Computation Fluid Dynamics

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Module No. # 01

Lecture No. # 42

Introduction to Turbulence Modeling (Contd.)

In the previous lecture, we introduced some concepts of turbulence and the whole objective was not to go into the details of turbulence, but to utilize those concepts in developing some strategies for modelling of turbulent flows. We will proceed further towards that and we will go ahead with the slides in order to achieve that objective.

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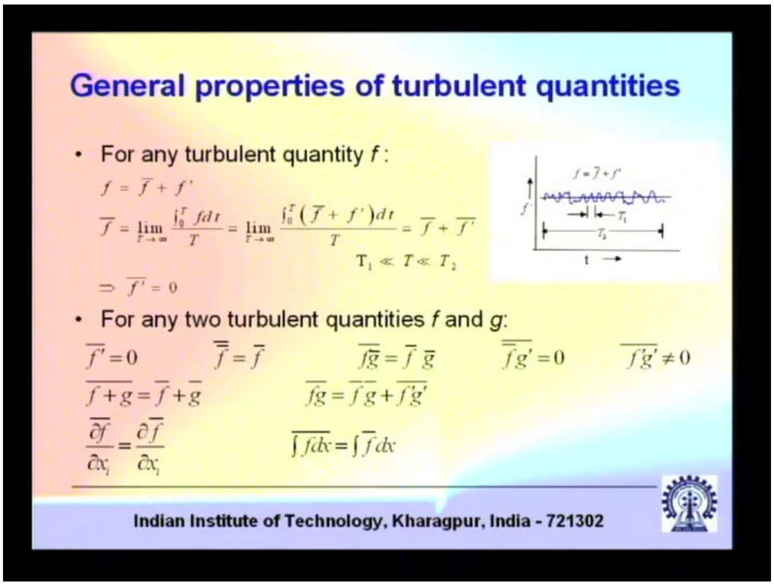

First, let us recall that for turbulent flows, we try to represent the turbulent flows with the aid of some statistics. And, the statistics of turbulent flow implied that any turbulent quantity say f can be decomposed into two parts. One is the mean of f or the average of f. We discussed about certain types of averages like time average, space average and assemble average. This is some sort of average; and then, a fluctuation component on the top of that. We defined the average in a particular way. For example, the time average of f, that is, f bar; we defined it as in the limit t tends to infinity integral of f dt by capital T;

where, capital T is the time interval over which the average is taken. If you look into this figure, capital T should be much greater than the turbulent fluctuation scale T 1. At the same time, it should be much smaller than the system characteristic T 2.

Now, what we can infer from here is that if you use this definition and plug in place of f bar and f prime, then you can get integral of f bar dt by T; you get f bar plus average of f prime; So, that means, f bar equal to f bar plus average of f prime, so that average of f prime equal to 0. So, any turbulent fluctuation quantity will have a 0 average. Now, there are certain related relationships for any turbulent quantities f and g in terms of their average for example, f prime. The fluctuation f average equal to 0; average of f bar is f bar itself, because f bar itself is an average. Then, average of f g bar is equal to average of f into average of g.

Similarly, average of f bar g prime is 0. Why? You can simplify it in this way. You can take f bar out of the average, because it is already average. So, it will become f bar into average of g prime; and, that will be 0. But, importantly, average of f prime g prime is not equal to 0. So, this is very important to keep in mind that average of f prime is 0, average of g prime is 0, but average of f prime g prime is not equal to 0. Average of f plus g is average of f plus average of g. Average of f g is equal to average of f into average of g plus average of f prime g prime. So, this you can easily get by decomposing f into f bar plus f dash and g as g bar plus g dash, and then multiplying. Similarly, you can write the corresponding rules for derivatives and integrals.

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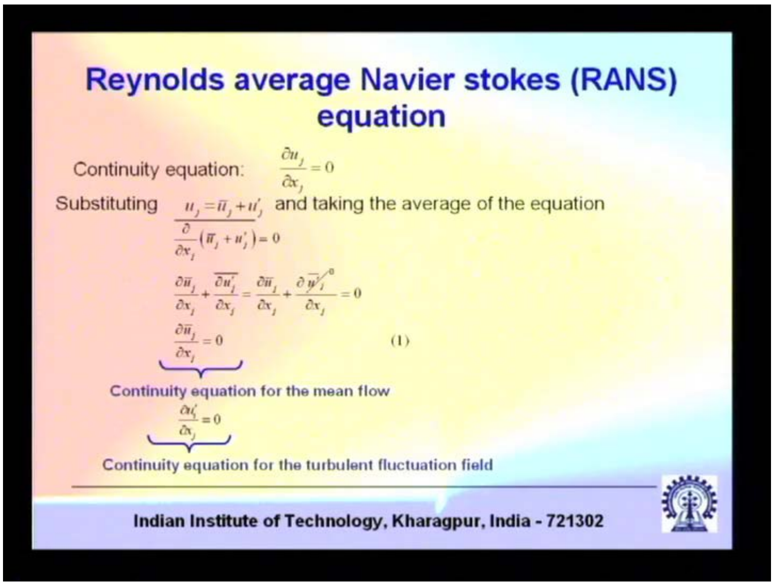

Now, with this elementary background, let us go into the Reynolds average Navier-Stokes equations or RANS. Now, we have already discussed that what is the motivation behind this; that the Navier-Stokes equations for turbulent flows exhibit certain characteristics like they are highly sensitive to initial conditions and they must address a wide range of length scales and time scales. On the other hand, you can reduce those complexities by considering the statistical average form of the Navier-Stokes equation, which are known as Reynolds average Navier-Stokes equation.

Now, before going into the momentum equation, we go into the continuity equation. So, you have del u j by del x j equal to 0. What we do is we substitute u j equal to u j bar plus u j prime. When you do that, you can take the average; and, within the average, there is derivative term. So, you can write the sum of these two; del u j bar by del x j plus del u j prime bar by del x j. So, del u j prime bar by del x j is 0, because u j prime bar is equal to 0. So, from here, what you can see that the mean flow continuity equation will be del u j bar by del x j equal to 0. Remember, we are considering a constant density flow. Now, if you have the continuity equation for the mean flow like this, now, as a next step, what we can do? We can subtract the equation 1 from the continuity equation. So, what we get? If you subtract these two, you get the continuity equation for the turbulent flow fluctuation. So, that is u j minus u j bar will be u j prime. So, del u j prime by del x j will be equal to 0. That is the continuity equation in terms of the turbulent fluctuation flow.

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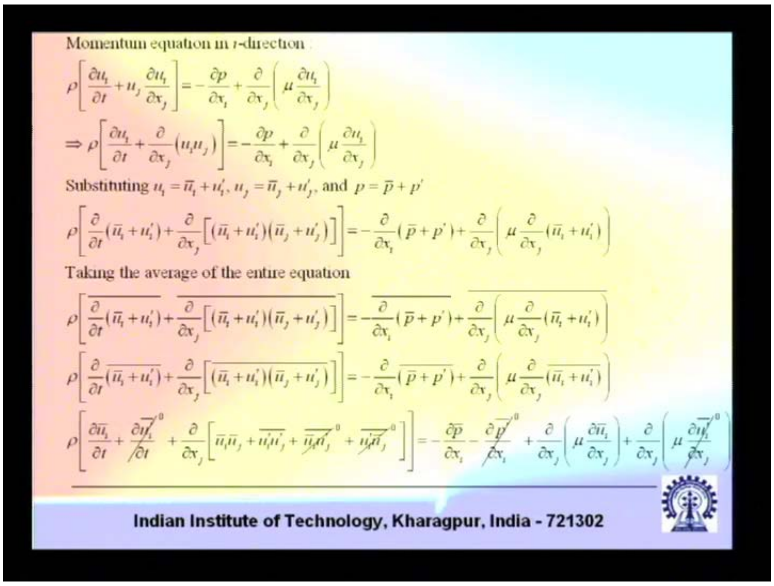

Now, let us consider the momentum equation in the i-direction. First, we write the momentum equation just like any other equation; we will be substituting u equal to u bar plus u prime. But, before that, we make a slight manipulation. What we do is that we have u j del u i by del x j; we add u i del u j by del x j with it, so that the terms together can be written as del by del x j of u i u j. And, when we add u i del u j by del x j, that is equal to 0, because del u j by del x j equal to 0 by continuity equation. So, it is a sort of going back to the conservative type of form by using the continuity equation.

Now, you substitute u i equal to u i bar plus u i prime; u j equal to u j bar plus u j prime; and, p equal to p bar plus p prime. So, if you do that and then expand, next step is to take the average of the entire equation. So, that we indicate by a bar at the top. So, the basic thing what we are doing, we are decomposing u as u bar plus u prime; p as p bar plus p prime; and then, with that expanded equation, we are taking the average. So, when we take the average, we should keep in mind that the average of the fluctuation is 0. So, out of four terms u i bar u j bar – that term will remain; u i bar u j prime – the average of that will be equal to 0, because that will be u i bar into u j prime average and u j prime average is 0. Similarly, u i bar u j prime average will equal to 0, but u i prime u j prime average will be not equal to 0. So, there will be extra term u i prime u j prime average. Right-hand side – similarly, you can decompose the pressure into two parts and the velocity into two parts and the fluctuation components will go away.

