Fluid Mechanics

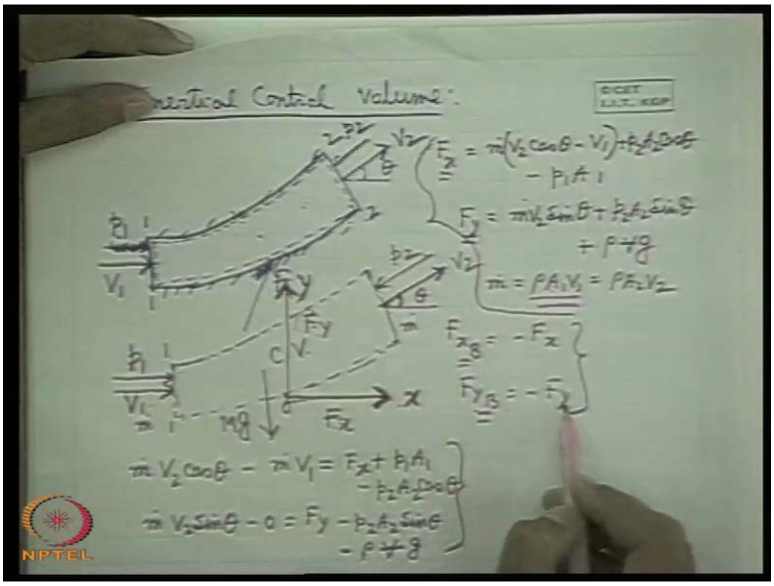
Prof. S. K. Som
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 18

Conservation Equations in Fluid Flow Part - VI

Good morning I welcome you all to this session. Well in the last class we started the discussion on flow of fluid through pipe bends, the analysis of flow of fluid through pipe bends which is a typical application of analysis of inertial control volume. So we also defined 2 types of control volume, inertial and non-inertial. So let us come back to the earlier discussion, inertial control volume problem.

(Refer Slide Time: 00:48)



So let us first see practically the problem is like this that if the fluid flows through a pipe bend, you know in practice the pipe bends are like this which is either expanding or reducing; that means the cross section of the pipe is changing and the pipe bend means that is a pipe where the axis changes it is direction. So this is a part of a pipe bend where the fluid flows through it, these are practical situations where we find the fluid flowing through a bend to change it is direction and pipe cross section is also changing.

In such a case now let us first find out practically without knowing any fluid mechanics we can tell that when the fluid flows with some velocity and pressure into the pipe and goes out then a force is experienced by the pipe bend; that means, we require some force to support the pipe bend in a static position.

So practically the problem is posed that in type, this type of flow through pipe bend what is the force required to make the bend at static position? Now think of physical problem; physically if we think when the fluid flows through this bend because of the reaction between the surface of the solid bend and the fluid the a force is exerted on the fluid body as you can think of a system approach as the fluid flows through this or within the fluid body or fluid body within this bend. So bend exerts some force because of the reaction at it is surface, this in terms of the normal force and if the fluid is viscous frictional force.

So as a result what happens this force is exerted on the fluid by virtue of which fluid is capable of changing its momentum, changing velocity vector though the mass flow rate remains same. Then in turn by Newton's third law the same force is exerted by the fluid body to the bend and we can find out, if I can find out that force we can tell this is the force acting on the bend.

So how this is analysed now, in this case it is always better to analyse the problem with respect to a control volume. Now you can appreciate that what is the advantage of analysing it to a control volume, without going for different fluid particles or system flowing through it better we take a control volume; that means, a fixed region in the bend which is just inscribing the bend, that is with the bend surface that means internal portion of the bend as it is shown by this red dotted portion this fluid this region as a control volume which at any instant composes of fluid.

Fluid may be of different identities but some fluid consisting within this volume; that means we tell if that this is a control volume of fluid taken. Now we apply the equation of motion to this control volume as we have already derived, to do this we fixed to axis's as shown that x and y; their positive directions are shown in the direction of the arrow.

So now if we recall the momentum theorem, before that let us prescribe the parameters; let at inlet the pressure is p 1 the velocity of the fluid is p 1, later the outlet the velocity is V 2 and we define this angle that means the normal to this cross section because this is the velocity vector normal to this cross section. And we do not consider any variation in this direction either velocity or pressure.

So these 2 dimensional problem we make it 1 dimensional. So therefore this axis, that is a perpendicular to the cross section at the exert makes an angle theta. So if we prescribe the hydrodynamic parameters at inlet and outlet and take the axis coordinate axis like that, then we can recall the momentum theorem at steady state. This is a steady flow condition.

