Computation Fluid Dynamics
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Lecture No. # 43
End Semester Questions Review

Today, we will discuss about some of the end semester questions, and try to review the answers of these questions. So, the first questions that we will like to address are some short answer type questions. So, let us go through the first question, we need to answers these questions in brief. First question is for flow through porous medium, the distributed resistance to flow is expressed by the source term.

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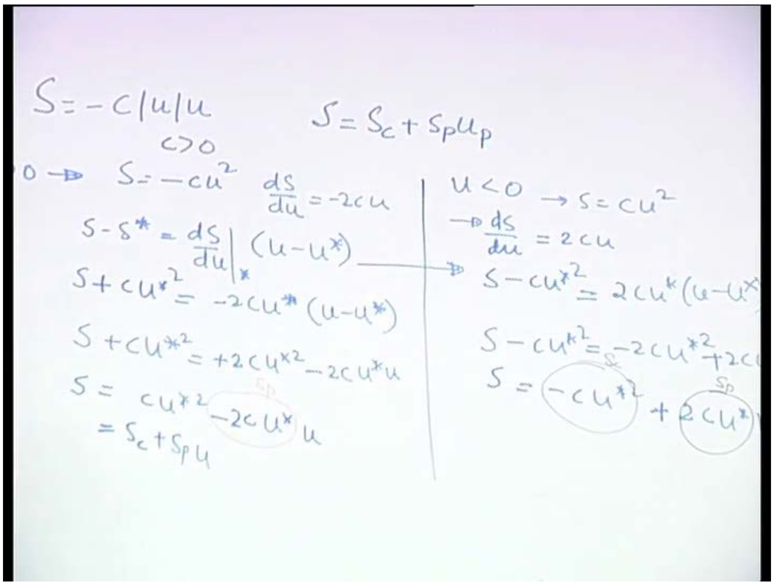

So, this is a source term for the x direction momentum equation, which represents a resistance against flow through a porous medium. Here c is a positive constant and u is the velocity component in the x direction. The question is write the best linearization of the source term by giving the expressions for s c and s p; that means, you want to express s is equal to s c plus s p u p. So, you have to find out what is s c and what s p? So, let us quickly see how to do that? Of course, it is quite straight forward. But we have to consider, because mod u is there, we have to consider two different possibilities one is u greater than 0 another is u less than zero.

So, if you have u greater than 0 then s is equal to minus c u square, because mod u is minus u mod u is u. So, it becomes minus c into u into u. Now, for source term linearisation what we can do? We can find out tangent to the s u curve that laniarises the source term. So, s minus s term is equal to d s d u s at the point star into u into u minus u star, this is the formula that we are going to use. So, at star point c is equal to c u star, c u square is c u start square and remembers that c is given greater than 0 it is given in the problem that c is a positive constant. Now, s minus s star is s plus c u star square, what is d s? d u is equal to minus 2 c u, minus 2 c u star into u minus u star. So, s plus c u star square is equal to plus 2 c u star square minus 2 c u star u.

So, s is equal to c u star square minus c minus 2 c u star u. So, you can see that you can write is that s c plus s p u. So, then what is s p? S p is equal to minus 2 c u star by the basic one of the basic rules you must have s p equal to what? you must have s p as negative. So, you can see here that we have assumed u as greater than 0 so, u star is also greater than 0 and c is greater than 0 and minus 2 c u star is less than 0. So, that will what fine? So, this is the linearization for u greater than 0 for u less than 0 you will have d s. So, for u less than 0 you have s is equal to c u square, because mod u is minus u for u less than 0. So, d s d u is equal to 2 c u.

So, from the equation s minus s star equal to d s d u star into u minus u star, you have s minus c u star square is equal to 2 c u star into u minus u star. So, s minus c u star square is equal to minus 2 c u star square plus 2 c u star u. So, s is equal to minus c u star square plus 2 c u star u. So, if you write this as again s c plus s p 5 s p u p then this is what is s c c does not matter whether s c is negative or positive. So, the sign criticality is there with s p. So, this is s p and because u is less than 0 u star is less than 0 c is greater than 0 so, s p is less than 0. So, that means this source term linearization is also fine. So, for both u greater than and u less than 0 you can get expressions, you can combine these 2 expressions of course, by expressing the entire thing in terms of mod of u again. But we just do not go into that details for this review.

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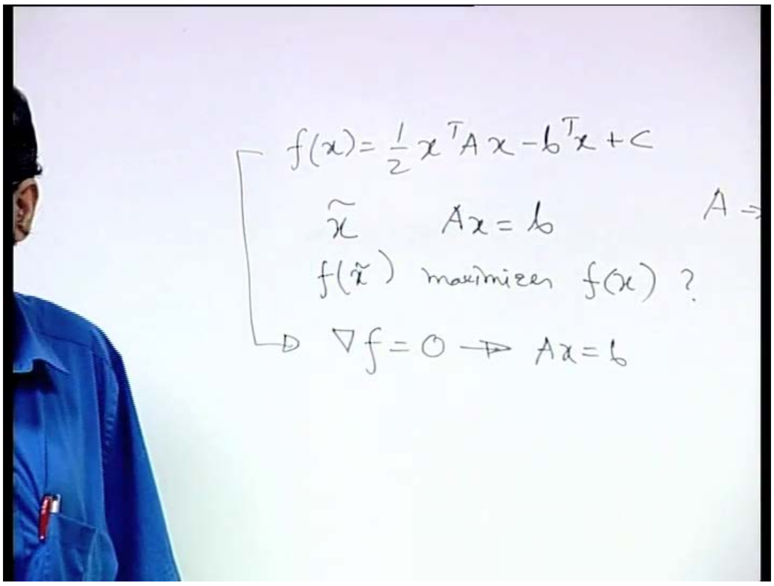

Now, next question, next question is considered the following quadratic form, we will express the quadratic form what is the quadratic form? The quadratic form is f x equal to half x transpose A x minus b transpose x plus c. Where A is a square symmetric matrix, x and b are vectors and c is a scalar. If x till the be the solution of the system A x equal to b. Then is it possible that f x till the maximises f x? So, is it possible that f x till the maximises f x? Let us try to answer this we will not go into detailed derivation; we have shown that for a symmetric matrix A. It is given that A is square and symmetric it is very, very important. So, what is A? A is square and symmetric matrix we have derived in 1 of our lectures that if A is square and symmetric, then gradient of f equal to 0 will lead to A x equal to b.