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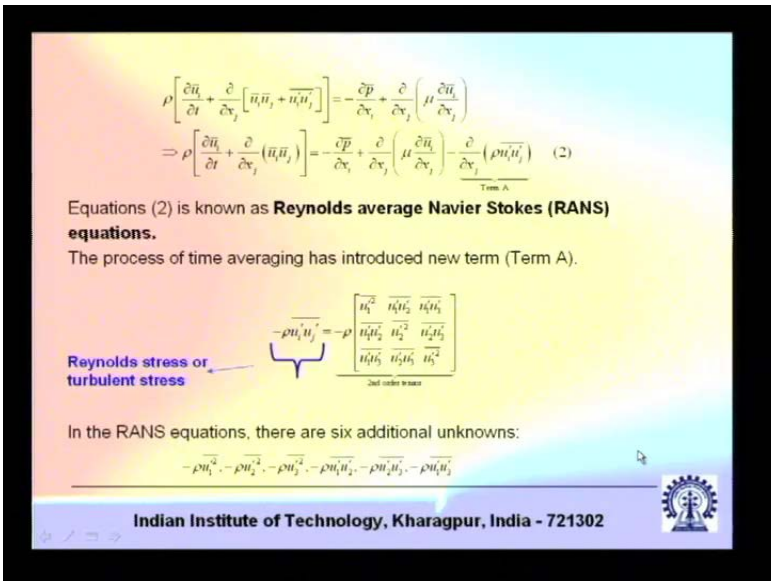

Then, come up with the governing equation in terms of the mean flow. When you write the governing equation in terms of the mean flow, it is like rho del u i bar by del t plus del by del x j into u i bar u j bar. So, this is like... The form is the same as that of the original momentum equation, except there has been an extra term, which we have now dumped into the right-hand side. So, there was a plus del by del x j of rho u i prime u j prime bar; that term we have taken to the right-hand side and it has become minus. So, that we call as term A in equation number 2.

The first observation is that the Reynolds average... So, this process is known as Reynolds averaging. The Reynolds average Navier-Stokes equation in terms of the mean quantities – they look like in terms of structure, the original Navier-Stokes equation in terms of the original quantities, except for the fact that now you have an extra term A in the right-hand side. And, it is important to understand how to treat this extra term. This extra term... At first, if you look at this extra term, it is del by del x j of something. If you recall, that the right-hand side in the Navier-Stokes equation has the form del by del x j of tau i j, so that del by del x j of something that what we have written, minus rho u i prime u j prime bar; that has a unit of tau or stress. So, it is something which has a unit of stress and it has some physical consequence. What is the physical consequence? u i prime u j prime.

Let us say that you have a velocity fluctuation along x-direction; now, that can interact with the velocity fluctuation along the y-direction. And, in that way, there can be an exchange of momentum, so that there is an additional stress. So, this physically we can talk about in terms of a stress, which originates because of this Reynolds averaging or Reynolds decomposition. This we call as Reynolds stress or turbulent stress. And, that is denoted by minus rho u i prime u j prime bar. So, if you write it in terms of a stress tensor, it is like a second order stress tensor, because it requires two indices for specification. But, it is not physically the stress that we consider for the momentum equation in its unperturbed form or like undecomposed form.

Now, you can see that how many independent components are here; of course, there are nine components; out of which you have six independent components of this stress tensor, because it requires two indices: i and j. And, because it is symmetric, the Reynolds stress tensor... Also, you can see that because u i prime u j prime is same as u j prime into u i prime, it is symmetric. So, you can swap i and j; still get the same term that makes the renders six independent terms in the second order stress tensor. So, in the Reynolds average Navier-Stokes equation, there are six additional unknowns. See now, this equation you try to derive on the basis of the mean flow; all quantities came in terms of the mean flow, except the last quantity, which itself is not a single unknown, but it involves six extra unknowns.

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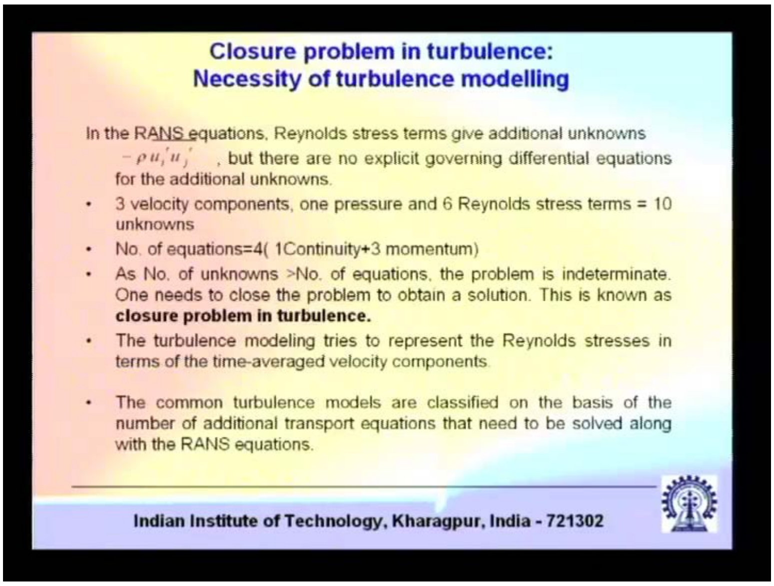

That is one of the important challenges that brings to something called as closure problem in turbulence. So, what is a closure problem? In Reynolds average Navier-Stokes equation, Reynolds stress terms gives additional unknowns, but there are no explicit governing equations for this additional unknowns, that is, minus rho u i prime u j prime bar. So, these extra unknowns appear, but till now, we have not come across any explicit governing equations for that. So, let us make an accounting of the number of equations with number of unknowns. We have 3 velocity components, 1 pressure and 6 Reynolds stress terms. So, we have total 10 unknowns out of this system after Reynolds averaging. Numbers of equations – you have 1 continuity equation and 3 components of the momentum equation. So, 4 equations. As the number of unknowns is greater than number of equations, the problem in this form is indeterminate. And, one needs to close the problem; that is, obtain additional or derive additional equations or model additional equations if necessary so as to come up with a matching condition in terms of number of equations and number of unknowns. So, this is known as closure problem in turbulence.

This is a very important concept, because it tells us that why do we require turbulence modelling. So, you have more number of unknowns in terms of Reynolds average quantities. And, those unknowns need to be closed with the aid of suitable equations. And, one needs to mathematically model that to come up with those additional equations. So, the turbulence modelling – what it tries to do? It tries to represent the Reynolds stresses in terms of the time-average velocity components. The common turbulence models are classified on the basis of the number of additional transport equations.

Now, before we get into the details of different types of turbulence models, let us try to consider one particular aspect; that is, why do we require a closure in terms of a very simple example?

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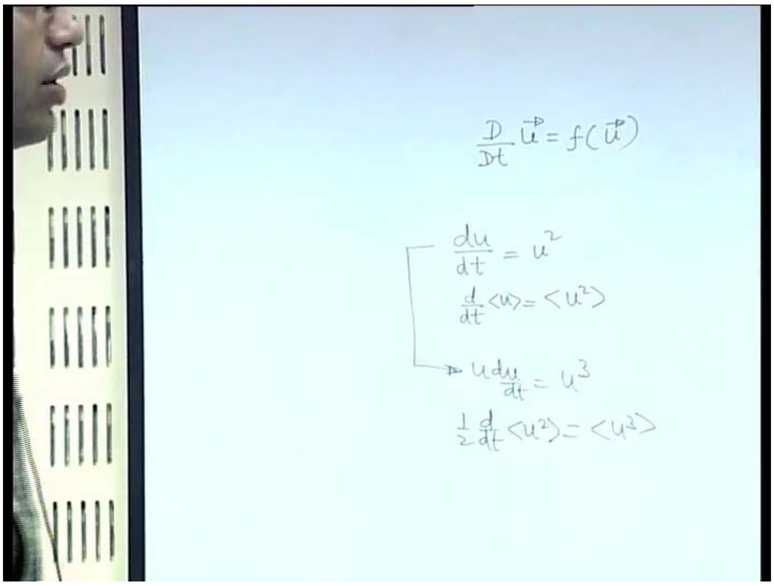

Let us say that you have a governing equation, du by dt equal to u square. Remember that in a very general form, the Navier-Stokes equation can be written as D by Dt of velocity vector is a function of velocity vector, because the left-hand side of the momentum equation is capital D by Dt of rho into capital D by Dt of u; that is, the acceleration term is there. Right-hand side you have pressure gradient and velocity; and, pressure – you can write in terms of velocity. We have earlier seen through one of our derivations in stream function what is (( )) equation that you can derive a pressure poisson equation, where you can express the pressure gradient in terms of the velocity. Therefore, in principle, write Du by Dt as a function of u. So, we have taken a very simple example; nothing to do with the Navier-Stokes equation, but with just a simple mathematical analogy that du by dt equal to u square.

Now, let us say that this equation is highly sensitive to initial condition, so that we want make a statistical averaging of it. So, if we make a statistical averaging of it, this is what we come up with. Now, once we make a statistical averaging, the tractability becomes an issue. Why? See earlier, only u was a variable before statistical averaging. Now, you have two variables: one is average of u; another is average of u square. So, you can say that I have average of u square. So, I should now introduce a new equation, which is new governing equation for average of u square. How you can do that? You can multiply this equation by u. If you write u du by dt, that will be u cube. So, half d by dt of u square and then you take the average. See what has happened now, you have got a governing equation in terms of u square average, but a new unknown u cube average has come up. So, this is essentially what happens when you try to average the Navier-Stokes equation.

