Computational Fluid Dynamics

Prof. Dr. Suman Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture No. # 41

Introduction to Turbulence Modeling

Today, we will start discussing on turbulence modeling. Now, turbulence is a very fascinating topic, at the same time it is not very easy topic to discuss about. So, the motivation of studying turbulence modeling in this course is to bring entire thing in prospective of CFD what we have learnt rather than going into details of studying turbulence, because turbulence itself is an involve topic for on which separated courses can be deal, so our objective will not be going into too much of details, but first to appreciate what are the important physical features of turbulence flows that lead to some of requirements of turbulence model.

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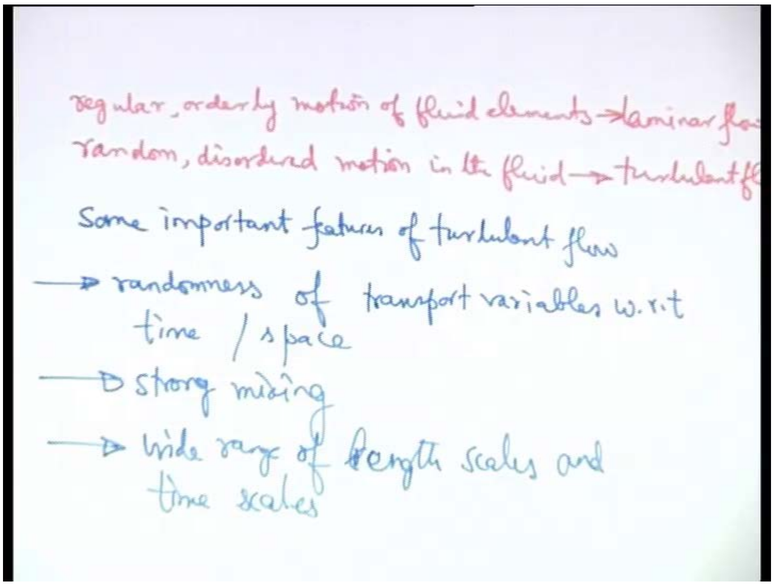

So, to first appreciate that like what are the basic feature of turbulence follows we can very quickly re visit the Reynolds famous Reynolds experiment in which there is a tube, and there is supply of fluid through the tube, and a color dye is injected with a fluid screen; at a very low velocity, what happens? The color dye forms layers one over the other, because of orderly motions of the fluidly element. So, you have for low fluid velocities regular orderly motion of fluid elements. This is what we usually call as laminar flow this are not formal definitions this are just qualitative understandings of what we are talking about.

Now, suddenly as you increase the velocity of flow what is seen is that, these dye liens no more remain like in a layered manner, and they come out get diffused and mix in the flow, so we then say that there is the random disordered motion in the fluid, which we qualitatively call as turbulence flow. Remember this is not a definition of a turbulence flow neither we are attempting to give a definition turbulence flow, because it is not possible to define a turbulence flow within a particular guideline only we can say that turbulence has certain important characteristics.

So, what we can see what some of characteristic we can learn from this very simple experiment that act very low velocities whatever is the perturbation in the flow, what how can a perturbation be there can be slight fluctuation in the inlet velocity, there can be perturbation due to the effect of the wall, so there can be perturbation or disturbances flow, but the disturbances do not get amplified the disturbances dye down, on the other hand if the velocity is large then because of the dominance of the inertial effect as compare to the viscose effect, viscose effect here there try to do they try to dampen out the amplification of perturbation, on the other hand the inetial effect try to amplified the perturbation.

So, if you have higher velocities you will have amplification of perturbation and that will ensure that there is a strong mixing a fluid elements that will come to that issue later on, but that will ensure that first there is no regular order motion of the fluid element, so that motion becomes random and disordered, because the perturbation trying to grow, so in that turbulent flow whatever, may be the perturbation in the flow that even very small perturbation till to grow, where as in the laminar flow those perturbations tends to lie down.

Now, we all know that we characterize this transaction behavior from laminar flow to turbulent flow with aid of the Reynolds number, which qualitatively may represent the ratio of inertia to discuss forces. Now, we will not go much into details of that, but with this qualitative understanding, we will try to identify some of the important features of the turbulence flow. Randomness of the transport of the transport variables with respect to time and also space, so we can say that this some average, on an average the quantities are fluctuating with respect to space and time, the quantity may be velocity with time whatever this are fluctuating with position and time. So, when they are fluctuating it is possible that because of fluctuating of velocity there is a significant amount of momentum transport between various flow fluid elements, because they are fluctuating in the velocities their interacting with their fluctuating components velocities and in that process there are having an exchange of momentum and therefore, it ensures that there is strong mixing in a flow.

