

Basics of Mechanical Engineering-2

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Week 06

Lecture 21

Tutorial-3 Forming (Part 1 of 2)

Welcome back to the course Basics of Mechanical Engineering II, where we are discussing the manufacturing sciences and manufacturing processes here. The first part is Basics of Mechanical Engineering. I discussed the basics of engineering, which included the theory of machines, strength of materials, etcetera. Here, in this week, we are discussing metal forming. In the last week, you have seen certain metal forming basics and the processes.

I will again walk you through a tutorial session on metal forming. We will review the various relations that we saw in metal forming processes, such as forging, rolling, extrusion, etcetera. I will try to examine certain problem statements, such as how to design the radius, the size, and determine the time, etc., for the different processes in metal forming.

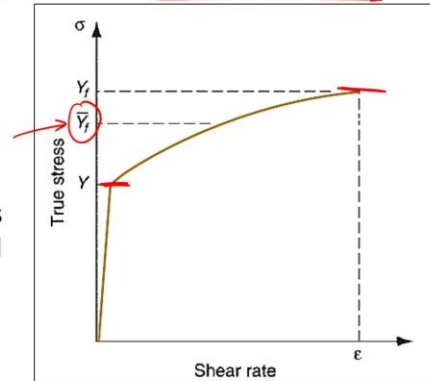
Forming Mechanism

- As the metal is deformed, its strength increases due to strain hardening.
- Flow stress is defined as the instantaneous value of stress required to continue deforming the material—to keep the metal “flowing.” It is the yield strength of the metal as a function of strain, which can be expressed:

$$Y_f = K\epsilon^n$$

where Y_f = flow stress, MPa

- Except some cases generally forming force is analyzed on the basis of average flow stress and strains.



Just to recall, the forming mechanism is such that as metal is deformed, its strength increases due to strain hardening. Flow stress is defined as the instantaneous value of stress required to continue deforming the material.

So this is Y_f , that is flow stress, which is taken in MPa.

$$Y_f = K\epsilon^n$$

This is there to keep the material flowing. It is the yield strength of the metal as a function of strain, which can be expressed as this relation. So, yield strength as a function of strain is expressed. Except in some cases, generally, forming force is analyzed on the basis of average flow stress and strains.

Forming Mechanism



Average flow stress (or mean flow stress)

- It is the average value of stress over the stress-strain curve from the beginning of strain to the final (maximum) value that occurs during deformation

$$\bar{Y}_f = \frac{K\epsilon^n}{1+n}$$

Strain Hardening exponent

where \bar{Y}_f = average flow stress, MPa (lb/in²); and ϵ = maximum strain value during the deformation process.



If I say average flow stress, this is

$$\bar{Y}_f = \frac{K\epsilon^n}{1+n}$$

Where \bar{Y}_f is average flow stress. And ϵ is maximum strain value during the deformation process.

And n is the strain hardening exponent. It is the average value of stress over the strain curve from the beginning of the strain to the final. That is the maximum value that occurs during deformation.

Forming Mechanism



Problem Statement: During a tension test, the tensile strength was found to be 340 MPa. This was recorded at 30% elongation, find the value of Strain hardening exponent (n) and strength coefficient(K).

Solution:

Engineering strain = 30% ; $e = 0.3$

True strain, $\epsilon = \ln(1+e)$
 $\epsilon = \ln(1+0.3)$
 $\epsilon = \ln 1.3$
 $\epsilon = 0.262$

at Ultimate Tensile Strength (UTS) $n = \epsilon$

True stress $\sigma_t = \sigma(1+e)$
 $= 340(1.3)$
 $= 442 \text{ MPa}$

$\sigma_t = K \epsilon^n$
 $442 = K (0.262)^{0.262}$
 $K = 627.8 \text{ MPa}$

Stress-Strain UTM (UTS) E **BME-1**



Let me come directly to a following statement. It is given that during a tension test, the tensile strength was found to be 340 MPa. It was recorded at 30% elongation. Find the value of strain hardening exponent N and strength coefficient K.

