

Solar Energy Technology
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Lecture - 27
Exercise I (Contd.)


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Lecture 27 Exercise 1 (Contd.)

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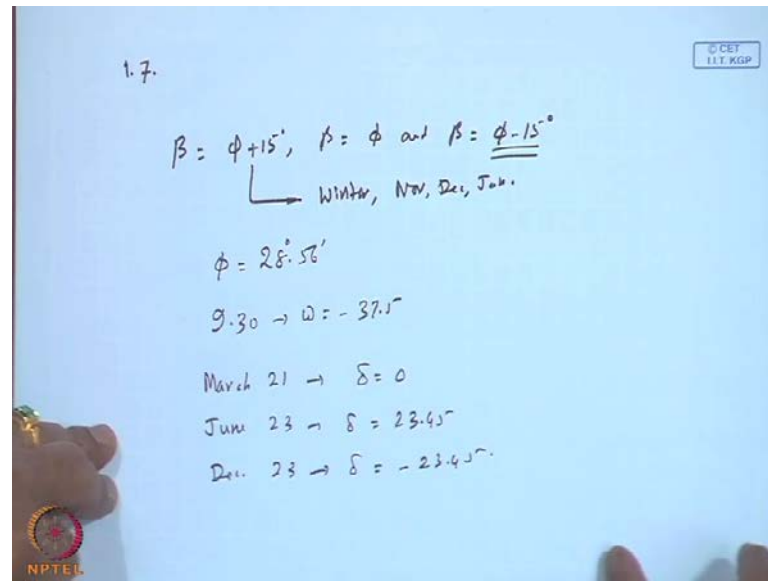
Note: All times specified in the following problems are SOLAR TIME except when specified

1.7 Estimate R_b for a collector with a slope of $\beta = \phi + 15^\circ$, $\beta = \phi^\circ$ and $\beta = \phi - 15^\circ$ from horizontal with $\gamma = 0^\circ$, at New Delhi ($\phi = 28^\circ 34' N$ and $L = 77^\circ 07' E$) at 9.30 AM on March 21st, June 23 and Dec. 23.


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We were trying to solve some problems in the last class, what I called exercise 1. We shall continue with that exercise and the problem is 1.7, possibly I have covered it, but nevertheless I should go through it quickly. The idea is to estimate the R_b the conversion factor from horizontal radiation to the tilted radiation. For a collector with the slope of $\phi + 15$ degrees or equal to ϕ degrees or $\phi - 15$ from horizontal or south facing in New Delhi location at 9.30 am on March twenty first, twenty third and December twenty third. So, these dates were chosen the first one is the () day declination is 0, and June has the maximum positive declination and December has the maximum magnitude by its negative declination.

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So, in general we want beta equal to phi plus 15 degrees and beta equals to phi and beta equal to phi minus 15 degrees. So, you can find out that the slope is plus 15 degrees, I can expect this to be a better orientation for winter in general or let us say November, December, January because the Sun's rays are pretty much lower and if you have a higher slope, then they will be near normal incidence whereas this will be desirable for summer.

Beta is equal to phi is sort of a compromise for the broad year list collection, which we shall do or calculate later. It is given for Delhi phi as 28.56 degrees minutes I think and time 9:30 corresponds to omega of minus 37.5 and March dates are plus or minus does not matter, delta is 0 and June delta is positive 23.45 and December 23, you have declination of minus 23.45. So I will go through these numbers quickly, so that we can conclude something out of this exercise.

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$$R_b = \frac{\cos(\phi - \beta) \cos \delta \cos \omega + \sin(\phi - \beta) \sin \delta}{\cos \phi \cos \delta \cos \omega + \sin \phi \sin \delta}$$

(A) March $\rightarrow \delta = 0$

(i) $\beta = \phi + 15^\circ$, $\phi - \beta = -15^\circ$
 $R_b = 1.09$

(ii) $\beta = \phi$

So, R_b for a south facing surface is the angle of incidence for the south facing surface $\cos \phi$ minus β , $\cos \delta$, $\cos \omega$ plus $\sin \phi$ minus β , $\sin \delta$ over $\cos \phi$ $\cos \delta$, $\cos \omega$ plus $\sin \phi$ $\sin \delta$. First we take a March with δ equals to 0 and if β equal to ϕ plus 15 degrees ϕ minus β is minus 15 and R_b is I had done the calculation last time 1.09 sorry, 1.09. So, this is for β ϕ plus March and so you can fill in the numbers and find out that they are okay. Second case, so this one, two this is say may call it A March, so β equal to ϕ and R_b turns out to be 1.09 and this (()) with is the same. We can check out that algebra little later or say arithmetic. For different reasons it does not mean the same, you can just check because δ being 0 or ϕ minus β being 0.

