Solar Energy Technology

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Lecture - 08

Estimation of Daily/monthly Average Daily Tilt Factor Under Terrestrial Conditions

So, we made the assumption that the tilt factor for the day for the direct radiation can be calculated under extra terrestrial conditions and the numerical values agree very well with the data values almost; particularly for south facing surfaces. Now, given the symmetry not being there; for example, when the surface does not face south, you will have to evaluate what shall be the kind of difference one will encounter if it had been made under terrestrial conditions. Either you use the data or now we have the correlations due to Liu and Jordan and Collares-Pereira and Rabl to express Id and I. And, hence one can carry on either numerical integration or analytical integration, at least to evaluate the order of magnitude of errors that are likely to be encountered had the terrestrial values been used.

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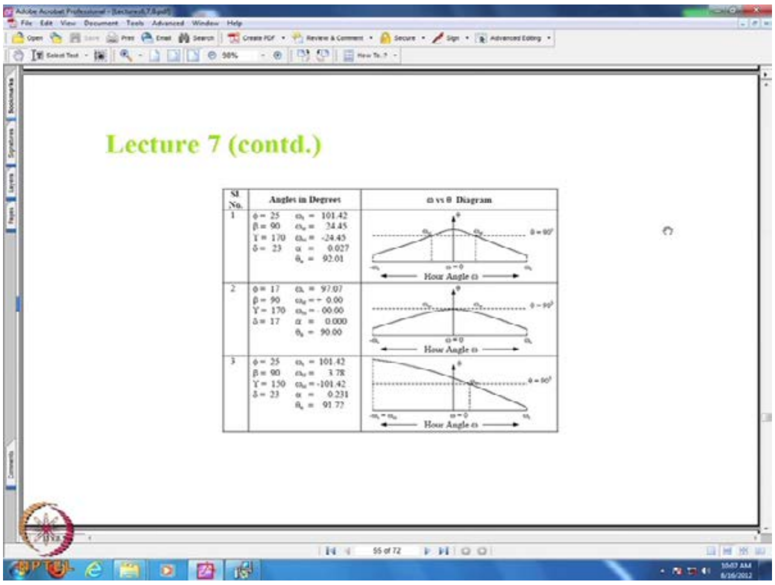

This is important, particularly when the uniform clearness index condition is broken, which shall be coupled with the added difficulty, in case if the surface is not facing south due to some sort of asymmetric distribution around solar noon time.

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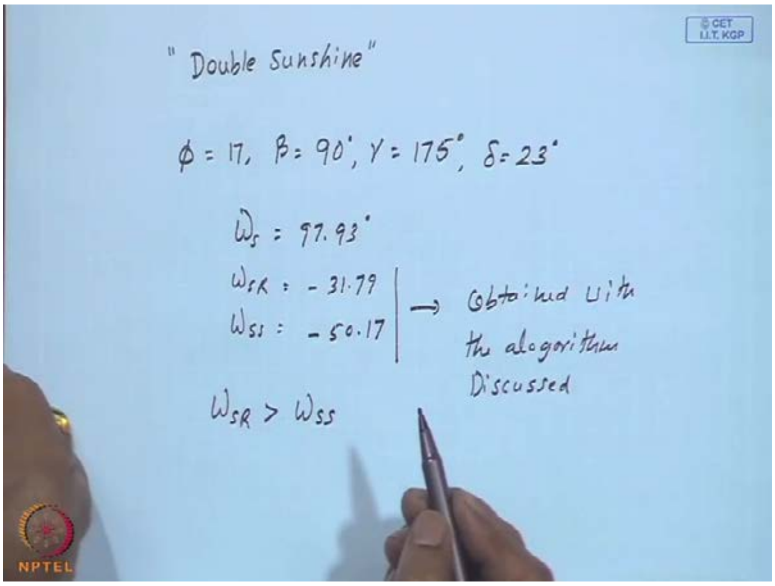

Before we go into that evaluation, last time we were talking about the cases of so called double sunshine. That is, the surface receives sometime during the day solar radiation with a no sunshine period in between and again receives sunshine for certain period. There can be n number of degenerate cases; both being only in the afternoon; both being only in the fore noon or distributed over forenoon and afternoon. We first identified a large number of cases where there is only single sunshine and I think the cases of one, two, three, where there is double sunshine, we have already discussed.

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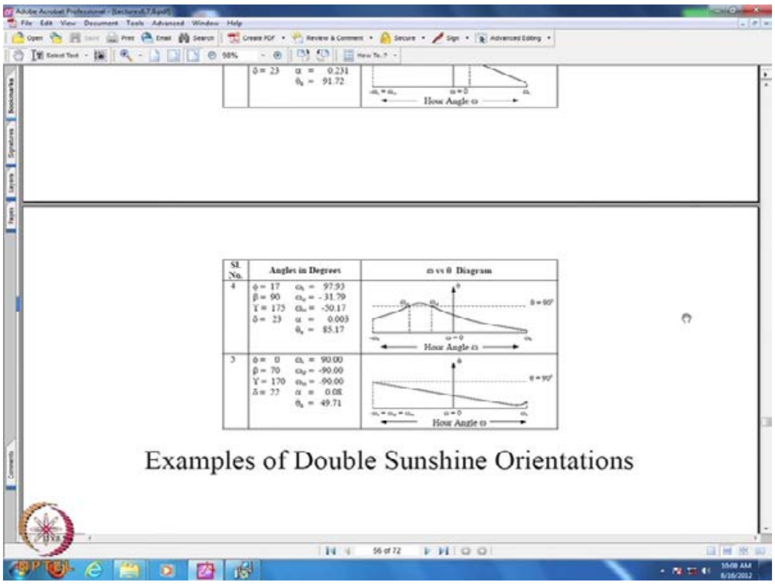

And, case four is having a phi latitude of 17 degrees; beta of 90 degrees with a gamma of 175 degrees and declination being 23 degrees. This is… something like that. So, you have a sunset hour angle of 97.93 degrees and again omega S R and omega S S; omega S R is minus 31.79 and omega S S is minus 50.17. Now, what is interesting is these are the numbers obtained with the algorithm, which we had discussed. That is, first you determine the magnitudes by using the cosine inverse, then determine the sine by using this sine root. Consequently, here it satisfies the condition mathematically. Omega S R is greater than omega S S; minus 31.79 is mathematically more than minus 50.17.

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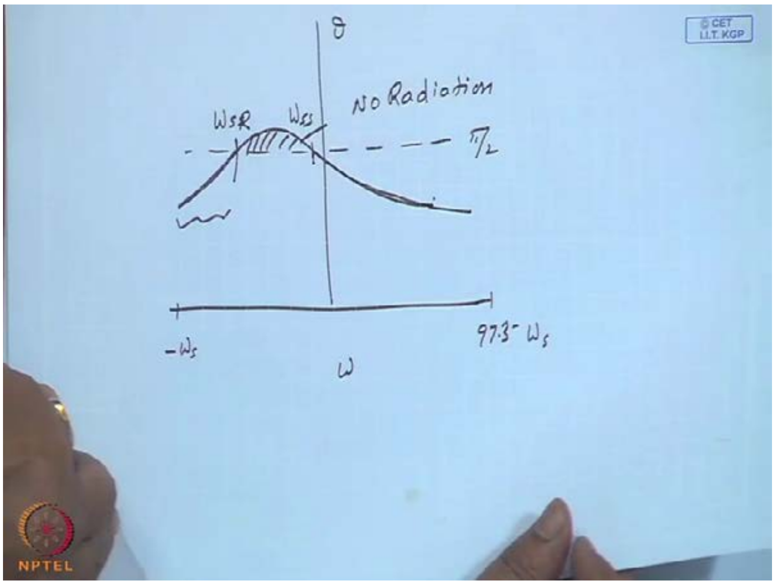

So, you have the theta verses omega curve. This is pi by 2; this is theta axis; omega; this may be your minus omega s which is 97.93 and this is plus omega s. So, now you have got something like this. So, this is your omega S S and this is… S R is minus 31. So, omega S R is greater than omega S S, because of the negative sign. And, actually if you have plotted you will have this theta less than pi by 2 and here theta is less than pi by 2, defining the surface should receive the radiation from the sunrise up to this sunrise omega S R and again from omega S S up to whatever is the physical sunset. So, this is the part; no radiation on the surface. That is where you do the piecewise integration minus omega s to omega S R and omega S S to omega s.