Engineering Strain = 30%; $e = 0.3$

True strain, $\epsilon = \ln(1 + 0.3)$

$$\epsilon = \ln 1.3 = 0.262$$

at UTS, $n = \epsilon$

True stress $\sigma_t = \sigma(1 + e)$

$$= 340(1 + 0.3)$$

$$= 442 \text{ MPa}$$

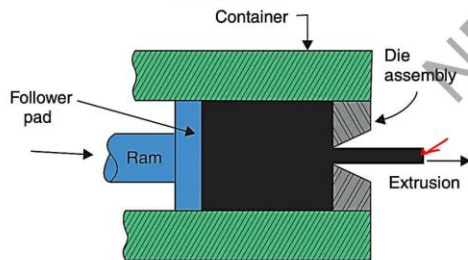
$$\underline{Y_f = K \epsilon^n}$$

$$442 = K (0.262)^{0.262}$$

$$K = 627.8 \text{ MPa}$$

Extrusion

- Extrusion is a compression process in which the work metal is forced to flow through a die opening to produce a desired cross-sectional shape.
- Extrusion is generally used to create long parts that have a uniform cross-section.



<https://extrudesign.com/what-is-an-extrusion-process/>
<https://makeagif.com/gif/extrusion-processes-lUzkjG>

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Let me now discuss one of the metal-forming processes, which is extrusion. Extrusion is a compression process in which the work metal is forced to flow through a die opening to produce a desired cross-sectional shape. Extrusion is generally used to create long parts that have a uniform cross-section. That means there is a die.

Die is $Di + e$. Die is two-dimensional. And e here stands for extrusion. Whatever pipes you see, as already discussed, those are generally extruded. If you see bullets, etc., those are all extruded.

So, a two-dimensional die is used. Nowadays, the word die is also used for the mold. The three-dimensional is also being used. But generally, it was invented or introduced as the word die because a two-dimensional cross-section is used. There is a ramp that pushes the material, and this is the extrusion that comes as output.

Extrusion Analysis

- Assuming both billet and extrudate are round in cross-section.

Extrusion ratio

- It is also called the reduction ratio

$$r_x = \frac{A_0}{A_f}$$

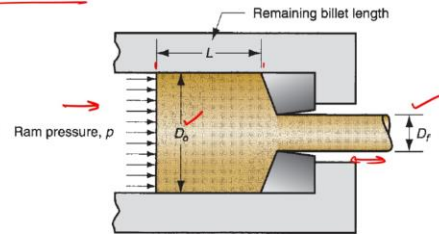
where r_x = extrusion ratio;

A_0 = cross-sectional area of the starting billet (mm^2)

A_f = final cross-sectional area of the extruded section (mm^2)

True strain(ϵ) in extrusion

Under ideal deformation occurs with no friction and no redundant work.



So, just to come to the extrusion analysis, assuming both billet and extrudate are round in cross-section. There is a ratio known as the extrusion ratio.

$$r_x = \frac{A_0}{A_f}$$

A_0 is the inlet area. A_f is the cross-sectional area of the extruded.

That is the outlet area. And r_x is our extrusion ratio. True strain in extrusion under ideal deformation occurs with no friction and no redundant work. No further work is required. It is the diameter given the initial diameter.

Extrusion Analysis

Ram pressure(P)

Under ideal deformation, the ram pressure required to extrude the billet through die hole is given by,

$$p = \bar{Y}_f \ln(r_x) = \bar{Y}_f \ln\left(\frac{A_0}{A_f}\right) \quad \text{where} \quad \bar{Y}_f = \frac{K \epsilon^n}{1+n}$$

Note: The average flow stress is found out by integrating the flow curve equation between zero and the final strain defining the range of forming

Where \bar{Y}_f is average flow stress, and ϵ is maximum strain value during the extrusion process.

The actual pressure for extrusion will be greater than in ideal case, because of the friction between billet and die and billet and container wall.

- Ram pressure to perform indirect extrusion (based on Johnson's Extrusion strain formula)

$$p = \bar{Y}_f \epsilon_x$$

And the final diameter, ram pressure, total length of the billet are given as inputs. And the length output can be calculated through volume. So, ram pressure can be calculated under ideal deformation. The ram pressure is required to extrude the billet through the die hole. It is given

$$p = \bar{Y}_f \ln(r_o) = \bar{Y}_f \ln\left(\frac{A_o}{A_f}\right)$$

And also, we can put

$$\bar{Y}_f = \frac{K\epsilon^n}{1+n}$$

which we saw in the metal forming mechanism. So, these relations we will use in solving the problem statements coming up. The actual pressure of extrusion will be greater than in the ideal case. Why is this greater?