Now, when we say that gradient of f equal to 0 will lead to A x equal to b. We imply that it is an extermination problem gradient setting gradient of f equal to 0 does not automatically, tell whether it is minimization or maximization. Later on we have shown that if A is also positive definite, then it leads a minimization problem and not a maximization problem. But if that constant is not given, then it could be a maximization or minimization problem setting gradient equal to 0 does not necessarily imply that it is minimization problem it could be a maximization or minimization problem. In the next step when we set that it to be a positive definite a to be positive definite matrix then we can show that it is in need a minimization problem and not a maximization problem.

The next question is what is the basic reason behind a numerical scheme to exhibit fox diffusion in a convection diffusion problem? That is the first part then can central differencing turn out to be superior over up (( )) in order to minimize such effects, you have to justify your answer.

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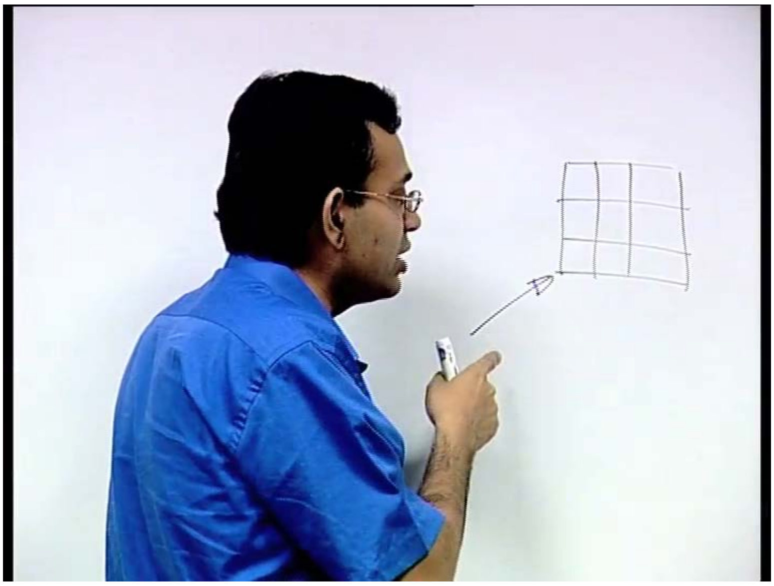

So, again we have discussed about this issue in details in one of our lectures, but just to summarize that. If you have a flow, which is miss oriented with respect to the grid lines that is not parallel to the grid lines and in client significantly with respect to the grid lines. Then that need orientation can give rise to an artificial diffusion that is even, if there is negligible, already low diffusion or 0 diffusion. It artificially represents diffusion between or mixing between two fluid streams. And that essentially shows that the false diffusion or numerical diffusion is mostly attributed to the orientation of the flow direction with respect to the grid lines. And that is the main cause behind false diffusion.

Now, whether the central differencing scheme can address, it is a debatable issue and there are cases when it cannot address it, because the central difference scheme will not work for very high peculiar number flows. And for very high peculiar number flows where the diffusion is insignificance in practice any numerical artefact in diffusion can their significant consequences. Because you are not having significant diffusion in physical reality but numerically, significant diffusion can creep up and for high peculiar number flows what we can see? is that the central differencing scheme itself is vulnerable. So, that is answer to this question.

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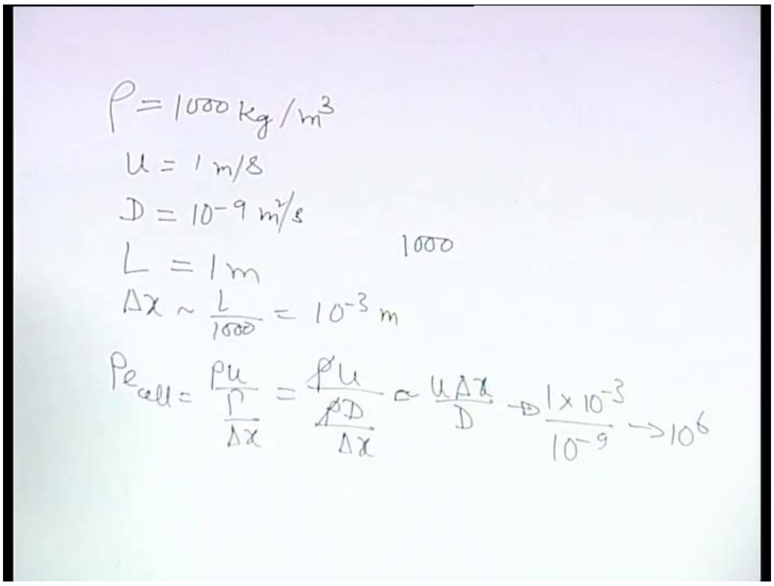

Next question for a one dimensional convection diffusion problem, it is given that the fluid density is 1000 kilogram per meter cube. Flow velocity scale is of the order of 1 meter per second, diffusion coefficient is 10 to the power minus 9 meter square per second and domain length is 1 meter. Question is will a central difference scheme work for numerical solution to this problem given that the dimension of the solution vector for t d m a should not exceed 1 thousand; that means, you should not use number of grid points of the order of more than 1 thousand, give reasons to your answer.

So, here as a very simple approach what we will try to do? We will try to calculate, what is the cell peculiar number, and see whether the cell peculiar number is less than 2 or not. So, the characteristic delta x is of the order of l by thousand. So, that is 10 to the power minus 3 meter that is the characteristic grid spacing. It is just an order of course, if you have non uniform grids; it is true that the grid spacing may differ significantly from this one, but this is just the order of magnitude of the grid spacing. Now, what is the peculiar number? Cell base peculiar number advection strength by diffusion strength. So, what is gamma? Gamma is rho into d, rho into the diffusion coefficient is rho into the, because the units if you want to match, this is d is a kinetic unit.