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(iii) June 23. $\delta = 23.45^\circ$

$\beta = \phi$

$R_b = 1.138$

(iii) $\beta = \phi - 15^\circ$

$R_b = \underline{\underline{1.09}}$

And then you have the third case, June twenty third delta is 23.45 beta. So, when you have got, let me repeat I am sorry, when beta is equal to phi your R b is 1.138, it cannot be the same as beta equals to phi plus 15. The third case is beta is equal to phi minus 15 for which R b is equal to 1.09. So, the phi plus 15 and phi minus 15, they give you the same arbitrary factor in the month of March, because declination is 0. This will not happen is declination is non-zero.

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June 23rd. $\delta = 23.45^\circ$

$\beta = \phi + 15^\circ, (\phi - \beta) = -15^\circ$

$R_b = 0.725$

(ii) $\beta = \phi$

$R_b = 0.878$

(iii) $\beta = \phi - 15^\circ$

$\rightarrow R_b = 0.9717$

So, we go to the next date June twenty third and declination is 23.45, first case being beta phi plus 15 or phi minus beta equal to minus 15 and R b you will have a low value less than 1.725. We have lots of these number and I am just repeating of some reason which you will come to know towards the end of this problem. For the second case of beta is equal to phi you have this slight liberator 0.785. Please go through this arithmetic calculation simple calculation, which we had done in detail last class so you can again check it out and you will find if there is a difference or qualitatively, I do not think there will be much difference.

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Dec. 23. $\delta = -23.45^\circ$

(i) $(\phi - \beta) = -15^\circ$
 $R_b = 1.79.$

(ii) $\beta = \phi$
 $R_b = 1.623$

iii) ~~$\phi - \beta = 0$~~ $(\phi - \beta) = 15^\circ,$
 $R_b = 1.34.$

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Phi minus 15 and this gives rise to R b 0.9717. One thing that has come out clearly compared to March twenty first, June twenty third has poorer R b for these orientations. If you go to December 23, twenty third with the definition of minus 23.45, for the first case of phi minus beta equal to minus 15 you will have R b 1.79 and then if the second case beta is equal to the latitude phi or you have R b equal to 1.63. If you have third one, phi minus beta equal to plus 15, then R b is 1.34. So, all this calculation we have done just it is essentially for continuity

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Summary.

Time: 9:30 AM, Location New Delhi:
 $\phi = 28^{\circ} 34' N$

Day	$\beta = \phi + 15^{\circ}$	$\beta = \phi$	$\beta = \phi - 15^{\circ}$
March 21	1.09	1.138	1.09
June 23	0.725	0.8785	0.9717
Dec 23	1.79	1.623	1.34



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If I set up a table in summary or the time is 9:30 am location New Delhi, so latitude is 20 sorry 28 degrees 34 minutes North and day we are considering March 21. Then June 23 and December 23. So, you have beta phi plus 15 beta equal to phi and beta equal to phi minus 15 with the corresponding values of 1.09, 1.138 and 1.09 and 0.725, 0.8785 and 0.9717 versus 1.79, 1.623 and 1.34. What comes out distinctly is that in tilt factors or negative declination or higher than those values for positive declination A and this is because delta is 0 phi plus 15 and phi minus 15 they have the same values, right?

Basically what it means is that, if this is a solar connector and if this is the out of normal, now phi plus 15 may have a ray like this and phi minus 15 may have a ray like this, but still this theta remains the same. Consequently you have the same answer for R b whether beta is phi plus 15 or phi minus 15 So, in two dimensional view it looks like there is a ray in one line, but that ray could be a pencil and a core disturbing the same angle. Of course, phi plus 15 and December declination you have the highest, 1.79. So, if you have set an horizontal radiation, even if it is 1.7 times, 1.2 times the December value, you have a pretty good inclination factor, if you choose the right slope.