So, turbulent flow we can say that has an enhanced effective diffusivity because diffusivity is a signature of mixing, so strong mixing in turbulent flow is there because of this fluctuation interaction of the fluctuation components of velocities, or may be other transport parameters like temperature, in terms of modeling there is another very important issue which is wide range of length scale and time scales that makes modeling and turbulence very complicated. To understand what are the ranges of this length scales for example, and how do they pose they challenge in terms of representing the turbulent flow through statically equation, we will see later on that why statically treatment is necessary, but even before that just to appreciate the different length scale and time scales, let us try to identify some of the basic entities in the turbulent flow.

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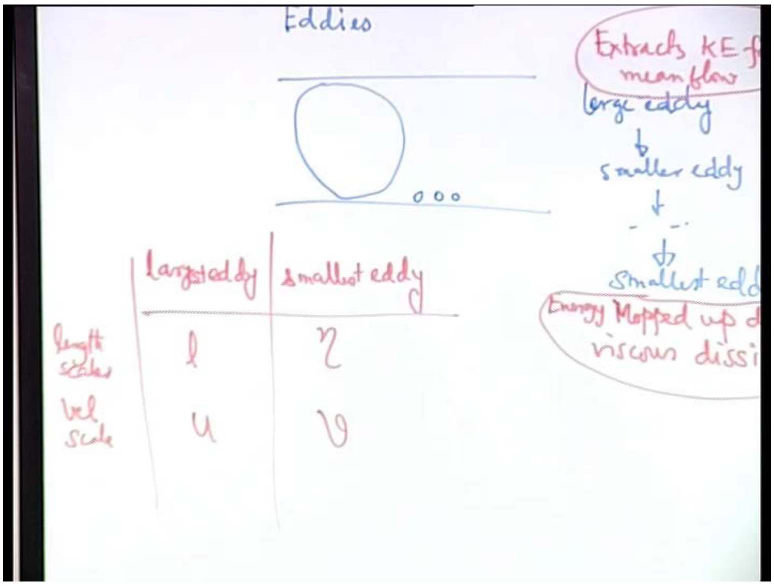

So, the basic entities in the turbulent flow are lams of rotating fluid masses called as Eddies, so these are, so these are essentially sum lams of fluids which are rotating. So, now there are Eddies of different sizes in a turbulent flow, so if you consider a channel like this you can have a large Eddy, a large the largest Eddy which is of the system length scale and you can have small Eddies which are of molecular length scale. So, what happens the large Eddy extract the energy from the mean flow because of the instabilities in the mean flow, so the mean flow has certain instabilities which is terracing the turbulence, because of the instabilities what happens is that the large Eddy can extract energy from the mean flow, once the large Eddy extract from the mean flow what it will do it will involve into smaller and smaller eddies to which the energies subsequently past and then.

So, energy will pass from the large Eddy to smaller Eddy in this way again to further smaller Eddy in this way to the smallest Eddy, so the large Eddy it extracts kinetic energy from mean flow, and that entire energy caskets through smaller and smaller Eddies till it is moped up by the smallest Eddy due to viscous dissipation. So, energy moped up due to viscous dissipation. The question is that why in the large Eddy energy cannot be moped up due to viscous dissipation and why do you have have to go to the smallest Eddy scale?

So, for that we have to understand what are the differentness in scale of large Eddy and small Eddy, so let us try to consider that you have large Eddy, when you say large we say largest Eddy and a smallest Eddy. Let us say the corresponding links scale, we just give length scale as a largest Eddy length scale as l and smallest Eddy scale as eta, large Eddy velocity scale as u and small Eddy velocity scale as v, if we known length scale and velocity scale using these two we can formulate what is the time scale. So, now what is the length scale, it is of the order of the system length scale, so if the Reynolds numbers is large what it means? It means that with respect to the system length scale the inertia forces dominant over the viscous force; that means, for the over the large Eddy length scale the inertia forces much, much more dominant over the viscous force that is why, whatever energy that the large Eddy extract from the mean flow that cannot be dissipated by the large eddy.