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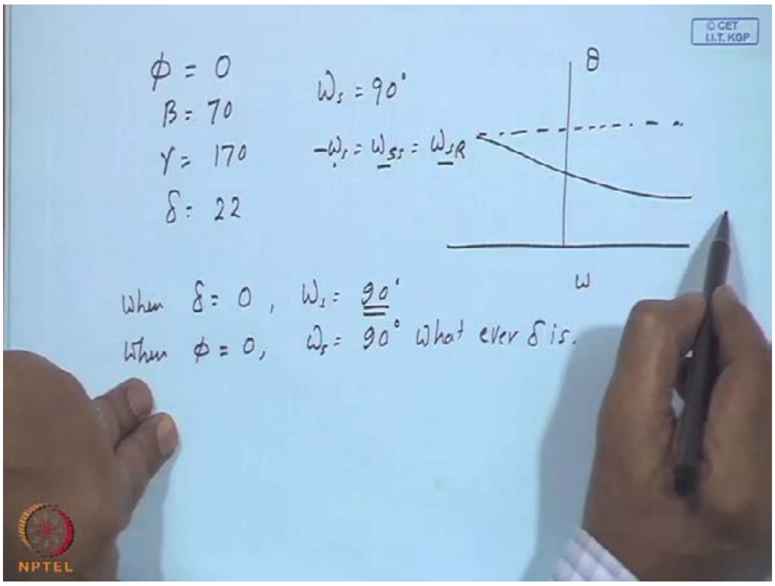

The last one is a peculiar situation. You have equator phi is equal to 0 and beta of 70, gamma of 170, that means almost towards north, and declination of 22. Now, first of all you can see this phi and delta 22 is phi. So, you have omega s 90 degrees. I guess we had mentioned, wherever you are on the earth on equinoxial day, that is, when delta is equal to 0 your omega s is 90 and phi is equal to 0; omega is equal to 90, whatever may be delta. Declination, you have the sunset hour angle to be 90 degrees on the equator and on the equinoxial day, whatever is a latitude you have got sunset hour angle to be 90 degrees.

So, this will have; you have minus omega s equal to omega S S equal to omega S R. right. You can just calculate. This turns out to be equal to minus omega s by limiting omega S S and omega S R to the physical sunrise and sunset hour angles. So, if you plot theta versus omega, we will have the satisfying like this. So, it looks like it is a single sunshine correctly, right, but it is because of omega S S and omega S R coinciding with minus omega s. And, this one has to calculate it and convince yourself. Basically what it means is, here really my theta will be less than pi by 2, but sunset has come about and consequently again it starts from seeing the same time. Right. The, one should exactly take these values and calculate theta and convince yourself that, that is a figure.

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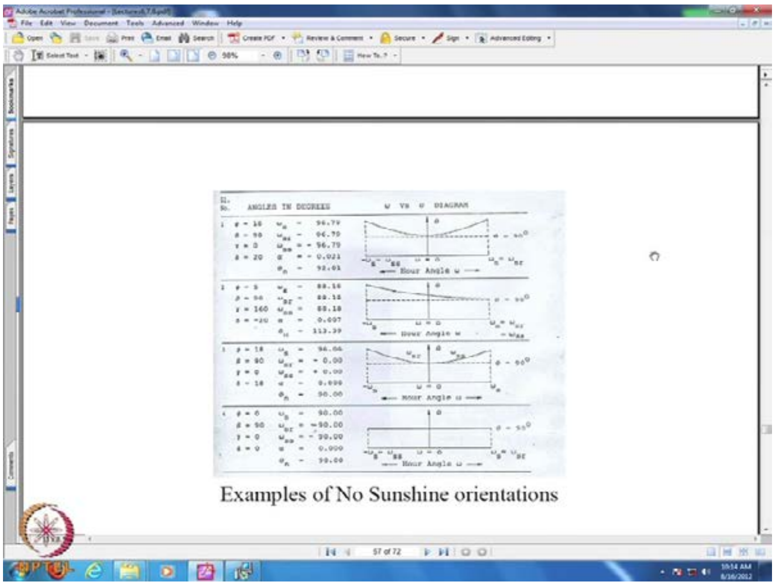

These are examples of no sunshine and under different cases.

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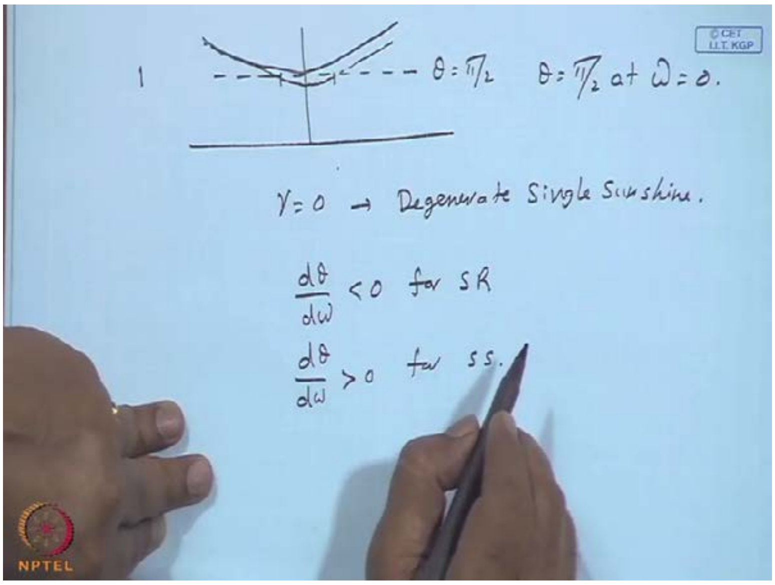

So, for example, first one which has got a figure like this; so, theta is equal to pi by 2 at omega is equal to 0. We have stated also this is gamma equal to 0. Consequently, it is symmetric case. So, it is a degenerate single sunshine; because if gamma had been nonzero or beta is not 90, 88 or something, I might have had a curve like this, which is normal with theta being less than pi by 2 over a period within the sunrise to sunset. And, it satisfies my d theta d omega less than 0 for sunrise. Here after, I will write in short hand; S R and d theta d omega greater than 0 for sunset. Then, again here a low latitude beta 90 degrees, gamma 160 north facing, delta negative, you can see more or less the number; case number one is inverted by choosing gamma 160, almost like 0 to 180, 22 to minus 25 and 18; both of them are less than the declination magnitude wise.

So, you will have certain situation where theta is equal to pi by 2 is satisfied, only beyond omega s. right. Throughout the day it is more than pi by 2. Consequently, you set it equal to omega s. But, really omega S R is equal to omega s equal to minus omega S S, after limiting to the magnitude of omega s. So, the real calculations show 88.18 is omega s, omega S R and omega S S. This is after limiting.

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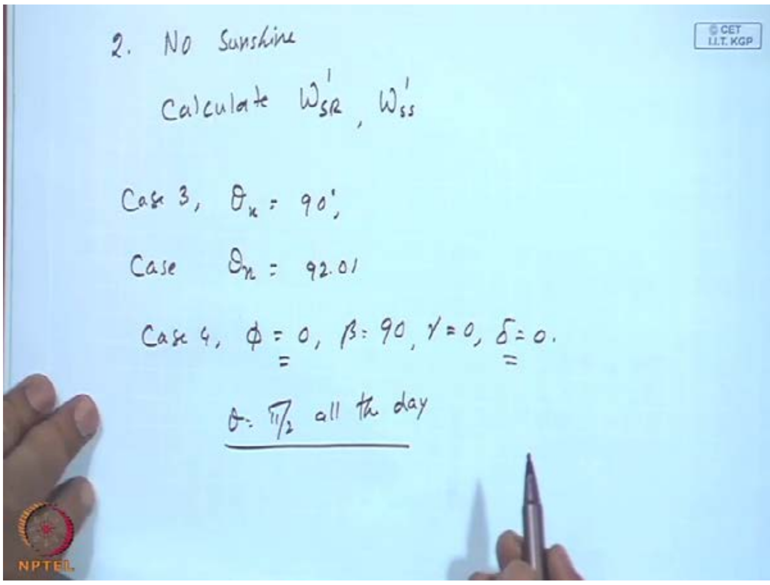

And, you can… case two; no sunshine. So, you can calculate whatever we are saying omega S R dashed and omega S S dashed. So, these numbers are given after doing this. Now, here is an interesting situation; phi 18, beta 90, gamma 0, delta 18. This exactly looks like the previous case number one. And you, ok right, in case number one… this is three. Theta at noon is exactly 90 degrees. Whereas case one, theta at noon is 92.01, graph here; by hand I could not show that. It is slightly more than 90. Does not matter; still it does not receive any radiation. As you can see, phi is equal to delta less than 23 degrees in the case number three.