So, you have converted it just like if you considered it as fluid flow problem, this become viscosity. So, you have to convert the kinematic unit into the dynamic unit. So, that is why you have multiplied it by rho. So, the 2 rho's get cancelled out so, it is of the order of u delta x by d. So, u is 1 delta x is 10 to the power minus 3 and d is 10 to the minus 9. So, it is of the order of 10 to the power 6. So, it is a very high self base peculiar number which implicates that the central difference scheme will not work; it is much greater than 2.

So, the only the key towards answering this question is one has to be careful, it is not the global peculiar number, but the cell peculiar number. So, it could happen that the l based peculiar number is greater than 2, but delta is base peculiar number is less than 2. Then, if such numerical data is there, then central difference scheme would still work. So, if you want to make central difference scheme work here, you have to use such a refine delta x, such that u delta x by d has to be less than 2. So, it has nothing to do with the total length of the domain, but it has something to do with the sale spacing.

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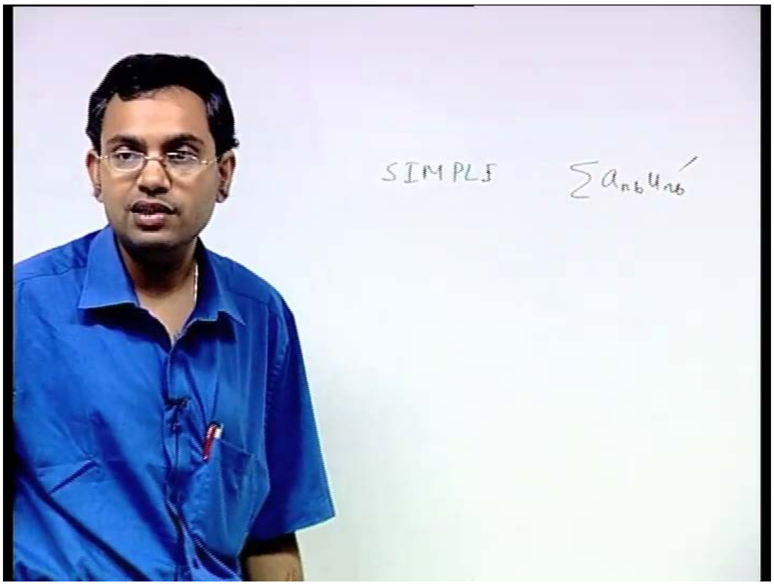

Next question, there is a statement given, simple algorithm is not fully implicit and therefore, it is conditionally stable. Is this statement correct? Justify you answer. I am repeating it again the simple algorithm is not fully implicit and therefore, it is conditionally stable, is the statement correct? Give reasons. So, again we have discussed about this extensively in one of our lectures. So, the semi implicit (( )) of the simple algorithm has nothing to do with time and descritization.

So, it has something to do with the omission of sigma a n b u n b prime the neighbouring velocity correction terms in the formula, for velocity correction and also, in the formula for pressure correction. So, that particular effect leads to the consideration of a semi implicitness of simple algorithm. It has nothing to do with the time descritization effects like implicit explicit crank Nicholson scheme. So, one need not confuse those with this effect therefore, there is no issue of conditional stability here it is unconditionally stable. Next question, what is the basic geometrical difference between structured and unstructured mess? Can you have rectangular sub domains in an unstructured mess give reasons?

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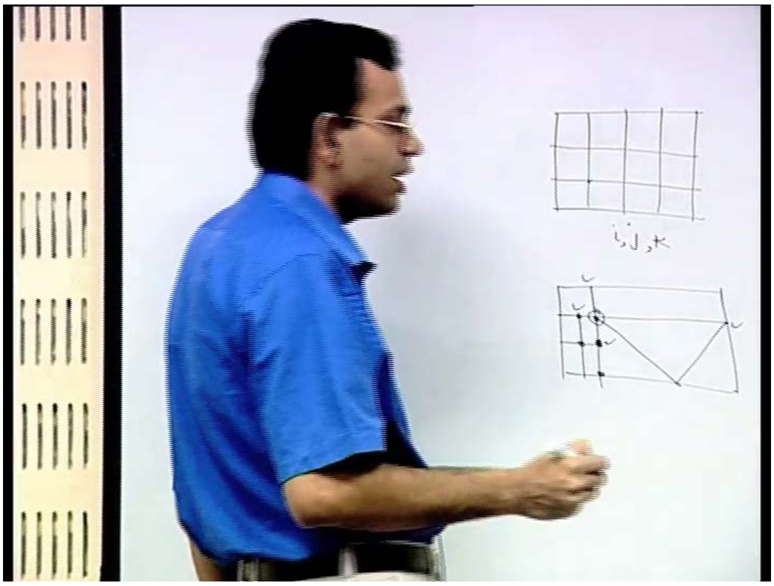

So, again we have discussed about this, but just to summarize if you have a structured mess, then you have the same connectivity of one vertex with the neighbouring vertices. Whereas, you have an unstructured mess, you may have different connectivity's between the vertices. So, one vertex may connect different number of neighbouring vertices depending on which vertex, you are considering. And, because such a structured layout is there for a structured grid, you can express the grid locations or the coordinates by using indices i j k like this depending on the dimensionality. Whereas, you have to explicitly specify the coordinates without referring to such grid line orientations i j and k to specify the coordinates, specify the locations of the vertices.

And, then you have established suitable connectivity's. If you consider about the issue of having a possibility of rectangular unstructured mess, so, if you can see for example, that here you have a vertex in this. So, this vertex connects to how many neighbouring vertices? 1 2 and 3 on the other hand if you consider these vertex, it connects how many neighbouring vertices? 1, 2, 3, 4. So, it is possible that you can have unstructured mess with rectangular grids you can of course, have also combinations like you can have combinations of triangular, rectangular and all sorts of combinations are there. So, it is very much possible that you can have unstructured mess with some control volumes that are rectangular.

Next question, for a flow under 0 pressure gradient necessity of a staggered grid will never arise. Is this statement correct, give reasons? So, I am repeating the question again for a flow under 0 pressure gradient necessity of a staggered grid will never arise. Is this statement correct, give reasons? See the necessity of staggered grid was mainly there, because of what because of interpolation of pressure at the faces of the main control volume, where pressure data is not directly available.