Eddy in the form of viscous dissipation because viscous effects are negligible as compared to inertial effects for large Eddies, but as come down to smaller and smaller

Eddy is the length, the size of the Eddy the length scale of eddy is small so that means, the inertial force with scales, with the length scale the inertial force will become smaller and smaller and as you go to the smallest Eddy, the smallest will be just good enough to dissipate all the energy that has been cascaded from the larger Eddy scale to the smaller eddy scale.

So, what we can say regarding the small Eddies, the smallest Eddy is will be just good enough to dissipate all the energy to viscous dissipation that means, the smallest Eddy will be characterized by a corresponding Reynolds number based on that length scale of the order of one, that is inertia force will just be balanced by viscous force, so that the Reynolds number will be the order of one. Now to understand this behavior, so this is known as energy cascading that is how the energy is cascaded down from the largest to the smallest Eddies.

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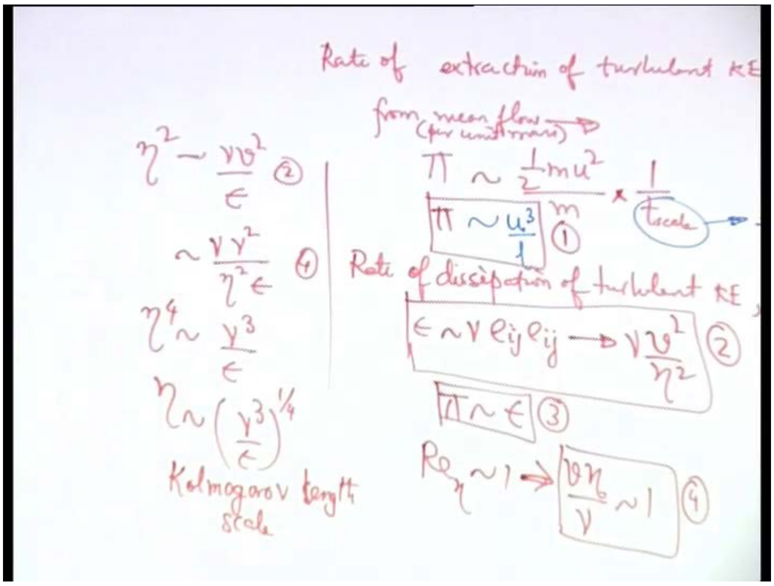

So, to understand that how different lane scales are involve with this energy cascading mechanism, let us first write what is the rate of extraction of turbulent kinetic energy from mean flow. So, let us call this as pi, so this is of the order of what half mu square this is per unit mass we write, so that divided by m this is that kinetic energy, so that divided by the time scale per unit time the rate. Now what is this time scale in terms of the velocity and the length scale, so this typically is called as the turn over time scale, that is a time scale characteristic to the large eddy turn over, so the large eddy because it will it evolves it is a dynamical structure it evolves into smaller and smaller Eddies and then again you have large Eddies appearing and so on.

So, it is a dynamically evolving situation, so the time scale is l by u so the this one the rate of extraction of turbulent kinetic energy is of the order of u cube by l ,what is the rate of dissipation of turbulent kinetic energy, that we call as epsilon; it is of the order of mu eij into eij, where eij is the rate of deformation tenser associated with the smallest Eddy, because the smallest Eddy is dissipating the entire kinetic energy, turbulent kinetic energy that has been extracted by the larger Eddies to viscous dissipation. So, this rate of deformation is given by the velocity gradient in terms of scale.

So, what is the velocity mu is the kinematic viscosity, so what is the rate of deformation in the scale of the smallest Eddy? v by eta, so mu v square by eta square, so we have an expression for pi of the order of this which is one expression then, mu the epsilon of the order of this one then for having a dynamic balance you must have pi of the order of the epsilon, so that there is no sort of storage at of energy at any intermittent condition, so whatever has energy that has been extracted the same energy is dissipated and this goes on in a cycle. So, pi is of the order of epsilon and the other constraint is that the Reynolds number based on the smallest Eddy lane scale is of the order of one, this we have just discussed why?

So, that will imply that v eta by mu is of the order of one, so based on these it is possible to relate what is the length scale of the system or the largest Eddy with what is the length scale of the smallest Eddy. So, let us try to do that it will just require some algebraic manipulation before doing that, let us try to characterize what is v, so v square is of the order of or let us first characterize what is eta, then we can characterize v on the basis of that, so eta square is of the order of mu v square by epsilon, so if you call this as 1, 2, 3, 4 this is from two, eta square is of the order of mu v square by epsilon. Now, from four you can replace v and write v is of the order of mu by eta, so mu into mu square by eta square epsilon this is using four.