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1.8

I_0 Chennai $\phi = 13^\circ N$

10:30 - 11:30

$$\delta = 23.45 \sin\left(360 \frac{284+n}{365}\right) = -20.9^\circ$$

I_0 (10:30-11:30)

$$\omega_1 = -22.5^\circ, \omega_2 = -7.5^\circ$$

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So, we will go to problem number 1.8. So, what is the extraterrestrial radiation on a horizontal surface I_0 at Chennai, whose latitude is 13 degrees North. June will be over 10:30 to 11:30. Now, I shall go back to the previous problem after finished this, this time reminded me of something that I can tell you, so declination for this Chennai will be 23.45 on that day rather any location sine 360 times 284 plus n by 365 or that equals to minus 20.9 or say as January 15 n is equal to 15, this is almost a mean declination. So, I_0 for 10:30 to 11:30 my omega 1 will be minus 22.5 and omega 2 will be minus 7.5 degrees, right? 11:30 corresponds to minus 7.5 half an hour before the solar noon; this is one and half hours before the solar noon. So, this will be minus 22.5 and 7.5 as per our convention.

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$$I_0 = \frac{12 \times 3600}{\pi} G_{sc} \left[1 + 0.033 \cos \frac{360n}{365} \right] \times$$

$$\left[\cos \phi \cos \delta (\sin \omega_2 - \sin \omega_1) + \frac{2\pi(\omega_2 - \omega_1)}{360} \sin \phi \sin \delta \right]$$

$$= \frac{12 \times 3600 \times 1353 \times 7}{22} \left[1 + 0.033 \cos \frac{360 \times 15}{365} \right]$$

$$\left[\cos 13 \cos(-20.5) \{ \sin(7.5) - \sin(-22.5) \} + \frac{2\pi(-7.5 + 22.5)}{360} \sin 13 \sin(-20.5) \right]$$

So, I 0 I will repeat the formula, so that you will remember a better, little better. So, tolerant is 3600 by pi times a solar constant G s c, tangent variation 0.033 cos 360 n by 365 times cos phi cos delta sine omega 2 minus sine omega 1 plus 2 pi omega 2 minus omega 1 by 360. So, this omega 2 and this omega 1 are degrees. Since, I am using pi phi variety or 2 pi 60 as the conventional type sine phi sine delta. So, this is if you this will be 12 times 3600 multiply by 1353 this is the value I used for G s c 1353 watts per meter square by pi 22 by 7, so it goes to the numerator multiplied by 1 plus 0.033 cos 360 into n is 15 by 3654 times cos sine pi cos delta times sine omega 2 minus sine omega 1 plus 2 pi into omega 2 minus, which becomes plus 22.5 omega 1 by 360 times sin phi times sine delta. So, you can check up each number and pause your videos and check up whether the formula is written or not and then you can calculate.

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$I_0 = 4.0747 \text{ MJ/(m}^2\text{-hr)}$

What is H_0 for Jan 15, ~~New Delhi~~ Chennai

$$\omega_s = \cos^{-1}(-\tan 13 \tan(-20.9))$$
$$\omega_s = 84.96^\circ$$
$$H_0 = \frac{24 \times 3600 \times 1353 \times 7}{22} \left[1 + 0.033 \cos \frac{360 \times 15}{365} \right]$$

So, if you make this long calculation answer will be I_0 , 4.0747 mega joules per meter square hour according to our rotation. So, this is what you will get, now we are also to calculate what is its ω_s for January 15 New Delhi? So, this I need the sunset hour angle ω_s will be $\cos \delta \sin \phi - \tan \phi \tan \delta$, which is minus 20.9. This is the location is Chennai so 13 degrees, so sorry.

