

Basics of Mechanical Engineering-2

Prof. J. Ramkumar

Prof. Amandeep Singh Oberoi

Department of Mechanical Engineering

Indian Institute of Technology, Kanpur

Week 10

Lecture 41

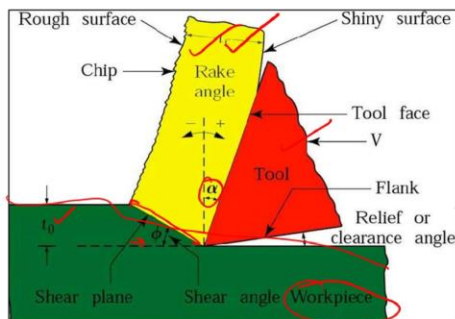
Tutorial-5 (Part 2 of 2)

Welcome to the second part of the tutorial on machining. We have discussed certain operations, lathe cutting operations, milling operations in the first part. In this part, I will cover the single point cutting tool that is the orthogonal cutting, the angles of the tool, the chip thickness ratio, etc. that will be covered. Then, we will try to see the numerical problems on the tool life equations. This is a tutorial session on machining. I am Dr. Amandeep Singh Oberoi.

Metal Cutting Problems and Solutions



Orthogonal Machining(2Dimensional Machining Analysis)



α : Rake angle

β : Frictional angle

ϕ : Shear angle

P_s : Cutting Force (Tangential force)

P_t : Thrust Force

F_s : Shear Force

F_n : Normal Shear Force

F : Frictional Force

N : Normal Frictional Force

V : Cutting velocity

V_c : Chip velocity

V_s : Shear velocity

L_s : Length of shear plane



<https://www.minaprem.com/machining/cutter/>

To recall the concepts on orthogonal machining, that is two dimensional machining analysis. This is what that we have discussed in the theory part that there is a chip that is

being removed from the workpiece. This green color is my workpiece and this is chip yellow color and red color is my tool. This is α that is my rake angle. It is given all the notations are given here.

β is the fictional angle that is given here. Shear angle is Φ . This is shear plane. The angle made by shear plane to the base is Φ .

And T_o and T_c are the uncut chip thickness and cut chip thickness respectively. Then they are cutting forces such as P_S is cutting force or we also call it as tangential force. Then, P_T is thrust force, we have shear force, normal shear force, frictional force F , normal frictional force, cutting velocity, chip velocity, shear velocity and length of shear plane. This is along the shear plane, what is the length. This all is discussed already in the previous lectures. You have seen the NRS, ORS, ASA, different kind of the cutting morphologies and NRS is more explained there.

Metal Cutting Problems and Solutions



Experimental Determination of Cutting Ratio

Shear angle (ϕ) may be obtained either from photo-micrographs or assume volume continuity (no chip density change)

$$\text{Since } t_o w_o L_o = t_c w_c L_c$$

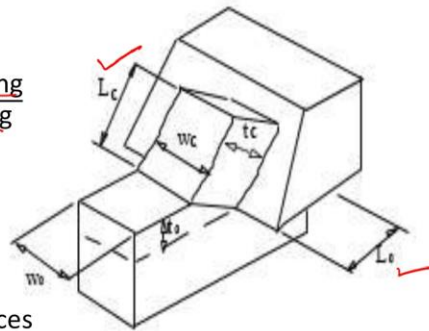
and $w_o = w_c$ (Experimental evidence)

$$\text{Cutting ratio, } r = \frac{t_o}{t_c} = \frac{L_c}{L_o} = \frac{t_1}{t_2} = \frac{\text{chip thickness before cutting}}{\text{chip thickness after cutting}}$$

Where: t_o = Uncut chip thickness (mm)

t_c = Cut chip thickness (mm)

A **higher cutting ratio (r)** means less chip compression, resulting in better surface finish and reduced cutting forces



<https://www.minaprem.com/machining/cutter/>

Let me now come to the relations that we will use to solve the problem statements. Shear angle may be obtained either from the photo micrographs or assume volume continuity, that is not chip density change. If it is considered that tip density remains same, that means the volume also remains the same. Volume = thickness x width into the length of the uncut chip = thickness x width x length of the cut chip.