So, eta 4 is of the order of mu cube by epsilon and eta therefore, is of the order of nu cube by epsilon to the power 1 by 4, see the smallest Eddy length scale does not depend on directly the features of the large Eddies, but it depends on the rate of dissipation of the turbulent kinetic energy and the kinematic viscosity and this length scale is known as Kolmogorov length scale.

Now, we can try to access that how this Kolmogorov length scale relates to the system length scale, so if you have the Kolmogorov length scale the corresponding velocity that you can obtain by using any of these relationships that is if you obtain v by using for example, v eta by mu of the order of 1, then from that you can obtain v that is known as Kolmogorov velocity scale, now if we want to relate it with the system scale see you have pi of the order of epsilon.

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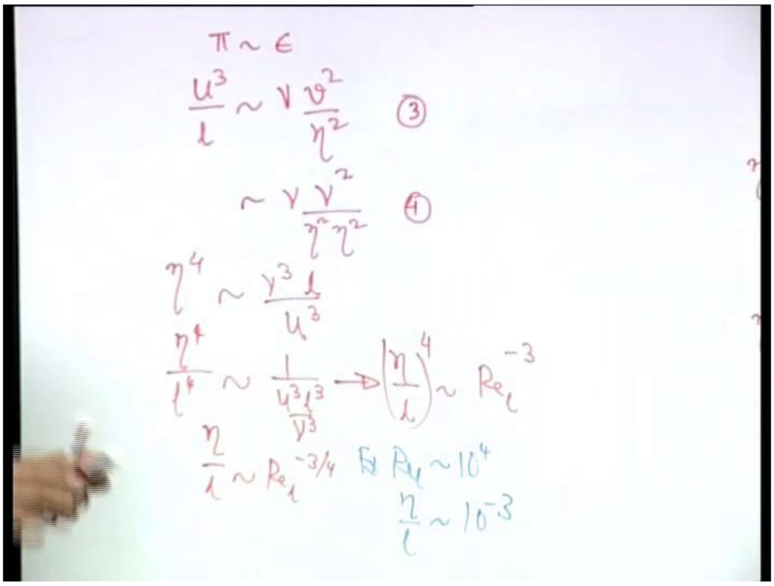

So, in the system scale you have u cube by l of the order of mu v square by eta square, so this is from three further v eta by nu is of the order of one, so you can eliminate v and write this as nu in place of v you can write nu by eta nu square by eta square eta square this is from four, so you have eta to the power 4 is of the order of mu cube l by u cube. So, if you want to relate eta with l you divide both sides by l cube, so eta to the power 4 by l to the power 4 will be of the order of u cube l cube by mu cube so that means, you have eta by l to the power 4 is of the order of Reynolds number l to the power minus 3 why we have defined the Reynolds number on the bias of this one is because this you have from the system level information that what is the system scale l large Eddy l will be of the order of system scale. So, that we can write eta by l is of the order of Reynolds number to the power minus 3 by 4 where this Reynolds number is based on the large Eddy scale.

So, now let us see that what we actually, let us try to have a feel of what do we actually mean by a wide range of length scales. So, if you have let us take an example say Reynolds number of the order of 10 to the power 4 then what is eta by l? 10 to the power minus 3. So, if it is 10 to the power minus 3 then we can see that the system the largest Eddy scale and the smallest Eddy scale, they are differing by this order which is quite large and greater and greater the Reynolds number this length will differ in a larger and larger manner.

So, what we can see that you have a large Eddy which has its own characteristics, so you need to capture that if you are trying to capture the physics of turbulent flow you would need to have the smallest eddy that you need to capture, those are like of the order of molecular scale and think about the continuum simulation of the navies oxicuation, where you are interested to capture such a small scale, so in a system you have various scales you need to capture all the scales to get the entire physics of energy cascading and capturing of all those length scales in single continuum simulation is very, very challenging, because over and above this multiple length scale and of course, multiple time scale it is faceted by the fact that there is a entire degree of randomness in the flow that means, what do we speak of? When we say that there is a randomness, it is highly sensitively dependent to the initial condition.