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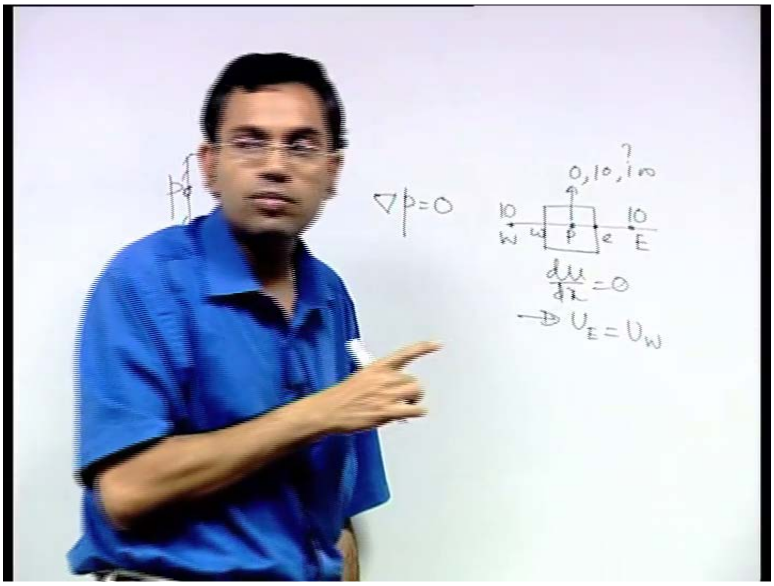

So, you have to interpolate. So, if there are situations, when the pressure gradient is 0 then the requirement of pressure interpolation is nullified pressure no more acts as a variable. But there is a catch word if you are going through the root of the continuity equation. Then you can have a checker boat of velocity field if you express the continuity equation in the normal grid sense that, we have discussed. So, it is issue of whether you need to go through the root of the continuity equation or not, if you need to go through the root of the continuity equation.

Then irrespective of how do you handle checker boat type of pressure fields, whether pressure field that is necessary or not. You also have to handle checker boat type of velocity fields if you have to go through the roots of continuity equation without staggered grids. Of course, that again can be rectified with some special interpolation schemes. But, if you do not use a special interpolation scheme, at just consider a linear zed interpolation. Then the continuity equation itself can give rise to (( )) or checker boat type of velocity fields even if the velocity is constant.

So, that means, if you consider, we have shown that d u d x in a 1 dimensional problem if you have d u d x equal to 0. It will be reflected in terms of u capital E is equal to u capital W the effect of p will be gone. So, that means, if you a velocity here as 10 and velocity here as 10 and velocity here as 0 10 100 whatever, all will be interpreted as same. So, by this is what we mean by the checker boat type of field or (( )) type of field. So, irrespective of the value here, the continuity equation here may be satisfied ambiguously with many possible values of u p.

So, that is one of the artefact so, the question remains whether we have to go through the root of the continuity equation, Many times we do not have to go through the root of the continuity equation. If you have unit relational flow you already know, what is the velocity distribution? And it is it is just a constant value that you know already. So, if you do not have to through the root of the continuity equation just have to solve the momentum equation, with d p d x equal to 0. Then you do not have to bother about staggered grid, because then it is issue of pressure of interpolation only and that needs to be need not be taken care of when the pressure gradient is 0. So, that is what we have to keep in mind.

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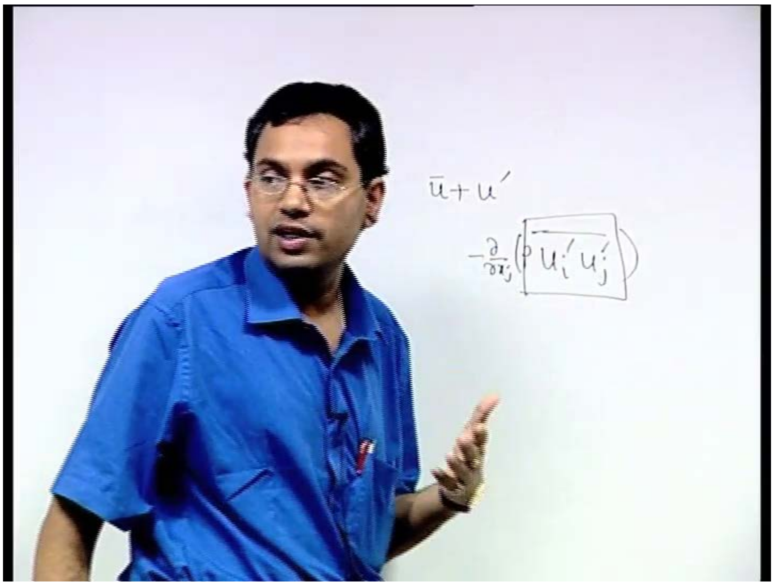

Next question, what is closure problem in turbulent modelling? So, this also we have discussed, but let us again summarize that in turbulence modelling we come up with the Reynolds average navies stroke equations. Where we basically, decompose the all the quantities in say the velocity in terms the mean and its fluctuation substitute that in the navies strokes equation and take the average. Once, you do that then new stress terms appear del x j of rho u i prime u j prime bar. So, this particular expression gives rise to new unknowns, what are these new unknowns? New unknowns are rho u i prime u j prime average.

So, when you consider this rho u i prime u j prime average these are new unknowns. So, because it is symmetric you have i ranges from 1 2 3, j ranges from 1 2 3 there could be 3 into 3 9 unknowns. But because i and j are interchangeable and symmetric, you have 6 independent unknowns. So, these new unknowns do not have matching number of governing equations and they need to close by suitable modelling. So, you do not have matching number of equations with unknowns, when you come up with a Reynolds average navies strokes equation. And, you need to close this extra number of unknowns by appropriate modelling effort and that is known as a closure problem in turbulence modelling.