So, at the noon time for a vertical surface, the sun's rays will be parallel; meaning, thereby the theta noon will be pi by 2 and other places it is more than pi by 2 and last one is phi 0, beta 90, gamma 0 and delta 0. That is the equinoxial day. So if you put it, you will get theta. Case 4; phi 0, beta 90 and gamma is equal to 0 and delta is equal to 0. So, it is a vertical surface; delta 0, phi is equal to 0. So, theta should be equal to pi by 2 at solar noon time, like in case number three. But since it is the equator, it, theta is equal to pi by 2 all the time. You can easily calculate cos phi, cos delta, cos omega. Let us see if we can get that.

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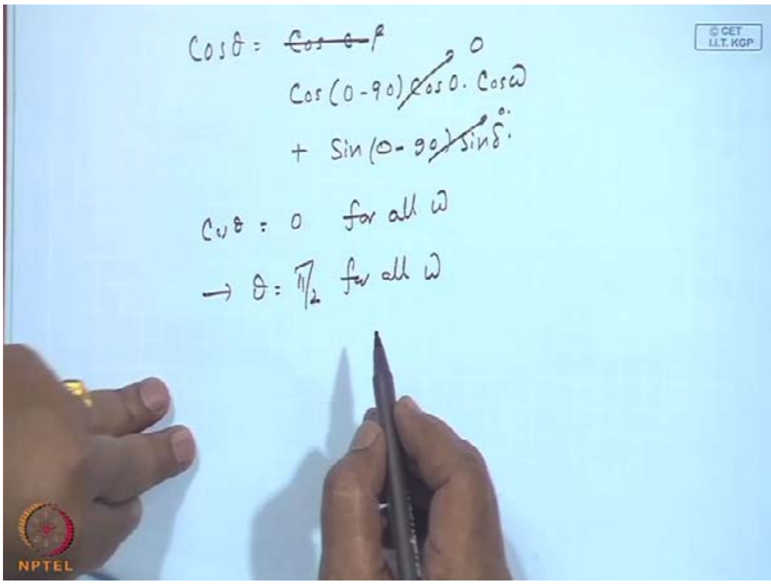

So, this is gamma 0, so cos theta is equal to cos phi minus beta; 0 minus 90, cos delta 0 cos omega plus sin 0 minus 90 sin pi sin delta equal to zero. So, this is 0 because delta is 0 and cos 90 is 0. So, cos theta is equal to 0 for all omega; this leads to theta is equal to pi by 2 for all omega. So, these are the types of situations one can encounter with the single sunshine or double sunshine, including no sunshine, which may be a degenerate case of single sunshine or double sunshine.

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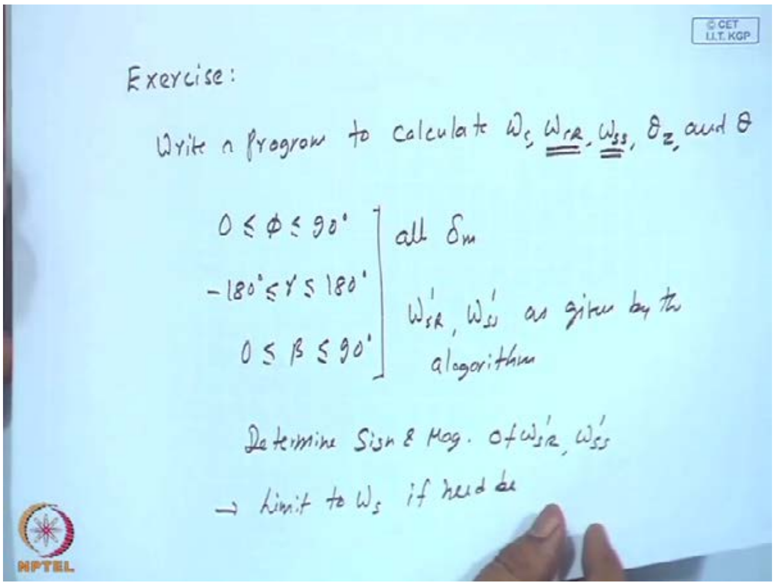

Then one can… I will leave it as an exercise. Write a program to calculate omega s, omega S R, omega S S, theta Z and theta; in the latitudes 0 less than or equal to phi less than or equal to 90 degrees and minus 180 gamma plus 180. And, you can take all twelve months of delta m, then 0 less than or equal to beta less than or equal to 90 degrees. So, in your computer output you can have those omega S R dashed and omega S S dashed as given by the algorithm. And then, determine the sign and magnitude of omega S R dashed and omega S S dashed and limit to omega s, if need be. In other words, what I am trying to point out, if you look at only the final numbers it may be a misreading and you may had got a correct answer, even if the calculation has been wrong. Suppose, if omega s is equal to 92 and you got 118 instead of 172, even then omega S R will show up as 92. So, that may be a correct answer, but may be a wrong logic in calculating either omega S R dashed or omega S S dashed. So, you can always ask it.

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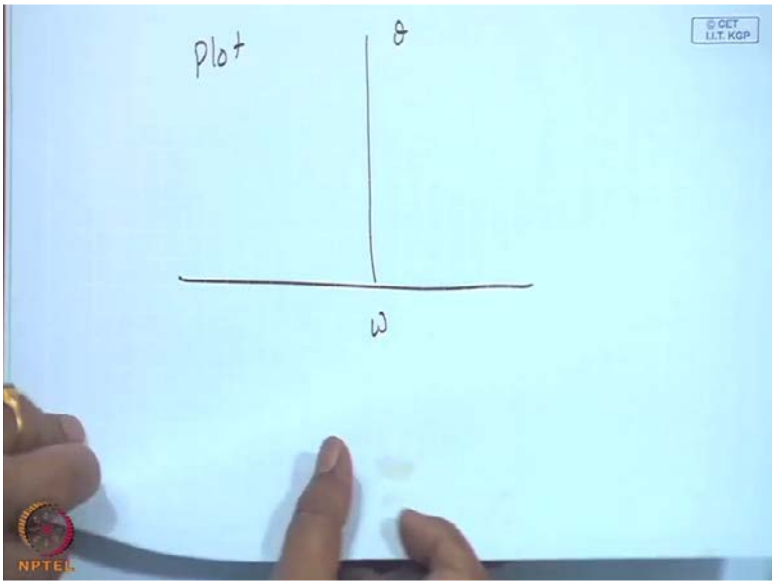

And then plot theta verses omega and if there is a double sunshine, single sunshine, the… Your values should justify the theta variation with respect to omega. So, what we have identified is that, at times calculation of R b bar without taking cognizance of whether it is a single continuous period or two periods with a no sunshine period in between, either it leads to an underestimate or may be a negative value. And, consequently some computer manipulation, like if it is negative take it as 0. And, that is one danger. The other thing is may be simple underestimate. Instead of 0.87, you may get the answer as 0.62, which looks deceptively close and correct. But, nevertheless one has to be verified in these things.

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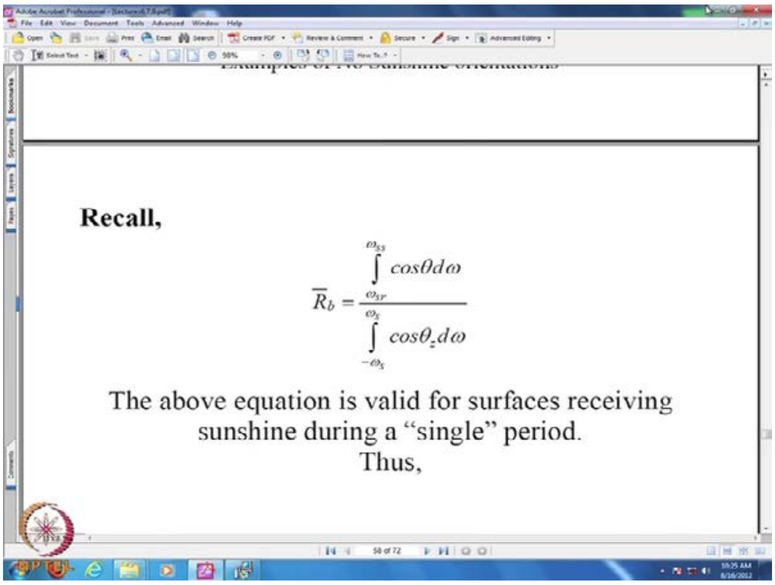

So, once again I will recall your equation for the R b bar as omega S R to omega S S cos theta d omega.