So, if you have a say let us say you want to find out u versus time, so at time equal to zero the particular point say this was u, so then as time evolves, so let us say that u evolves in this way, now if you slightly change the initial condition by a very small infinitesimal change which is nothing but something which can be within numerical errors, but that can lead it entirely to a different orbit, so it is a hallmark of something known as K O T advection and in fact, in a turbulent flow vortices at vex chaotically over space and time. We will see later on that what is the role played by vorticity in the in the dynamics of turbulent flow, but what we can see here is that this highly sensitive dependence to initial condition is also one of the hallmarks of turbulent flow and that is why it is very difficult to model it numerically, because if you have a slight change in the initial condition if you have a slight numerical perturbation it may entirely lead to a different solution. So, dealing with the actual instantaneous quantities in turbulent flow is not a very easy thing.

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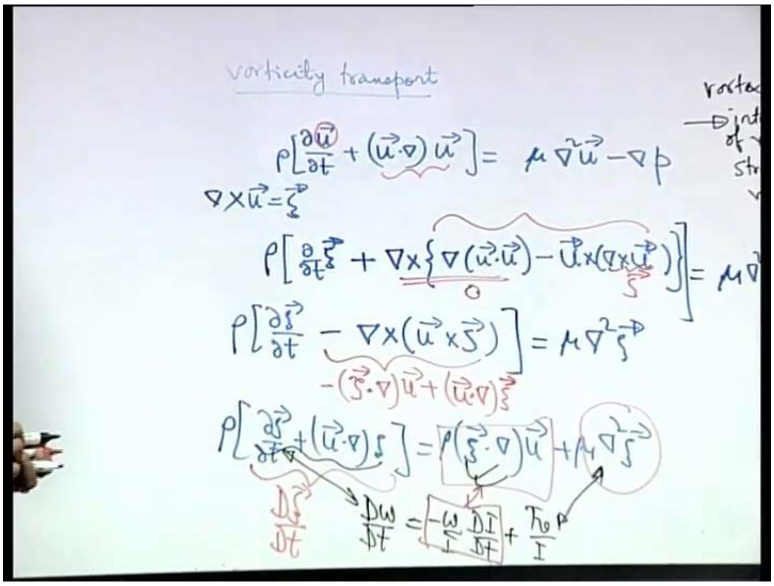

Now, we just mentioned about the vortices features and it is not a bad idea to talk about the vortices dynamics in turbulent flows, so vorticity transport to have mathematical feel on how to express or explain the vorticity transport, let us start with the navier's stokes equation. So, what we have written here is we have written the navier's stokes equation without anybody forester in a vector form, so it is like if you we have written the velocity vector u in a single term, you can obtain this by finding a vector sum of x component of Navier Stokes equation, y component of Navier Stokes equation, and zee component of navier stoke equation.

Now, in from velocity we want to get the vorticity, so what we can do simply we can take cal of both sides in this equation, so we take cal of both sides Del cross, so if you do that and we define Del cross velocity as the vorticity vector which you call a zeta plus now before taking the Del cross, you can simplify this particular term by using a vector identity, this a vector identity to write this corresponding term now ,the right hand side will become mu Del square zeta minus now recall from vector calculus that cal of gradient of a scalar is equal to zero, so this will be null because it is called of gradient of the scalar pressure similarly, here the first term it is cal of gradient of a scalar v that is u dot u square, so that will also be a null and hence what we will get you will get rho minus Del cross u cross zeta because Del cross u is zeta.

So, this is equal to zeta, is equal to mu del square zeta, now this term we can write again by a vector identity zeta dot del u and u dot del zeta one with the opposite sign as that of the other this includes the minus sign, so you can write rho Del zeta Del t plus u dot del zeta is equal to zeta dot del u plus mu del square zee, this incidentally is the total derivative of zeta the capital D D T of zeta. So, this gives a vector evolution of vorticity there is one rho here, now we can see that it is like a sort of advection diffusion equation with a source, so you have the left hand side as a transient term and the advection term, if you can recognize this is the diffusion term and this is something like a source now what is the source of vorticity?

So, that means there is some source of vorticity and what is that source of vorticity in a turbulent flow? This is a mathematical derivation not so complicated, but at the same time once we get a term as a source it is not so easy to interpret it physically that what does it do for turbulent flow, so let us try to use more simple analysis or analogy to figure out that what this source term is essentially signifying and then we will relate that with these derivations.