What is the momentum theorem at steady state? I tell that the net momentum a flux in any direction, let consider x direction is equal to the net force in the x direction acting on the control volume. Similarly the net momentum a flux from the control volume in y direction; we have taken this y; y is equal to the net force in y direction acting on the control volume. Now let us write this, now let F x and F y is the force we consider acting on the positive x direction and positive y direction on the control volume, let us consider F x and F y are the forces acting on the control volume. That is the force because of it is interaction with the bend from where it has been taken as an isolated control volume, sort of free body diagram. Then we can write that net momentum a flux in x direction is what? If m dot is the flow rate, m dot which is same at inlet and outlet into V 2 cos theta, this is the momentum a flux in x direction.

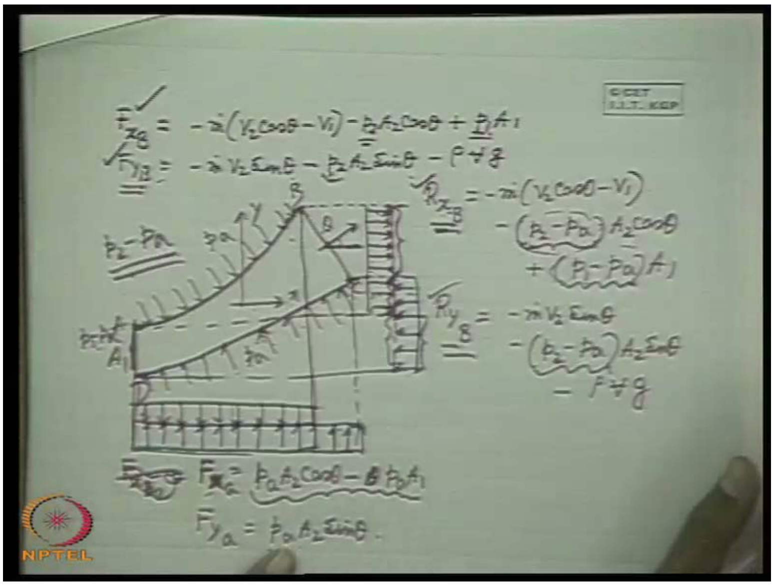
What is momentum a flux influx in x direction, positive x direction? Is m dot V 1; very simple is equal to, is equal to the force. That means we can write here, what is the force? Force is equal to F x any other force plus p 1 A 1 any other force minus p 2 please tell p 2, p 2 A 2 is the force and cos theta A 2 cos theta. In y direction what is the momentum a flux, in positive y direction? m 2 V 2 sine theta. What is the momentum influx in y direction? 0. So minus 0 is equal to F y, what is the component there is no contribution here; minus p 2 A 2 sine theta. Now if we consider also, if the pipe bend is in the horizontal plane and x y we take 2 mutually perpendicular axis in the horizontal plane this is sufficient, but if the pipe bend is in a vertical plane then definitely y direction will be the vertical direction because this x y plane will be again a vertical plane. So x will be any horizontal direction so the perpendicular to that in the vertical plane will be along the vertical direction. So, therefore, the weight of the control volume it cell; that means, the fluid element contained within that will come into picture and that will act downwards, that is m if m is the mass of the fluid within the control volume Mg; and this we can write as M is rho times if I consider V cut is the internal volume within the pipe bend then rho V into g this will have to take depending upon the plane in which the pipe bend exists.

So this is, these are the 2 equations from which I can write F x is equal to; F x is equal to m dot V 2 cos theta minus rather you write like this V 1. So this will be plus p 2 A 2 cos theta, p 2 A 2 cos theta, this will go there minus p 1 A 1. Similarly F y will be m 2 you see here V 2 sin theta plus p 2 A 2 sin theta and plus rho g. So, this is the force F x and F y in a positive x and y direction acting on the control volume.

In numerical, numerical values if it comes out to be positive then it is in this direction, if any of the force comes out to be negative; that means, it is in opposite direction. Along with that we have another equation for mass flow rate which is from the integrated continuity equation, that is either rho A 1 V 1 or is equal to rho A 2 V 2 for incompressible fluid rho is constant; that means, if we equate the mass flow rate at inlet it will be rho A 1 V 1 and if we equate it at the outlet rho A 2 V 2 and this will be equal. Because the mass flow coming in is equal to the mass flow going out, this is the continuity equation for the control volume under steady state.

So with these are the sufficient equations to find out F x F y. Now simple Newton's third law we can tell that the force acting on the bend; that means, let in x direction F x B it will be minus F x alright and F y be will be equal to minus F y; that means, this is the force acting on the bend. Again the force required to keep the bend, again will be negative of that; that means this is the force actually in magnitude and direction which is required to keep the bend stationary. Depending upon the problem we will find out, but negative of this is the force acting on the required acting on the bend; but apart from these there are some other forces acting on the bend because of the atmospheric pressure that will have to consider. Let us consider this, now F x B is minus F x F y B is minus F y.