Where w_0 and w_c are experimental evidence only. And cutting ratio

$$r = \frac{t_0}{t_c} = \frac{Lc}{Lo} = \frac{t_1}{t_2} = \frac{\text{chip thickness before cutting}}{\text{chip thickness after cutting}}$$

Where: t_0 = Uncut chip thickness (mm)

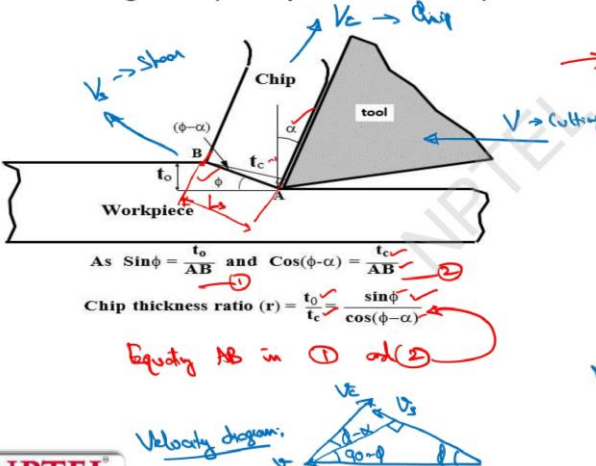
t_c = Cut chip thickness (mm)

The higher the cutting ratio means less chip compression. This results in a better surface finish and reduced cutting forces. This is Lc , which is the length of the chip, and this is Lo , which is the length of the uncut chip.



Metal Cutting Problems and Solutions

Cutting Ratio (or chip thickness ratio)



As $\sin \phi = \frac{t_0}{AB}$ and $\cos(\phi - \alpha) = \frac{t_c}{AB}$

Chip thickness ratio (r) = $\frac{t_0}{t_c} = \frac{\sin \phi}{\cos(\phi - \alpha)}$

Equating AB in ① and ②

Velocity diagram:

$$r = \frac{t_0}{t_c} = \frac{L_s \sin \phi}{L_s \cos(\phi - \alpha)}$$

$$\tan \phi = \frac{r \cos \alpha}{1 - r \sin \alpha}$$

For minimum shear strain condition:

$2\phi - \alpha = 90^\circ$

Mass continuity equation

$$V t_0 = V_c t_c \quad \left| \quad V_s = V \frac{\cos \alpha}{\cos(\phi - \alpha)} \right.$$

$$V_c = V \cdot r$$

https://www.minaprem.com/machining/cutter/

Here, you can also see it is again given the rake angle and the shear angle. We have the length of the shear plane here. This is my L_s , the length of the shear plane. Here,

$$\sin \phi = \frac{t_0}{AB} \text{ and } \cos(\phi - \alpha) = \frac{t_c}{AB}$$

So, the tip thickness ratio

$$\text{Chip thickness ratio } (r) = \frac{t_0}{t_c} = \frac{\sin \phi}{\cos(\phi - \alpha)}$$

So, how has it come? If I put this as equation 1 and this as equation 2. So, equating AB in 1 and 2, we get this equation. This is our relation that we will use to find the shear angle,

to find the rake angle, and sometimes to find the cut chip thickness that will be obtained. Also, the ratio that is

$$r = \frac{t_o}{t_c} = \frac{L_s \sin \phi}{L_s \cos(\phi - \alpha)}$$

$$\tan \phi = \frac{r \cos \alpha}{1 - r \sin \alpha}$$

So, these will be used in the calculation and also for the minimum shear strain condition. It comes when $2\Phi - \alpha = 90$ degrees. So that becomes when the shear is minimum.

And if we wish to calculate the velocity event. So I will put here, here in this direction we have the chip velocity V_c , then we have shear velocity in this direction V_s , then we have cutting velocity in this direction that is V . This is cutting, this is chip, and this is shear. These are velocities. So if I put here $V_{to} = V_{stc}$. This gives me $V_c = V_r$. This r is my cutting ratio.

This also gives me a relation that is

$$V_s = V \frac{\cos \alpha}{\cos(\phi - \alpha)}$$

So, where has this come from? It has come from the velocity diagram. It could have put something like this. For example, this is my velocity vector V , and this is my vector V_c , and here comes the shear vector, this direction V_s , and this is angle Φ . Here if I draw a perpendicular line here, this is suppose 90 degrees, this becomes $90 - \Phi$, and this becomes $\Phi - \alpha$.

