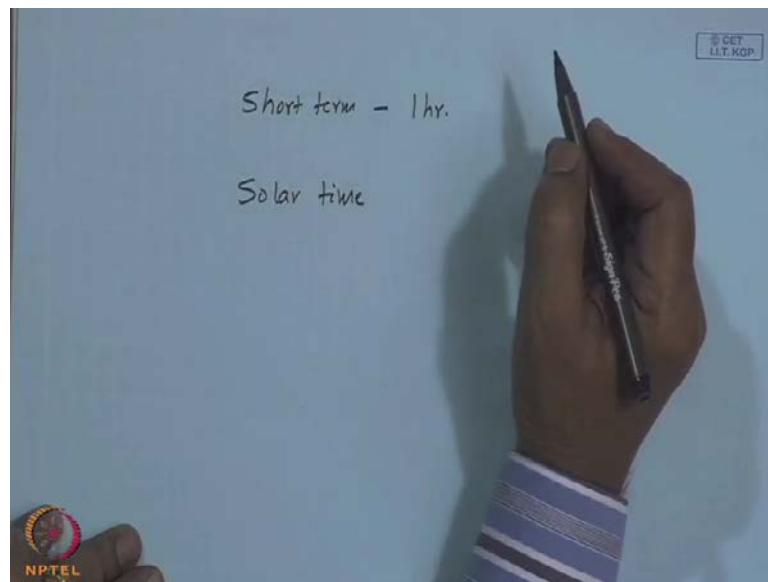


**Solar Energy Technology**  
**Prof. V. V. Satyamurty**  
**Department of Mechanical Engineering**  
**Indian Institute Technology, Kharagpur**

**Lecture - 06**  
**Radiation Processing – Long Term**

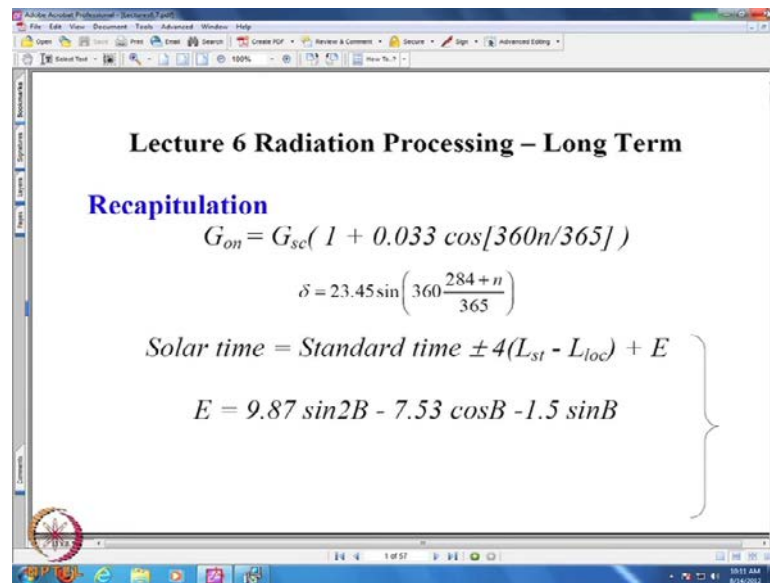
Last time, we were discussing about how to calculate the solar radiation received by surface of general orientation; that is, having a slope beta placed at an azimuthal angle of gamma. That is for a short period of one hour.

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Generally, we understand by short term as for a period of one hour. Now, we shall try to discuss how to evaluate the tilted radiation for a period of a day and then a monthly average day. Of course, it is not difficult if you make twelve calculations or round about by summation procedure, provided you have the actual data. The idea is to have a formula, similar to that of a short term like one hour and whether we can develop it and apply it for let us say a monthly or daily solar radiation on a horizontal surface to get the tilted radiation. However, we have listed a large number of equations.

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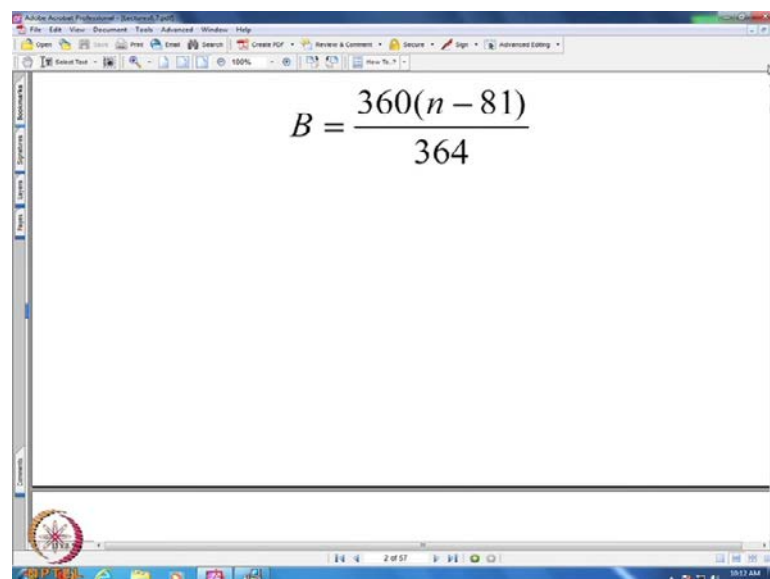
**Lecture 6 Radiation Processing – Long Term**

**Recapitulation**

$$G_{on} = G_{sc} (1 + 0.033 \cos[360n/365])$$
$$\delta = 23.45 \sin\left(360 \frac{284+n}{365}\right)$$
$$\text{Solar time} = \text{Standard time} \pm 4(L_{st} - L_{loc}) + E$$
$$E = 9.87 \sin 2B - 7.53 \cos B - 1.5 \sin B$$

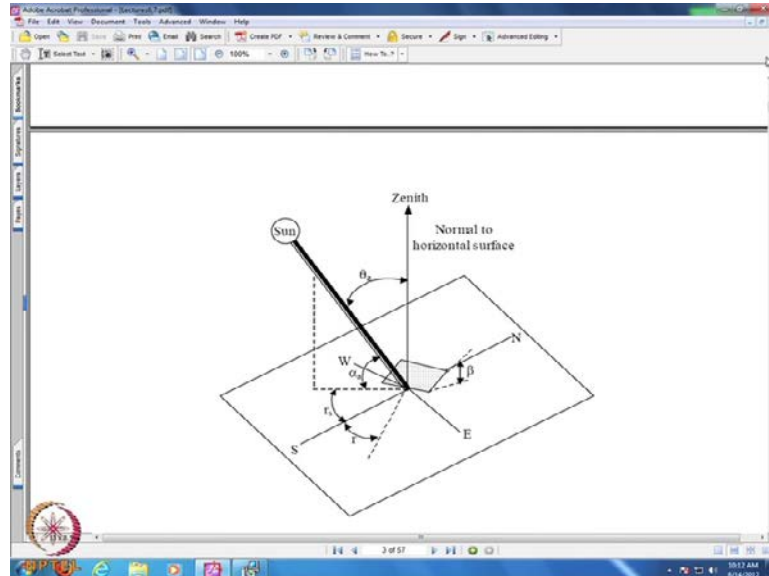
And so, I thought it is about time that we spent on recapitulating what we have started in the beginning. We define the solar constant  $G_{sc}$  as you can see on the screen. And then, the so-called radiation normal to the sun's ray at the current sun earth distance is  $G_{on}$ . And, next equation deals with the declination, that is, the angular difference between the equatorial plane and the plane of rotation in terms of the day of the year. Then, almost all the time insolar energy, we deal with solar time which needs to be corrected depending upon the location and the longitude on which your time is based; that is, in terms of  $L_{st}$  first and  $L_{loc}$ .

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$$B = \frac{360(n - 81)}{364}$$

And, a equation of time E expressed in terms of a parameter B defined as 360 times n intominus 81 by 364. This is purely an empirical relation.

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Just we recapitulate sothat, you will have idea of what we are talking. Next,we have a general surface of slope beta, oriented zoo east byazimuthal angle of gamma. And, the angle between the vertical and the sun's ray is zenith angle and the angle between the sun's rays and the outer normal to the surface underconsideration; we will call it the angle of incidence.

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$$\begin{aligned} \cos \theta &= \sin \delta \sin \phi \cos \beta - \sin \delta \cos \phi \sin \beta \cos \gamma \\ &+ \cos \delta \cos \phi \cos \beta \cos \omega \\ &+ \cos \delta \sin \phi \sin \beta \cos \gamma \cos \omega + \cos \delta \sin \beta \sin \gamma \sin \omega \\ \cos \theta &= A + B \cos \omega + C \sin \omega \\ A &= \sin \delta (\sin \phi \cos \beta - \cos \phi \sin \beta \cos \gamma) \\ B &= \cos \delta (\cos \phi \cos \beta + \sin \phi \sin \beta \cos \gamma) \\ C &= \cos \delta \sin \beta \sin \gamma \\ \cos \theta_z &= \cos \delta \cos \phi \cos \omega + \sin \delta \sin \phi \end{aligned}$$

That has got a long expression in terms of the latitude phi and the slope beta and the declination delta and the time of the day integrated by the hour angle omega. And of course, lastly the azimuthal angle gamma. This can be in a, formally written as in a simplified form as A plus B cos omega plus C sin omega; where the constants A, B and C are defined. If you put beta equal to 0, you will come across the horizontal surface or the correspondingly the zenith angle, which is cosine theta Z. Simply you will find that, if you put beta equal to 0 in the general equation for cosine theta, you will not find dependence upon the azimuthal angle, which you can expect because the projection of the outer normal to the surface. This surface being horizontal will be a point; consequently there will be no azimuthal angle.