Every time you try to average, new quantities appear. And, if you want to write governing equations in terms of new quantities, another set of new quantities appear. So, tractability becomes an issue. And, that is why you see that perhaps you can go on doing this; but, if you want to stop somewhere, you have to write a model equation, which closes the system of equations with number of equations are number of unknowns; otherwise, you will be going on adding new equations and at the same time new variables. It will never be closed. So, unaveraged equations may not be well-behaved; averaged equations may be well-behaved, but average equations may give rise to additional unknowns, which need to be closed. And, that is one of the very important essential concepts of the closure problem in turbulence modelling.

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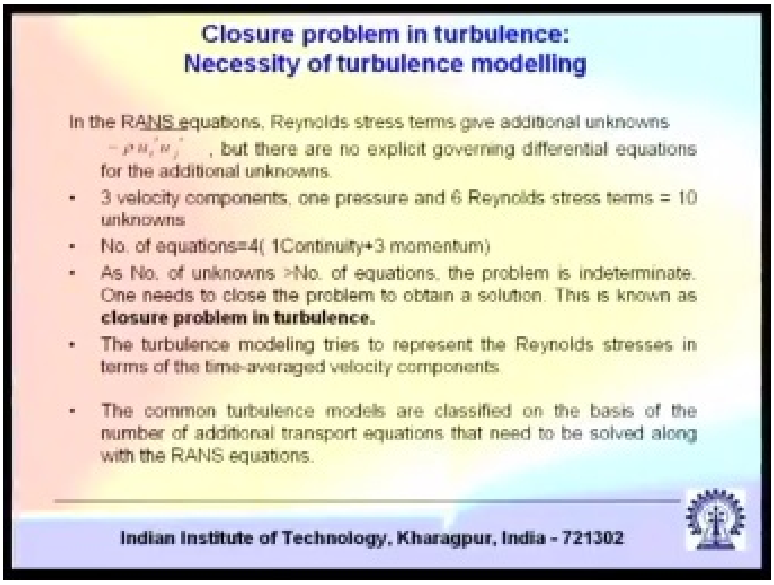

Now, with this understanding, what we will do is we will now get back to our basic discussions on turbulence modelling, which we are having. So, the common turbulence models we will now try to go through.

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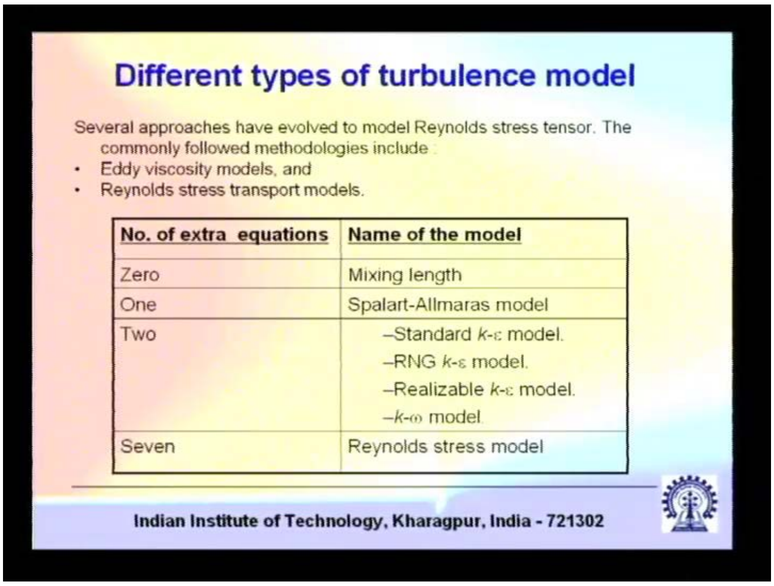

Remember, in these elementary codes, we will not go into the details of all the turbulence models, but we will try to catch up with some of the very important and very commonly used turbulence models. So, different types of turbulence models. Now, several approaches have evolved to model the Reynolds stress tensor. See the turbulence modelling is all about modelling the Reynolds stress tensor. So, several approaches have evolved to model the Reynolds stress tensor. The most commonly followed methodologies include the following: one is eddy viscosity model and that other is Reynolds stress transport model.

Now, let us just go through the names of some turbulence models and number of extra equations needed to frame those models. So, there is one model, which is called as zero equation model, which is known as mixing length model, which we will come across soon. Then, one additional equation, Spalart-Allmaras model; then, two equation model, standard k-epsilon model, RNG k-epsilon model, realizable k-epsilon model, k-omega model. These are the names of some of the models; then, you can have even seven equation models like the Reynolds stress model. And, we will also very briefly look into that.

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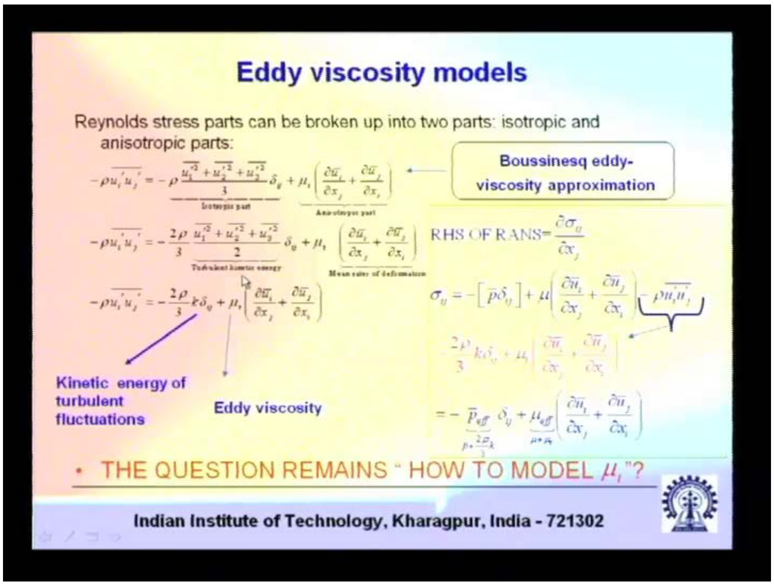

So, those were some examples. Remember that those were not the exhaustive leads of turbulence model. But, just to give you an idea that what can be the ranges of the number of additional equations. Now, we come to eddy viscosity models. So, let us see that you have the Reynolds stress. Now, the Reynolds stress can be divided or decomposed into two parts: one is isotropic and another in anisotropic part. So, just like if you recall that the stress tensor in the usual case can be decomposed into two parts: hydrostatic component and deviatory component. The anisotropic component is just like the hydrostatic component. So, it is minus rho u 1 prime square average plus u 2 prime square average plus u 3 prime square average by 3. So, the arithmetic average of the diagonal elements of the stress tensor that times delta i j, because it is isotropic component. So, we multiply it with (( )) delta delta i j, where delta i j is 1 if i is equal to j.

Now, with the isotropic part, we also add the anisotropic part, so that we get the total stress, Reynolds stress tenser. So, what we are doing is, we are mathematically treating the Reynolds stress tensor just like the regular stress tensor. Although physically they are not originating from the same physical behaviour, but mathematically, we are trying to treat them analogously. So, for the anisotropic part, what we are doing is we are trying to write the Reynolds stress tensor as a function of the rate of deformation in terms of the average quantity. Just like the deviatory component of the regular stress tensor, we write mu into del u i by del x j plus del u j by del x i. Here instead of mu, we use a different coefficient, mu t called as turbulent viscosity or eddy viscosity. And, in terms of the regular rate of deformation, it is a rate of deformation in terms of the average quantity. So, this is known as Boussiness eddy-viscosity approximation or eddy viscosity model.

Now, when you do that, you can now write the isotropic component in terms of the turbulent kinetic energy. How do you write the isotropic component in terms of the turbulent kinetic energy? You have u 1 prime square plus u 2 prime square plus u 3 prime square; their averages were appearing in the isotropic component. Now, what you do is multiply by 2 and divide by 2, so that the quantity that appears here is the turbulent kinetic energy. We have already defined turbulent kinetic energy earlier. So, you have the turbulent kinetic energy, which is defined as u 1 prime square average plus u 2 prime square average plus u 3 prime square average by 2; that into delta i j is the first term. So, we have an adjusting term minus 2 by 3 rho plus mu t into rate of deformation on the mean quantity. So, we can write minus rho u i prime u j prime average is equal to minus 2 third rho into k; where, k is the turbulent kinetic energy delta i j plus mu t into del u i bar del x j plus del u j bar del x i.

Now, if you consider the right-hand side of the Reynolds average Navier-Stokes equation, what are the terms that are there? One is that you can write the entire right-hand side, except if there is any additional body force. We are not considering any additional body force; the right-hand side – all the terms we can write in the form del sigma i j del x j; where, sigma i j is a sort of stress tensor, which includes the pressure that is minus p delta i j plus mu del u i bar del x j plus del u j bar del x i. These are like the regular features of the Navier-Stokes equation plus Reynolds stress tensor; that is, minus rho u i prime u j prime average. And, that has two parts. Now, one of these parts; the turbulent kinetic energy part you can dump with the pressure term and other part you dump with the viscous term. So, if you do that, what you get? You get this equal to minus P effective delta i j; where... What is this P average effective? It is p plus 2 third rho k. So, as if a new pressure, which is the mean pressure plus 2 third rho into the kinetic energy that delta i j plus...