So, this is known as the velocity diagram. So, let me come to the problem statements. So, that we understand this and try to see how we get the solutions, how we get answers to the questions that are asked.

Metal Cutting Problems and Solutions



Problem Statement: Find out the shear angle of a single-point cutting tool which has a rake angle of 12° . The depth of cut is 0.81 mm, and the chip thickness is 1.8 mm?

$$\begin{aligned}\alpha &= 12^\circ \\ d &= 0.81 \text{ mm} \rightarrow t_o \\ t_c &= 1.8 \text{ mm} \\ r &= \frac{t_o}{t_c} = \frac{0.81}{1.8} = 0.45 \\ \tan \phi &= \frac{r \cos \alpha}{1 - r \sin \alpha} \\ \tan \phi &= \frac{0.45 \cos 12^\circ}{1 - 0.45 \sin 12^\circ} \\ \tan \phi &= 0.4861 \\ \phi &= \tan^{-1}(0.4861) \\ \phi &= 26^\circ\end{aligned}$$



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The first problem is: Find out the shear angle of a single-point cutting tool which has a rake angle of 12° . The depth of cut is 0.81 mm, and the chip thickness is 1.8 mm?

Given:

$$\alpha = 12 \text{ degree}$$

$$d = 0.81 \text{ mm} = t_o$$

$$t_c = 1.8 \text{ mm}$$

Solution:

$$r = t_o/t_c = 0.81/1.8 = 0.45$$

$$\tan \phi = \frac{r \cos \alpha}{1 - r \sin \alpha}$$

$$\tan \phi = \frac{0.45 \cos 12^\circ}{1 - 0.45 \sin 12^\circ}$$

$$\tan \phi = 0.4861$$

$$\Phi = \tan^{-1}(0.4861)$$

$$\Phi = 26 \text{ degrees (ans.)}$$

Metal Cutting Problems and Solutions



Problem Statement: Determine the shear plane angle for the minimum possible shear strain condition when the rake angle of the single point cutting tool is 12°

Minimum shear strain condition:

$$\phi = ?$$
$$\alpha = 12^\circ$$
$$2\phi - \alpha = 90^\circ$$
$$2\phi - 12 = 90$$
$$\phi = 51^\circ$$



Determine the shear plane angle for the minimum possible shear strain condition when the rake angle of the single point cutting tool is 12°.

Given:

$$\Phi = ?$$

$$\alpha = 12 \text{ degrees}$$

Solution:

$$2\Phi - \alpha = 90$$

$$= 2\Phi - 12 = 90$$

$$= \Phi = 51 \text{ degrees (ans.)}$$

Metal Cutting Problems and Solutions



Problem Statement: In an orthogonal machining operation, the chip thickness and the uncut chip thickness are equal to 0.45 mm. If the tool rake angle (α) is 0 degree. Calculate the shear plane angle (ϕ)

$$t_o = t_c = 0.45 \text{ mm}$$

$$\alpha = 0^\circ$$

$$\phi = ?$$

$$r = \frac{t_o}{t_c} = 1$$

$$\tan \phi = \frac{r \cos \alpha}{1 - r \sin \alpha}$$

$$\tan \phi = \frac{1 \cdot \cos 0^\circ}{1 - 1 \cdot \sin 0^\circ}$$

$$\tan \phi = \frac{1}{1}$$

$$\phi = \tan^{-1}(1)$$

$$\phi = 45^\circ = \frac{\pi}{4} \text{ radians}$$



In an orthogonal machining operation, the chip thickness and the uncut chip thickness are equal to 0.45 mm. If the tool rake angle (α) is 0 degree. Calculate the shear plane angle (ϕ).

Given:

$$t_o = t_c = 0.45 \text{ mm}$$

$$\alpha = 0 \text{ degree}$$

$$\phi = ?$$

Solution:

$$r = t_o/t_c = 1$$

$$\tan \phi = \frac{r \cos \alpha}{1 - r \sin \alpha}$$

$$\tan \phi = \frac{1 \cos 0^\circ}{1 - 1 \sin 0^\circ}$$

$$\tan \phi = 1/1$$

$$\phi = \tan^{-1}(1)$$

$$\phi = 45 \text{ degrees} = \pi/4 \text{ radians}$$

To have more details, you can maybe visit courses on metal cutting which talk specifically about orthogonal cutting.