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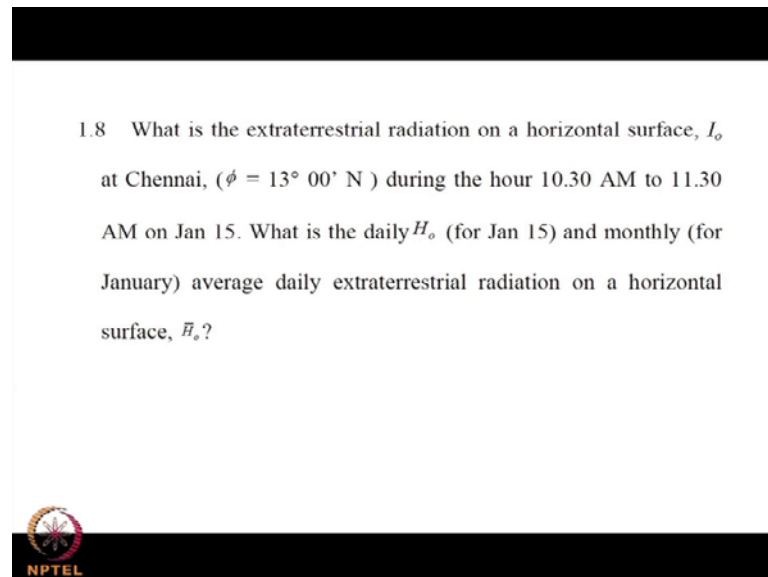
$$\left[\cos 13 \cos(-20.9) \sin 84.96 + 2 \times \frac{22}{7} \times \frac{84.96}{360} \sin(13) \sin(-20.9) \right]$$
$$= \underline{\underline{30.201 \text{ MJ/m}^2\text{-day}}}$$

So you have got this ω_s is point sorry 84.96 degrees, so you can understand that this is January, but it is low latitude, so you almost have 90, but less than 90 it should be.


H_0 will be 24×3600 multiply by solar constant 1353 by π^2 by 7 or times $1 - \cos 0.033 \cos 1160$ into 15 that is n by 365 multiplied by $\cos \phi \cos \delta \sin \omega s$ plus twice 22 by π times ωs by 362 π by $360 \sin \phi \times \sin \delta$.

So, this comes true, 30.20ϕ mega joules per meter square take. So, you can check up from standard textbooks or the extraterrestrial deviation tabulated at intervals of 10 degrees might be $10, 20, 30, 40$ and by interpolation between 10 and 20 . You can check whether this is or not for Chennai, which is 13 degrees. The name is location is Chennai we are choosing, otherwise what really matter only is the latitude 13 degrees and the day we have considered not the physical date or location.

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1.8 What is the extraterrestrial radiation on a horizontal surface, I_0 at Chennai, ($\phi = 13^\circ 00' N$) during the hour 10.30 AM to 11.30 AM on Jan 15. What is the daily H_0 (for Jan 15) and monthly (for January) average daily extraterrestrial radiation on a horizontal surface, \bar{H}_0 ?



So, we will go to problem 1.9.

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1.9 If the daily horizontal radiation for Jan 15, in the above problem has been measured to be 19.8 MJ/(m²-day), what is the daily clearness index?



This is if the horizontal daily radiation in the above problem as usual measure to be 19.8 mega joules per day, what is the daily clearance index?

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1.9


Given as per our notation

$$H = 19.8 \text{ MJ/m}^2\text{-day}$$

Daily Clearness Index K_T

$$K_T = \frac{H}{H_0}$$

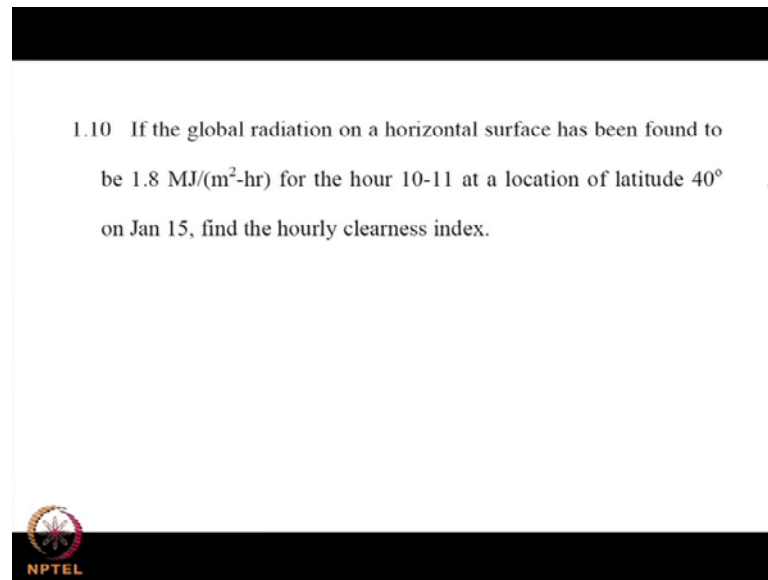
From, 1.8 problem, $H_0 = 30.205 \text{ MJ/m}^2\text{-day}$