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The screenshot shows a presentation slide with the following mathematical formulas:

$$\omega_s = \cos^{-1}[-\tan \phi \tan \delta]$$

$$N_s = 2\omega_s/15, \omega_s, \text{ is in degrees}$$

$$G_o = G_{on} \cos \theta_z$$

$$I_o = \int_{t_1}^{t_2} G_o dt$$

From this, we define the so-called sunset hour angle and which is nothing but, when the sun appears at the horizon, which is obtained by setting theta Z is equal to pi by 2. And, if you solve cosine theta Z is equal to 0, then you will get the corresponding angle omega s as cosine inverse of minus tan phi tan delta. We had of course discussed in detail. The difficulty is that one phases. If latitude is more than 66.45 degrees and declination plus the latitude turns out to be more than 90 degrees, considering the magnitude of the declination. The number of sunshine hours are related to the sunset hour angle, which is twice of omega s upon 15; because 24 hours for the earth's rotation and it will take 15 degrees for each hour.

So,  $N_s$  will be the number of sunshine hours that the location will receive. And then, from the  $G$  on that is the plane being normal to the sun's rays, we calculate what is the horizontal radiation under extra-terrestrial condition; which is nothing but geometrically is  $G$  on times cosine of theta  $Z$ . There is a reason for particularly defining the horizontal radiation because most of the measurements that are available will be on a horizontal plane.

So, you will have a comparison point on the extra-terrestrial conditions that, what will be the radiation if the atmosphere transmittance has been unity. Over a period of time between  $t_1$  to  $t_2$ ,  $I_0$  generally defined for a period of 1 hour is obtained by integrating  $G_0$  in the time interval of  $t_1$  to  $t_2$ .

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$$I_o = \frac{12 \times 3600}{\pi} G_{sc} (1 + 0.033 \cos[360n / 365]) x$$

$$[\cos \phi \cos \delta \sin(\omega_2 - \sin \omega_1) + \sin \phi \sin \delta (\omega_2 - \omega_1)]$$

$$H_o = \frac{24 \times 3600}{\pi} G_{sc} [1 + 0.033 \cos(360n / 365)] x$$

$$[\cos \phi \cos \delta \sin \omega_s + \sin \phi \sin \delta \omega_s]$$

$$H/H_o = a + b (N_b / N_s)$$

Orgil and Hollands [7]

And, in terms of the corresponding hour angles  $\omega_1$  and  $\omega_2$ , you can express  $I_0$  by this following expression; where you have put  $G_0 \cos \theta Z$  and  $G_0$  is related to the solar constant and  $1 + 0.33$  etcetera and, this is what will you get  $I_0$ . Then, if you integrate over a day that is minus  $\omega_s$  to plus  $\omega_s$ , you will get the corresponding daily horizontal radiation is  $H_0$ ; which is nothing but the expression that is given here, which we also have derived in the previous classes.

Then, as a relation to the bright sunshine hours and the number of possible sunshine hours at a location on a given day, originally Armstrong and subsequently Page have correlated the ratio of the daily radiation to the extra terrestrial radiation in terms of two

constants a and b, which will depend upon the particular location under consideration. In general if the climate is similar, then a and b are varied. The number of sunshine hours is measured by what we discussed in detail; the sunshine recorder, along with other instruments for solar radiation measurements.

Then, in order that to fill certain missing information or the data that is not recorded, several correlations have been proposed, which in retrospect may be called the synthetic data generation. The idea is, if you have a broad, measured or estimated entity like monthly average daily radiation, can we reconstruct or can we construct hourly radiation values which will mimic the actual data, though not equal to the exact actual data.

(Refer Slide Time: 08:06)

The image shows a presentation slide with the following content:

Recall, ( $k_T = I / I_0$ )

$$\frac{I_d}{I} = \begin{cases} 1.0 - 0.249k_T & \text{for } k_T < 0.35 \\ 1.557 - 1.84k_T & \text{for } 0.35 < k_T < 0.75 \\ 0.177 & \text{for } k_T > 0.75 \end{cases}$$

$$\bar{K}_T = \bar{H} / \bar{H}_0$$

Collares-Pereira and Rabl [7]

First of the correlations is due to one of the Orgil and Hollands, which expresses diffuse radiation to the global radiation as a function of the clearness index. And, you may recall that clearness index is nothing but the radiation that is received on the earth surface by the extra-terrestrial radiation  $I_0$ . Correspondingly you may define a  $\bar{K}_T$ , the monthly average daily clearness index as the corresponding ratio of terrestrial radiation to the extra-terrestrial radiation.

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Recall,  $(K_T = H/H_o)$

$$\frac{H_d}{H} = \begin{cases} = 0.99 & \text{for } K_T \leq 0.17 \\ = 1.188 - 2.272 K_T + 9.473 K_T^2 - 21.865 K_T^3 + 14.648 K_T^4 & \text{for } 0.17 < K_T < 0.75 \\ = -0.54 K_T + 0.632 & \text{for } 0.75 < K_T < 0.80 \\ = 0.2 & \text{for } K_T \geq 0.80 \end{cases}$$

Recall,  $\bar{K}_T = \bar{H} / \bar{H}_o$

$$\frac{\bar{H}_d}{\bar{H}} = 0.775 + 0.00653(\omega_s - 90) - [0.505 + 0.00455(\omega_s - 90)] \cos[115\bar{K}_T - 103]$$

And, Collares-Pereira and Rabl have given the relations for the daily diffuse fraction, in terms of daily clearness index; that is  $H_t$  by  $H$ . And, also a  $\bar{H}_t$  by  $\bar{H}$ , which is nothing but the monthly average daily diffuse radiation to the monthly average global radiation.

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Liu and Jordan [8]

$$r_d = I_d / H_d$$

$$r_d = \frac{I_d}{H_d} = \frac{\pi}{24} \frac{\cos \omega - \cos \omega_s}{\sin \omega_s - (2\pi \omega_s / 360) \cos \omega_s}$$

Collares-Pereira and Rabl [7]

$$r_t = I / H$$

And subsequently Liu and Jordan, not necessarily in the chronological order of subsequently, it is a matter of detail. The ratio of hourly diffuse radiation to the corresponding total diffuse radiation over the day  $I_d$  by  $H_d$  has been simply expressed as

a trigonometric function, in terms of the hour angle omega and the sunset hour angle omega s. And, in the class when we are dealing with this lecture in detail, I pointed out that this rd is nothing but I 0 upon H0. So, it is a bit surprising that the ratio of the diffuse radiation to the total diffuse radiation is analytically has been set equal to the corresponding extraterrestrial values. But, this works within a certain percentage error of 4 to 10 percent on root mean square basis. Collares-Pereira and Rabl more recently, relatively recently, proposed a correlation for a corresponding global radiation values hourly I, upon the corresponding daily value H Which is the same expression, but multiplied by a plus b cos omega; where the constants a and b have been derived using a large database and correlate it to the sunset hour angle omega s and certain other constants.

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$$r_t = \frac{I}{H} = \frac{\pi}{24} (a + b \cos \omega) \frac{\cos \omega - \cos \omega_s}{\sin \omega_s - \left( \frac{2\pi\omega_s}{360} \right) \cos \omega_s}$$

Where,

$$a = 0.409 + 0.5016 \sin(\omega_s - 60)$$

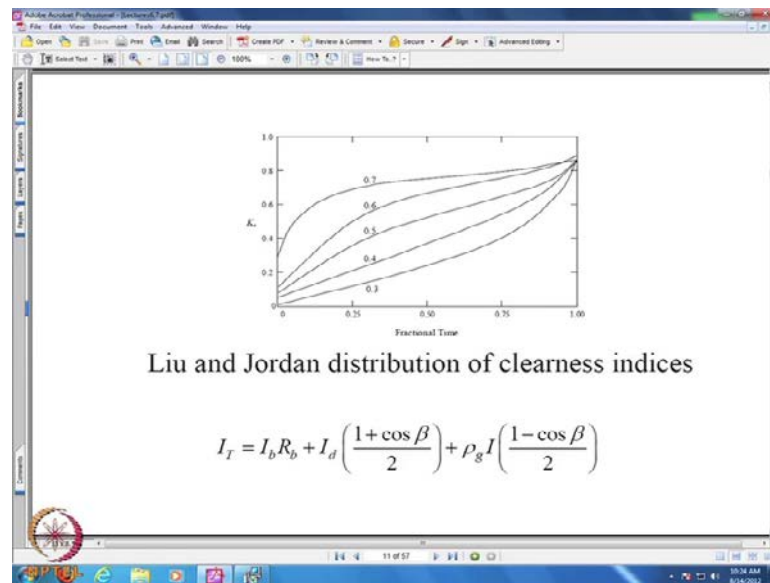
$$b = 0.6609 - 0.4767 \sin(\omega_s - 60)$$

**Distribution of Clearness Indices**  
Liu and Jordan [8]

But, you should remember that this correlations for  $r_t$  and  $r_d$  work extremely well, if you applied for the monthly average day. And, if you applied for an individual day, the errors are can be expected to be larger and over larger in general. A vital piece of information is the distribution of clearness indices. In other words, in terms of a cumulative frequency  $f$  for every given average clearness index 0.7 to 0.3, what are the clearness index values are plotted in terms of this?