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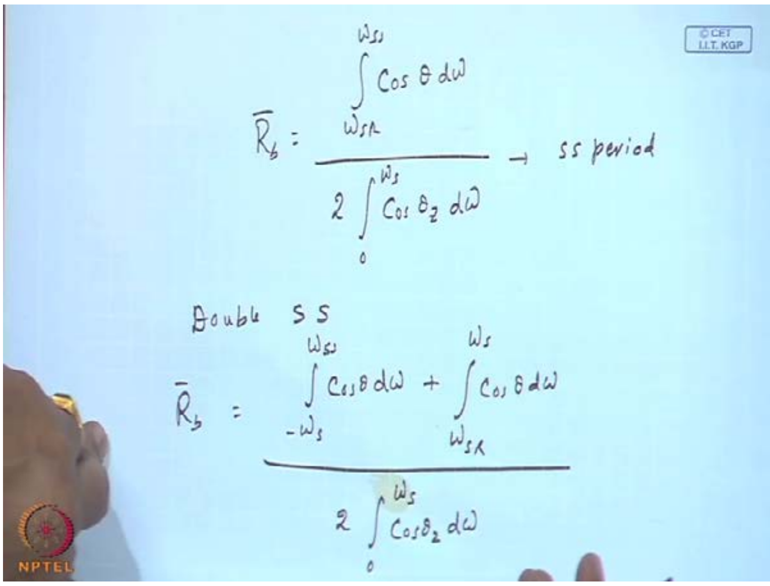

It is minus omega s to plus omega s; is twice of 0 to omega s because it is a symmetric function. This is true. We said that single sunshine period. At that point of time, we did not have this wisdom of how do you distinguish between single sunshine period and double sunshine period.

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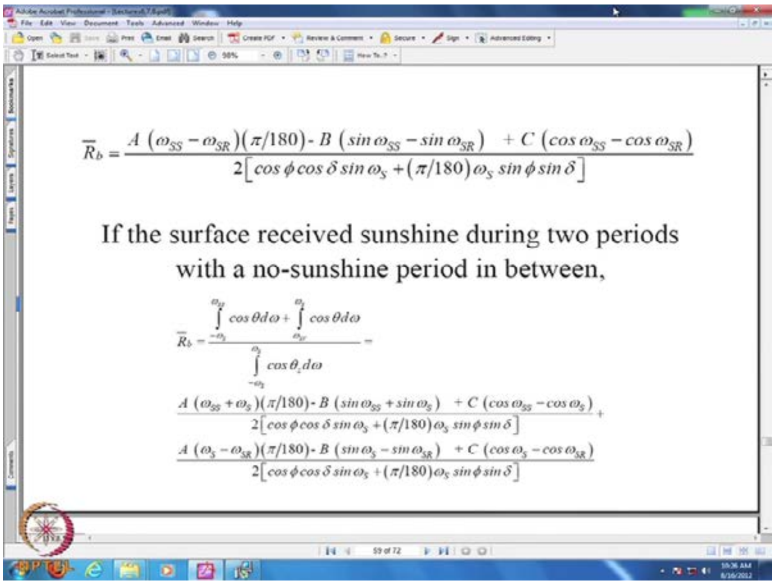

And, we generally assumed it will be from sunrise to sunset, apparent or otherwise. So, if you integrate it under integral, terrestrial, extraterrestrial conditions, this is what you will have; that A into omega S S minus omega S R. And, I am emphasized with pi by 180. If you use omega S S and omega S R in degrees, it should be converted into radiants. Now, if it receives double sunshine, R b bar should be piecewise evaluated; minus omega s to omega S S cos theta d omega plus omega S R to omega s cos theta d omega upon twice 0 to omega s of cosine theta Z d omega. It is only a formal integration. It is, just your cos theta is nothing but A plus B cos omega plus C sin omega, where A, B, C are independent of omega. So, consequently at this stage they do not bother us.

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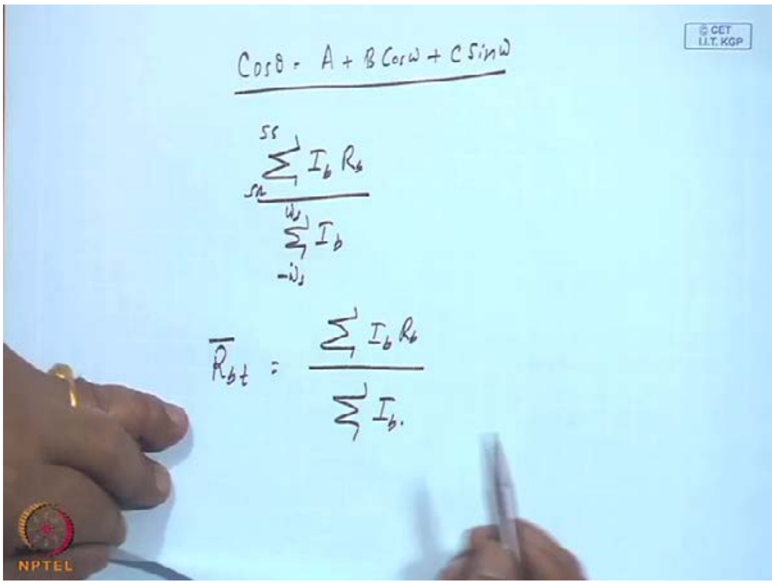

So, it gives rise to two terms; which is, first term integral is cos theta is A, that is A into omega S S, minus minus omega s will be plus omega s. Like that you have two terms. So, one can evaluate R b bar under extra terrestrial conditions, even if the surface diffuse sunshine during two periods. So far, we assumed that the evaluation can be done under extra terrestrial conditions and the accuracy had been reasonable at least for south facing surfaces by comparing with the data.

And, now we will see if we can evaluate under terrestrial conditions. Of course, if you have the data you can always do it as a summation of direct radiation from sunrise to sunset. And, this is minus omega s to plus omega s.

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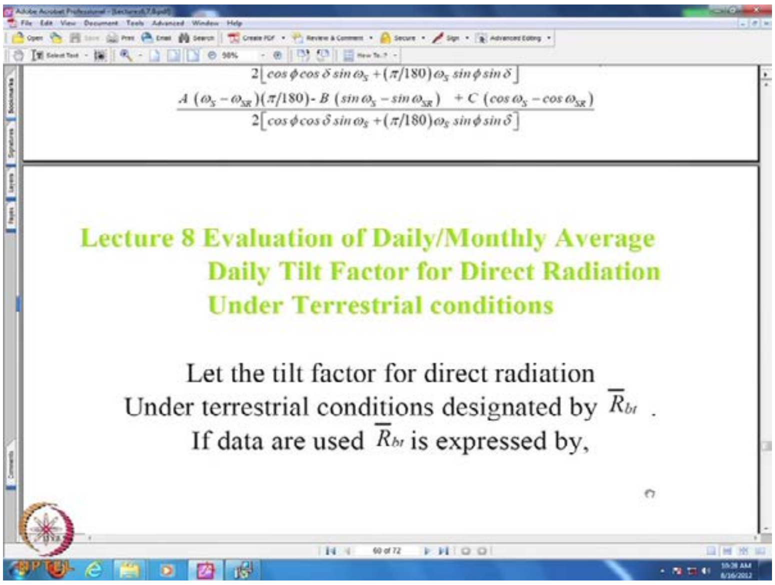

But this difficulty is calculating twelve calculations or so. And then further you need the data.