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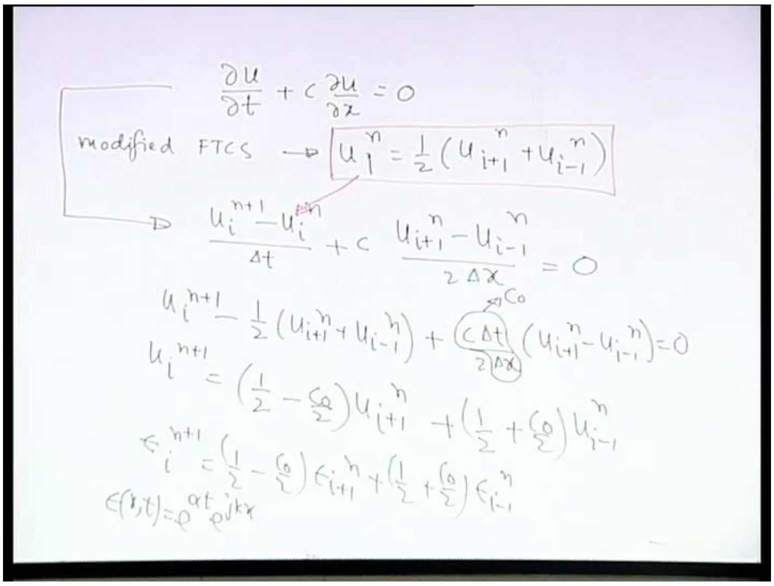

Now, next let us go to some questions, which are bit longer answer type. The next question, consider the following template one dimensional wave equation. del u del t plus c del u del x equal to 0, you are using a modified f t c s scheme, where in the time descritization you express the term u i superscript n as half of u i plus 1 n and u i minus 1 n summation. Where the index i represents special descritization and the superscript n represents temporal descritization. So, this is the problem you have to examine the stability of the scheme by 1 (( )) stability analysis that is the question of the problem. So, let us try to express this del u del t as.

So, it is a forward differencing in time u i n plus 1 minus u i n divided by delta t plus c del u del x is a central space; so, u i plus 1 n minus u i minus 1 n by 2 delta x. So, first order central differencing equal to 0. Now, next is that you are given that u i n is equal to this 1 which needs to be substituted here. So, once that is substituted let us write what we get so, we get u i n plus 1 minus half u i plus 1 n plus u i minus 1 n plus c delta t by 2 delta x into u i plus 1 n minus u i minus 1 n. So, now see delta t by delta x we define as coherent number.

So, you can write u i n plus 1 is equal to half minus coherent number by 2 into u i n plus 1 n plus half plus coherent number by 2 u i minus 1 n. So, you can write the corresponding perturbations if epsilon i n plus 1 is equal to half minus coherent by 2, into epsilon i plus 1 n plus half plus coherent by 2 epsilon i minus 1 n. Next, what we will do? we will substitute epsilon x t is equal to E to the power alpha t into E to the power j k x where j is square root of minus1.

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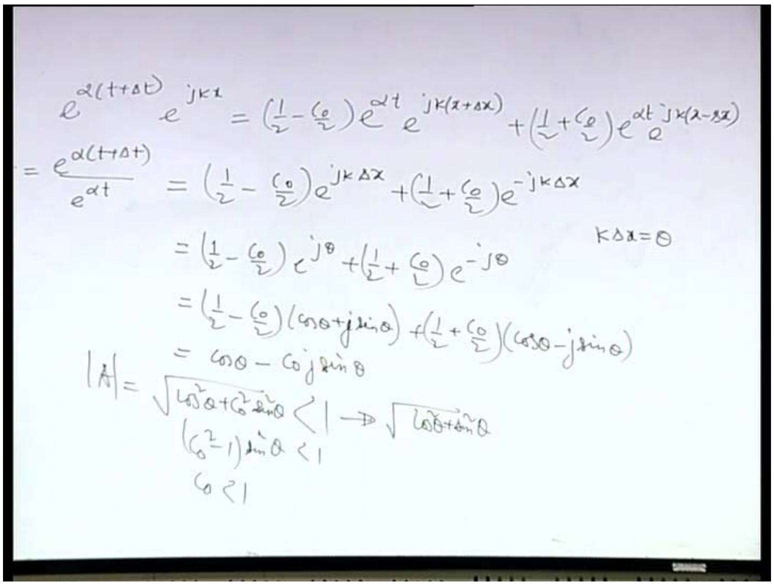

So, you have e to the power alpha t plus delta t we are doing it for the left hand side into e to the power j k x is equal to half minus coherent number by 2 e to the power alpha t e to the power j k x plus delta x plus half plus coherent number by 2 e to the alpha t into e to the power j k x minus delta x. So, we divide the left hand side by e to the power alpha t. So, that the amplification be factor becomes e to the power alpha t plus delta t by e to the power alpha t. So, that will be equal to the right hand side we divide by e to the power j k x. So, it becomes half minus coherent number by 2 e to the power j k delta x plus half plus coherent number by 2 e to the power minus j k delta x.

So, you can define k delta x equal to theta. So, this is half minus coherent number by 2 e to the power j theta plus half plus coherent number by 2 e to the power minus j theta; so, half minus coherent number by 2 cos theta plus i sin theta plus half plus coherent number by 2 cos theta sorry plus j sin theta cos theta minus j sin theta. So, you can write this as half cos theta plus half cos thetas so, cos theta minus coherent by 2 cos theta plus coherent by 2 cos theta. So, that cos theta goes then half j sin theta minus half j sin theta cancels minus coherent number by 2 j sin theta minus coherent number by 2 j sin theta; so, minus coherent number j sin theta. So, amplitude of A square root of cos square theta plus coherent square sin square sin square theta.

This must be less than 1 for a stable scheme and 1 you can write as root over cos square theta plus sin square theta. So, from here you can write that coherent number square minus 1 sin square theta must be less than 1. So, that means coherent number must be less than 1 to make the strictly valid for all possible values of sin theta. So, you have to remember that coherent number cannot be negative. So, we are only dealing with the less than 1 sin we are not considering the mod case here, because it is we are considering it to be any number 0 onwards. Because c delta t y delta x c is positive, delta t is positive and delta x is positive. So, the summary is that it is a scheme which is conditionally stable it depends on the coherent number that must be less than 1 we go the next question.