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To give an example, it is rather to fix the idea; it is something like a percentile score of marks. If the remarks of the students in a class are known how many people got below ten percent and twenty percent, thirty percent that becomes the cumulative frequency  $f$ . Hundred percent of them will get less than the highest mark. So, fractional time will vary between 0 to 1 and the clearness index of course varies maximum 0 to 1. And, you can see when the average clearness index is very low, lower values are more and when the average clearness index is high, the higher values are more. If you discretize it in terms of thirty one days, and you may have day number one and day number thirty one and each ordinate will represent clearness index of the day.

So, given the monthly average clearness index as 0.5 or 0.6 or 0.7 whatever, one can find out thirty clearness index values pertinent to the each day of the month what you really do not know is the sequence, in which it occurs. But, nevertheless we are likely to, we get the likely daily clearness index values which will occur have the average clearness index being something. So, this is a very good information. And, this also has been found to be applicable even for this so-called monthly hourly distribution of clearness indices. In other words, replace that capital  $K_T$  with small  $k_t$  and capital  $K_T$  with small  $k_t$ .

So for a particular hour, 7 to 8 let us say, if you have a hourly clearness index of average of 0.7, I will have 30 or 31 daily clearness index values. And, the corresponding hourly clearness index values, which we can be read of from this. However, subsequent

investigators have noted that there could be differences between hourly distributions and monthly daily distributions. The reason being, the variation is likely should be larger at hourly time scale, rather than at the hour daily time scale.

Next object; in terms of the utility and in order to predict the performance of solar collectors, we have to estimate the radiation received by a solar collector, which in general is kept at a slope of beta; generally, oriented towards south if it is a solar collector. But nevertheless, if there are other applications where you would like to calculate the solar radiation on a surface received by arbitrary orientation of slope beta, set at an azimuthal angle of gamma and that  $I_T$  has been expressed in terms of three components. One is the direct radiation component and other one is the sky diffuse component and the third one is the ground reflected component.  $\rho_g$  is the ground reflect T, which has the usual value of 0.2 and for a snow covered ground, it is as I has 0.7.

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$$\text{If } I_T = R I$$

$$R = \left(1 - \frac{I_d}{I}\right) R_b + \frac{I_d}{I} \left(\frac{1 + \cos \beta}{2}\right) + \rho_g \left(\frac{1 - \cos \beta}{2}\right)$$

$$R_b = \cos \theta / \cos \theta_z$$

$$\cos \theta = A + B \cos \omega + C \sin \omega$$

where

$$A = \sin \delta (\sin \phi \cos \beta - \cos \phi \sin \beta \cos \gamma)$$

The factors 1 plus cos beta by 2 is arrived at, if you assume this sky diffuse radiation to be isotropic. It will be the spectral part that is intercepted by the radiation falling on the tilted surface with the slope beta. You can see if beta is set equal to 0, it will be 1 plus 1 by 2 equal to 1. That means, all the radiations diffuse isotropic will be received by horizontal plane.

And, similarly if beta is equal to pi by 2 a vertical surface, half of it will be received. And, of course again if beta is set equal to 0, this will be 0. In other words, say a horizontal plane will not receive any reflected radiation from the ground, which is also horizontal. So, if in general the whole of I T is expressed as the factor R multiplied by I, we have R in terms of R b and the other quantity, simply we can obtain it by dividing R. And, you will notice that I try to avoid I b. And, everywhere it is written as a ratio I d by I because we have relations for I d by I and most of the time the measurements are on a horizontal plane.

I is available and diffuse radiation might not be measured at as many meteorological stations as the global radiation has been. So consequently, in solar energy it is customary to write in terms of the diffuse fraction, rather than in terms of direct radiation and global radiation separately. Now, this R b is nothing but the ratio of cosine angle of incidence to the cosine of the zenith angle; that from a simple geometry we can get it. Where, of course again I repeat this equation;  $\cos \theta$  is  $a + b \cos \omega + c \sin \omega$ ; where the constants a and b and c we have been defined.

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$$B = \cos \delta (\cos \phi \cos \beta + \sin \phi \sin \beta \cos \gamma)$$

$$C = \cos \delta \sin \beta \sin \gamma$$

$$R_b \text{ for a south facing surface simplifies to,}$$

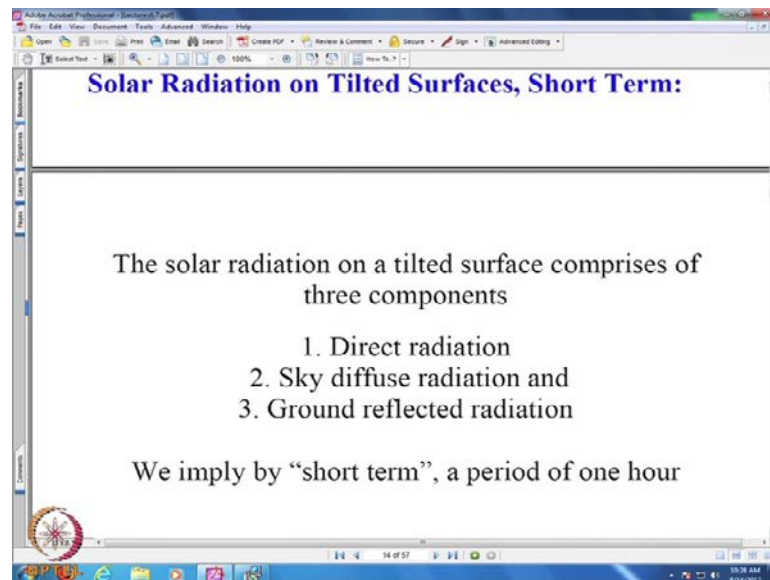
$$R_b = \frac{\cos(\phi - \beta) \cos \delta \cos \omega + \sin(\phi - \beta) \sin \delta}{\cos \phi \cos \delta \cos \omega + \sin \phi \sin \delta}$$

**Recapitulate**  
**Solar Radiation on Tilted Surfaces, Short Term:**

And, the factor for a south facing surface simplifies to simply  $\cos \phi - \beta \cos \delta \cos \omega + \sin \phi - \beta \sin \delta$  upon the corresponding zenith angle cosine. So, this is; however if it is a surface of general orientation, simply  $\cos \theta$  upon  $\cos \theta_z$ ; where  $\cos \theta$  is known and  $\cos \theta_z$  is nothing but the expression over

here in the denominator. So, before we proceed to the long term or the so-called daily or monthly average daily solar radiation received by tilted surface, in brief again we will recapitulate.

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**Solar Radiation on Tilted Surfaces, Short Term:**

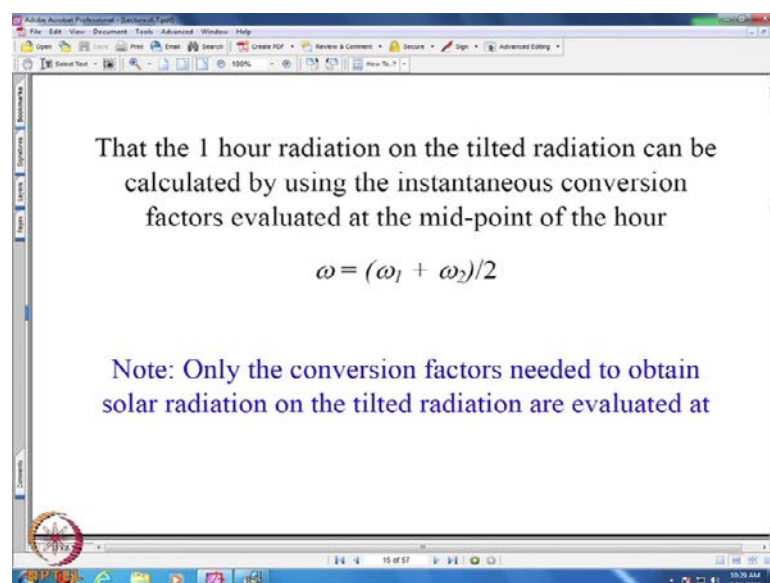
The solar radiation on a tilted surface comprises of three components

1. Direct radiation
2. Sky diffuse radiation and
3. Ground reflected radiation

We imply by “short term”, a period of one hour

The short term values or details; though solar radiation on tilted surface, in general comprises of three components. Direct radiation, sky diffuse radiation and ground reflected radiation.

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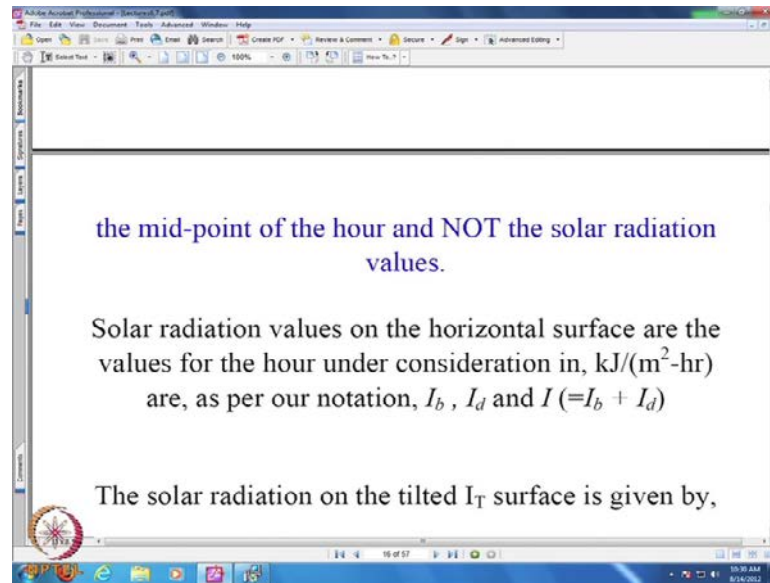


That the 1 hour radiation on the tilted radiation can be calculated by using the instantaneous conversion factors evaluated at the mid-point of the hour

$$\omega = (\omega_1 + \omega_2)/2$$

Note: Only the conversion factors needed to obtain solar radiation on the tilted radiation are evaluated at

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We imply by short term a period of one hour. And, that the one hour radiation on the tilted radiation can be calculated using instantaneous conversion factor at a point equal to them id point of the hour. If the hour is designated as duration between omega 1 and omega 2, you calculate R b at omega, given by omega 1 plus omega 2 by 2. In other words, even though the intensity is varying, the accuracy is sufficient. If you evaluate R b at the midpoint of the hour, of course a question comes whether you can use the midpoint of the day to calculate for the day little factor. Obviously, that leads to a larger error which we will see little later. Now, you have to remember that only the conversion factor need to needed to obtain solar radiation on the tilted radiation are evaluated at the midpoint, not the radiation. It is not the intensity of the radiation. It is I, during the period omega 1 and omega 2, whereas my R b z omega is given by omega 1 plus omega 2 by 2. So, what we have here is the kilo joules per meter square hyphen hour; are I b and I d and I are expressed for the period of one hour.