Because there is always friction between the billet die and the container wall. This has already been discussed again. The actual pressure is higher because of the friction between the billet die and the container wall, etc. Ram pressure to perform indirect extrusion, based on Johnson's extrusion strain formula, is given by

$$p = \bar{Y}_f \epsilon_x$$

That is the average flow stress times the strain. This is the ram pressure in extrusion.

Extrusion Analysis



Ram pressure in direct extrusion:

- In Direct extrusion due to friction ram pressure is greater than for indirect extrusion.

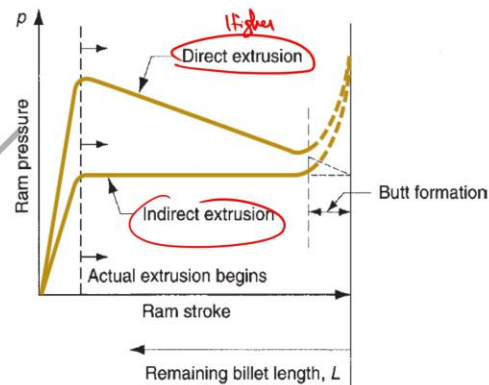
$$p = \bar{Y}_f \left(\epsilon_x + \frac{2L}{D_o} \right)$$

here ,

$2L/D_o$ = Additional pressure due to friction at the Container-billet interface.

L = Portion of the billet length remaining to be extruded,

D_o = original diameter of the billet



Ram pressure in direct extrusion. You can see the difference between direct and indirect extrusion. Generally, for direct extrusion, the pressure is higher. Indirect extrusion due to friction, which is given here.

The ram pressure is greater than the indirect extrusion;

$$p = \bar{Y}_f \left(\epsilon_x + \frac{2L}{D_o} \right)$$

Extrusion Analysis



Ram force in indirect or direct extrusion-

$$F = p A_0 \quad (\text{here } A_0 = \text{area of billet})$$

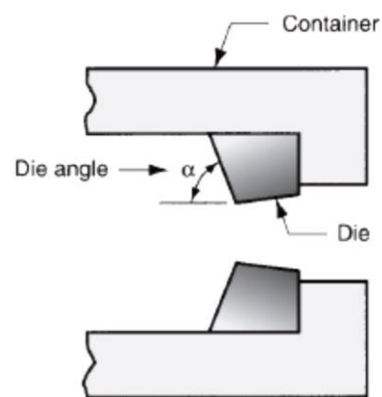
Power required to carry out the extrusion operation

$$P = Fv$$

Here, P = power (J/s)

v = ram velocity (m/s)

- The shape of the initial pressure build-up depends on the die angle.
- Higher die angles cause steeper pressure buildups.



Ram force in indirect or direct extrusion is given by $F = PA_0$. Area is the initial area of the billet. Power required to carry out the extrusion operation $P = Fv$, where v is ram velocity. And force is calculated from here.

The shape of the initial pressure buildup depends on the die angle. Higher die angles cause steeper pressure buildups. This was all a review of the things that we already have covered in the previous lectures.

Extrusion Analysis



Problem Statement: A cylindrical billet of 40 mm diameter and 100 mm length is reduced by backward extrusion to a 15 mm diameter.

If the Johnson's equation has $b = 1.5$ and strength coefficient $a = 0.8$ of material 750 MPa strain hardening coefficient $n = 0.15$

Determine:

- 1) Extrusion ratio
- 2) True strain
- 3) Extrusion strain
- 4) Ram force

$$\frac{\pi}{4} D_o^2 L_o = \frac{\pi}{4} D_f^2 L_f$$

$$(40)^2 (100) = (15)^2 L_f$$

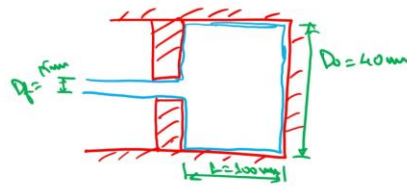
$$L_f = 711.11 \text{ mm}$$

$$D_o = 40 \text{ mm} \quad K = 750 \text{ MPa}$$

$$L = 100 \text{ mm} \quad n = 0.15$$

$$D_f = 15 \text{ mm} \quad a = 0.8$$

$$b = 1.5$$



3) Extrusion strain:

$$e = \frac{L_f - L_o}{L_o}$$

$$= \frac{711.11 - 100}{100} = 6.11$$



Extrusion Analysis



Solution:

2) True strain = $\epsilon = \ln(1 + e)$

$$\epsilon = \ln(1 + 6.11)$$

$$\epsilon = \ln(7.11)$$

$$\epsilon = 1.96$$

1) Extrusion ratio = $R = \frac{A_o}{A_f} = \frac{\frac{\pi}{4} D_o^2 L_o}{\frac{\pi}{4} D_f^2 L_f}$

$$= 7.11$$

4) $\sigma_o = \bar{Y} = \frac{K \epsilon^n}{n+1} = \frac{750 (1.96)^{0.15}}{0.15+1} = 721 \text{ MPa}$

$$\sigma_a = \sigma_o [a + b \ln R] = 721 [0.8 + 1.5 \ln(7.11)] = 2698.16 \text{ MPa}$$

✓ Ram force = $\sigma_a \times \frac{\pi}{4} [D_o^2 - D_f^2]$

Ram force = 2.9 MN ✓



Let me now come to a problem statement where it is given a cylindrical billet of 40 mm diameter and 100 mm length. Which means my D_o is 40 mm and my length. This is reduced by backward extrusion to a 15-millimeter diameter by DF. That is, the final diameter is 15 millimeters. If the Chancel equation has B equal to 1.5 and strength coefficient A as 0.8 of material, 750. MPa strain hardening coefficient N as 0.15. Determine the extrusion ratio, the true strain, extrusion strain, and ram force.

$D_o = 40 \text{ mm}$

$$L = 100 \text{ mm}$$

$$D_f = 15 \text{ mm}$$

$$K = 750 \text{ MPa}$$

$$n = 0.15$$

$$a = 0.8$$

$$b = 1.5$$

Solution:

$$\pi/4 D_o^2 L_o = \pi/4 D_f^2 L_f$$

$$(40)^2 (100) = (15)^2 L_f$$

$$L_f = 711.11 \text{ mm}$$

3) Extrusion Strain:

$$e = \frac{L_f - L_o}{L_o} = \frac{711.11 - 100}{100} = 6.11$$

2) True strain = $\epsilon = \ln(1 + e)$

$$\epsilon = \ln(1 + 6.11)$$

$$\epsilon = \ln(7.11)$$

$$\epsilon = 1.96$$

$$1) r_x = A_o/A_f = \frac{\pi/4 D_o^2 L_o}{\pi/4 D_f^2 L_f} = 7.11$$

$$4) \sigma_o = \boxed{\bar{Y}_f = \frac{K \epsilon^n}{1+n}} \frac{750(1.96)^{0.15}}{0.15+1} = 7.21 \text{ MPa}$$

$$\sigma_d = \sigma_o [a + b \ln r_x] = 721 [0.8 + 1.5 \ln(7.11)] = 2698.16 \text{ MPa}$$

$$\text{Ram Force} = \sigma_d \times \pi/4 [D_o^2 - D_f^2]$$

$$\text{Ram force} = 2.9 \text{ MN}$$

Drawing Analysis

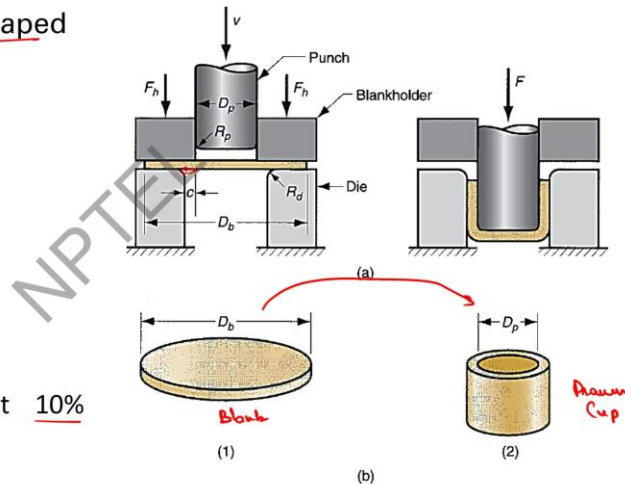
In case of drawing of a cup shaped part

c = Clearance
 D_b = Blank diameter
 D_p = Punch diameter
 R_d = Die corner radius
 R_p = Punch corner radius
 F = Drawing force
 F_h = Holding force