Now, let us say that you have large Eddy in a turbulent flow the Eddy is a lump of fluid with certain rotationality, say rotating lamp of fluid, so to say in very simple language, so it will have its movement of inertia and an angular velocity omega, now if you consider D DT of I omega this is what it is I d omega d t plus omega DI Dt, because we are considering a fluid flow type of analysis we are considering the total derivate, but it essentially behaves in the same way as that of the ordinary derivative with regard to the rules of differentiation, so we can use the product rule now this is what, this is basically the time rate of change of angular momentum.

So, this is equal to what? This is equal to the net torque that is acting on the system and here the torque that acts on the system is to dampen out the surface that is the viscous torch, so we can write this as T v, where T v represents the viscous torque. So, we can write D omega Dt, let us write together with a vorticity transport equation, so that we can get one to one analogy. So, D omega Dt is equal to minus omega by I DI DT plus T v by I so, omega is like angular velocity and we know that angular velocity has some relationship with the vorticity of the flow, so these two terms have sort of analogies not this single term does come together that is capital D D T this we know that, it is a viscous term mu because viscosity is present therefore, we can conclude these term and these term must have identical physical meaning.

So, the term rho zeta dot del u has a qualitative physical meaning of the term equivalent to minus omega by I DI DT, so let us try to find out the physical meaning of that, so to do that let us consider a large Eddy scale as an example. So, if you consider a large Eddy scale large Eddy scale, so if you consider a large Eddy scale what will happen? If you consider a large Eddy scale what happens to the viscous torque? In the large Eddy scale the viscous torque is negligible because inertia force dominates much, much more over the viscous effects.

So, for the large Eddy scale capital D D T of I omega is equal to zero I omega is conserved an angular momentum is conserved, so this is one of the peculiarities of the large Eddy where the angular momentum is conserved, so then for the large Eddy scale we can write D I Dt is equal to minus I by omega D omega Dt, now consider an Eddy, so Eddy is going to, a large Eddy is going to extract energy from the mean flow similarly, a little bit of smaller energy smaller Eddy will extract energy from the larger Eddy in this way energy is cascading.

So, when the little bit of smaller Eddy extracts energy from the larger eddy what will happen to its angular velocity? It will increase because its rotational kinetic energy will increase, so when thSat is increasing that means, capital D D T of omega will be positive because with time the angular velocity of that it is increasing. So, with time if that is increasing, now I am momentum inertia which is positive and omega remember here we are writing it in the same sense, so whatever is the omega sense here the same omega sense is here, so that if d omega d t is positive then this right hand side will be negative, if this is positive then DI DT is negative.

So, DI DT is negative means what? The vortex element now will have a reduce movement of inertia how can it do? So, if the vortex element is like this it is not changing its volume say it is say it is an incompressible vortex element, so what it is doing? It is evolving into a structure where it is bit of elongated, so that its effective radius of direction goes down that is how its moment of inertia can go down, so its earlier radius was this scale now its radius is roughly this scale, so what happens the vortex element appears to be stretched or elongated and this vortex stretching is a very interesting thing, because of this elongation you have a greater chance of interaction of one vortex with the other.

So, that is one physical aspect, but the interesting physical aspect with respect to vorticity transport is that there is intensification of vorticity due to stretching of vortex elements or associated with the stretching of vortex elements because of stretching of vortex elements you have the DI Dt negative and that will make d omega DT positive or if you think terms of the other way if d omega DT is positive then DI DT is negative, so you, all in all we can say that there is a very interesting phenomenon in turbulent flow called as vortex stretching, by virtue of which you have intensification of vorticity that is angular velocity increases in terms of magnitude with stretching of vortex elements.

So, what we can conclude that whatever is that intensification of vorticity due to stretching vortex elements that effect of vortex stretching is must be given by this term because this term is associated with DI DT with sort of indicates the effect of change of moment of inertia which occurs due to vortex stretching. So, the source term here it appears in the vorticity transport is because of stretching of vortex elements and this is one of the important physical issues in turbulent flow.