$$K_T = \frac{H}{H_0} = \frac{19.8}{30.205} = 0.6555$$



1.9 given as per our notation which equal to 19.8 mega joules per meter square a day and you are required to calculate daily clearness index capital K T and capital K T is H upon H 0 from 1.8 problem H 0 is 30.205 mega joules per meter square K. So, your clearance index K T will be H by H 0, which is 19.8 by 30.205 equals to 0.6555.

So, it is not that it is a a great calculation or problem, but the problems in your course or in reality or in the exams can be formatted in different ways. For example, without 1.8 and given that the global radiation horizontal surface 19.8 was the clearance index? The whole idea is you will have to calculate 80 also to get the clearance index K T.

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1.10 If the global radiation on a horizontal surface has been found to be $1.8 \text{ MJ}/(\text{m}^2\text{-hr})$ for the hour 10-11 at a location of latitude 40° on Jan 15, find the hourly clearness index.



So, now we go to the next problem 1.10, so if the global radiation on a horizontal surface has been found to be 1.8 mega joules per meter square per per hour 10 to 11 at a location of latitude 40 degrees on January 15, find the hourly clearance index. Like I was mentioning this problem has been modified to take care of the hourly factor rather than the daily.

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$I = 1.8 \text{ MJ}/(\text{m}^2\text{-hr})$
 10-11 AM \rightarrow
 at $\phi = 40^\circ$
 day \rightarrow Jan 15th.

$$I_0 = \frac{12 \times 3600}{\pi} G_{sc} \left[1 + 0.033 \cos \frac{360n}{365} \right]$$

$$\times \left[\cos \phi \cos \delta (\sin \omega_2 - \sin \omega_1) + \frac{2\pi(\omega_2 - \omega_1)}{360} \sin \phi \sin \delta \right]$$

So, as per our rotation we are given I equal to 1.8 mega joules per meter square hour, the day time is 10 to 11 am. This we will come to our angles at a location of latitude 40 degrees and day is January fifteenth. So, the formula is first you have to find out as I_0 should be equal to 12 into 3600 by pi G s c and 1.033 cos 360 n by 365 times cos phi cos delta times sine omega 2 minus sine omega 1 plus 2 pi omega 2 minus omega 1 by 360 times sin phi sine delta. Same formula we have used.

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$\phi = 40^\circ; \delta = -20.9$
 $\delta = 23.45 \sin \left(\frac{281+n}{365} \cdot 360 \right)$ check!!
 $\omega_1 = -30^\circ, \omega_2 = -15^\circ$

$$I_0 = 2.167726 \text{ MJ}/\text{m}^2\text{-hr.}$$

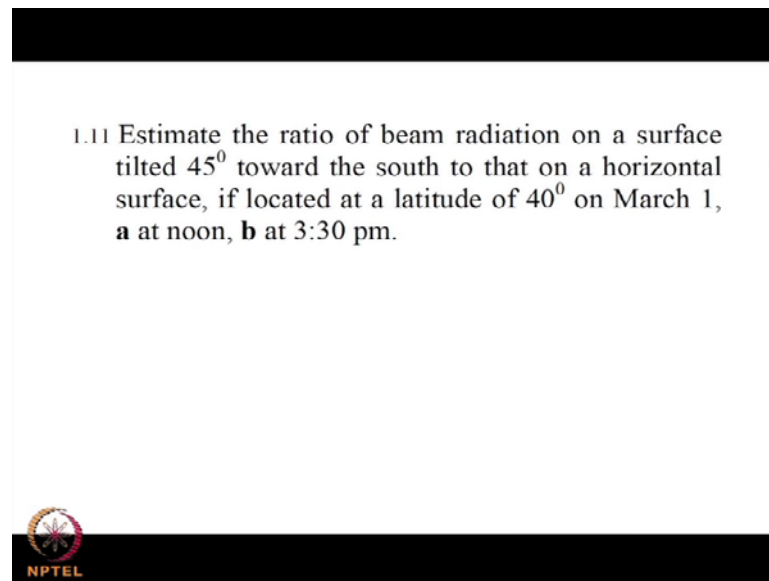
$$\sim 2.17 \text{ MJ}/\text{m}^2\text{-hr.}$$