Clearance

Clearance in drawing is about 10% greater than the stock thickness.

$$c = 1.1t$$



Let me just go through the theory of drawing once. In the case of drawing a cup-shaped part, c is the clearance. That is, here, D_b is the blank diameter, D_p is the punch diameter, r_d is the corner radius, R_p is the punch corner radius, and F is the drawing force. And F_h is the holding force. Clearance in drawing is about 10 percent greater than the stock thickness. That is, $c = 1.1t$, as you have already seen. So, this is a plate that is drawn into a cup—this plate. In technical terms, it is known as a blank. So, this is the drawn cup.

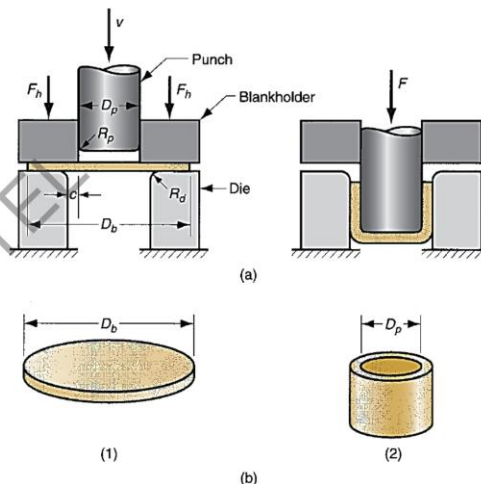
Drawing Analysis

Drawing ratio

Defined as ratio of blank diameter(D_b), to punch diameter(D_p), greater the ratio, the more severe the drawing operation.

$$DR = \frac{D_b}{D_p}$$

The limiting value of $DR (\leq 2)$ depends on punch and die corner radii, friction conditions, draw depth, and quality of the sheet metal like ductility, degree of directionality of strength properties in the metal.



Here, we find a ratio called the drawing ratio. This is defined as the ratio between the blank diameter and the punch diameter. The greater the ratio, the more severe the drawing operation. Which means where a bigger blank is used.

The drawing would be more deep. It would be more severe if blank diameter is higher. If punch diameter is higher, blank is smaller, this ratio would be smaller. Limiting value of DR as less than equal to 2 depends upon the punch and die corner radii. Friction conditions, draw depth and quality of sheet metal like ductility, degree of directionality or strength properties in the metal etc.

Drawing Analysis

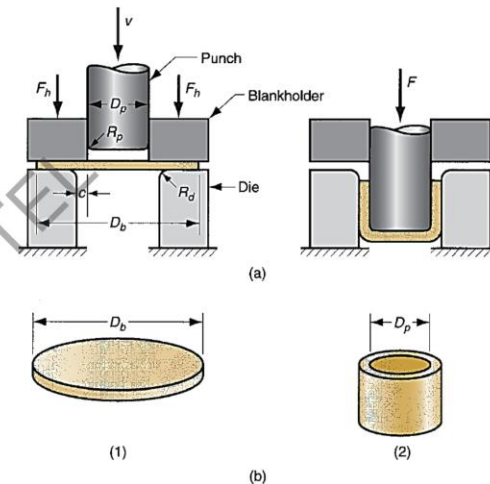
Reduction (r) in deep drawing-

$$r = \frac{D_b - D_p}{D_b}$$

Value of reduction $(r) \leq 0.5$

Thickness-to-diameter ratio (t/D_b)

- t/D_b ratio should be greater than 1%.
- As t/D_b decreases, tendency for wrinkling increases.



Here also reduction is there. Reduction r in deep drawing is

$$r = \frac{D_b - D_p}{D_b}$$

This is the value of reduction which is typically less than equal to 0.5. Thickness to diameter ratio that is t/D_b . Thickness ratio should be greater than 1%.

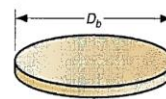
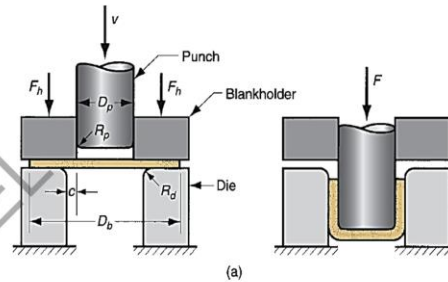
As t/D_b decreases the tendency for wrinkling increases. Because its thickness is very less. It just looks like to be a very thin sheet or just like a paper that will wrinkle if you try to draw it. So, this thickness ratio is generally greater than 1 percent.