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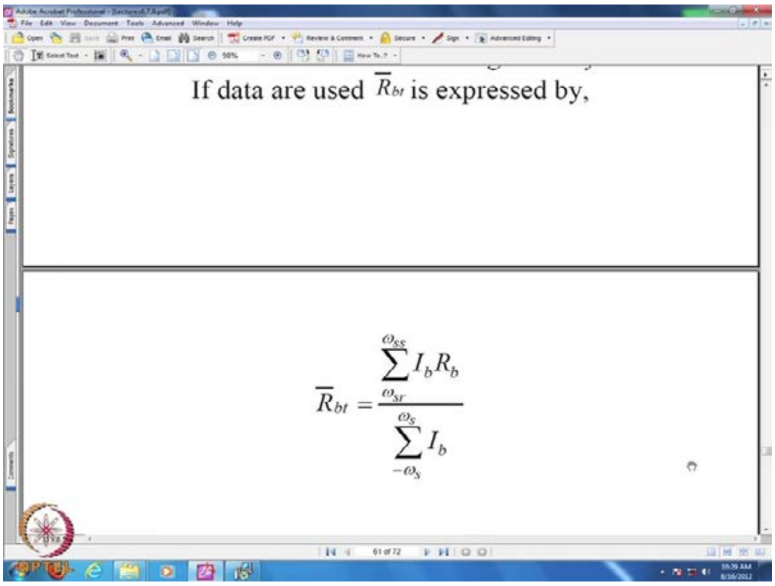

So, if I use it for the terrestrial conditions I have to, I am designating it with R b bar t which will depend upon the, of course I b R b by sigma I b. These are either data values or we will see what simplification can be made.

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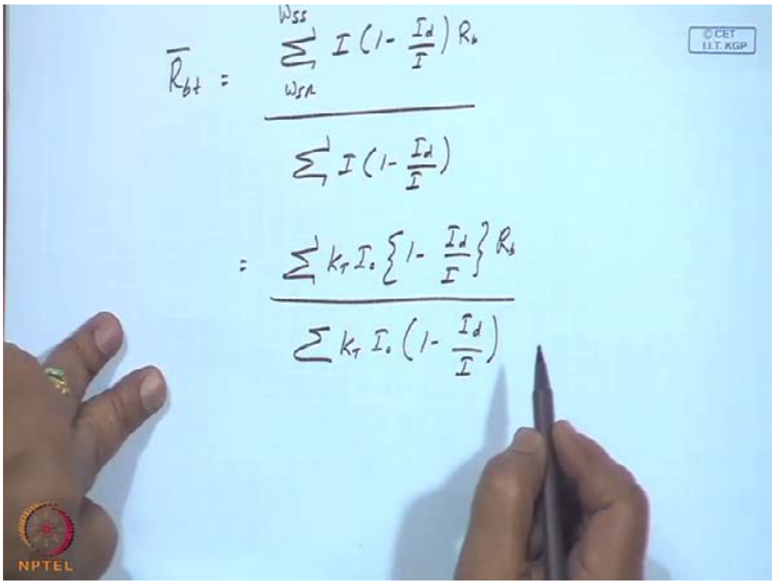

So, this R b bar t is expressed in terms of the diffused fraction. Just, I is taken outside common, so that I d by I is a known function. In fact, we have written it like this earlier, when we are trying to prove that the extra terrestrial calculation is close to the terrestrial calculations, if the daily clearness index is uniform throughout the day.

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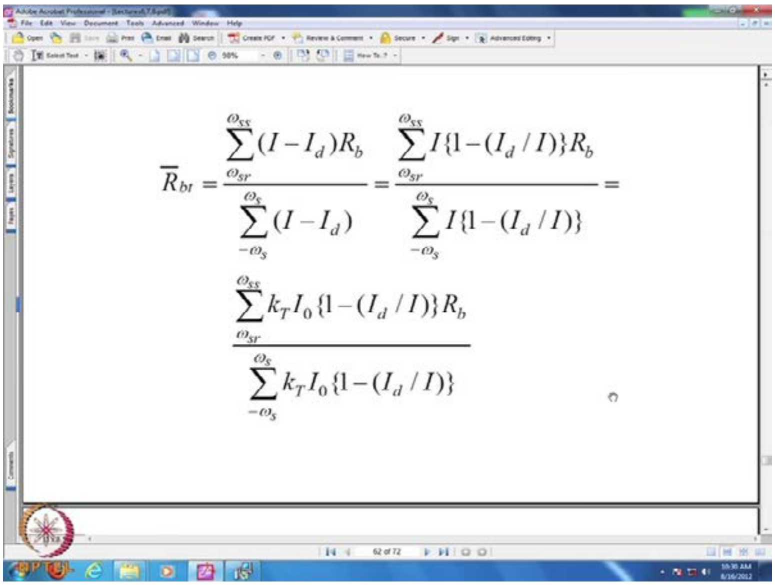

So, this can be rewritten as in terms of clearness index in the extra terrestrial radiation. It is a similar step that we had followed earlier so far.

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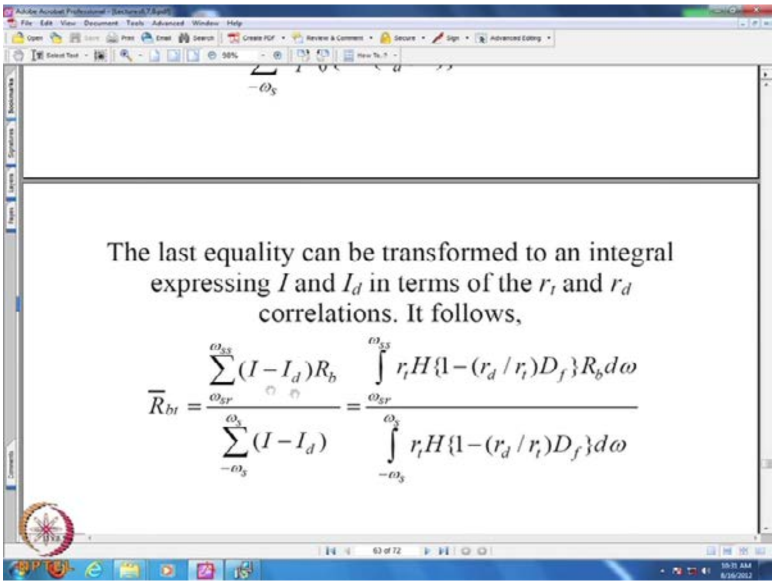

Now what I can do is, though for the sake of our understanding we first started with a summation of the hourly values, this can be rewritten as a continuous function.

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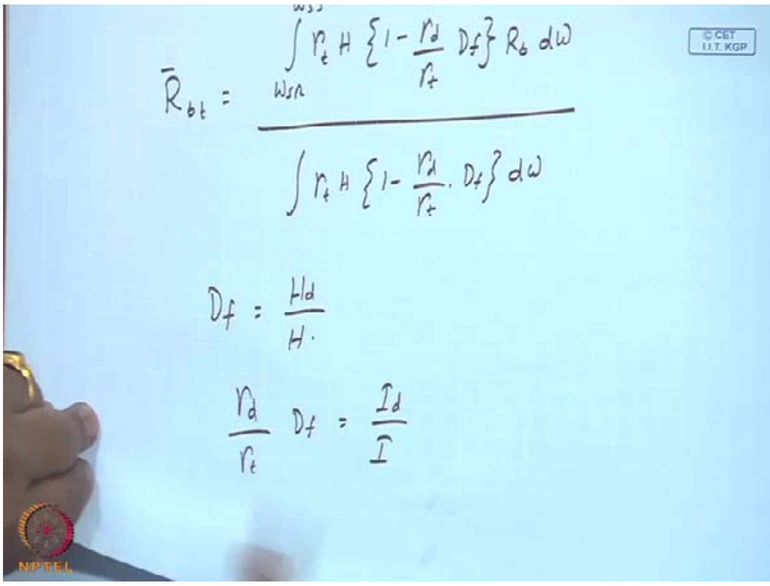

Since, we have the correlations of Collares-Pereira and Rabl and Liu and Jordan, so integrate it. I is nothing but r t in to H and this Id is r d by r t into diffuse fraction D f times R b d omega by... So, where this D f is for simplicity; H d by H. R b is I d by H d by I by H into H d by H. What I will be having is I is r t in to H. I if it is taken out, I d by I; that is what I will be having here. So, r d by r t into D f is nothing but Id by I.