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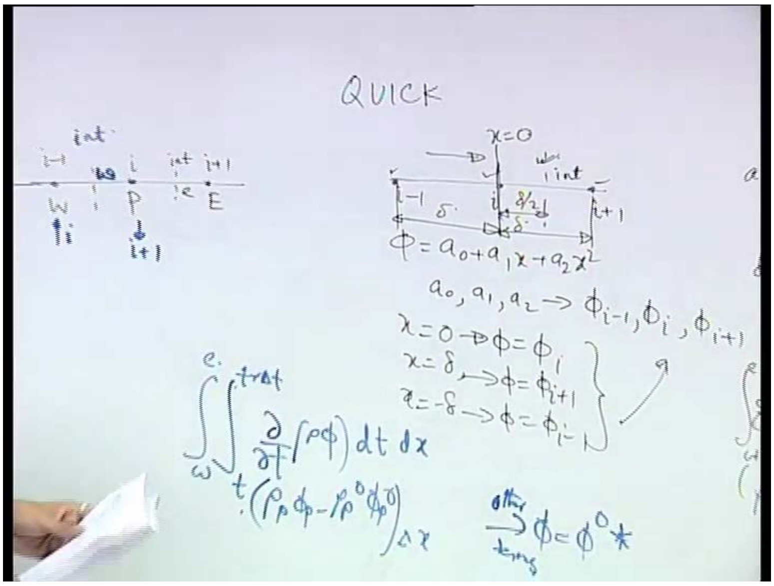

The next question is considering a 1 dimensional steady state convection diffusion problem without any source term derives a profile assumption for variation of dependent variable. I am repeating derive a profile assumption for variation of the dependent variable in the advection term following the quick scheme. And considering the same profile assumption for the diffusion term derives the complete descritization equation for the convection diffusion problem that is the first part. So, this we have done in the class in our previous lectures just to summarize the quick scheme refers to quadratic upstream interpolation scheme for convective kinetics.

So, if you have grid points like i minus 1 and i plus 1. And, if here is the interface, then you use the interface interpolation, you use a profile for interface interpolation, where you use two upstream points and 1 downstream point. So, you can write phi as a 0 plus a 1 x plus a 2 x square for your convenience you can select x equal to 0 here. And, then you can write a 0 a 1 a 2 in terms of phi i minus 1, phi i and phi i plus 1 by noting that when x equal to 0 phi equal to phi i then let us say this is delta by 2 and this is delta. So, when x equal to delta phi equal to phi i plus 1, and when x equal to minus delta phi equal phi i minus 1. So, if you use this conditions you will get a 0 a 1 a 2 in terms of phi i minus 1 phi i phi i plus 1.

And, then for the advection terms what is the difference between the advection and diffusion term? For the advection term you require rho u phi at the interface. So, the interface is located at x equal to delta by 2. So, once you know a 0 a 1 a 2 then you substitute x equal to delta by 2 to get phi at the interface. This is phi at x equal to delta by 2 this is a generic way of finding phi at the interface, what is for the diffusion term? you require d phi d x at the interface. So, what is d phi d x? D phi d x is a 1 plus 2 a 2 x at x equal to delta by 2. So, this is the only slight difference. But important difference between the derivation that we made in the class and these derivation in the class for the diffusion term from the beginning we assume that it is piece wise linear profile.

Now, we are not assuming, that it is a piece wise linear profile for the diffusion term, we are using the same profile for interpolating the advection term and the diffusion term. So, then you can assemble the advection and the diffusion terms, in the advection diffusion descritization equation that is d d x of rho u phi is equal to d d x of gamma d phi d x. So, you can have integrate this one from small w to small e remember small w and small e are interfaces. So, rho u phi at small e minus rho u phi at small w is equal to gamma d phi d x at small e minus gamma d phi d x at small w. So, if you consider a grid layout you have p capital E capital W small e small w. And, you have to keep in mind that if this is the interface, then this is i this is i minus 1 and this i plus 1.

On the other hand, if this is the interface, then you require another grid point w w where you say that this is i this is i plus 1 and this is i minus 1. So, it depends on what is the interface. So, the with the blue colour I have represented the situation where w is the small w is the interface. And, with a black colour I have represented a situation where small e is the interface. So, these 2 interfacial conditions you require for evaluating these terms. And then you can assemble if some of the coefficients may be negative or some of the coefficients may be of opposite side. And other coefficients, then you have to conclude that the scheme is not the scheme is conditionally stable. And, you can find out the stability criteria.

So, from the basic rules you can evaluate that it is simple algebra remaining after this exercise. And, you can extend this so, the next part of the question is extending the derivations to 1 dimensional unsteady state convection diffusion problem with fully explicit time descritization. So, if you add an unsteady term to the problem, then it requires the 2 important considerations. What are those two important considerations? One is the integration of this with respect to time and then with of course, with respect to x. So, it will become from time t to t plus delta t small w to small e. So, rho p phi p minus rho p not phi p not by delta t into delta x by delta t will come. If you divide both the right hand side and left hand side, by delta t let us not do that at this stage and do it straight forward.

So, rho p phi p minus rho p not phi p not times delta x. So, this is the transient term, what will be the consequence of fully explicit scheme in the other terms? the consequence will be that everywhere in all other terms phi will be the phi at the beginning of the time step phi with superscript 0 this is very critical. So, that is the difference between the explicit and implicit scheme. If the, if it was an implicit scheme, then in the right hand in the other terms phi would be phi at the end of time state phi at t plus delta t. Now, because it is fully explicit scheme that is asked in the question it should be phi at t not phi at t plus delta t. So, all other phis expressed in terms of the descritization that already we have derived there you have substitute phi as phi not that is the only difference.