Metal Cutting Problems and Solutions



Taylor's Tool Life Equation

The tool life is mainly affected by cutting speed, means higher the cutting speed the smaller the tool life. Taylor gave the relation between cutting speed and tool life that is

$$V T^n = C$$

Where, V = cutting speed

T = tool life

C = Machining constant or Taylor's constant

n = Tool life exponent (depends on both tool and work material)

- Taylor's exponent 'n' depends only on Tool Material.
- Taylor's constant depends on Tool Material, Workpiece Material and Cutting Conditions.
- The dependency of tool life on various parameters in decreasing order is given by:

Cutting speed > Feed > Depth of Cut > Nose Radius



Now, I will come to the simple round statements in Taylor's tool life equation. To recall Taylor's tool life equation, tool life is mainly affected by cutting speed, meaning the higher the cutting speed, the smaller the tool life. Taylor gave the relation between the cutting speed and tool life, that is $V T^n = C$, where V is cutting speed. T is tool life. C is a machining constant, or it is also called Taylor's constant. n is the tool life exponent that depends upon the tool type and work material, Taylor's exponent, and depends only on tool material majorly. Taylor's constant depends on tool material, workpiece material, cutting conditions, etc.

The dependency of tool life on various parameters in decreasing order is given as cutting speed having the highest effect. Then comes feed, then comes depth of cut, then comes nose radius.

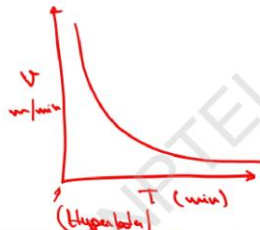
Metal Cutting Problems and Solutions

The tool life is mainly affected by cutting speed, means higher the cutting speed the smaller the tool life .

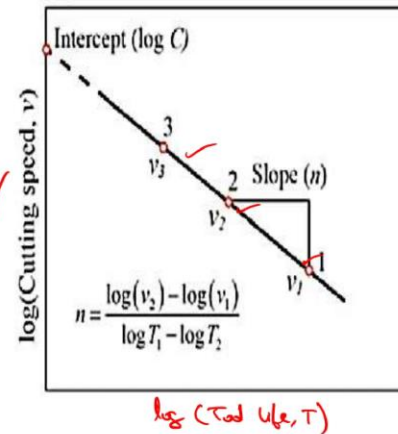
$$V T^n = C$$

$$\log V + n \log T = \log C$$

$$\log V = -n \log T + \log C$$



Tool Materials	Taylor's Exponent (n)
High Speed steel (HSS)	0.08 – 0.2
Cast Hard Alloys	0.2 – 0.15
Carbides	0.2 – 0.6
Ceramics	0.5 – 0.8



So here, tool life is mainly affected by cutting speed, meaning high cutting speed, smaller the tool life, as it is given here. If we take log on both sides, $\log V + n \log T = \log C$. This is generally calculated. This becomes a simple equation of a linear line.

You see here. This is the log of cutting speed intercept. So, this is V1, V2, V3; 3 points are given, and here we can see

$$n = \frac{\log(v_2) - \log(v_1)}{\log T_1 - \log T_2}$$

So, this is cutting speed, and here we have the tool life. This is the log of tool life, that is T. And there are certain Taylor exponent numbers which are given for high-speed steel because it is dependent majorly on the tool material; it varies from 0.08 to 0.2 for high-speed steel. And as you go harder down, the higher is the exponent value.

So, it is harder or highest for ceramics. It is from 0.5 to 0.8. This is the value of n in general, that is cutting speed versus tool life, which you have already seen. If this is cutting speed, this is tool life T, suppose it is given in minutes, and cutting speed if I give it in meters per minute, it is something like this, because you can see an exponent $V T^n = C$. So, as the cutting speed is decreased, the tool life increases. At higher cutting speed, the tool life is lower. So, this is our hyperbola here. So, let me come to the concept slightly.

