculate the tensile strain applied to the body.

$$Original\ length = L$$

$$Change\ in\ length = L - \frac{L}{4} = \frac{3L}{4}$$

$$Longitudinal\ Strain = \frac{length\ change\ in}{original\ length} = \frac{\Delta L}{L} = (\frac{3L}{4})_L$$

$$= 0.75$$



Let us try to solve this problem. The longitudinal strain for a wire is 0.02 and its Poisson's ratio is 0.6. Find the lateral strain in the wire.

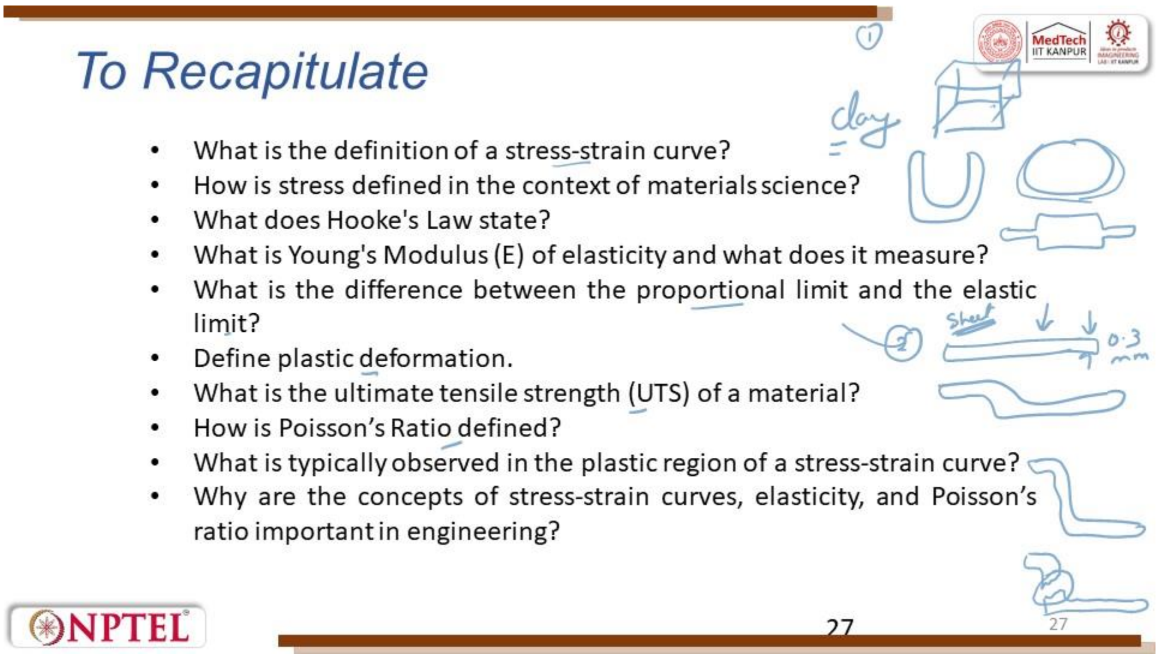
$$Longitudinal\ Strain\ of\ wire = 0.02$$

$$r = 0.6$$

$$The\ Poisson's ration = \frac{Lateral\ Strain}{Longitudinal\ Strain}$$

$$0.6 = \frac{Lateral\ Strain}{0.02}$$

$$Lateral\ Strain = 0.6 \times 0.02 = 0.012$$



To recapitulate what we have seen in this lecture, we saw Stress-Strain curve. We saw how is the stress defined in the context of material science. What does Hooke's law state? What is Young's modulus of elasticity? What does it measure?

What is the difference between the proportionality limit and the elastic limit? Define plastic deformation we saw. What is UTS of a material? How is Poisson's ratio defined? What is typically observed in the plastic region of a stress-strain curve?

What are the concepts of stress-strain curve, elasticity and Poisson's ratios important in engineering? Let us take a simple example. You are trying to take a clay, a cuboidal clay. It can be plastic clay which is used for making china clay. we say where in which this clay you have taking it and then you are trying to make shapes like cup and you are trying to make flat plate from this and then we are also trying to make rollers, right.

From the clay whatever you have, so now for each of these applications you should try to figure out what is the load you apply and how do you get it uniform. So, this is one example, you will try the second example is you will try to take a thick sheet. And you will try to beat the sheet and then try to make a shape like this. The thickness of the sheet can be 0.3 mm. You will try to beat.

So, when you try to beat, you will try to see to get the shape, what are the loads you apply. And you will try to beat it to such a level. First you do first step, then you will do a very high aspect ratio step. At some point of time, it will try to fracture. Try to do up to this level.

When it fractures, you try to see what happens at the fractured portion. Now here you are trying to do experiments in trying to understand the stress-strain curve. So here the plastic deformation will start, it will continue up to ultimate tensile and here comes the fracture. So please try to do these two examples so that you will start appreciating what you studied in the lecture with real time application. This will be clay, this will be a thin sheet of metal, right.

So, with that, I would like to thank you. As usual, we have used the following books as references such that you can try to understand the concepts more clearly. Thank you very much.