$$k_T = \frac{1.8}{2.17} \rightarrow \text{please calculate.}$$


So, the latitude ϕ is 40 degrees and δ is the mean declination for the month of January is minus 20.9 or you can calculate δ is equal to $23.45 \sin$ or 284 plus n by 360 ϕ whatever multiply it by 360 , check. So, declination for January 15 is minus 20.9 you can plug it in this, if this formula slightly wrong you correct it. I think it is right. So, ω_1 will be 30 degrees minus and ω_2 is minus 15 degrees. So, the time is 10 to 11, 11 o'clock will correspond to minus 15 and 10 o'clock will correspond to minus 30. So, ω_1 is minus 30 and ω_2 is minus 15.

So, I_0 if you plug in all this numbers comes out to be 2.1677 mega joules per meter square hour. So, we are also required to find out the clearance index, so and I guess this is even if you say 2.17 mega joules per meter square hour. In fact my professor used to direct some of us if you give too many decimals because there is no way of measuring so accurately, simply because the calculator is giving do not put 8 to 12 decimal places for solar m_g 2 to 3 are more than adequate. Your K_T is your I 1.8 by 2.17. Please calculate this, I just thought I (()), it but forgot to do this simple division.

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1.11 Estimate the ratio of beam radiation on a surface tilted 45° toward the south to that on a horizontal surface, if located at a latitude of 40° on March 1, **a** at noon, **b** at 3:30 pm.



Now, we shall go to problem 1.11. Estimate the ratio of beam radiation on a surface 45 degrees towards to the South to get on an horizontal surface, if your created at a latitude of 40 degrees or March first a at noon time and b at 3:30, so it is a straight forward calculation of R_b , right?

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1.11
 $\phi = 40^\circ N$
 $\beta = 45^\circ N$
March 1 at noon and at 3.30 PM.
$$\delta = 23.45 \sin \frac{284 + (31 + 28)}{365} \times 360$$
$$= 23.45 \sin 338.30$$
$$\rightarrow -8.6^\circ$$

So, this is going to tell us something 1.11 latitude is 40 degrees and slope beta is 45 degrees and March 1 and at noon and at 3:30 pm. So, declination is 23.45 sine 284 plus 31 plus 28 because this is March first 31 days for the Month of January and 28 for the month of February, so it will be I can even put 29 it does not matter upon 365 times 360. I think in the previous example, I was forgetting this 284, but you can correct it.

So, 23.45 times sin 338.30 which comes to minus 8.6 degrees. This is quite because this is March first before the equinoctial day, it should be negative and we got a reasonable low negative number minus 8.6 degrees.

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Handwritten derivation on a whiteboard showing the calculation of R_b at noon time. The derivation starts with $\omega = 0 \rightarrow$ Noon time. The formula for R_b is given as:

$$R_b = \frac{\cos(40-45) \cos(-8.6) \cos 0 + \sin(40-18) \sin(-8.6)}{\cos 40 \cos(-8.6) \cos 0 + \sin 40 \sin(-8.6)}$$

The result is calculated as 1.538 .

And at noon time, which means omega jouse 0 for noon time, so my R b will be cos phi minus beta cos delta cos omega plus sine phi minus beta sine delta upon cos phi, which is 40 cos delta times cos 0, which is omega plus sine phi sine delta. This comes to 1.538. Now, you find that a beta is close to 40, omega is of course noon time, the best south oriented surface and the declination is pretty low in the sense that it is close to equinoctial date. So, we found in the (()) phi minus beta is equal theta is an optimal orientation. So, similarly here you have got phi pretty close to beta and pretty close to the equinoctial day, so you have got a fairly high R b at noon time.