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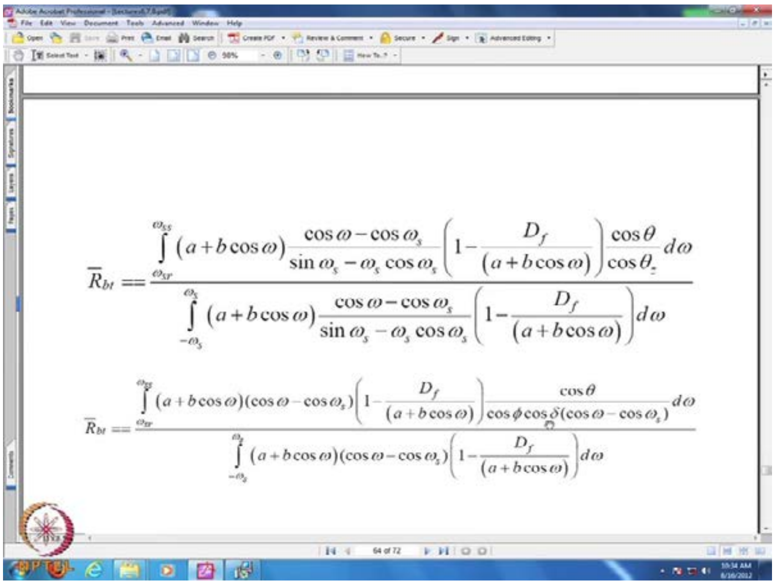

So, now this is a function, one you should have a little bold heart to look at this integration. And, r t is a plus b cos omega times cos omega minus cos omega s sin omega s minus omega s cos omega s multiplied by pi by 24. I reduced one step that gets cancelled with the denominator. And, the other thing is 1 minus I d by I; that is, r d by r t is nothing but 1 upon a plus b cos omega.

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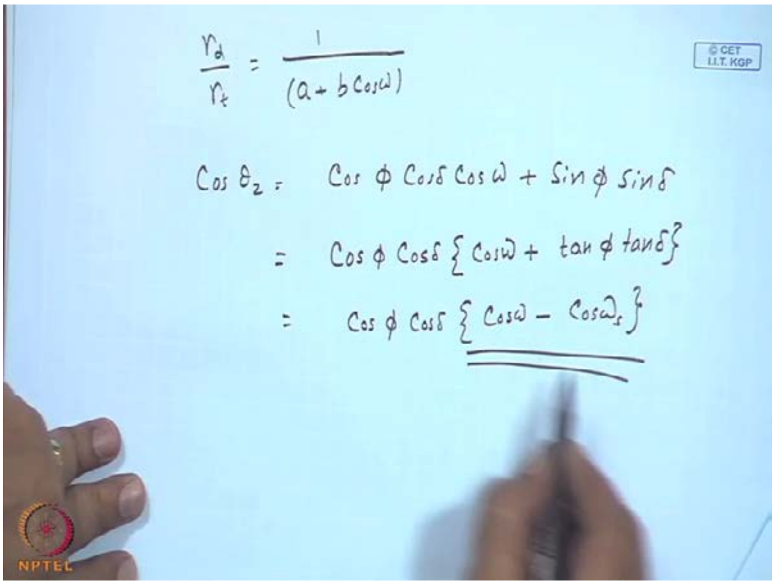

R d by r t is nothing but one by a plus b cos omega. And, you know that a and b are the two constants in the Collares-Pereira and Rabl’s correlation for r t, which have been expressed in terms of omega s. Multiplied by r b, which is cosine theta upon cos theta Z into d omega. So, the similar thing in the denominator; except this cos theta by cos theta Z is not there.

Now, this is all one of the grad students was working on this sometime back. The, why people did not do, I do not know. It is a quite simple integration, except the success lies in this sin omega s minus omega s cos omega s should be written, can be written as cos phi cos delta cos omega minus cos omega s. That is, this is cos theta Z. Let us see. Just if you take out cos phi cos delta common, this is the term which gives you the simplification; gets cancelled with this cos omega minus cos omega s.

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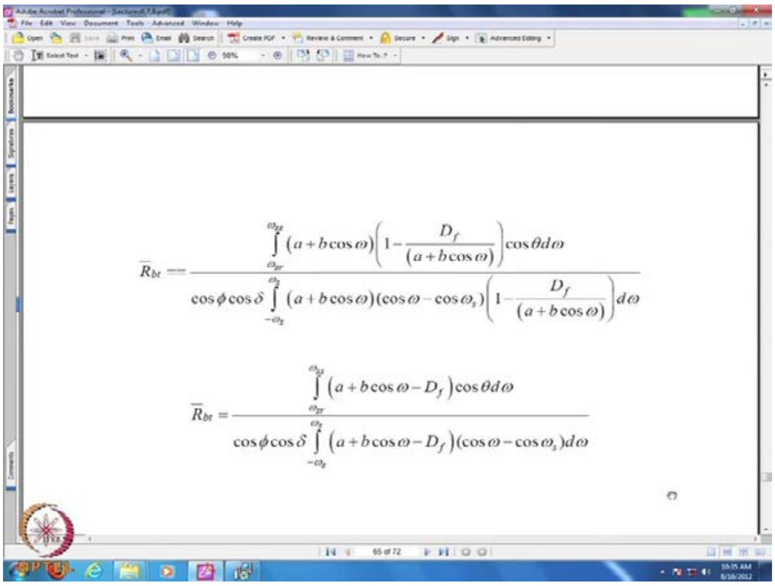

So, now this starts shrinking. It becomes a smaller integral; a plus b cos omega into 1 minus D f by a plus b cos omega into cos theta d omega. And, this cos theta Z 1 by cos phi cos delta comes to the denominator, but cos omega s minus cos omega s is cancelled with this. Then, you have got further simplification; if you multiply a plus b cos omega minus D f times, cos theta d omega upon cos phi cos delta times cos omega cos omega s.

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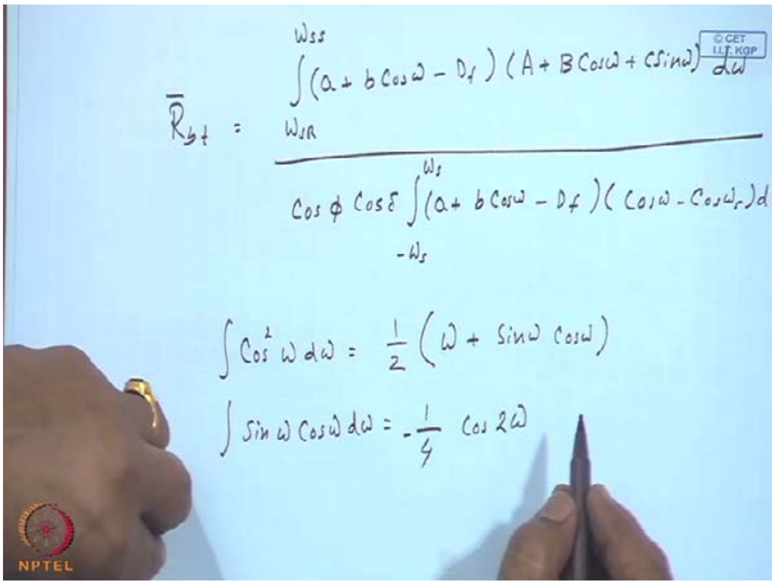

Now, if you express; so in a neat form, this goes on the c e t symbol by… Mind you, as far as we are concerned, this integration is concerned, a is a constant; b is a constant and D f is a constant, of course cos omega s is a constant.

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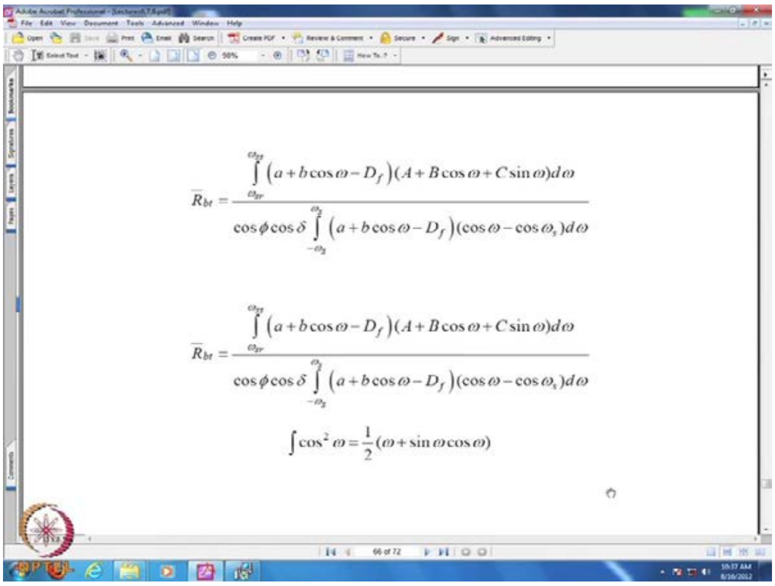

So, we have written this, again the same thing. Now, the highest order complexity is only cos squared omega. Integration will be b into this b cos omega will be giving me cos squared omega. To make you feel comfortable, cos squared omega has a simple integral. It is omega plus sin omega cos omega. The other thing somewhat less complicated is, sin omega cos omega d omega; which is half of sin 2 omega d omega is minus one-fourth of cosine 2 omega. If you want to note down, I will write down; cos squared, that d omega is missing there. And, you can have quite a good exhaustive table on the web, even Wikipedia. So, you can see other functions also.