Metal Cutting Problems and Solutions



Problem Statement: Determine the value of exponent and constant of Taylor's tool life equation for a cutting speed of 60 m/min and the corresponding tool life is 81 minutes, and for a cutting speed of 90 m/min and the corresponding tool life is 36 minutes.

$$V_1 = 60 \text{ m/min}$$

$$T_1 = 81 \text{ min}$$

$$V_2 = 90 \text{ m/min}$$

$$T_2 = 36 \text{ min}$$

$$V_1 T_1^n = V_2 T_2^n = C$$

$$\left(\frac{T_1}{T_2}\right)^n = \frac{V_2}{V_1}$$

$$\left(\frac{81}{36}\right)^n = \frac{90}{60}$$

$$n \ln(2.25) = \ln(1.5)$$

$$n = 0.5$$

$$C = V_1 T_1^n = 60 \times (81)^{0.5}$$

$$C = 540$$



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The first problem says: Determine the value of exponent and constant of Taylor's tool life equation for a cutting speed of 60 m/min and the corresponding tool life is 81 minutes, and for a cutting speed of 90 m/min and the corresponding tool life is 36 minutes.

Given:

$$V_1 = 60 \text{ m/min}$$

$$T_1 = 81 \text{ min}$$

$$V_2 = 90 \text{ m/min}$$

$$T_2 = 36 \text{ min}$$

Solution:

$$V_1 T_1^n = V_2 T_2^n = C$$

$$(T_1/T_2)^n = V_2/V_1$$

$$(81/36)^n = 90/60$$

$$n \ln(0.25) = \ln(1.5)$$

$$n = 0.5 \text{ (ans.)}$$

$$C = VT_1^n = 60 \times (81)^{0.5}$$

$$C = 540 \text{ (ans.)}$$

Metal Cutting Problems and Solutions



Problem Statement: Following is the data available on cutting speed and tool life.

$$V = 150 \text{ m/min}, T = 60 \text{ min}$$

$$V = 200 \text{ m/min}, T = 23 \text{ min}$$

Determine the Taylor's constant and tool life exponent.

$$VT_1^n = VT_2^n = C$$

$$n = ?$$

$$C = ?$$

$$n = 0.3$$

$$C = 512.0$$



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Similarly, there is another problem statement. You can do this by yourself. Simply again,

$$VT_1^n = VT_2^n = C$$

$$n = ?$$

$$C = ?$$

$$n = 0.3$$

$$C = 512.0$$

(These above two quantities are the answers.)

Metal Cutting Problems and Solutions

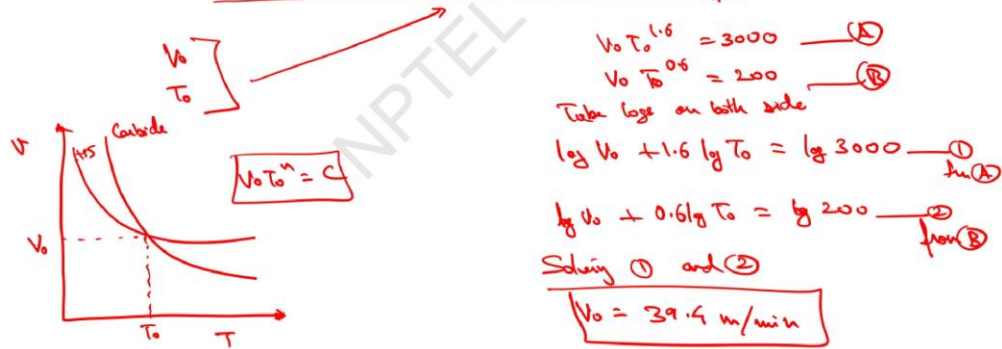


Problem Statement: The tool life equations for the two cutting tools are

For tool 1 it is given by $V T^{1.6} = 3000$

For tool 2 it is given by $V T^{0.6} = 200$

Determine the cutting speed at which both the tools provide same life?



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Now, let me try to see another problem statement, The tool life equations for the two cutting tools are

For tool 1 it is given by $V T^{1.6} = 3000$

For tool 2 it is given by $V T^{0.6} = 200$

Determine the cutting speed at which both the tools provide same life?