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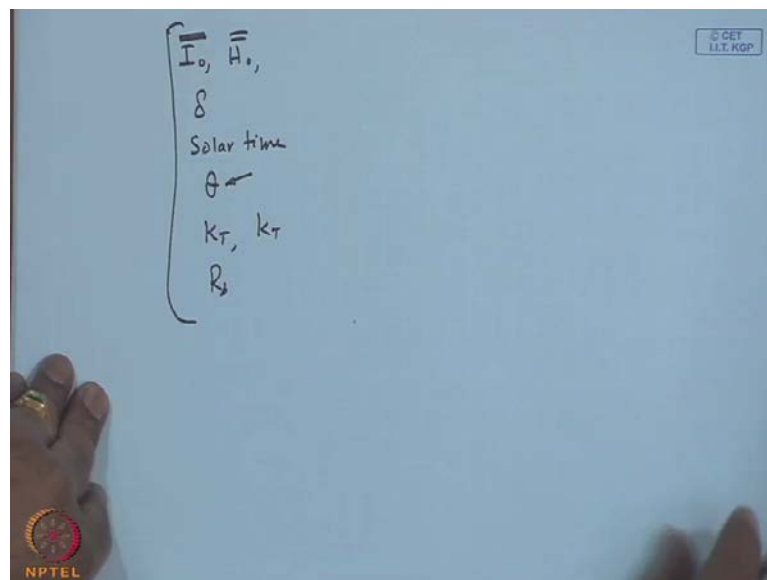
Handwritten derivation on a whiteboard showing the calculation of R_b at $\omega = 52.5^\circ$. The derivation starts with $\omega = 52.5^\circ$. The formula for R_b is given as:

$$R_b = \frac{\cos(40-45) \cos(-8.6) \cos(52.5) + \sin(40-45) \sin(-8.6)}{\cos 40 \cos(-8.6) \cos(52.5) + \sin 40 \sin(-8.6)}$$

The result is calculated as 1.66 , with a note: "Please check the ~~math~~ calculation." and a checkmark.

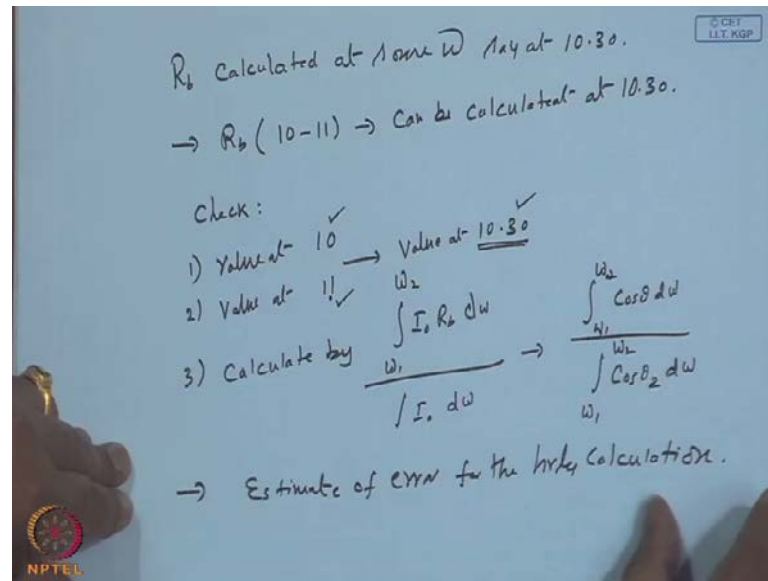
So, now the next one is omega 3:30, which comes to 52.5 so 1 o'clock is 15, 2 o'clock is 30, 3 o'clock is 45 plus 7.5 for 3:30 So, R b same step $\cos \phi \sin \beta \cos \delta \cos \omega$ plus $\sin \phi \sin \beta \sin \delta$ upon $\cos \phi \cos \delta \cos \omega$ plus $\sin \phi \sin \delta$ this is 1.8. So, the R b factor is pretty high at this omega is equal to 52.5. In fact it is more than 1.538. So, please check the calculation once again, it may be quite right, but I believe at 3:30 it may not be higher than what it is at the noon time, unless your 10 minus 45 that phi minus beta is making a different situation, all right? You just check it out or I also have a number here 1.66, so I will give you a clarification in the next class, which one is the correct one? Whether 1.66 or 1.88?