Instead of the laminar viscosity, you have an effective viscosity, which is laminar viscosity plus the turbulent viscosity: mu plus mu t into the rate of deformation. So, as if we have derived a constitutive relationship. Remember, this is not a fundamental derivation. That is why I am saying as if... So, it appears that as if we have derived the constitutive relationship. The whole idea, the objective is to cast this equation in sigma i j in a constitutive form that we are commonly familiar with for laminar flows as well. So, to do that, what we have seen now is that the pressure is modified to a different parameter. And, in place of the laminar viscosity, it is the laminar viscosity plus the turbulent viscosity. So, that will make the governing equations, the Navier-Stokes equation.

The average Navier-Stokes equation looks exactly the same as that of the unaverage Navier-Stokes equation, except mu being replaced by mu effective. But, the question is it is an illusion. It illusively appears that as if it is closed; but, it is not yet closed, because we do not know what is mu t. Like nobody tells us that mu t will be a particular value. So, it needs to be modelled and the big question remains how to model mu t. So, the eddy viscosity turbulence models, post this or try to answer this question that how to model the turbulent viscosity. The turbulent viscosity is also called as eddy viscosity, because eddies are involved in the transfer of the momentum due to fluctuation components.

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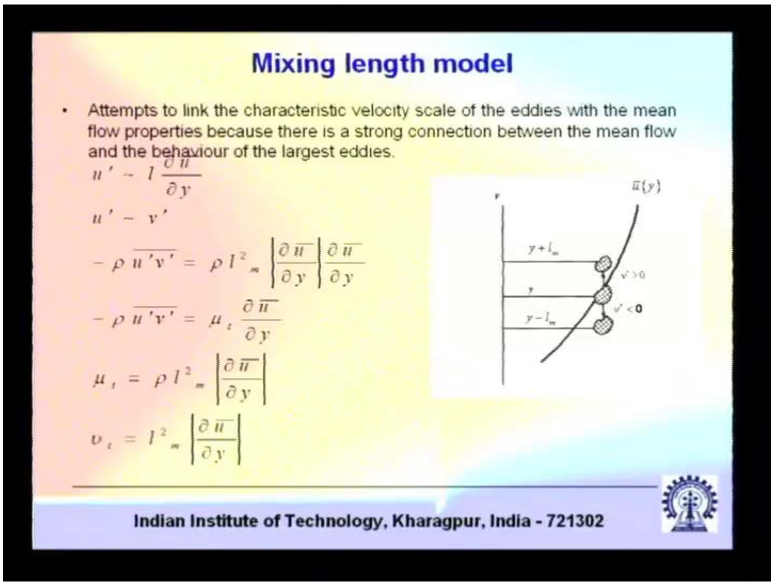

The first model, which is a very simple model that we will discuss is called as the mixing length model. So, in this mixing length model, what one tries to do is one tries to draw an analogy with the kinetic theory of gases. That is an analogy of the behaviour of eddies in turbulent flows with molecules in kinetic theory of gases. So, let us say that you have eddies with certain fluctuation velocity component. So, if you look at this figure, there is a velocity profile. So, you have the x component of velocity varying with y.

Now, you may have a particular velocity fluctuation scale. So, what is the velocity fluctuation scale? The velocity fluctuation scale is the fluctuation in velocity between two adjacent layers when they interact. So, what is that? That is the length over which they interact between two successive layers times the velocity gradient. So, del u by del y into some delta y; that is equal to the total delta u, so that delta y we call as l, which in a formal language will be related to something called as mixing length; that we will come subsequently. So, this l – what is the molecular analogy of this l? If these were not eddies, but molecules, what would have been this l? This l would have been the mean free path. So, it is a distance traversed by a molecule before colliding with another molecule. So, here it is not molecule, but like eddies, having a particular length scale being traversed, before having the interaction with another set of eddies with which it can have momentum exchange through fluctuation components. So, u prime is of the order of l into del u by del y. And, u prime and v prime in terms of order of magnitude, they can be stated as equal. So, minus rho u prime v prime average. Remember, this is a simplified approximating model. So, it is not a reality, but it simplifies many cases nicely. So, minus rho u prime v prime; there is an interesting thing to see about or infer about minus rho u prime v prime average.

Let us say there is a layer of molecules, which is moving horizontally. Now, there is a fluctuation component. Let us say that there is a positive v prime. So, because of positive v prime, instead of molecules, now, we consider eddies for turbulent flow. So, because of positive v prime, these eddies will now interact with the eddies in the upper layer. And, when they interact with eddies in the upper layer, what new u prime they result in, positive u prime or negative u prime? See they are coming from a lower velocity group. So, they are coming lower velocity group and they are coming towards the higher velocity group, because v prime is positive. So, once they are interacting with another group, they are now trying to reduce their fluctuation velocity components, because they are coming from a group with a lower velocity. So, a positive v prime is associated with a negative u prime, so that you can see that minus rho u prime v prime. In that, this minus sign is as if absorbed with the opposite signs of u prime and v prime, so that we write this as mu t; where, it is a turbulent viscosity times the velocity gradient.

See this is just like we are trying to represent the physics through mu t and we are trying to represent the law as if it is we are writing a Newton's law of viscosity. So, it is like the stress of the momentum transfer term is equal to mu times the velocity gradient. But, this mu is not the laminar viscosity that we are talking about; here it is a momentum exchange, because of fluctuation components in the eddies. So, we call it the turbulent viscosity. So, now, if you write this minus rho u prime v prime average and instead of this, in place u prime and v prime, you write l du by dy. So, to make sure that you come up with the correct sign, one of the du by dy you write with mod, so that you allow it to be achieving its correct sign. So, if du by dy is positive, then left-hand side will essentially become positive, because positive u prime will be associated with a negative v prime. That is why one du by dy we keep free and the other du by dy we keep in the mod.

And, the proportionality constant – when we say that it is of the order of l in to du by dy, it may be some k into l into du by dy. All those ks are absorbed into this and we call it l m; where, l m is equal to l into k. And, this l m is called as mixing length. So, mixing length is qualitatively analogous to the mean free path for molecular flow. Instead of the mean free path, it is like the distance traversed by one of the eddies before interacting its fluctuating component with the fluctuating component of another eddy. So, that is what is the mixing length.

When we do that, if you compare now these two equations, then you get mu t equal to rho l m square into mod of du by dy average. And then, you can write mu t, the corresponding eddy kinematic viscosity that is equal to mu t by rho. So, you can see that if you know what the mixing length, then you can specify the turbulent viscosity, because the entire expression is written in terms of the mean. So, it does not require any additional equation. It is just a formula for eddy viscosity, where you have to substitute a correct value for the mixing length. The beauty of this model is its simplicity. And, although it is very simple, this model works very nicely in many cases. So, it should not be discarded by considering this as a very primitive model. Maybe it is not one of the very advanced turbulence models, but it is illusively simple in a way that although it is simple, it works efficiently in cases much beyond what we can imagine.

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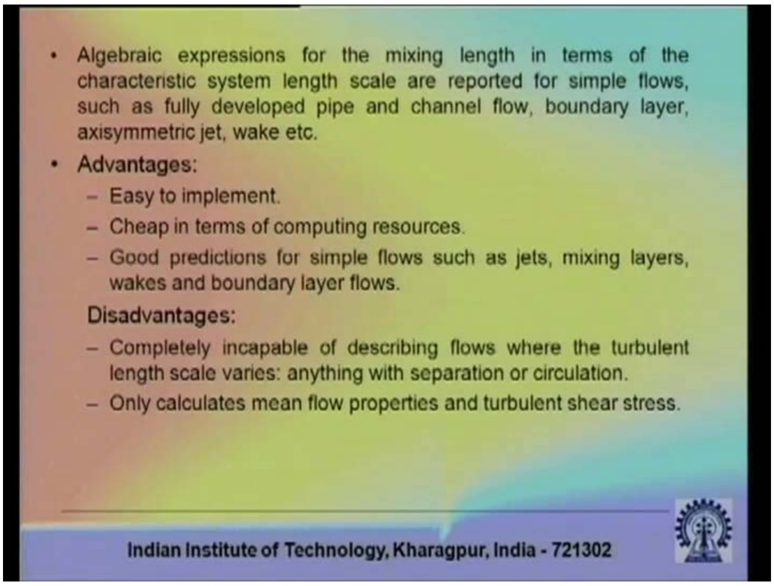

Now, it is important to remember that algebraic expressions for mixing length in terms of the characteristic system length scale are reported for simple flows, such as fully developed pipe and channel flows, boundary layer flows, axisymmetric jets, wakes, etcetera. So, what you know, you know l m in terms of the system length scale for certain simple flows. Therefore, you can specify the eddy viscosity in terms of the mean velocity gradient very effectively. And, using that, you can dump that in the Reynolds average Navier-Stokes equation and that makes the equation closed. What are the advantages? We have seen that it is very easy to implement, because you do not require really additional equations; and, cheap in terms of computing resources, because you do not have to solve additional governing transport equations; it gives good predictions for simple flows such as jets, mixing layers, wakes and boundary layer flows.

What are the disadvantages? Completely incapable of describing flows where the turbulent length scale varies. So, what it tries to do? It tries to sum up the effective length scale in terms of one single length scale, that is, the mixing length scale. But, if you have flow where the turbulent length scale varies anything associated with separated flow or flows with circulation, in those cases, it is not capable of describing the flow in an appropriate manner.