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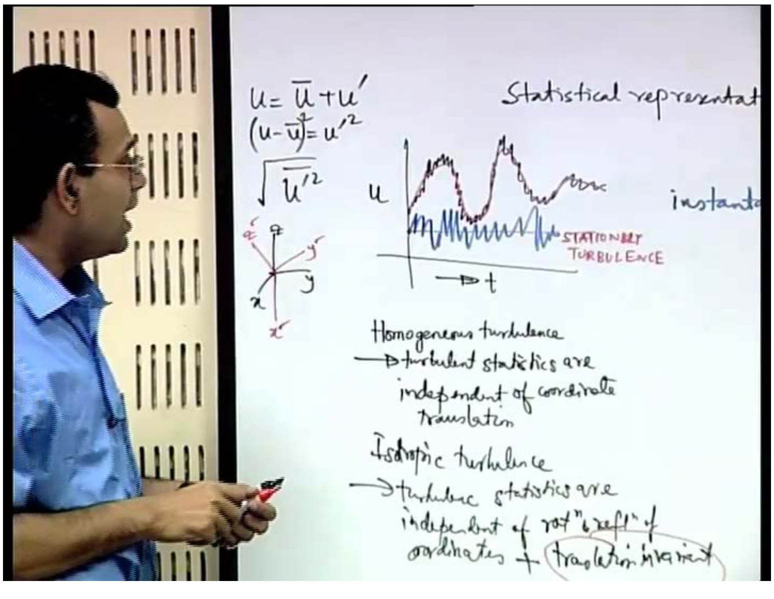

So, with this background we will now go to the statistical representation of turbulent flows the question is why should we go to a statistical representation of turbulent flows? What is the need? We have seen already that there is a high level of fluctuation local fluctuation, the fluctuation may be time dependent, the fluctuation may be specially dependent, but high level of fluctuation of the transport quantity in a turbulent flow and they are highly sensitive to initial conditions, so it all makes a deterministic treatment of the turbulent flow very difficult and one has to go for a statistical analysis.

So, the randomness in turbulent flow for example, if you consider a turbulent flow velocity at a particular location as a function of time you can get such a scenario, so because of random fluctuation at each and every location at an at each and every time and these are truly random and highly sensitively dependent to the initial condition, it is very difficult to treat them deterministically, so that is a numerical challenge in fact, a theoretical challenge not just a numerical challenge.

Now, what you can do is you can for the time being considered it as a super position of a mean behavior and a fluctuation on the top of that, so this red line let us say is a mean behavior and whatever fluctuates with respect to that is a fluctuation, so we can write that the instantaneous quantity is equal to mean plus fluctuation now it may, so happen we have to remember one thing the turbulent flow is by nature always three dimensional and unsteady, but when you statistically average it may be possible to have a statistically average steady behavior how it is possible? Let us take another example, let us say that this is the mean and may be this is the variation of the instantaneous velocity at a given location, so you can see that it is actually unsteady, but when you take the mean statistically the mean is steady such a turbulence is called as stationary turbulence. So, when we have a mean, we also have a standard deviation with respect to mean, so you can represent a turbulent flow with respect to mean standard deviation codilation coefficient very much similar to what you do for any statistical analysis, now when you do that there are certain important special types of turbulent flows that appear which let us discuss for the sake of completeness one is called as a homogenous turbulence.

So, what is a homogenous turbulence? The turbulent statistics are independent of coordinate translation that means, in other words they are position independent that is why if you translate the coordinate, you will get the same turbulent statistics by turbulent statistics if you may mean the R M S or the mean like this. So, how do you evaluate the

R M S? See u bar is the average of u, then u is equal to u bar plus a fluctuation, we will come into this in more details as we consider the turbulence models, now when you consider u equal to u bar plus u prime then u minus u bar this is deviation from mean is u prime square of that is u prime square and mean of that if we give it by a symbol then that is mean square deviation and square root of that is root means square deviation or R M S.

Now, if we consider something called as isotropic turbulence, here it means that the turbulence statistics are independent of rotation and reflection of coordinates in other words it means that it is orientation independent because rotation what it does? It changes the orientation, so if you had x y z and you have some description of R M S of u, R M S of v and R M S of w, then if you have a rotation of this x y z to mu x, mu y mu zee then with respect to this rotated axis you will get the same statistics of R M S of u, R M S of v and R M S of w. So, you can say that the turbulence statistics are independent of rotation and also reflection plus translation that means, isotropic turbulence by definition must be homogenous, because when you say it is translation independent in variant that means, you are essentially talking of a homogenous characteristic, so these are some terminologies.