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$$I_T = I_b R_b + I_d \left( \frac{1 + \cos \beta}{2} \right) + \rho_g I \left( \frac{1 - \cos \beta}{2} \right)$$

Expressing,

$$I_T = RI$$
$$R = \left( 1 - \frac{I_d}{I} \right) R_b + \frac{I_d}{I} \left( \frac{1 + \cos \beta}{2} \right) + \rho_g \left( \frac{1 - \cos \beta}{2} \right)$$

Where,

This of course, now the expression to calculate the solar radiation received by tilted surface comprising of the direct radiation plus the sky diffuse radiation plus the ground reflected radiation. Again, overall factor R can be written in terms of the diffuse fractions.

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$$\cos \theta_z = \cos \phi \cos \delta \cos \omega + \sin \phi \sin \delta$$

$R_b$  for a south facing surface simplifies to,

$$R_b = \frac{\cos(\phi - \beta) \cos \delta \cos \omega + \sin(\phi - \beta) \sin \delta}{\cos \phi \cos \delta \cos \omega + \sin \phi \sin \delta}$$

Thus

$I_T$ , the hourly solar radiation received by a tilted surface can be calculated

Where, of course  $R_b$  is this much and this is a repetition. So, for a south facing surface it is simpler. Now, you will find that if you put beta equal to 0, obviously these two things will be equal. It is the zenith angle that is for a horizontal surface. So, in other words a

tilted surface at a latitude equal to phi given by phi minus beta behave like the horizontal surface at another phi star is equal phi minus beta.

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**Radiation Processing – Long Term**

**The Daily Tilt factor:**

**For the hour,**

$$I_T = I_b R_b + I_d \left( \frac{1 + \cos \beta}{2} \right) + \rho_g I \left( \frac{1 - \cos \beta}{2} \right)$$

Summing up the above equation from sunrise to sun set, it follows,

Now, we will come to the radiation processing on a long term basis, which will be a day to start with. This is for the hour which we already stated.

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$$H_T = \sum I_T = \sum_{\omega_s} I_b R_b + \sum_{-\omega_s} [(1 + \cos \beta) / 2] I_d + \sum_{-\omega_s} [(1 - \cos \beta) / 2] I \rho_g$$

**The sunset hour angle, as already introduced, is given by,**

$$\omega_s = \cos^{-1} [ - \tan \phi \tan \delta ]$$

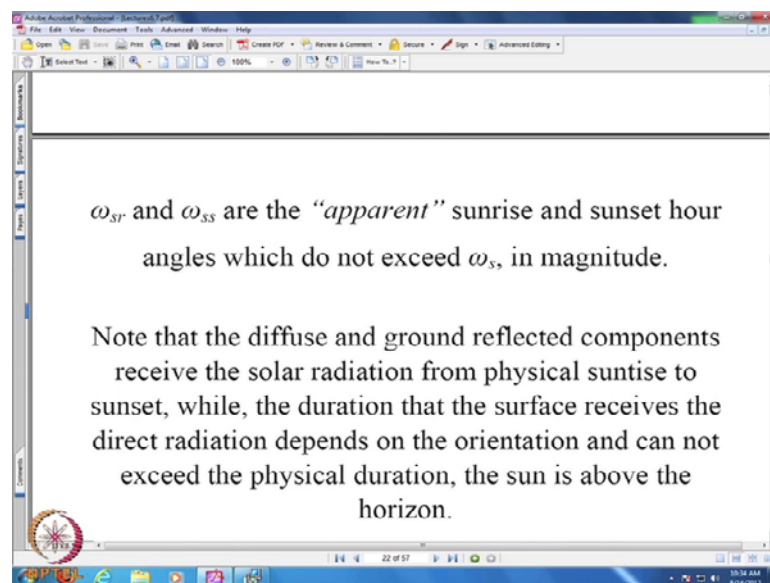
And, if we sum up from sunrise to the sunset, this is what you will get which we designated by H T, the solar radiation received by the tilted surface over the day as the summation of the hourly components, which comprises of summation of the direct



radiation plus the summation of the sky diffuse component plus the summation of the ground reflected component. You will notice here in the sky diffuse component, the summation is from  $\omega_{sr}$  to  $\omega_{ss}$ , not from sunrise to sunset in the case of diffuse and ground reflected radiations; because the direct radiation depending upon the orientation of the surface will be received by the surface, a part of the sunshine or utmost equal to the sunshine duration and never always equal to the sunshine duration.

We shall come to the evaluation of  $\omega_{sr}$  and  $\omega_{ss}$  a little later. There are quite a few issues involved in that. Though it is quite simple for a south facing surface, but for non-south facing surfaces owing to the nature of the transcendental equation, multiple roots are possible. Then, you may have to have a method to decide which multiple, which root is the correct root.

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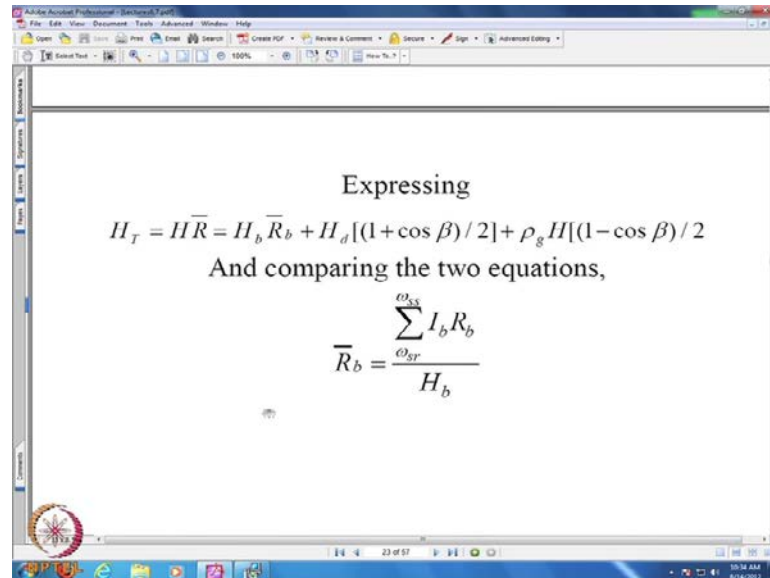


Now, we already know that  $\omega_s$  is the physical sunset hour angle given by cosine inverse of minus tan phi tan delta. Of course,  $\omega_{sr}$  and  $\omega_{ss}$  are the apparent sunrise and sunset hour angles which do not exceed  $\omega_s$  in magnitude. In other words, if according to the calculation  $\omega_{sr}$  and  $\omega_{ss}$  in magnitude are more than  $\omega_s$ , we have to limit it to  $\omega_s$ . In other words, surface will not and shall not receive solar radiation, beyond which the sun is below the horizon. I mean, so this already I have emphasized that the diffuse component and ground reflected component shall be received all the while, whereas the direct radiation will be received depending



upon the orientation of the surface; a part of the sunshine duration or utmost equal to the sunshine duration.

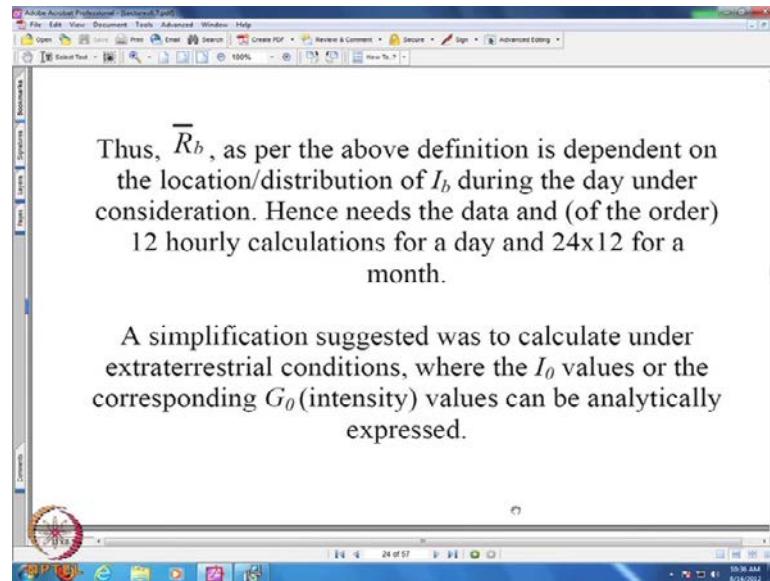
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Again expressing it as a single factor  $\bar{R}_b$ , we can write  $H_T$  as comprising of the  $H_b \bar{R}_b$ ; this is written analogously to the hourly value. But, now we should realize that  $\bar{R}_b$  should be a weighted average of  $I_b$  and  $R_b$  upon  $H_b$ . In other words, whatever the previous equation that we have shown over here, if we equate  $\sum I_b R_b$  to  $H_b \bar{R}_b$ , it follows  $\bar{R}_b$  is summation of  $I_b R_b$  over  $H_b$  or  $\sum I_b R_b / H_b$ . You can write it in any way you like.