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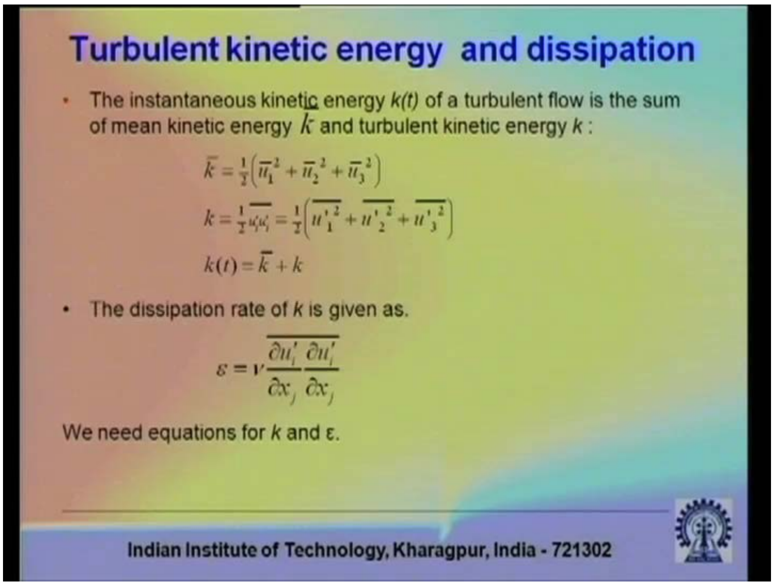

Now, we need to improve upon this model. And, before doing that, we try to introduce two important concepts: the turbulent kinetic energy and its dissipation. We already had introduced these concepts while talking about the various length scales in a turbulent flow. Now, to mathematically define, the instantaneous kinetic energy of a turbulent flow is a sum of the mean kinetic energy and the turbulent kinetic energy. So, the turbulent kinetic energy is associated with the fluctuating components. So, we can write the total kinetic energy as the mean kinetic energy plus the kinetic energy due to turbulent fluctuation. And, the dissipation rate of turbulent kinetic energy is defined as mu del u i bar prime by del x j into del u i prime by del x j. So, what you have to keep in mind here is that the energy cascading mechanism by virtue of which these quantities become physically important. So, the large eddies extract kinetic energy from the mean flow because of instabilities. And, that energy is cascaded into smaller and smaller eddies and till they are totally mocked up by the smallest eddies by virtue of viscous dissipation. And, the corresponding length scale of those eddies is known as the (( )) length scale. This is the preliminary understanding that we had.

Now, what one can do is, one can try to utilize these two important physical quantities. So, k and epsilon turn out to be important physical quantities for a turbulent flow. So, one can in principle try to utilize these two quantities and come up with governing transport equations for k and epsilon to model mu t or the turbulent viscosity. So, we are trying to go for a model different from the mixing length model.

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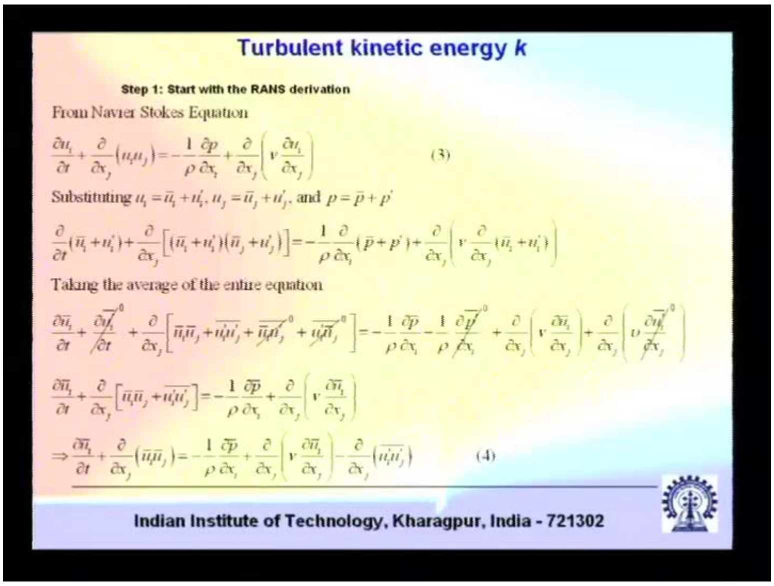

Now, the turbulent kinetic energy – what is the objective of this analysis? This derivation is to go for the derivation of a governing transport equation for turbulent kinetic energy. So, what is the step 1? You start with the Reynolds average Navier-Stokes equation derivation. So, what you do? You substitute u equal to u bar plus u prime and p equal p bar plus p prime and take the average of the entire equation. This we have already done. Just to summarize, we get the Reynolds average Navier-Stokes equations with an extra term in the right-hand side in terms of the appearance of the non-averaged equation; we have an extra term. And, what is that extra term? That is the Reynolds stress term.

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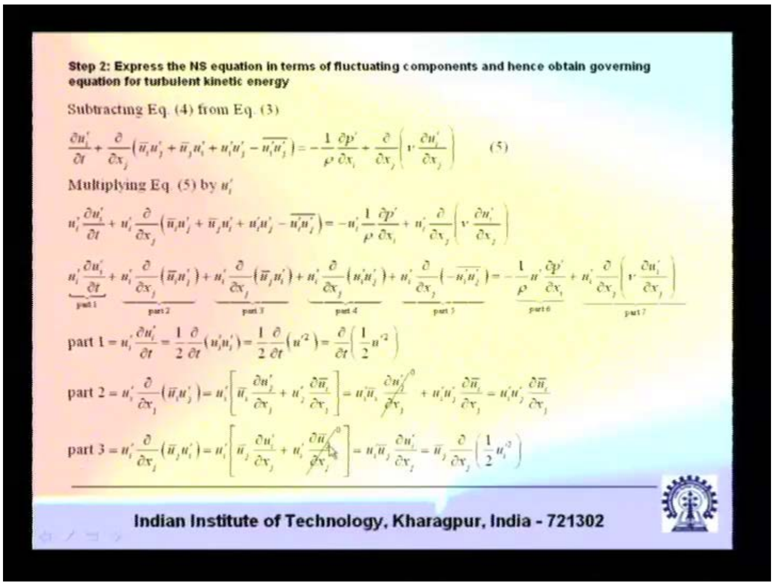

Now, the step 2 – express the Navier-Stokes equation in terms of fluctuating components and hence the governing equation for turbulent kinetic energy. So, let us try to go back with the previous equation. See the previous equation, where equation number 4 in the slide; in equation number 4, you have the governing equation in terms of u bar. The original Navier-Stokes equation is the governing equation in terms of u. So, if you subtract these two; that is, if you subtract 4 from equation number 3, you will get a governing equation in terms of u prime. That is whatwe will be doing in the next step. So, that is very straight forward; here subtract those two.

Now, you do not want a governing equation in terms of u prime; you want a governing equation in terms of turbulent kinetic energy. So, u i prime square by 2. So, what you do to get that? You multiply the equation 5 with u i prime, because u i d u i by dt is like d by dt of u i square by 2. So, that is how you get the kinetic energy. That is why the manipulation. See always when we do some manipulation, it is important to understand a query that why we are doing that manipulation. The objective of multiplying equation 5 by u i prime is to get the turbulent kinetic energy as the governing parameter. So, you multiply that.

Once you multiply that, you get different terms. So, first you get the term... So, these terms we have identified as different parts: part 1, part 2, part 3, part 4, part 5, part 6, part 7, just for clarity in showing the algebraic derivation. So, the part 1 is u i prime into del u i prime by del t. So, that you can write half of del by del t into u i prime into u i prime. So, that is del by del t of u i prime square by two. So, you have got the turbulent kinetic energy then the part 2. So, if you go through similar derivations, this is a bit tedious, but let us try to go through it for completeness. So, u i prime into del by del x j of u i bar into u j prime. Then, what you do? You use a product rule for the second two terms. And, when you use the product rule, there you use the fact that del by del x j of u j prime equal to 0. So, you come up with a simplified term. Similarly, part 3 – also, you use the product rule of differentiation. And, again in the product rule, now, you use the fact that the continuity equation in terms of the mean, that is, del u j bar del x j; that is equal to 0, so that you come up with simplified form for the part 3.

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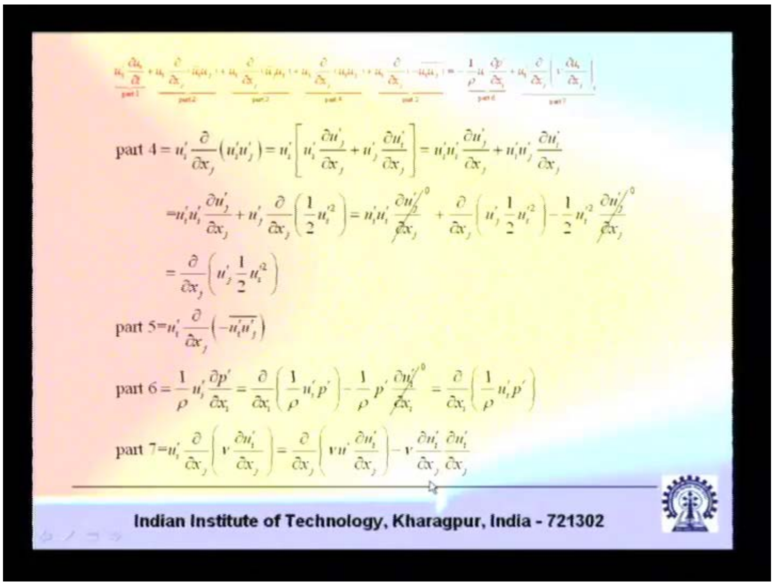

Then, you come to part 4. Again, you use the product rule. And, in the product rule, you again simplify it with a consideration that the continuity equation based on the fluctuation component is 0. So, for this simplification of these terms, you are basically using the continuity equation: one is the continuity equation in terms of mean flow that is 0; another is continuity equation in terms of the fluctuating component is equal to 0. Part 5 is as usual already simplified; we do not simplify it further, because it is already in a compact form. Part 6 – again, we simplify with the consideration that in terms of the fluctuation component, the continuity equation comes out here, which is 0. And, part 7 – we try to use the product rule and observe the term, which is their outside the derivative to inside the derivative. So, we observe the term inside the derivative; there is a minus term that appears. So, that is also by using the product rule of differentiation.