The hyperbolic curve that we draw, this is cutting speed, this is tool life. It is one tool here, then we have second tool here. Let me say, for example; this is carbide, this is HSS. There is a point here of the cutting speed V_o with a tool life is same, this we need to find. So, here $V_o T_o^n = C$, This we need to determine. So, for both the equations which are given,

$$V_o T_o^{1.6} = 3000 \quad \text{(A)}$$

$$V_o T_o^{0.6} = 200 \quad \text{(B)}$$

Taking log on both sides:

$$\log V_o + 1.6 \log T_o = \log 3000 \quad \text{(1) from (A)}$$

$$\log V_o + 0.61 \log T_o = \log 200 \quad (2) \text{ from (B)}$$

Solving 1 and 2:

$$V_o = 39.4 \text{ m/min (ans.)}$$

Metal Cutting Problems and Solutions



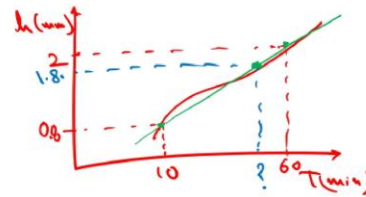
Problem Statement: During Machining the wear land (h) has been plotted against machine time (T) as given below. Find tool life for a critical wear land of 1.8 mm.

$$\begin{aligned} h_1 &= 2 & T_1 &= 60 \\ h_2 &= 0.8 & T_2 &= 10 \\ h_0 &= 1.8 & T_0 &= ? \end{aligned}$$

$$\frac{h_1 - h_2}{T_1 - T_2} = \frac{h_0 - h_2}{T_0 - T_2}$$

$$\frac{2 - 0.8}{60 - 10} = \frac{1.8 - 0.8}{T_0 - 10}$$

$$T_0 = 51.67 \text{ min}$$



Let me now take a problem During Machining the wear land (h) has been plotted against machine time (T) as given below. Find tool life for a critical wear land of 1.8 mm.

For example, if you need to find a tool life for a critical wear land of 1.8 millimeter during the machining the wear land h has been plotted against the machining time. So, there is a plot given here that this h land in millimeter and this is tool life T in minutes. It is plotted something like this. At the value of h_2 the tool life is 60 minutes and at the value of h 0.8 the tool life is 10 minutes.

Solution:

$$\frac{h_1 - h_2}{T_1 - T_2} = \frac{h_0 - h_2}{T_0 - T_2}$$

$$\frac{2 - 0.8}{60 - 10} = \frac{1.8 - 0.8}{T_1 - 10}$$

$T_o = 51.67 \text{ min (ans.)}$

Metal Cutting Problems and Solutions



Problem Statement: An HSS tool is used for turning operations. The Tool life is 1 hr when turning is carried at 30 m/min. The tool life will be reduced to 2.0 min if the cutting speed is doubled. Find the suitable speed in RPM for turning 300 mm diameter rod, so that tool life is 30 min.

$$\begin{aligned} T_1 &= 1 \text{ hr}, & V_1 &= 30 \text{ m/min} \\ T_2 &= 2 \text{ min}, & V_2 &= 60 \text{ m/min} \\ T_o &= 30 \text{ min}, & V_o &= ? / N_o = ? \end{aligned}$$

$$V_1 T_1^n = V_2 T_2^n = V_o T_o^n$$

$$n = 0.203$$

$$V_o = 34.532 \text{ m/min}$$

$$V = \frac{\pi D N}{1000}; \quad N = 36.64 \text{ rpm}$$



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This is the last problem statement in this tutorial. An HSS tool is used for turning operations. The Tool life is 1 hr when turning is carried at 30 m/min. The tool life will be reduced to 2.0 min if the cutting speed is doubled. Find the suitable speed in RPM for turning 300 mm diameter rod, so that tool life is 30 min.

Given:

$$T_1 = 1 \text{ hr}$$

$$V_1 = 30 \text{ m/min}$$

$$T_2 = 2 \text{ min}$$

$$V_2 = 60 \text{ m/min}$$

$$T_o = 30 \text{ min}$$

$$V_o = ?$$

$$N_o = ?$$

Solution:

$$V_1 T_1^n = V_2 T_2^n = V_o T_o^n$$

$$n = 0.203$$

$$V_o = 34.532 \text{ m/min}$$

$$V = \pi D N / 1000$$

$$N = 36.64 \text{ rpm (ans.)}$$

So, with this, I am closing the tutorial on machining. In the next lecture, I will talk about the demonstration. Demonstration—that is, the virtual lab demonstration on machining. In the next weeks—the two weeks which are left—we will be covering non-conventional machining in week 11 and sustainable manufacturing in week 12.

Thank you.