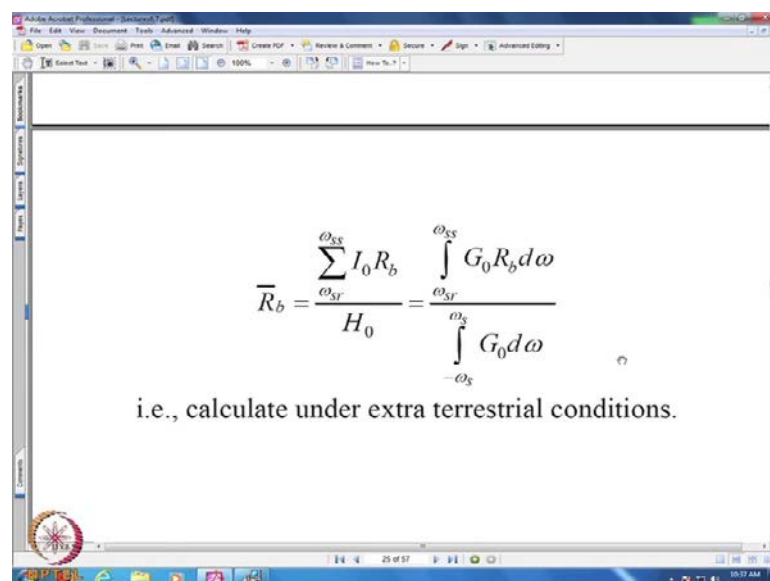
So, now this leaves us a little uncomfortable position because you have to have the data of  $I_b$  from sunrise to sunset, in order to evaluate  $\bar{R}_b$ ; not only that, it requires an average of twelve calculations for the day. Even if we have the data, it requires twelve calculations.

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So, the spirit of directly extending the hourly type of calculation to a day is not satisfied. Even though you can calculate the diffuse component and the sky ground reflected component as a single calculation, this direct component has to be done as twelve calculations. And if you want for a month, it is approximately 12 into 24 calculations. And, it has been suggested; a simplification is that do not use  $I_b$  or the terrestrial values, but use the extraterrestrial values like intensity values  $G_0$  or the hourly integrated quantity wise  $I_0$ .

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In other words, we replace  $I_b$  bar is equal to  $I_b R_b$  by  $H_b$  with  $I_0 R_b$  upon its 0. There is a justification. Later on, we will give even a better justification. One justification is I am making a mistake in the numerator as well as in the denominator, so to a large extent it may get cancelled and I may have a reasonably accurate value subject to the validation.  $I_b$  is replaced with  $H_0$  and  $H_b$  is replaced with  $H_0$ . Consequently, this ratio is likely to be reasonably correct. So, in terms of intensity I can go to my analytical integration procedure because  $G_0$  is an analytical value; you have an analytical expression for it. So, we can do that under so-called, we will call it extraterrestrial conditions. So,  $R_b$  bar which is a weighted average of  $I_b R_b$  to  $I_b$  has been calculated as a simple integral under extra terrestrial conditions.

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$$\bar{R}_b = \frac{\int_{\omega_{sr}}^{\omega_{ys}} Gsc(1 + 0.033 \cos[360n/365]) \cos\theta_2 (\cos\theta / \cos\theta_2) d\omega}{\int_{-\omega_s}^{\omega_s} Gsc(1 + 0.033 \cos[360n/365]) \cos\theta_2 d\omega}$$

If we do that, expand  $G_0$ ,  $G_s c$  in terms of that and it is an integral quantity, very easily integral quantity.

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Which simplifies to,

$$\bar{R}_b = \frac{\int_{\omega_{sr}}^{\omega_{ss}} \cos\theta d\omega}{\int_{-\omega_s}^{\omega_s} \cos\theta_z d\omega}$$

Simplifies to surprisingly  $\cos\theta d\omega$  by  $\cos\theta_z d\omega$ , integrated of course from apparent sunrise to apparent sunset by sunset to sunrise. This is surprising, whereas  $R_b$  is  $\cos\theta$  by  $\cos\theta_z$ , whereas  $\bar{R}_b$  is integral  $\cos\theta$ . That is not a perfectly valid Mathematics, but it tend out to be so because of the relation between  $G_o$  and  $\cos\theta_z$ ; where this  $\cos\theta_z$  gets cancelled. Consequently, it looks as if the numerator and the denominator are separately integrated to get the integrated value of the function, but this is a very simple thing.

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Using the relevant expressions for  $\cos\theta$  and  $\cos\theta_z$ ,

$$\bar{R}_b = \frac{A (\omega_{ss} - \omega_{sr}) (\pi/180) - B (\sin \omega_{ss} - \sin \omega_{sr}) + C (\cos \omega_{ss} - \cos \omega_{sr})}{2 [\cos \phi \cos \delta \sin \omega_s + (\pi/180) \omega_s \sin \phi \sin \delta]}$$

IT IS TO BE NOTED THAT IN WRITING THE ABOVE EQUATION FOR  $\bar{R}_b$  IT IS INHERENTLY ASSUMED THAT

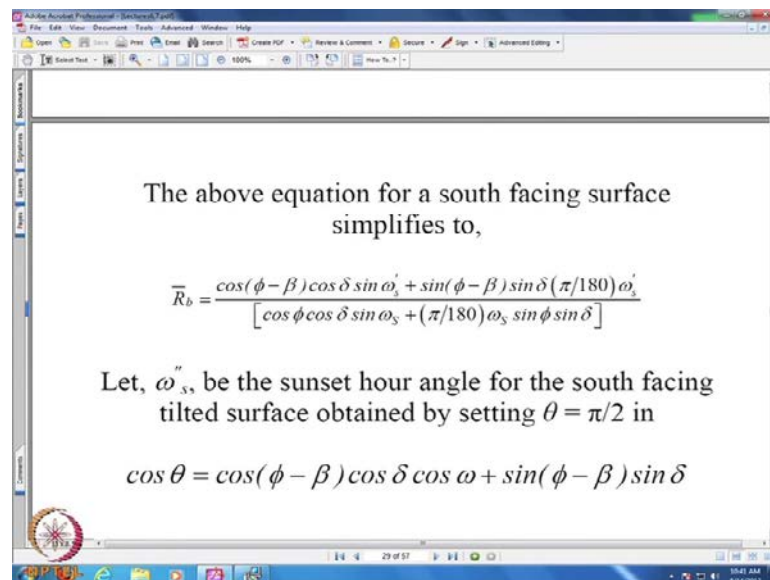
$-\omega_s \leq \omega_{sr} < \omega_{ss} \leq \omega_s$

OR Simply,  $R_b > 0$  IN  $\omega_{sr} < \omega < \omega_{ss}$

Now, I can put down that in terms of  $\cos \theta$  is equal to  $A + B \cos \omega + C \sin \omega$  as a into  $\omega_s$  minus  $\omega_s$  R into  $\sin C$  into  $\cos$ . And, I have put this  $\pi$  by 180 to emphasize that  $\omega_s$  and  $\omega_s$ . If they are in degrees, you have to use the conversion factor of  $\pi$  by 180. Now we make without explicit mention or knowledge, an assumption that  $\omega_s$  to  $\omega_s$ ; that  $\omega_s$  is lower than  $\omega_s$  or minus of  $\omega_s$  less than or equal to  $\omega_s$ , less than  $\omega_s$ , less than or equal to  $\omega_s$ . Let me give a counter example. Suppose, my sunrise hour angle is 32 plus and sunset hour angle is 38 minus, then my integration procedure would have given me would give me a wrong value. right

Or, simply what it really means is  $R_b$  should be positive in the interval of my integration. If by chance,  $R_b$  turns out to be negative during  $\omega_s$  to  $\omega_s$ , there will be a negative contribution which may subtract, even though there is otherwise some positive contribution and it may lead to an underestimate or maybe even an overall negative quantity. We will definitely discuss a lot more when we are evaluating about  $\omega_s$  and  $\omega_s$ .

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The above equation for a south facing surface simplifies to,

$$\overline{R_b} = \frac{\cos(\phi - \beta) \cos \delta \sin \omega_s' + \sin(\phi - \beta) \sin \delta (\pi/180) \omega_s'}{[\cos \phi \cos \delta \sin \omega_s + (\pi/180) \omega_s \sin \phi \sin \delta]}$$

Let,  $\omega_s''$  be the sunset hour angle for the south facing tilted surface obtained by setting  $\theta = \pi/2$  in

$$\cos \theta = \cos(\phi - \beta) \cos \delta \cos \omega + \sin(\phi - \beta) \sin \delta$$

To be simple minded, you assume  $\omega_s$  is close to the sunrise. Suppose, you are having an east facing that  $\omega_s$  will be closer to minus  $\omega_s$  and if it is slightly towards west that will be little less than minus  $\omega_s$  magnitude wise. And hence, so for these deviations it seems to be perfectly logical. Most of the time you may have that

assumption is not violated, but there will be equally important situations where that assumption may not be valid. So, for a south facing surface again, this simplifies and this  $\omega_s$  dashed now should be symmetric, instead of  $\omega_s$  r and  $\omega_s$  s an equal values. If  $\gamma$  equal to 0, have a minus  $\omega_s$  dashed and plus  $\omega_s$  dashed just like minus  $\omega_s$  and plus  $\omega_s$  for the horizontal surface.