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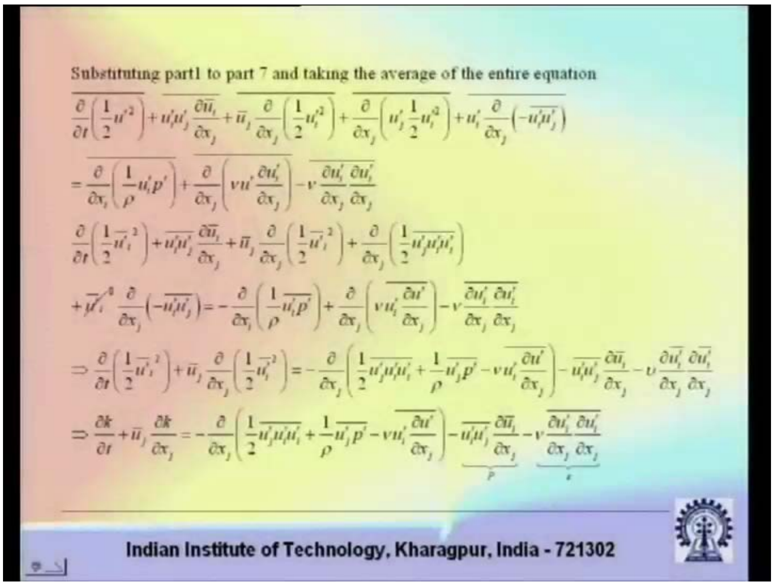

So, once we do these manipulations, we substitute all the parts in the governing equation and take the average of the entire equation. So, when you take the average of the entire equation, this is how it looks. So, you have different terms assembled together and you are getting the governing equation in terms of the turbulent kinetic energy. So, in terms of the turbulent kinetic energy, you get the governing equation; where, see the left-hand side del k by del t plus u j bar del k by del x j equal to the right-hand side is minus del by del x j of something plus a source term. So, these source terms – I mean, basically, two types of source terms: one is u i prime u j prime bar del u i by del x j; and then, there is a term which is minus of mu del u i prime by del x j into del u i prime by del x j.

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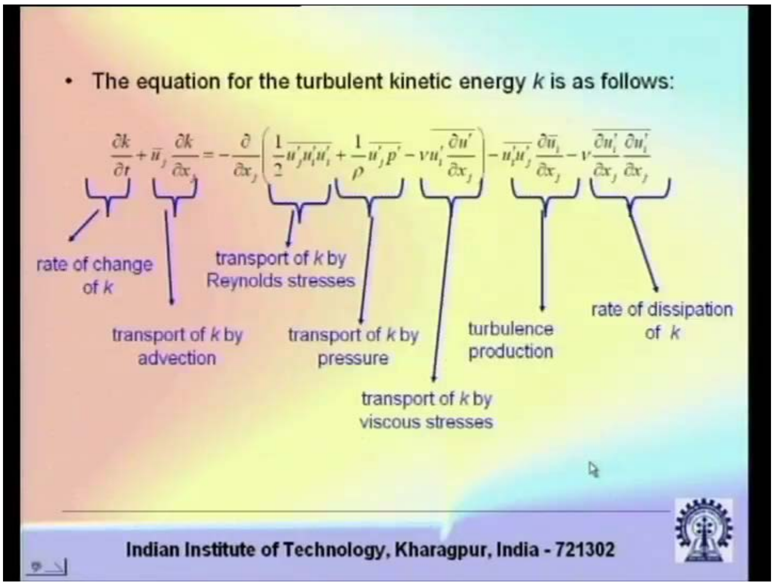

When you come with this, if you see, now, you have that equation for the turbulent kinetic energy described as follows. This equation is a modelled equation. So, this will be continuous bars, not separate bars. Just the typing is like this; you take it as continuous bars. Now, del k by del t plus u j prime del k by del x j is equal to minus del by del x j of the transport of k by the Reynolds stress. We will come into the physical significance of these terms subsequently, but minus del by del x j of some tau i j plus some source term. So, you can see that this equation.

First of all, from CFD prospective, this equation looks like the standard convection diffusion equation. So, the left-hand side, you have the transient term, the advection term; right-hand side, you have the diffusion term and a source term. So, what you have here is the rate of change of k, the transport of k by convection or we can say that transport of k by advection also; then, this is a transport of k by Reynolds stress, transport of k by pressure, transport of k by viscous stress. So, diffusive transport of k has three contributions: one is transport of k by Reynolds stress; another is transport of k by pressure; another is transport of k by viscous stress. Then, there is a turbulent production term – see u i prime u j prime – when they interact with each other, they generate or produce some turbulent kinetic energy. Similarly, you have the viscous dissipation term, mu del u i prime by del x j. These are terms for dissipation of turbulent kinetic energy.

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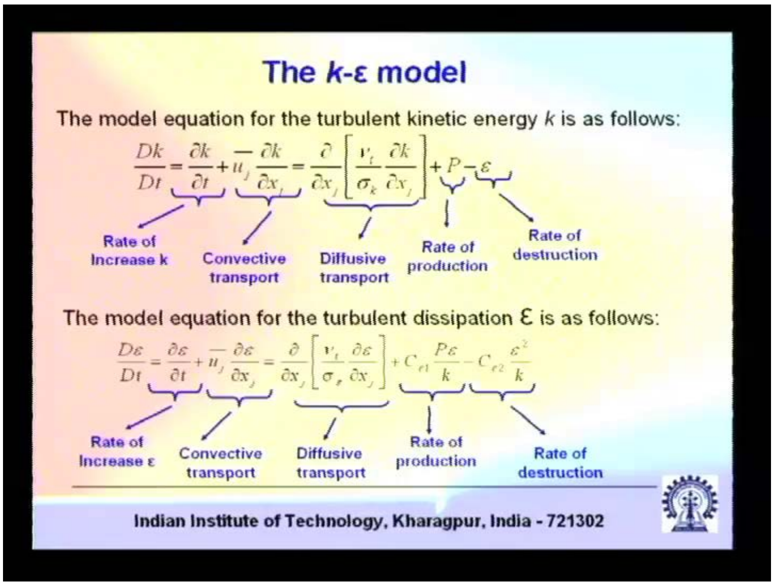

So, we can consider the last two terms; we can model, because the last two terms again will give raise certain additional unknowns. You have u i prime, u j prime; all those terms again will appear. So, what we do is we do not try to represent them actually in actual, but try to represent them in an approximate form through a model. So, what we do? We consider something called as k-epsilon model. So, first, we write the model equation for k. So, capital D by Dt of k is the left-hand side, which is nothing but the rate of change of k with time, which may be increased or decreased plus advective transport of k; that is, u j bar into del k by del x j. The righ-hand side is the diffusive transport. So, the right-hand side, what we do? You can see that the diffusion term we have modelled. There is no apparent similarity in the form of the diffusion term that we had just in the last slide and this one. So, this is just with a conceptual understanding that this is like diffusion. So, it is like del by del x j of some diffusion coefficient time, del k by del x j. So, that diffusion coefficient we call as mu t time sigma k. That we call as a diffusive transport.

And, the next two terms, we try to physical interpret. What are these? These are production and destruction of turbulent kinetic energy. So, it is a balance between production and destruction. So, production because of interaction between eddies with the mean flow; the velocity gradient of the mean flow interacts with the eddies. That is the production and the dissipation because of viscous effects. So, production minus dissipation. Now, similarly, with just an analogy, not with any rigorous derivation, the model equation for the turbulent dissipation is also written as rate of change of epsilon, the advective transport, the diffusive transport; then, rate of production of the dissipation and rate of destruction of the dissipation. You can see here that some while attempting this. What is the advantage of this? Now, you can see that you are able to write it in a standard convection diffusion form without incurring any additional unknowns, because otherwise, averaging will involve additional unknowns. But, to do that, you have to consider some fitting coefficients, fitting model constants.