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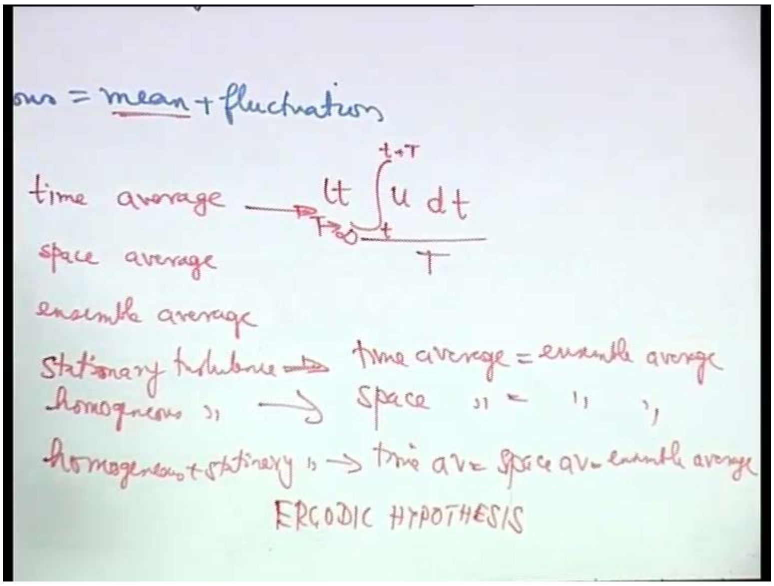

Now, next is that we have talked about the average or the mean question is what type of mean? Or what type of average? There are different types of averages which are possible and let us try to understand those. Those are called as time average, space average and ensemble average. So, what is time average? What you do? At a given location, you measure the property as a function of time and then find out an average, so when you say you find out an average how you write a time average? So, let us say you write, so when you write an average that means, you have u versus time, so to write the average you must integrate u with respect to time over a time period divide by the total time period take the limit as this time period t, so may be this from t to small t to small t plus capital T we take a limit which technically we say capital T tense infinity this is just a representation.

What we physically mean by that? So, now, let us consider the issue of wide range of time scales, we had earlier discussed about the issue of length scales, now let us focus on the issue of time scales. So, if you see that in a turbulent flow there are some turbulent fluctuation length scales times scales like this one's, so these are very small time scales over which the turbulent quantity fluctuates, on the other hand there is a system time scale, so you can see that here with respect to system, it does not change on an average, so in the example curve shown above the bottom one what we can see, we can see that with respect to time there is a system characteristic time scale, where you have a sort of a periodicity over the system and that time scale is much larger than the turbulent fluctuation length scale.

So, now when we consider capital T tense to infinity we mean that it is a time scale which is much larger than the turbulent fluctuation time scale, but it must also be much smaller than the system time scale otherwise, let us say if you literally interpreted as t tense to infinity without considering what physics it talks about then you take the total time as the starting time to the end time an average over that you lose all the transient information in between. So, it is not the total large time scale it is a time scale which is definitely much larger than the individual fluctuation scale, but much smaller than the overall system characteristic time scale that we call as capital T, so this we call as time average.

Now, what is space average? Space average is like the similar thing you replace t with x special coordinate, so what you do is at a given time you make an average over position for a time average at a given position you are making an average over time, for special average given time you make an average over position and the third one is ensemble average. So, what you do here is you do a large number of repeated experiments, hugely large number of repeated experiments and then once you do that when you find out the statistical average of those repeated experiments under different conditions then that average is called as, so you basically have the say similar it is a stochastic type of experiment, so similar boundary condition similar arrangement, but many different experiments.

So, when you have many different experiments all those experiments will give you some average and that is known as ensemble average, so if you have a stationary turbulence what will that imply? It will imply the time average is equal to ensemble average because if you have stationary turbulence what you are having? If you are having stationary turbulence; that means, turbulence statistics do not change with time, so doing getting a reading at a different time is as good as getting a reading for a different experiment, so averaging over time is as good as averaging over different random experiments, so time average equal to ensemble average similarly, for homogenous turbulence you have space average equal to ensemble average, so for homogenous plus stationary turbulence you have time average, equal to space average, equal to ensemble average and this in statistical analysis is known as ercodic hypothesis.

So, from hence forth we will assume that the erotic hypothesis will be valid, so when we will be doing an averaging will assume that we are either doing time average space, average or a ensemble average, but if erocodic hypothesis is the is valid one can be replaced in terms of equivalence of the other and so we will call it as a averaging, but we that averaging may be time space or ensemble. we stop here today we will continue again in the next class thank you.Thank you