So we will say, let us just to be clear,  $\omega_s$  double dashed be the sunset hour angle for the south facing tilted surface obtained by setting  $\theta$  is equal to  $\pi/2$  in the relevant equation for cosine  $\theta$ .

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$\omega_s'$ , the apparent sunset hour angle for the south facing tilted surface is defined by,

$$\omega_s' = \text{Min} \left\{ \begin{aligned} \omega_s &= \cos^{-1} [ - \tan \phi \tan \delta ], \\ \omega_s &= \cos^{-1} [ - \tan(\phi - \beta) \tan \delta ] \end{aligned} \right\}$$

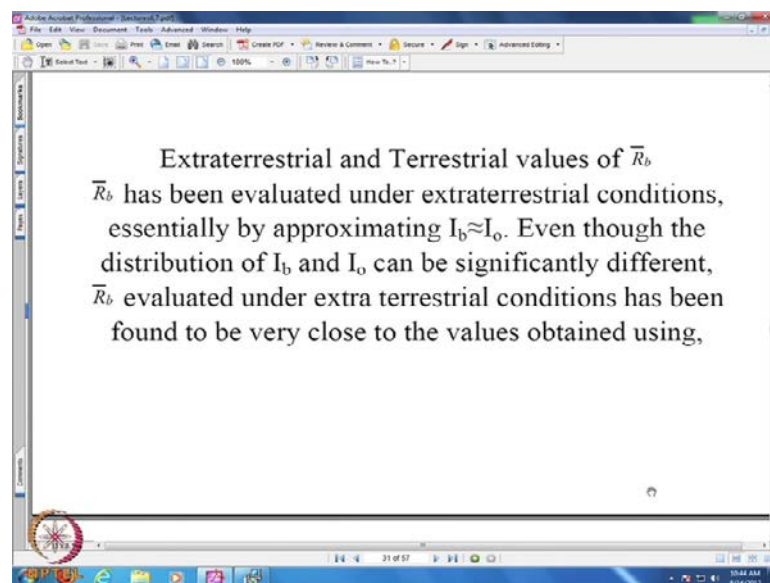
It is straight forward to envisage that in the northern hemisphere ( $\phi > 0$ ),

$$\omega_s' = \left\{ \begin{aligned} \cos^{-1} [ - \tan \phi \tan \delta ] &\text{ for } \delta < 0 \\ \cos^{-1} [ - \tan(\phi - \beta) \tan \delta ] &\text{ for } \delta > 0 \end{aligned} \right.$$

So, now  $\omega_s$  dashed is limited to  $\omega_s$  and  $\omega_s$  double prime. So, this is what the surface sees; that means, the sun's ray is expected to be parallel to the surface of the, parallel to the surface at this hour angle. But, this is the way this sun goes below the horizon. So if this is higher, I have to limit it to this. In other words, you are trying to look into the well, your angle at which you are able to see is possible and is feasible, but if the wall of the welcomes in the way and we cannot look into the well; so, that is something like that, this sun's rays if the sun is existing above the origin will be parallel to the given tilted surface. But, you should be limiting it to this one. Otherwise this is, mathematically you will get a cos inverse of minus tan phi minus beta tan delta; which may be a larger number or a smaller number than  $\omega_s$ . And, the surface cannot receive the radiation beyond which the sun is not shining.

So, now you can use little bit of trigonometry of delta being negative, positive; phi minus beta being positive negative, and it also always...that your omega s dashed is given by the simple sunset our angle for negative delta and given by the sunset hour angle of the tilted surface for positive delta. In other words, when delta is positive the surface receives a part of the day's duration of the sunshine, whereas in winter or rather if declination is negative, it is all the time the sun is above the horizon. So, this is an advantage of the negative declination, though in general the duration of the sunshine is smaller, is lesser.

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Now, people have compared the values of  $\bar{R}_b$  so-called, evaluated under extraterrestrial conditions like this approximation, which we have already discussed and compared with the values calculated using the data. And, they have found that they are reasonably close and sometimes remarkably accurate. So a question arises, why is this so?



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$$\bar{R}_{bt} = \frac{\sum_{\omega_{sr}}^{\omega_{ss}} I_b R_b}{H_b}$$

In the above equation  $\bar{R}_{bt}$  has been used to distinguish from  $\bar{R}_b$  evaluated under extraterrestrial conditions. It is to be noted that  $\bar{R}_b = \bar{R}_{bt}$  if the hourly clearness index  $k_T = 1$  from sunrise to sunset. The daily clearness

So, you can expect  $I_b$  distribution to be considerably different from  $I_0$  distribution. Apart from the errors being cancelled, there may be a cloud cover; there may be a bright hour etcetera. So  $\bar{R}_{bt}$ , just to distinguish for the sake of discussion; under terrestrial conditions will be  $I_b R_b$  upon  $H_b$ . Now, this value of  $\bar{R}_b$  evaluated under extraterrestrial conditions shall be identically equal to the value under terrestrial conditions, if the clearness index is equal to unity throughout the day. If  $K_T$  equal to 1 for all the hours or if capital  $K_T$  equal to 1, it ensures that for small  $k_t$  equal to 1 for the all the hours you have  $\bar{R}_{bt}$  is identically equal to  $\bar{R}_b$ .

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index  $K_T = 1$  ensures  $k_T = 1$  from sunrise to sunset and vice-versa.

$$\bar{R}_{bt} = \frac{\sum_{-\omega_s}^{\omega_{ss}} (I - I_d) R_b}{\sum_{-\omega_s}^{\omega_{sr}} (I - I_d)} = \frac{\sum_{-\omega_s}^{\omega_{ss}} I \{1 - (I_d / I)\} R_b}{\sum_{-\omega_s}^{\omega_{sr}} I \{1 - (I_d / I)\}} = \frac{\sum_{-\omega_s}^{\omega_{ss}} k_T I_0 \{1 - (I_d / I)\}}{\sum_{-\omega_s}^{\omega_{sr}} k_T I_0 \{1 - (I_d / I)\}}$$



So, now I am trying to prove sort of why the terrestrial calculation is so close to the extraterrestrial or extraterrestrial approximation is reasonably good. So, I write that  $I_b$  as  $I_0$  minus  $I_d$ , which can be written as  $I_0(1 - I_d/I_0)$ , which can be written as clearness index  $I_0(1 - I_d/I_0)$ . Once again, you can see that I am trying to avoid  $I_d$ , everywhere  $I_d/I_0$  and the analytically calculable  $I_0$ .

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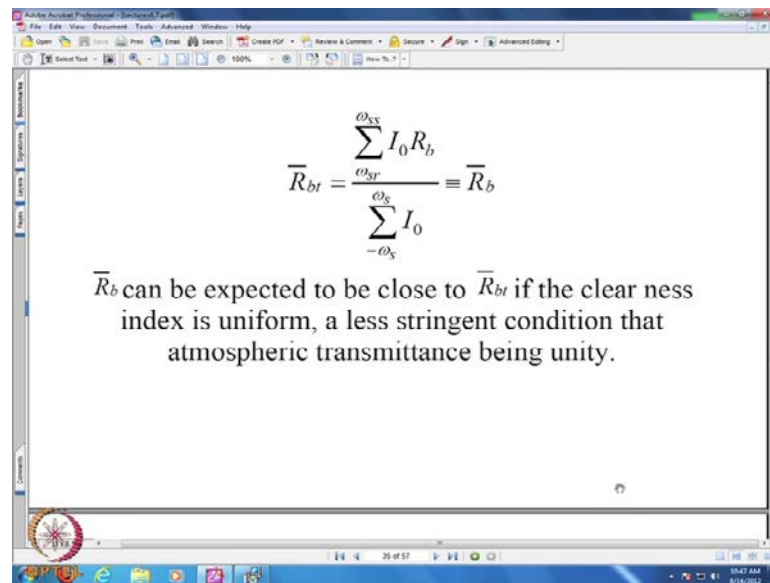
Realizing that  $(I_d/I_0) = f(k_T)$ , the last equality can be rewritten as,

$$\bar{R}_b = \frac{\omega_s \sum F(k_T) I_0 R_b}{\sum F(k_T) I_0}$$

If  $k_T$  is uniform (not necessarily equal to unity) the above equation reduces to,

Now according to Orgil and Holland's correlation,  $I_d/I_0$  is dominantly a function of clearness index. So, consequently  $k_T$  is any way the clearness index. So, I can write the above equation as  $\bar{R}_b$ ; simply some function of  $k_T$  times  $I_0 R_b$  by some function of  $k_T$  into  $I_0$ ; that means, it is converted into the extra terrestrial calculation augmented or attenuated or changed, modified, according to some distribution function which depends upon the clearness index. Now, this is important note; if  $k_T$  is uniform, not necessarily equal to unity, the above equation reduces to  $I_0 R_b$  because  $F$  of  $k_T$  is constant, if  $k_T$  is constant; not necessarily equal to 1.