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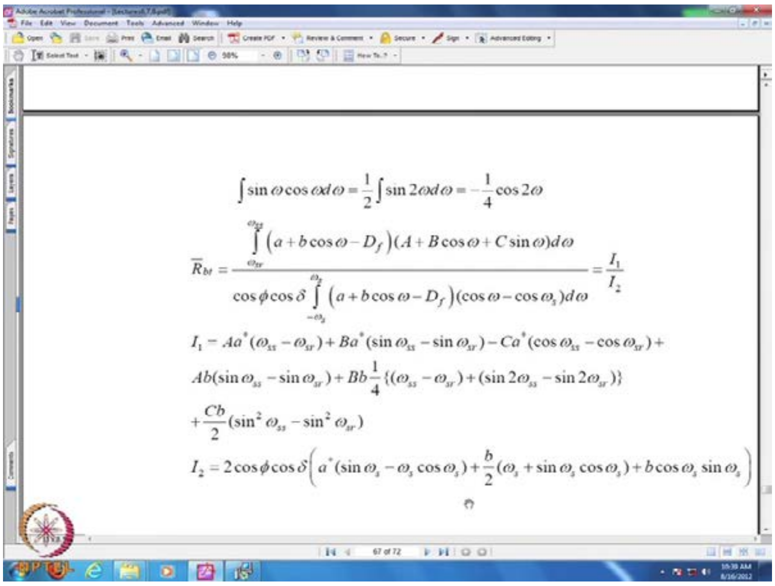

So, I have struggled. And, numerator is expressed as I 1; that is integral 1. I 1 is the numerator and I 2 is the denominator.

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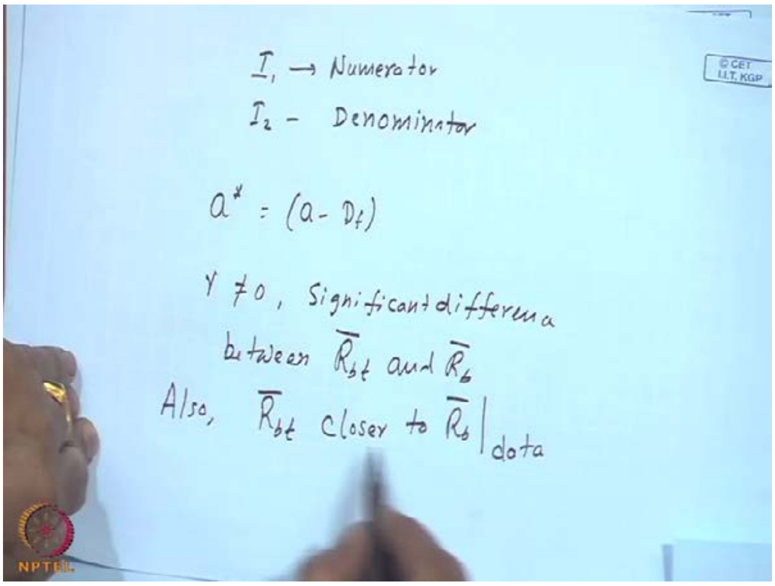

So, they are both in terms of omega S S and omega S R. You can check the signs. I tried because I was doing this bit of algebra directly on the screen and I did not have the benefit of a old documentary word to directly take it. So, this was done; that a star which is the simplification, is nothing but a minus D f. It acts like a single number. Now, when the surface receives sunshine at two time intervals, that is the so-called double sunshine, it is straight forward to write the integrals I 1 and I 2; as is been done in expressing r b bar under extra terrestrial conditions. I mean, I did not want to write another one year long relation for double sunshine periods.

So, this gives a difference of about good ten to fifteen percent compared to the extra terrestrial calculations. Particularly, when gamma is not equal to 0 significant differences exists. Also, a fortunate does the redeeming feature; is R b bar t, which we have analytically evaluated is always almost closer to R b bar data.

So, you take a location of Kharagpur, Calcutta, whatever and use the data hour by hour sum up I b R b upon I b, then calculate R b bar t with this formula. That will be closer to the data value compared to R b bar, which we have earlier calculated under extra terrestrial conditions. So in other words, the Collares-Pereira, Rabl and Liu and Jordan, though they are analytical representations, captured the somewhat the realistic daily variation. And, the success is not so much because of the distribution. But, it contains the term D f, difference fraction. Somehow, we could bring in that any amount of radiation is not the same. What is the diffuse fraction? Consequently, it will be affecting your tilt factor; how much is the directly radiation received by the surface.

So, this is because if you look at it, a plus b cos omega in to some term and that cos omega minus cos omega s, they are almost I 0 by H 0 with a modification by a plus b cos omega, which is also a geometric factor; nothing to do with the distribution of the data.

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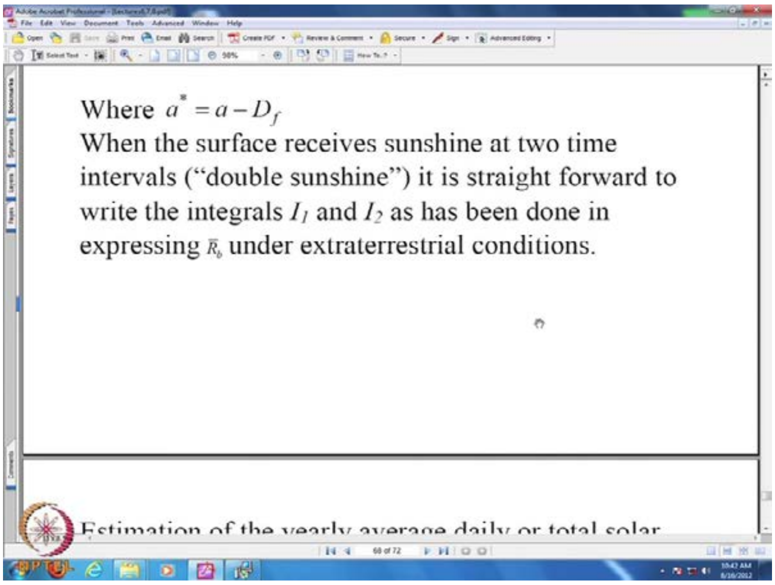

Now, however this D f that comes into the picture will attenuate or enhance R b bar evaluated under terrestrial conditions. Compare it to the extra terrestrial value.

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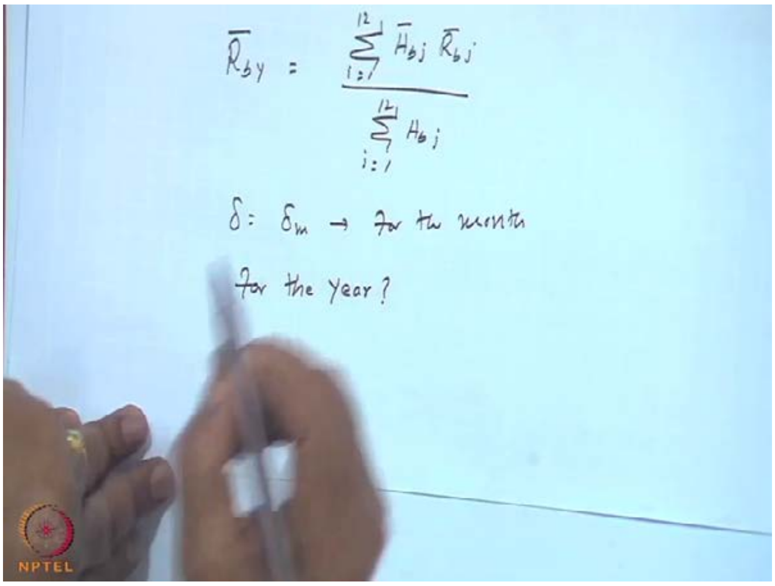

Now, somebody can be little greedy. Can we estimate for the year? In other words, we can always estimate given the data and given a computer and make 365 into 12 or even technically 24 hours calculations or 365 daily calculations or at least 12 monthly calculations.