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So, from these exercises what we get is calculation of extra terrestrial I_0 or H_0 or the declination for a given day and solar time and the angle of incidence for different orientation and clearance index on a deviated list on a hourly basis. Then the R b factors, so which we found a pattern that you are in general.

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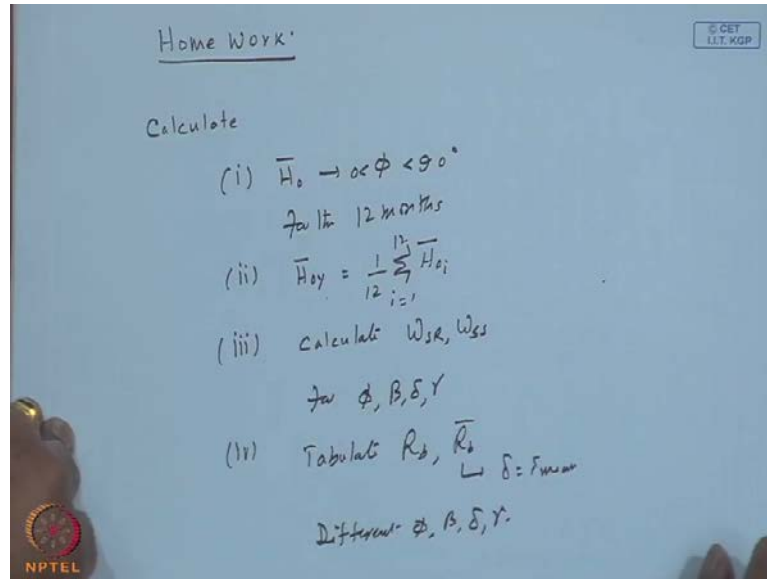
If you appropriately orient your collector R_b will be high in winter. Of course, clearance index has something to do with this solar radiation. This is similarly, angle of incidence will be favorable in December when R_b is high, a declination we know it changes negative to positive to negative from January to December. As you go ahead in H_0 all the measures of the expectance to variation ω on an hourly times scale and a daily times scale. Regarding the angles and R_b I was mentioning, R_b we calculated at some omega right 10:30 or whatever it is. Say at, so we have been emphasizing that this R_b for the hour 10 to 11 can be calculated at 10:30.

Check one value at 10 o'clock 2 value at 11 o'clock and compare with the value at 10:30, is it sort of a linear average or approximately average of these two values at ten 10:30. The third thing that you can do is calculate by using the formula omega 1 to omega 2 at R_b d omega by I_0 d omega, which will ultimately turn out to be integral omega 1 to omega 2 of $\cos \theta$ d omega by integral $\cos \theta_2$ d omega omega 1 to omega 2, right?

Like for a day if I do for an hour in the interval omega 1 to omega 2, I will get the weighted average R_b factor and we would like to compare what will be the difference between this weighted average and the value at the midpoint of the hour, right? I have no clue to this difference, but you can calculate simply as if it is a day the two times being omega 1 omega 2 instead of twice of 0 to omega s and here also omega 1 to omega 2,

right? So, you calculate these three values at 10 o'clock and 11 o'clock and at 10:30 giving the corresponding values and you check of course, obviously the value at 10:30 will lie between 10 and 11 o'clock and how far it is from the average that you can see and make an exact calculation, between omega 1 to omega 2, so that this will give a estimate of error for the hourly calculation.

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So, these are the exercises, which we can do and as some sort of a homework you can do calculate \bar{H}_0 for ϕ 0 to 90 for the 12 months. Let us say its \bar{H}_0 , so let it simple $2 \bar{H}_0$ \bar{H}_{0y} equal to summation of \bar{H}_0 of 8 months upon 12. I equal to 1 to 30, so that you will have an idea with the formula that we have given, how much it is off? Third thing you can do is you can calculate, I have already said that $\omega_s R \omega_s s$ for different ϕ, β, δ and γ 4 tablet R_b and \bar{R}_b that is a monthly average or the daily this should say only the δ equal to $\delta = F_{mean}$ for or different ϕ, β, δ and γ . So, if you do these exercises it almost covers all the formulae. We have done in the first three to four topics or lectures, which we have covered through the problems.

Thank you.