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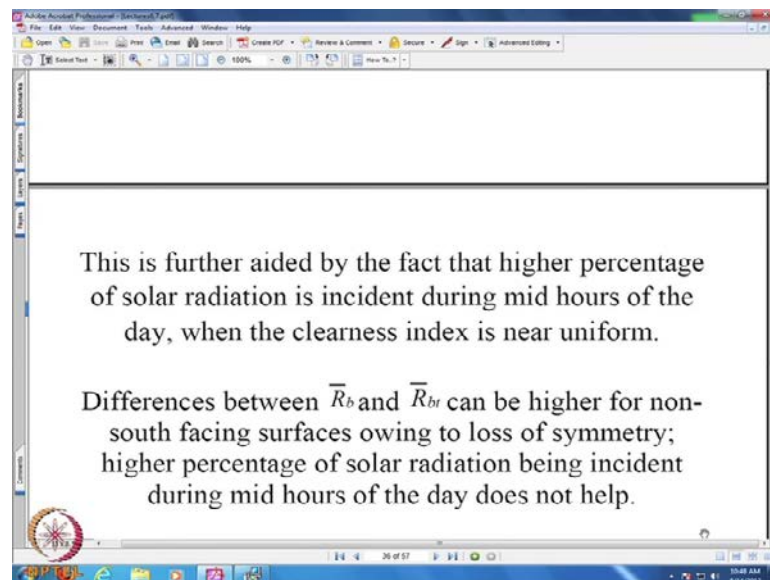
The screenshot shows a presentation slide with the following content:

$$\bar{R}_{bt} = \frac{\sum_{\omega_{sT}} I_0 R_b}{\sum_{-\omega_s} I_0} \equiv \bar{R}_b$$

$\bar{R}_b$  can be expected to be close to  $\bar{R}_{bt}$  if the clearness index is uniform, a less stringent condition than atmospheric transmittance being unity.

So,  $\bar{R}_{bt}$  will be identically equal to  $\bar{R}_b$ , if the clearness index is uniform; not necessarily unity, but just uniform. So, this is a much less stringent condition, than saying that the atmospheric transmittance is unity or the clearness index is hundred percentage.

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The screenshot shows a presentation slide with the following text:

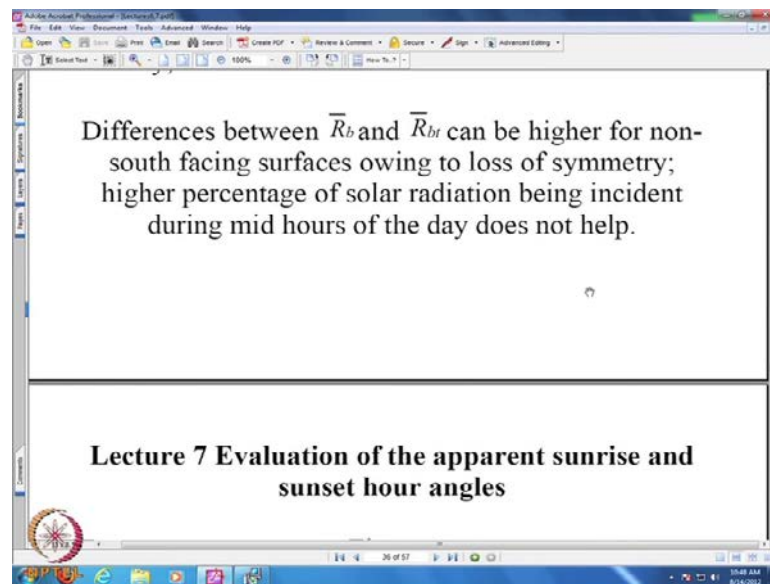
This is further aided by the fact that higher percentage of solar radiation is incident during mid hours of the day, when the clearness index is near uniform.

Differences between  $\bar{R}_b$  and  $\bar{R}_{bt}$  can be higher for non-south facing surfaces owing to loss of symmetry; higher percentage of solar radiation being incident during mid hours of the day does not help.

And, the success or why this  $\bar{R}_b$  is reasonably close to  $\bar{R}_{bt}$  is, a significant percentage of the radiations something like seventy percentage occurs during the mid-four hours and mid-five hours, where your clearness index is reasonably uniform.

Consequently, that uniform clearness indexes satisfy when the solar radiation is higher, when the angles of incidents are favorable, when the  $R_b$  is higher. Consequently, the error that may be arising from non-uniformity in the clearness index is very small, which is likely to occur at low intensity of radiation times. And this is particularly true, if the surface is in south facing.

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And if the surface is non-south facing, this symmetry is broken and you may not have the uniformity. And, people have found that there will be some significant differences; rather effaceable differences between terrestrial values and the extraterrestrial values.

Now, one word we have to say here; is it possible to evaluate  $\bar{R}_{bt}$ ? With data of course, you require a summation process. Like without summation, but we may recall that you have the correlations for  $R_t$  and  $r_d$ . They are analytical expressions, which are  $I_d$  by  $H_d$ . Let me go back. Yes, this Liu and Jordan,  $I_d$  by  $H_d$  and Collares-Pereira and Rabl was  $I$  by  $H$ . Now,  $I$  can express in principle;  $I$  minus  $I_d$  in terms of  $r_t$  into  $H$  minus  $R_d$  times  $H_d$  that will give me an analytical expression for  $I_b$ . Though these themselves are not accurate, but they are representing some sort of a distribution on the terrestrial surface, a. And b, eventually you will bring in diffuse fraction in the calculation of  $\bar{R}_{bt}$ , if you use these correlations. Right that is one part. We shall discuss in detail how to evaluate under terrestrial conditions and what are the types of errors that will be encountered or compared to terrestrial conditions. And, which you will in general find

that the errors are larger, if the surface are non-south facing and errors are smaller, when the surface are south facing.

So, this brings us to the end of lecture six. In general, we briefly reviewed at the equations so far we have derived or the empirical relations. And then, the short term calculation of solar radiation on a tilted surface has been extended to a long term like, for example, a day and that resulted in or technically, strictly speaking a summation process or an integration process. However, with certain approximation of calculating under extraterrestrial conditions, you could evaluate  $\bar{R}_b$  for the day with as a analytical expression of complexity analogous to the ... calculation; not more complicated. Except that, one thing that is not known to us and has not yet been decided determined is the apparent sunrise on sunset hour angles. In principle, it is only you set theta equal to pi by 2 in the transcended equation; the cos theta equal to a plus b cos omega plus c sin omega and solve it.

We have talked about estimating the conversion factor from solar radiation in a horizontal surface to any tilted surface on a daily basis. The calculations with a suitable approximation, we could do it with one single multiplication factor to the solar radiation in a horizontal surface without calling for hour by hour calculations. Now, can we do something similar to estimate the monthly average daily radiation on tilted surfaces?

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MONTHLY AVERAGE DAILY

$$H_T = H \bar{R} = H_b \bar{R}_b + H_d (1 + \cos \beta) / 2 + P_g H (1 - \cos \beta) / 2$$

$$\bar{R} = \bar{R}_b \left(1 - \frac{H_d}{H}\right) + \frac{H_d}{H} (1 + \cos \beta) / 2 + P_g [1 - \cos \beta] / 2$$

$$\sum_{i=1}^N H_{Ti} = \sum_{i=1}^N H_{bi} R_{bi} + \dots$$

The image shows a whiteboard with handwritten mathematical equations. The title is 'MONTHLY AVERAGE DAILY'. The first equation is  $H_T = H \bar{R} = H_b \bar{R}_b + H_d (1 + \cos \beta) / 2 + P_g H (1 - \cos \beta) / 2$ . The second equation is  $\bar{R} = \bar{R}_b \left(1 - \frac{H_d}{H}\right) + \frac{H_d}{H} (1 + \cos \beta) / 2 + P_g [1 - \cos \beta] / 2$ . The third equation is  $\sum_{i=1}^N H_{Ti} = \sum_{i=1}^N H_{bi} R_{bi} + \dots$ . There is an NPTEL logo in the bottom left corner.

So, if you recall the tilted radiation on a surface, average day is given by this equation; where  $H_b$  is the beam component of the radiation and  $H_d$  the diffuse component of the radiation and  $\rho_g$  the ground reflectivity,  $\beta$  is the tilt of the surface. So, the overall factor is given by... So, this consists of the beam radiation component  $\bar{R}_b$  and this sky diffuse component and ground reflected component. Now, if you sum it up over all the days in the month  $i$  varying from 1 to  $N$ , you will get several terms  $H_{bi} R_{bi}$ ;  $i$  is equal 1 to  $N$  plus soon and so forth.

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$$\bar{H}_T = \bar{H}_b \bar{R}_{bm} + \left[ \frac{1 + \cos \beta}{2} \right] \bar{H}_d$$

$$+ \rho_g \frac{1 - \cos \beta}{2} \bar{H}_d$$

$$\bar{H}_T = \frac{1}{N} \sum_{i=1}^N H_{Ti}$$

$$\bar{R}_{bm} = \frac{\sum_{i=1}^N H_{bi} \bar{R}_{bi}}{\sum_{i=1}^N H_{bi}}$$

Suppose, I write this equation in a compact form; that is, I divide throughout with the number of days in the month, then to distinguish between the daily tilt factor and the monthly average daily tilt factor, I use the symbol  $\bar{R}_b m$ ; where  $\bar{H}$  is the global radiation or the total radiation over the day. You will find the corresponding averages, for example,  $\bar{H}_T$  is defined as  $\frac{1}{N} \sum_{i=1}^N H_{Ti}$ ;  $i$  is equal to 1 to  $N$ , where  $N$  is a number of days in the month. So, consequently we define the monthly average tilt factor indicated by  $\bar{R}_b m$ ; should be a weighted average of all the day, values of the direct radiation by the total for the month of the direct radiations. So,  $i$  is equal to 1 to  $N$ .