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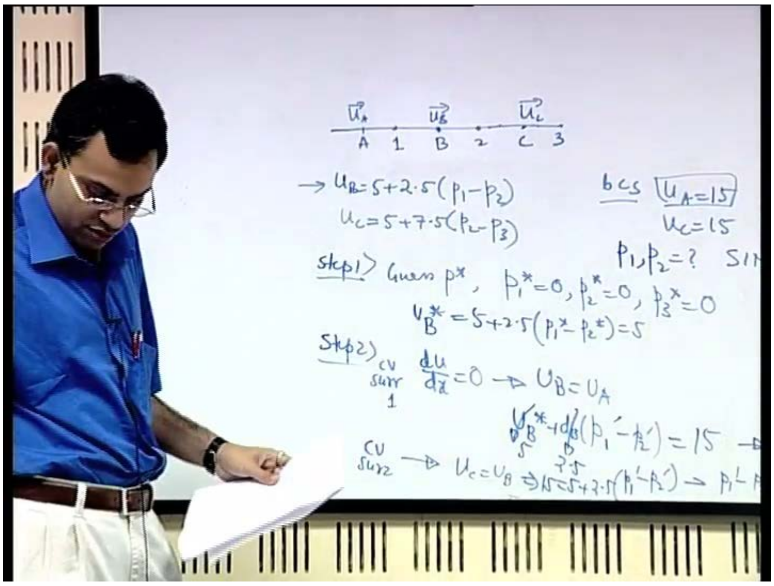

Let us go to the next question, we consider a one dimensional constant density flow where the momentum. So, the constant density flow situation is shown in the figure, where you have uniformly space grid points. The velocity grid points and the main grid points shown separately its staggered grid arrangement. The momentum equations for u B and u C are given. So, u B is equal to 5 plus 2 point 5 into p 1 minus p 2 and u c is 5 plus 7 point 5 into p 2 minus p 3. The boundary conditions are given u A equal to 15 u C equal to 15, all values are given in consistent units obtain the values of question is what are p 1 and p 2? Using simple algorithm based calculation based procedure comment on the uniqueness of your solution.

So, if you want to follow a simple algorithm based procedure what is the step 1 you have to guess a pressure field. Let us say, p 1 star equal to 0 p 2 star equal to 0 p 3 star equal to 0 we know that these are not correct, but just guess is a guess we can always make incorrect guesses. So, based on that see what momentum equation we will solve? we will u B momentum equation to get u B star, we will not solve u C, because u C is already given why we should solve it. So, u B star is equal to 5 plus 2 point 5 into p 1 star minus p 2 star. So, that is equal to 5.

Next is you had to solve the pressure correction equation? So, you have to derive the pressure correction equation. So, what you have to do? you have to consider the continuity equation. So, for the continuity equation for considering the gird point 1 the control volume surrounding 1, d u d x equal to 0 is the 1 dimensional continuity equations that will give you u B equal to u A. What is u B? U B equal to u B star plus d B into p 1 dash minus p 2 dash. And, that is equal to u A; because u A is given there is no question of writing it in terms of correction formula. So, let us substitute values u B star equal to 5, then d B is 2 point 5 that is given here in the momentum equation itself. The correction equation will look like the momentum equation, with differences as the correction terms.

So, what will be p 1 dash minus p 2 dash? That is equal to 4. Then, if you can write the momentum equation for the grid point 2 also, control volume surrounding two. So, what will that give that will give u C equal to u B, u C is given as 15. So, you have fifteen equal to again 5 plus 2 point 5 into p 1 dash minus p 2 dash. So, it will give rise to same equation p 1 dash minus p 2 dash equal to 4. So, you can see that by considering the 2 different grid points, you are basically getting only 1 independent equation not 2 independent equation it shows, that this pressure correction is a relative quantity.

So, what will be the values of p 1 dash and p 2 dash? Depending on what you chose as an initial guess for one? So, for example, if you chose p 1 dash equal to 0, then what will be p 2 dash? That will be equal to minus 4. So, what will be then p 1? So, then you can also calculate what is new u b based on this correction? So, steps 3 u B is equal to u B star plus d B into p 1 dash minus p 2 dash. So, u B star is 5 d B is 2 point 5 and p 1 dash minus p 2 dash is 4. So, this makes it 15 without testing anything you can readily check for this problem that u A equal to 15 u B equal to, u C equal to 15. And now, u B equal to15 so, for 1 dimensional flow u is constant. So, in 1 step it has just converged and what will be the value of p 1 and p 2? So, step 4 p 1 is equal to p 1 star plus p 1 dash.

So, that is 0 and p 2 is equal to p 2 star plus p 2 dash that is equal to minus 4. So, you can see here, is that depending on the choice of p 1 dash, you could get different values of p 1 and p 2 combinations, but the difference will always be 4. So, is the pressure difference that remains preserved, but absolute values of pressures depend on the choice of the initial guess? So, it is not a unique solution of a pressure it depends on what is the guess that? You take for one of the pressure corrections. Now, we have two more questions, which we will go through a bit quickly, because these questions we have already discussed in full details in some of our lectures. So, the next question, you have a steady hydro dynamically fully develop flow in a parallel plate channel.

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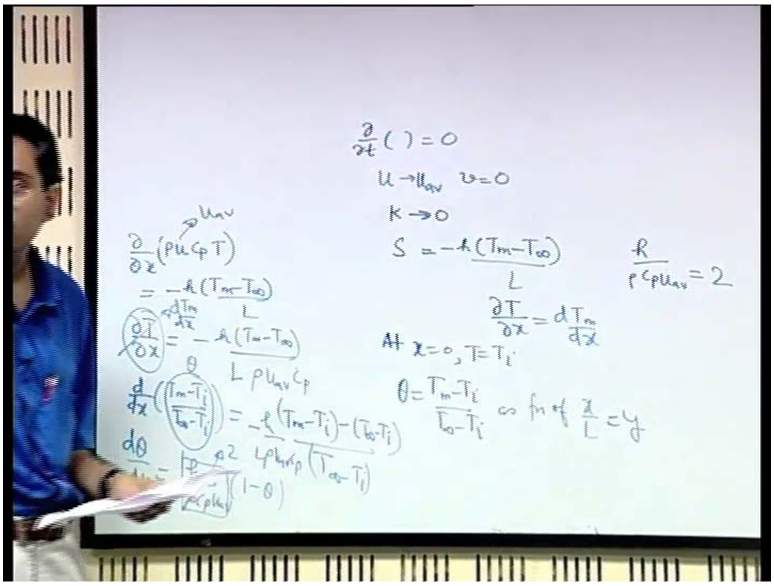

So, first it is a steady flow so, del del t term is a 0, hydro dynamically fully develop flow in a parallel plate channel means you have only u, but v equal to 0. It is a two dimensional flow with a uniform velocity over the cross section. So, u equal to u average which is uniform over the cross section. So, it is a flock flow, thermal conductivity of the fluid is so small that the heat diffusion effects can be neglected. That means, the thermal conductivity the conduction term in the advection diffusion equation you can neglect. For that, you have a volumetric sink term which can be expressed as h into T m minus T infinity by l is a heat loss term. And, what is this T m? This T m is the bulk mean temperature of flow, and it is also given that h by rho CPU average is equal to 2. Where h is the heat transfer coefficient and l is the channel length and T infinity is the ambient temperature and for solving this problem.