So, my R b bar y for the year should be H b bar j R b bar j by H b bar j. This is j equal to 1 to 12 for the 12 months. Just like we have extended I b R b by I b for R b bar and H b R b by H b for R b bar month. So for the year, I am satisfied with the monthly average values or one can construct in terms of the right up to the hourly value.

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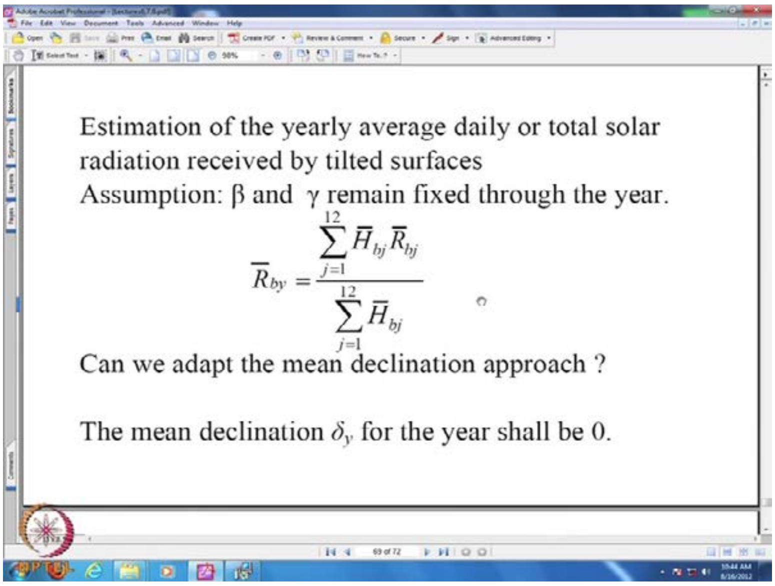

Now we, can we be encouraged? We used delta equal to delta m for the month.

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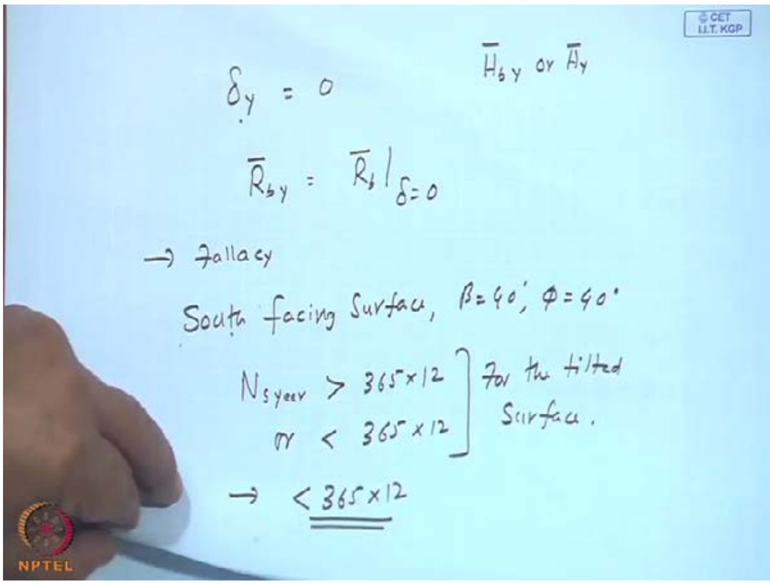

For the year if I try to do that a simple minded, delta y will be 0 because it changes from minus 23 to plus 23. I take the average and I will get a 0; seems to be simple. And in other words, I am given; let us say H b bar y or H y bar from which I may be able to calculate diffuse radiation and then the direct radiation.

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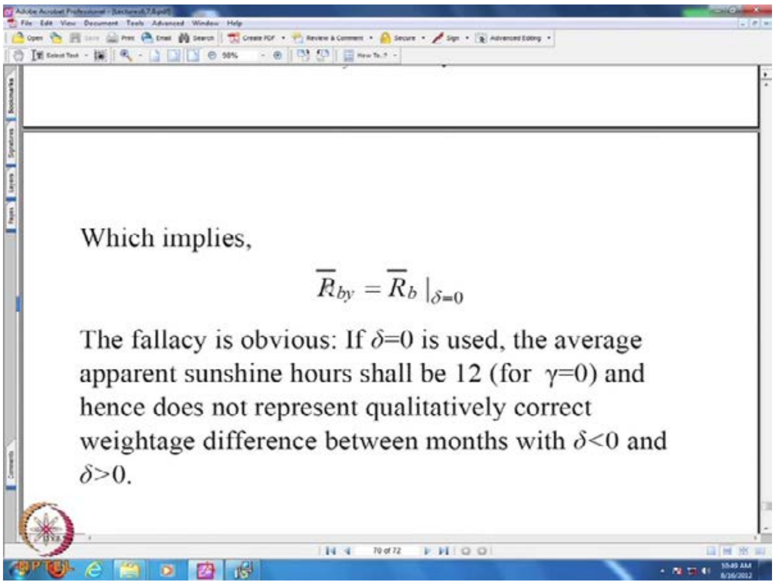

So, this means I am trying to do this. As if it is the single day, I will calculate R b bar with delta equal to 0. Now, one will readily say the fallacy in this. What do you expect the… suppose a simple south facing surface, right, and some beta 40 degrees, latitude 40 degrees or you can have anything; does not matter. I do not want to go to 66 beyond or less than 23, but any mid range latitude and the corresponding slope. Can you expect the number of sunshine hours will be more than 12 into 365 or less than 12 into 365 for the tilted surface? So, let us say N s year greater than 365 into 12 or less than 365 into 12 for the surface trigonometry. We can readily say it should be less than 365 into 12.

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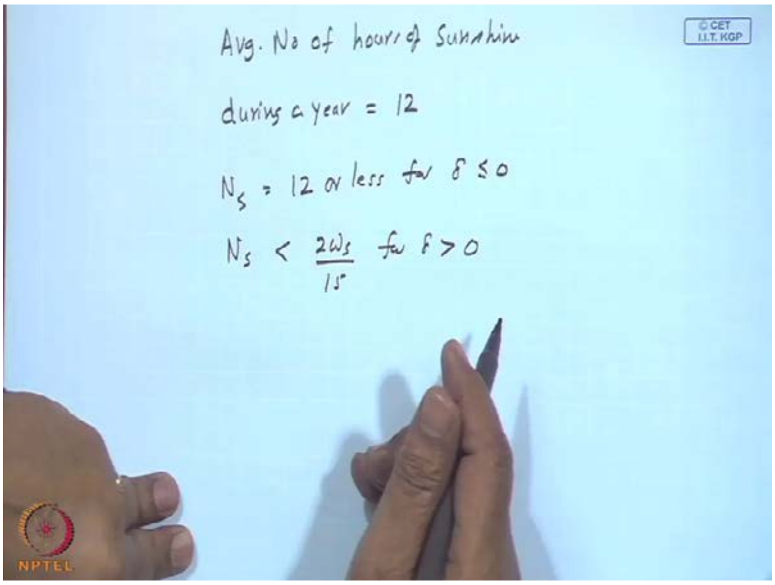

I will give you an additional piece of information, just through Mathematics. Anywhere on earth, average number of hours of sunshine during a year is 12; where you go to North Pole or the equator. At the equator it is 12, all the days and at the North Pole it is 24 hours into 6 months plus 0 into 24, 0 into 6 months. So, the average is again 12 months. Given this fact and comparing with the tilted surface, my N s is 12 or less for delta less than or equal to 0. That means if I am having a negative declination, my physical sunrise and sunset hour angles or 12 or less than 12.

And I restricted to depending upon the value of phi minus beta to whatever is the value that you obtained by calculation. It is not more than the N s. Whereas, N s is less than twice omega s by 15 for delta greater than 0 because we restricted according to the tan phi minus beta tan delta. So, my average will turn out to be less than 12, since the benefit of having a higher number of sunshine hours during delta greater than 0 is not allowed for the tilted surface.

So if I take delta equal to 0, my average sunshine hours, total number of sunshine hours should be 12 into 365 because every day we will be having 12 hours and we put delta equal to 0. It becomes independent of phi; independent of phi minus beta. Consequently, I am very likely to get a incompatible value, if not a wrong value.