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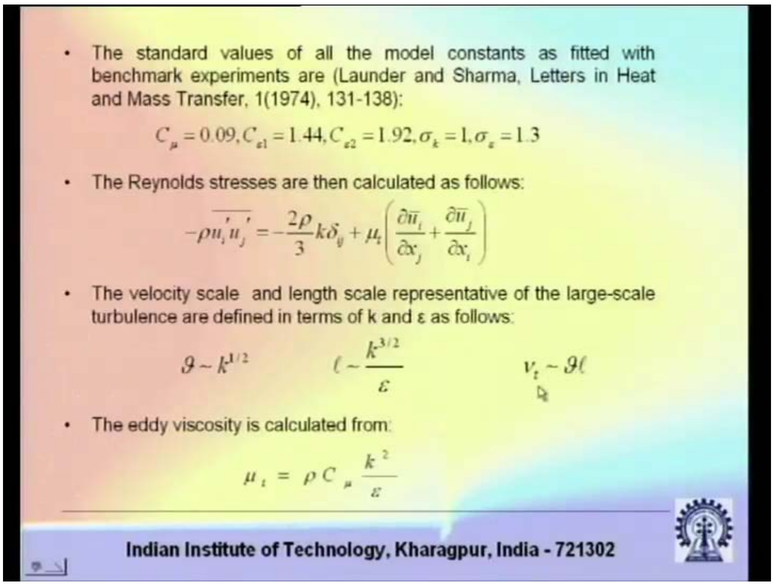

And, the standard values of these model constants have been obtained by fitting with benchmark experiments. So, how do you get these ones? So, what you do is, you have benchmark experimental results and you have numerical results fitting with this benchmark experiments; and, values of these constants in the k-epsilon model were proposed by Launder and Sharma in 1974. And, these are the standard models, standard constants; these have been used successfully till today. And, once you have that, the Reynolds stresses are then calculated as follows. These Reynolds stresses are then calculated with this regular formula. So, if you know the k, you have to know mu t.

Now, what is mu t? We have got k; we have got epsilon. So, our objective now is to express mu t in terms of k and epsilon. So, to do that, we have to understand that the velocity scale and length scale representative of the large scale turbulence are defined in terms of k and epsilon as follows. So, the velocity is k to the power half and the length scale is k to the power 3 by 2 by epsilon. This you can do very easily by going through the dimensional analysis of k and epsilon. And then, mu t scales as the velocity scale times the length scale. So, from that, the very important relationship comes up in terms of the eddy viscosity mu t, which scales as rho into k square by epsilon. And, the fitting constant is something a parameter C mu. So, C mu rho k square by epsilon. See how this k-epsilon model is closed. So, if you know k and epsilon through the transport equations, then using k and epsilon, you can get mu t by using the formula for mu t equal to C mu rho k square by epsilon.

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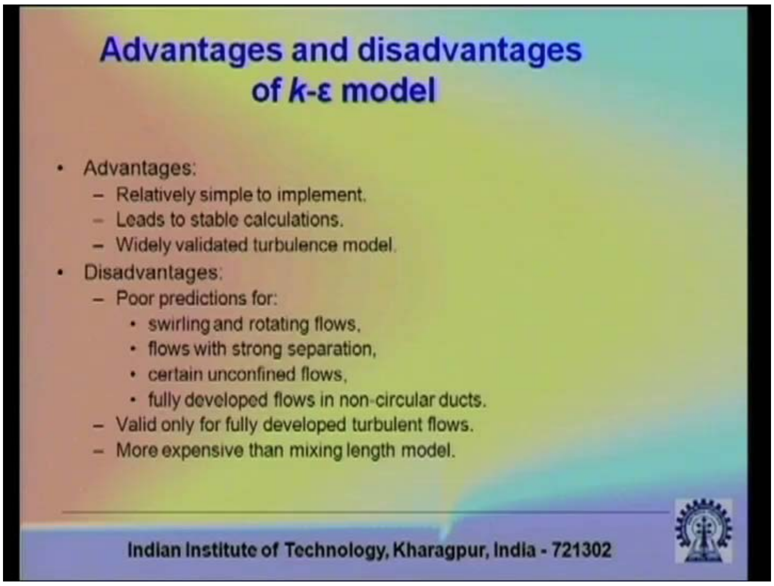

Advantages and disadvantages of k-epsilon model – advantages – they are relatively simple to implement; leads to stable calculations; and, these are widely validated turbulence models. Disadvantages – they are poorly predicting for swirling and rotating flows; flows with strong separation; certain unconfined flows; fully developed flows in non-circular ducts; valid only for fully developed turbulent flows; and, more expensive than mixing length model.

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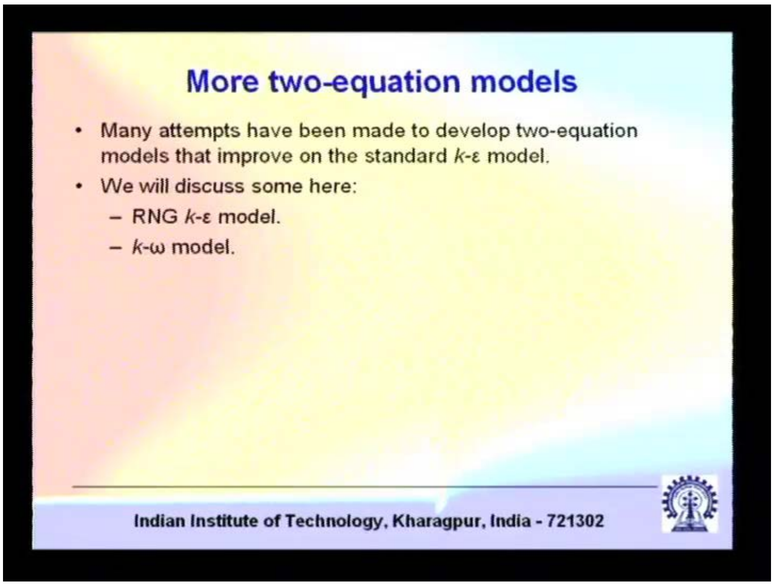

There are some more two-equation turbulence models. Many attempts have been made to develop two-equation models that improve on the standard k-epsilon model. We will briefly discuss a couple of those.

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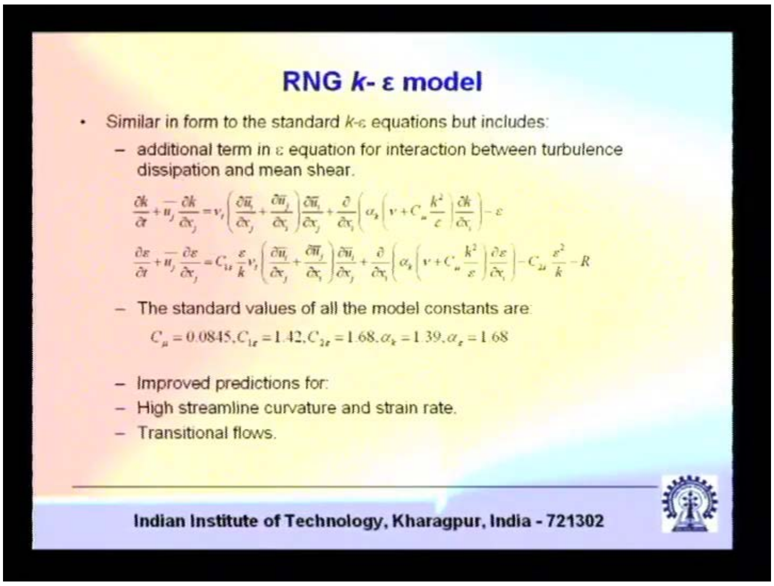

One is RNG k-epsilon model. Now, we will just go through the concepts of the model rather than going through the details, because details are just algebraic expressions. It is in similar form as the standard k-epsilon model, but includes additional term in the epsilon equation for interaction between turbulent dissipation and mean shear. So, you can see that there is an additional term R, which is for interaction between turbulent dissipation and mean shear. And, the standard model constants have also been validated with experiments. It has improved predictions for high streamline curvature and strain rate for which the standard k-epsilon model may not work that efficiently; and, also for transitional flows, not just fully developed flows.

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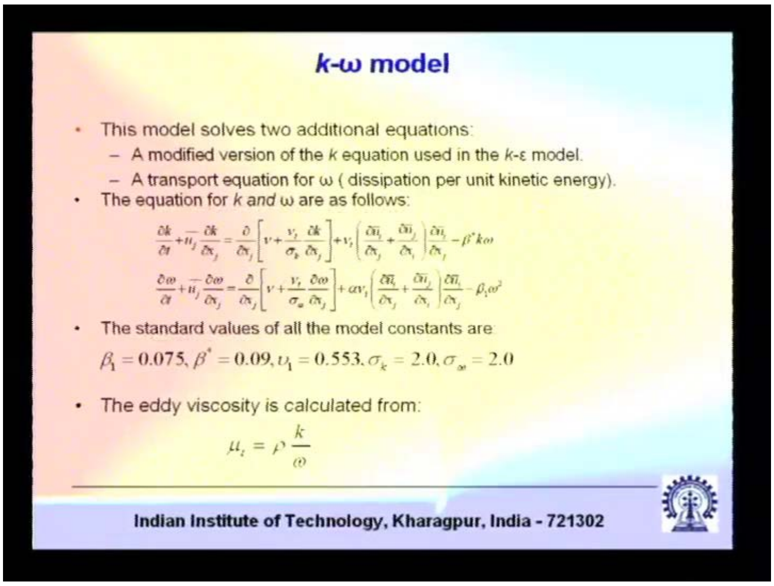

Then, k-omega model introduced by Wilcox – this model solves two additional equations: a modified version of the k equation used in the k-epsilon model and the transport equation for the omega. What is omega? Omega is dissipation per unit kinetic energy. So, it is not a dissipation, but dissipation per unit kinetic energy; so, normalized dissipation. The equations for k and omega are given as follows. From CFD perspective, again, you can see that these equations can be cast in a general conservative form and there are standard values of the model constants. And, the eddy viscosity can be calculated from k and omega from dimensional analysis.