You can assume that del T del x is equal to d T m d x, here rho is density and c p is specific heat. So, what is the problem? The problem is that if the inlet temperature that is at x equal to 0 if t is T i then obtain the variation of theta is equal to T m minus T i by T infinity minus T i as function of x by l. So, we will just quickly derive the governing equation the unsteady term is not there. So, d d x rho u C p T is equal to there is no diffusion term, the diffusion because the thermal connectivity is 0. And, there is a source term minus h into T m minus T infinity by l given that u is u average.

So, you have, and all the other properties we assume as constant. So, d d x of so, we have d T d x. So, we cannot write really d T d x because T could be a function of both x and y although u is a function of x only, but T could be a function of both x and y. So, better write as del del x instead of d d x. So, we can write del T del x is equal to minus h by t m minus infinity by l rho u average c p. Now, instead of del T del x you can write as d T m d x that is given in the problem. Now, if you define theta is equal to T m minus T i by t infinity minus T i then what we will get. So, d d x of t m minus T i by T infinity minus T i. So, here you can write T m minus T i minus T infinity minus T i by T infinity minus T i in the right hand side minus h by l rho u average c p and you observe this l by expressing a non-dimensional x that is small x by l.

So, if you consider say y is equal to small x by l, then this is defined as theta. So, d theta d y is equal to h by rho C P U average into 1 minus theta and this is given as 2. Now, recall this is exactly the same (( )) governing equation with which we delta in 1 of our walked out examples during the during solving the convection diffusion problem d theta d y is equal to 2 into 1 minus theta. So, based on this you can find out you can integrate this by using the up winding scheme which is given in the question. So, the question is determine the non-dimensional temperature theta as a function of y using the up winding scheme take 5 uniformly space grid points for your computation and solve the resultant system of algebraic equations by t d m a.

So, we have solved this problem exactly in one of our lectures. So, I just formulated the problem, because this formulation of this problem is different from the problem we derived our we discussed with in one of our lectures, But once, this problem is formulated the final form of the equation is same as the equation that we delta with in of our lectures. So, you can use the same derivations for solving this problem using an upwind scheme. So, here you have an upwind scheme where you have an advection term source term, but 0 diffusion terms that is what we have to keep in mind.

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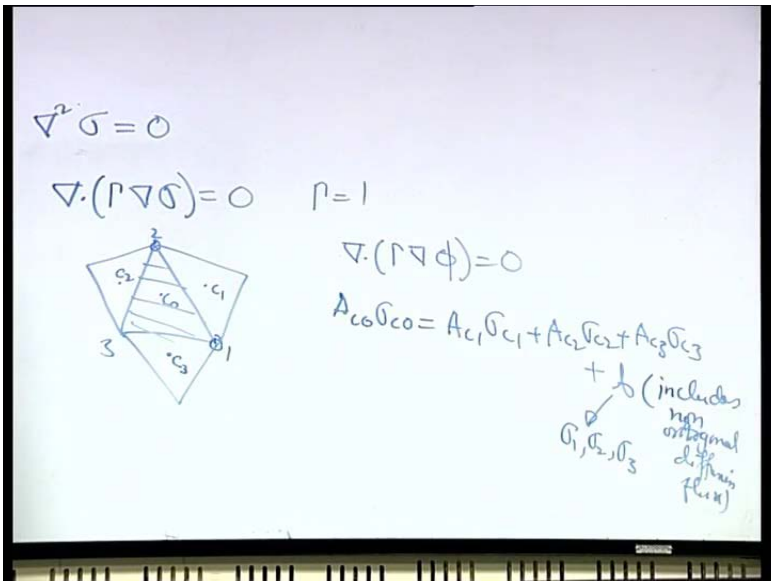

Then, the final question let us go through the variant of nutrient concentration sigma outside and within a malignant tumour colony. That is a domain of cancerous sales is governed by an approximate equation characterising diffusive equilibrium. So, let me repeat the variation of nutrient concentration sigma outside and within a malignant tumour colony that is a domain of cancerous sales is governed by an approximate equation characterising diffusive equilibrium as del square sigma equal to 0. Taking an unstructured triangular mess simulating and approximate 2 dimensional version of this problem derive appropriate descritization equation for solving the distribution of sigma.

So, I repeat taking and unstructured triangular mess simulation and approximate 2 dimensional version of this problem derive appropriate descritization equations for solving the distribution of sigma. So, we need not go through any more details of this derivation, because this is of the form del dot gamma del sigma equal to 0 where gamma is equal to 1 it is a pure diffusion problem. And, if you consider an unstructured mess like this, with the control volume layouts around at triangular shape control volume the neighbouring control shown here. So, you can we have seen that how to descriptive a general problem? del dot gamma del 5 equal to 0. In the class we derived where we have phi equal to u for the velocity.

So, and gamma was the viscosity. So, here gamma is 1 and instead of velocity we are now dealing with the sigma as the variable. But in mathematical terms the problem does not differ anymore from what we derived in our class. So, you can use the same derivations to decompose it in the form a p c p sorry a p sigma p or instead of p we will call it c zero. So, a c 0 sigma c 0 is equal to a c 1 sigma c 1 plus a c 2 sigma c 2 plus a c 3 sigma c 3 plus b where in b you have terms involving sigma 1 sigma 2 sigma 3. And, the corresponding fluxes are known as no orthogonal diffusion fluxes. So, these fluxes are functions of the difference between the values of sigma of the vertices of the triangle whereas, these are the values of sigma at the cancroids of the main control volume.

So, you can write it in a similar way as that of the standard convection diffusion standard diffusion equation for a structured mess. Where you have the values expressed in terms of the differences of the values at the centroids of the neighbouring control volumes. But because of the unstructured nature of the mess, you also have some non orthogonal diffusion flux which will appear in this. So, this includes non orthogonal diffusion flux. So, considering this it is a straight forward derivation and this derivation can be done by following our lecture notes on unstructured mess. So, we stop here today. Thank you very much.