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And the monthly average daily tilt factor for direct radiation is defined  $\bar{R}_{bm}$  as,

$$\bar{R}_{bm} = \frac{\sum_{i=1}^N H_{bi} \bar{R}_{bi}}{\sum_{i=1}^N H_{bi}}$$

Expressing the monthly average daily radiation received by the tilted surface as,

Again we have here, last certain simplicity. You need thirty calculations, even if you had already approximated each day's value of  $\bar{R}_{bi}$  by calculating through suitable approximation, which we discussed earlier.

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$\bar{H}_T = \bar{R}_{month} \bar{H}$

$\bar{R}_{bm} \rightarrow$  Weighted Avg.

$\bar{R}_{bm} = \bar{R}_b \Big|_{\delta = \delta_m}$

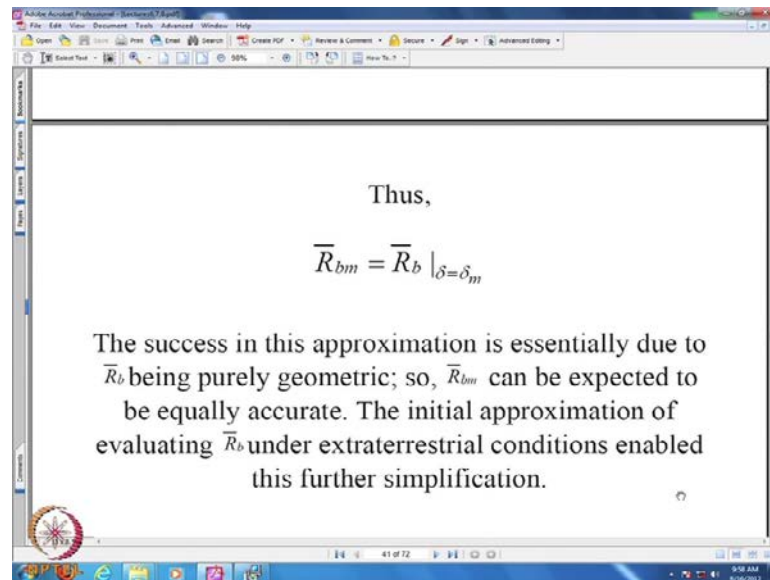
$\bar{R}_{bm}$  close to  $\bar{R}_b \Big|_{\delta = \delta_m}$

$\bar{R}$  on a day with  $\delta = \delta_m$

So, the overall tilt factor can be expressed as  $\bar{H}_T$  into  $\bar{R}_{bm}$  into  $\bar{H}$ . That is, basically we are trying to estimate the monthly average daily radiation received by a tilted surface in terms of a single factor, multiplied by the monthly average daily horizontal radiation. Exactly analogous to the way, we have done for the month.

However, we noticed that  $\bar{R}_b$  for the month is a weighted average. Fortunately, this  $\bar{R}_b$  can be calculated just like  $\bar{R}_b$  with  $\delta$  is equal to  $\delta_m$ . That is we have already earlier defined the mean declination for the month is the one with the closest extraterrestrial radiation equal to the average for the month.

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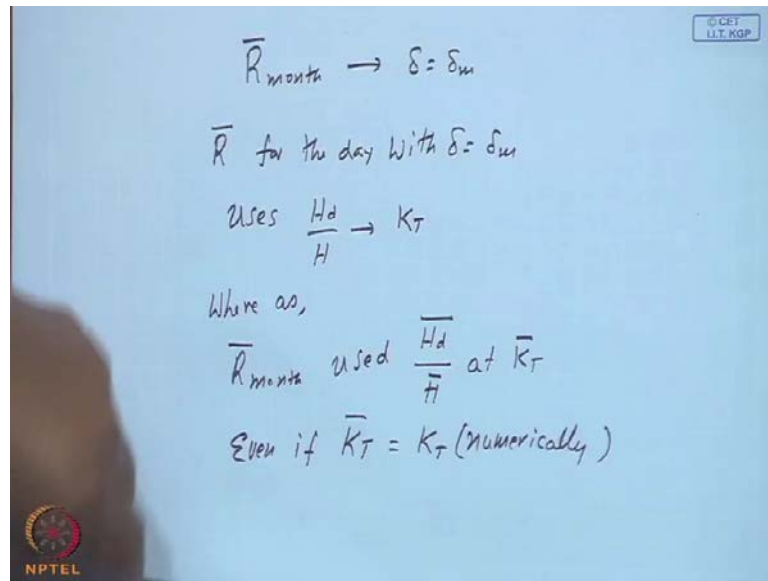
Thus,

$$\bar{R}_{bm} = \bar{R}_b |_{\delta=\delta_m}$$

The success in this approximation is essentially due to  $\bar{R}_b$  being purely geometric; so,  $\bar{R}_{bm}$  can be expected to be equally accurate. The initial approximation of evaluating  $\bar{R}_b$  under extraterrestrial conditions enabled this further simplification.

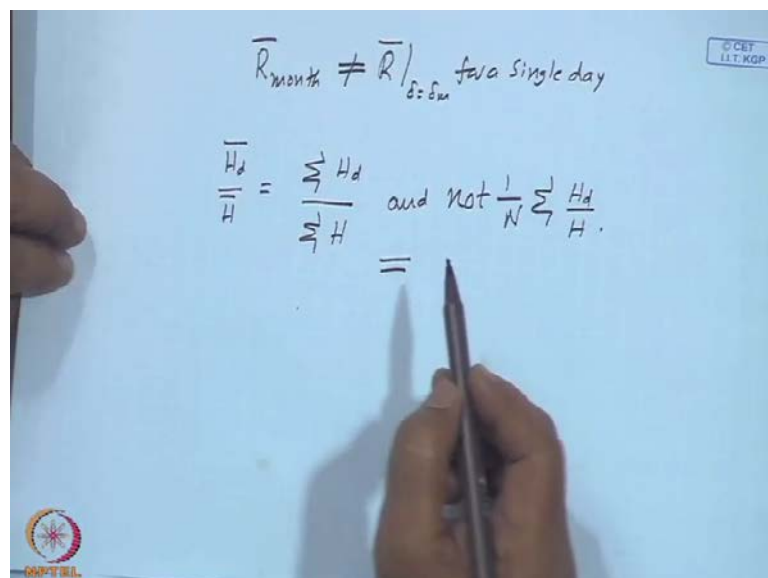
Now, this  $\bar{R}_b$  is close to  $\bar{R}_b$  be revaluated with  $\delta$  equal to  $\delta_m$ , which has been verified with the data of thirty or thirty one days. And you, in other words accentually is sum of the earlier relation, get  $\bar{R}_b$  and compare with the value that you obtained as if it is a single day, but with declination being  $\delta_m$ .

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Now, you have to notice what is the difference between  $\bar{R}$  on a day with delta equal to delta m and  $\bar{R}$  for the month; here also delta equal to delta m. The difference is  $\bar{R}$  for the day with delta equal to delta m uses  $H_d$  by  $H$ ; that is the daily diffuse fraction corresponding to daily clearness index  $K_T$ . Whereas,  $\bar{R}$  month uses  $\bar{H}_d$  by  $\bar{H}$  at  $\bar{K}_T$ ; that is the monthly average daily clearness index used calculating the monthly average diffuse fraction.

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So even if you have numerically, let us say this  $\bar{R}$  month is not equal to  $\bar{R}$  bar at delta equal to delta m for a single day. The reason is,  $\bar{H}_d$  by  $\bar{H}$  is equal to summation of  $H_d$  by summation of  $H$ , and not  $1$  by  $N$  summation of  $H_d$  by  $H$ . It is a total diffuse radiation in the month by the total global radiation over the month, not the average of the diffuse fractions of the each day. Even if it fortunately coincides, there is no need for, if numerically equal  $K_T$  and  $\bar{K}_T$  values are there,  $\bar{H}_d$  by  $\bar{H}$  will not be equal to  $\bar{H}_d$  bar by  $\bar{H}$  bar. This is sort of a mathematical reason. Physical reason is, if there is a clearness index 0.5 for a particular day and if there is a monthly average clearness index of 0.5, this comprises of diffuse fractions varying depending upon the individual day's clearness indices. They do not vary linearly with  $K_T$ . Consequently, the average shall not be corresponding to the single day's value at the average clearness index numerically equal.

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Note the difference between  $\bar{R}$  for the day with  $\delta = \delta_m$  and  $\bar{R}_m$

$\bar{R}$  for the day with  $\delta = \delta_m$  is calculated with  $\frac{H_d}{H}$ , the diffuse fraction for the day.

$\bar{R}_m$  for the month is calculated with  $\frac{\bar{H}_d}{\bar{H}}$  the monthly average diffuse fraction

$\frac{H_d}{H} \neq \frac{\bar{H}_d}{\bar{H}}$  even if  $K_T = \bar{K}_T$  numerically

So in common substance, we can calculate the monthly average daily tilt factor. Also, as if it is a single day's value using mean declination of the month and the monthly average clearness index and the corresponding diffuse fraction.

So in summary, we have a simple formula which you have already obtained under extraterrestrial conditions. We realized why  $\bar{R}_b$  evaluated under extraterrestrial conditions is accurate enough at least for south facing surfaces, compared to the terrestrial calculations. And then, the success in using the same formula for the monthly

average daily is essentially taking care of the non-linearity in delta variation from day number one, today number thirty one in a particular month which is around three to four degrees. So, the functions cause delta or sine delta do not vary all that much in this span of four to five degrees declination.