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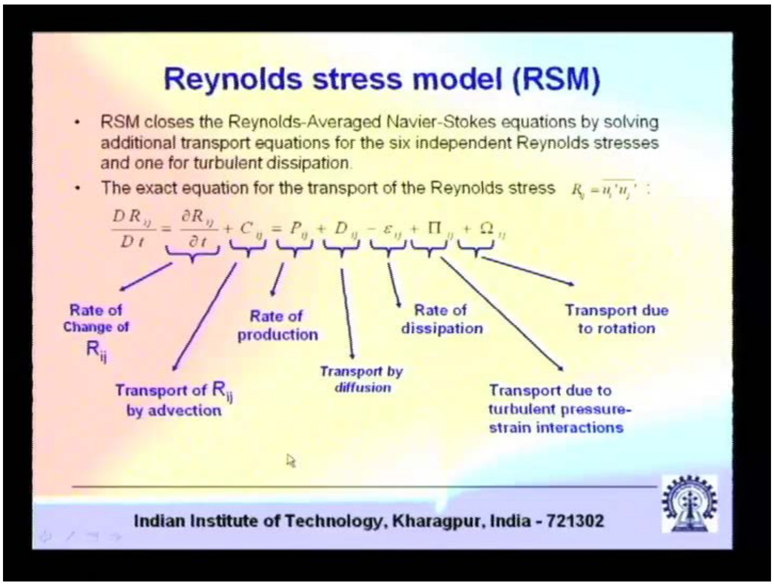

Now, if you want to really modelling more details; that is, if you want to get the individual Reynolds terms from each of the governing equation; for that, you have the Reynolds stress model. It closes the Reynolds average Navier-Stokes equation by solving additional transport equations for the six independent Reynolds stress and one for turbulent dissipation; so, seven additional equations. So, for each component of the Reynolds stress, you have one additional equation, where you have the rate of production, transport by diffusion, rate of dissipation, transport due to turbulent, pressure-strain interactions and transport due to rotation. So, rotation, pressure-strain interaction – all these features combine together to give rise to the dynamical evolution of the Reynolds stress for which you have the governing equations.

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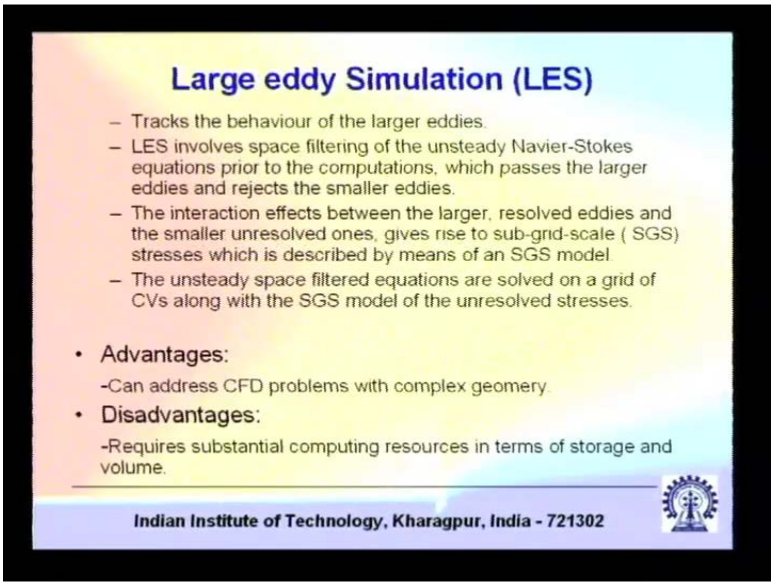

Now, if you want to still improve upon these, large eddy simulation or LES – LES tracks the behaviour of the larger eddies. So, the LES what it does, it involves space filtering of the unsteady Navier-Stokes equation prior to the computations. So, there is a filtering, which passes the larger eddies and rejects the smaller eddies, because most of the anisotropy is associated with the larger eddies. And, smaller eddies are more or less isotropic; of course, they are not totally isotropic, but they are much more isotropic than the larger eddies. So, the interaction between larger... So, the larger eddies are resolved. As if you are sitting with the filter, you are filtering of the smaller eddies and representing them by an equivalent model, but capturing directly the behaviour of the larger eddies. So, the interaction effects between larger resolved eddies and smaller unresolved one give rise to sub-grid-scale stresses, which is described by a sub-grid-scale model. And, the unsteady space filtered equations are solved on a grid of control volumes along with sub-grid-scale model of the unresolved stresses. The LES can address CFD problems with complex geometry, but it requires substantial computing resources in terms of storage and volume.

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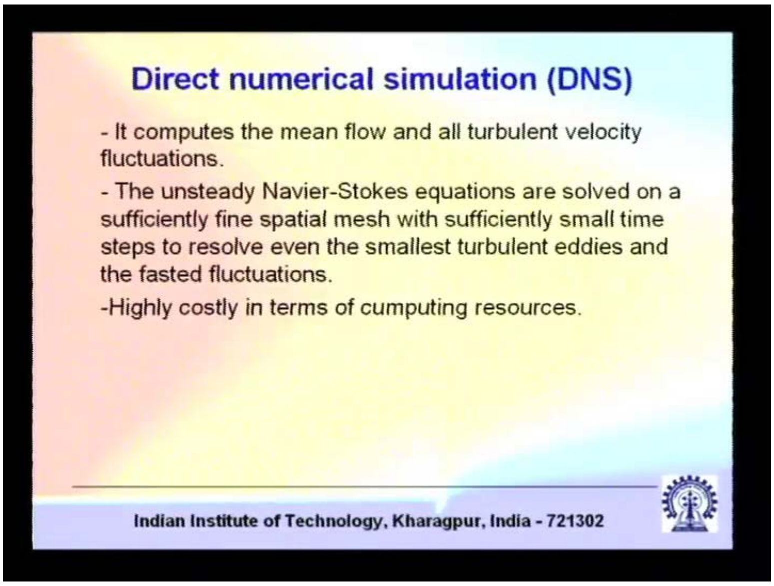

Then, direct numerical simulation – see we have to understand that the Navier-Stokes equations are valid for turbulent flows. We go for statistical averaging because of certain approximation that we need to do through statistical averaging, so that we do not have to capture all the length scales and time scales. So, in direct numerical simulation, what we intend to do? We intend to capture all possible length scales and time scales explicitly. So, it computes the mean flow and all turbulent velocity fluctuations directly. The unsteady Navier-Stokes equations are solved on a sufficiently fine highly refined spatial mesh with sufficiently small time steps to resolve even the smallest turbulent eddies and the fastest fluctuations. So, you directly resolve everything. So, it requires very fine grid and very fine time step. And therefore, even till today, only for very simple problems you have DNS solutions; you do not have DNS solutions for very complicated problems.

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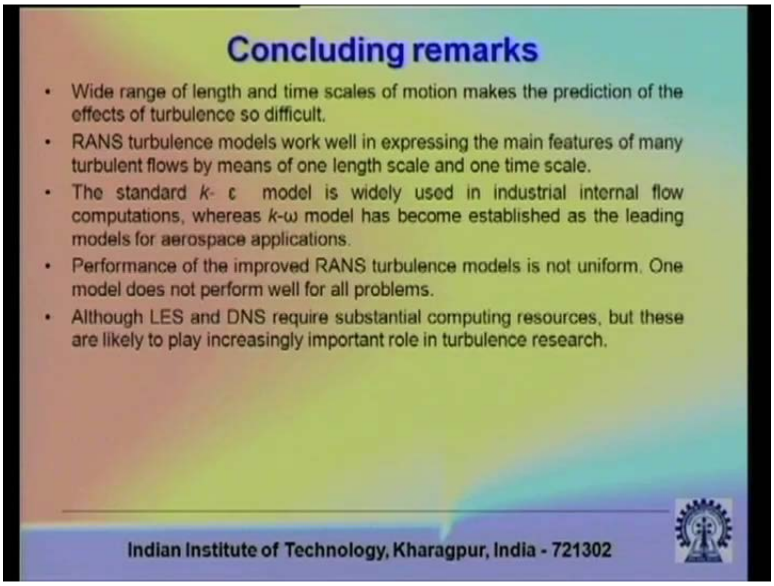

Concluding remarks – the turbulent flows have wide range of length scales and time scales; it makes their predictions so difficult. Reynolds average Navier-Stokes equation based turbulence models work well in expressing the main features of many turbulent flows by means of one length scale and one time scale. The standard k-epsilon model – why we have discussed it in more details? It is widely used in industrial internal flow computations; and, the k-omega model has become established as the leading model for aerospace applications or external flow application. So, generally, k-epsilon mode for internal flows and k omega mode for external flows for aerospace

Performance of improved Reynolds average Navier-Stokes based turbulence models – these performances are not uniform. One model does not perform well for all predictions. So, you can have different turbulence models working well for different cases. So, one single model cannot work for all cases. And, although LES and DNS require substantial computing resources, these are likely to play increasingly important role in turbulence research. So, these are some of the very key features or key aspects of turbulent flow modelling. So, with that we stop here.

Thank you.