Drawing Analysis

The maximum drawing force, F , can be estimated approximately by the following equation.

$$F = \pi D_p t \sigma_{UTS} \left(\frac{D_b}{D_p} - 0.7 \right)$$

Correction factor for friction



(1)



(2)

The holding force, F_h , is given by,

$$F_h = 0.015 \sigma_{ys} \pi \left\{ D_b^2 - (D_p + 2.2t + 2R_d)^2 \right\}$$

$$F_h = \frac{F}{3} \quad (\text{approx. holding force is one-third of drawing force})$$



M.P. Groover, Fundamental of modern manufacturing Materials, Processes and system

Now, to calculate the forces, certain relations. Such as

$$F = \pi D_p t \sigma_{UTS} \left(\frac{D_b}{D_p} - 0.7 \right)$$

Maximum drawing force and holding force is

$$F_h = 0.015 \sigma_{ys} \pi \left\{ D_b^2 - (D_p + 2.2t + 2R_d)^2 \right\}$$

Here generally $F_h = F/3$ that is approximate holding force is one third of the drawing force using these relations.

Drawing Analysis

Problem Statement: A Cylindrical cup without flange is to be drawn from a 2 mm thick sheet.

The cup shall have 15 mm diameter and 40 mm height.

Reduction ratio in the first and subsequent draws may not exceed 40% and 15% respectively.

Determine the blank size and the number of draws necessary.

Handwritten solution:

$t = 2 \text{ mm}$
 $D_b = ?$

Blank: $\frac{\pi}{4} D_b^2$
 Cup: $\frac{\pi}{4} d^2 + \pi d h$

$\frac{\pi}{4} D_b^2 = \frac{\pi}{4} (15)^2 + \pi (15)(40)$
 $D_b = 51.23 \text{ mm}$

Ratio (overall) = $\frac{D_b}{d} = \frac{51.23}{15} = 3.41$



Drawing Analysis

Solution:

d_1 = diameter of cup after 1st draw (blank diameter for second draw)
 d_2 = " " " " 2nd " (" " " " 3rd " ")

$$\underbrace{\frac{D_b}{d_1}}_{1^{st} \text{ draw}} \times \underbrace{\frac{d_1}{d_2}}_{2^{nd} \text{ draw}} \times \dots \times \underbrace{\frac{d_{n-1}}{d_n}}_{(n-1)^{th} \text{ draw}} = 3.41$$

$$d_1 = (1 - 0.4) D_b = 0.6 D_b$$

$$d_2 = (1 - 0.15) d_1 = 0.85 d_1$$

$$\frac{1}{0.6} \times \left(\frac{1}{0.85}\right)^{n-1} = 3.41$$

$$\left(\frac{1}{0.85}\right)^{n-1} = 2.049$$

Taking log (ln) :

$$(n-1) \ln\left(\frac{1}{0.85}\right) = \ln(2.049)$$

$$n = 5.49 \quad \boxed{n = 6}$$



Let us try to see a problem statement. Here it is given a cylindrical cup without flange is to be drawn. From a 2 mm thick sheet, 2 mm thick sheet, which means I have been given a blank. If I try to draw it in a two-dimensional. Here, this thickness T , T , its diameter is D . A cup shall have 15 millimeter diameter and 40 millimeter height. And subsequent draws may not exceed 40 percent and 15 percent. Respectively, determine the blank size and number of draws necessary.

To find: D_b

$$\frac{\pi}{4} D_0^2 = \frac{\pi}{4} d^2 + \pi d h$$

$$\frac{\pi}{4} D_0^2 = \frac{\pi}{4} (15)^2 + \pi (15)(40)$$

$$D_b = 51.23 \text{ mm}$$

$$\text{Ratio (overall)} = D_b/d = 51.23/15 = 3.41$$

So, with this solution, I will take a break. I have discussed forming mechanisms, extrusion, and drawing in this part of the tutorial. The next part of the tutorial on forming, I will continue where I will discuss further metal forming processes such as rolling and so on.

Thank you.