(Refer Slide Time: 10:11)



That means, if we write the minus of this thing we can see that we can write F x B then minus of this things you can see from your note now that minus of this thing is minus m dot V 2 cos theta minus V 1, well minus p 2 A 2 cos theta well plus p 1 A 1. Similarly F y B will be minus m dot V 2 sin theta, just a negative sin, minus p 2 A 2 sin theta minus rho V g well. Please minus rho v g, this is a F y B that is acting on the bend x direction, force acting on the bend in the x direction force acting on the bend in the y direction; that means, negative of this minus p 2 A 2 cos theta plus p 1 A 1 and F y B is minus m dot V 2 sin theta minus p 2 A 2 sin theta minus rho V g, alright.

Now apart from this there is a atmospheric pressure forces acting on the bend, how is it let us show this; now this is the bend, now on the bend body throughout the atmospheric pressure forces acting which is P a throughout the body surface of the bend. So this gives rise to a pressure net force in x and y direction, in x and; that means, this is the x direction this is the y direction, but do not confuse it with the boiling with the ((boincy)) do not confuse it with the ((bioncy)) forces because P a is same we are not this is not varying from this point to this point even if this is in a vertical plane; that means, if this could have been closed body then the effect would have been 0, there is no net force acting in any direction if the pressure is uniform throughout that. Because of the shape of the bend we can see that there is a net force acting in x and y direction; how can we find it? Let us make a projection like this.

So we will see that here, for example this force on this surface; let A B surface, let this is C D, gives rise to a force gives rise to a force like this, gives rise to this force which is equal to atmospheric pressure times the projected area of this surface. Similarly the force on C D gives rise, force on C D gives rise; so let us make this gives rise to a force which is the projected area of this C D surface in this direction times the P a alright. Similarly in horizontal, in vertical direction also if we want to find out the force, you have to make the same thing that the pressure force due to A B is like this. Try to understand this very simple, similarly due to C D the pressure forces will be sorry like this, due to C D I can tell that pressure force is like this; that means same intensity of pressure P a, but multiplied with a area which according to this figure is much more, that is the projection area of C D on a horizontal plane.

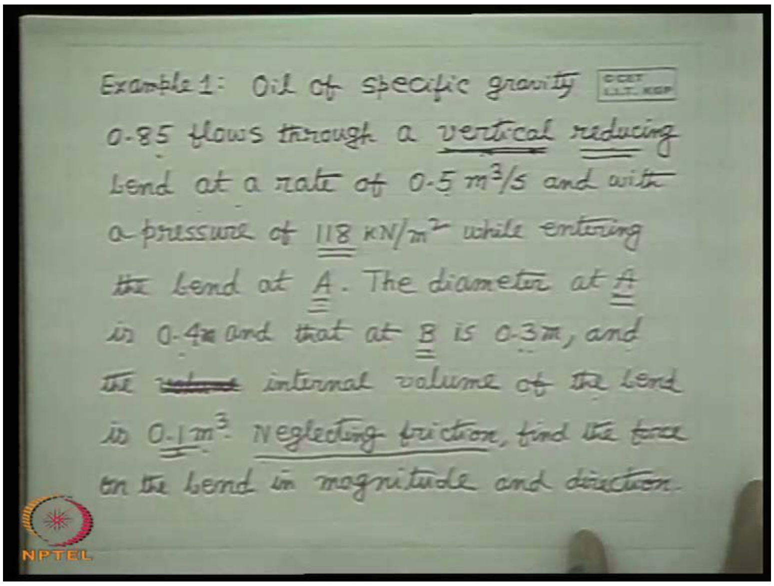
So now therefore, we see an interesting thing that this part cancels out in this projection; that means, ultimately this force minus this force. So therefore the x direction force due to atmospheric effect; that means, x F B due to atmospheric effect in another suffix I write a; that means, x direction force on the bend due to atmospheric force, let us simply write F a. So if you know that we can define the nomenclature it is alright; that means F a is the force due to the atmospheric pressure in the x direction F x a, F x a in the bend that can be written as P 1 P a into this much; that means A 2 if this is theta. So it is A 2 cos theta. So this projection on this plane is A 2 cos theta, so P a atmospheric pressure A 2 cos theta minus this much because this part cancelled out from 2 sides; so minus this is what this is A 1 for example, A 1 area sorry first I write P a A 1 the A 1 simply remains same. So this is P a A 2 cos theta minus p a A 1, so this is the force due to atmospheric pressure in the x direction.

Similarly the force y component of force due to atmospheric pressure on the bend will be positive y in this direction; that means this minus this, so up to these it is cancelled. So only remaining part is this one; that means this is nothing but the projection of A 2 on this horizontal planes time P a. That means P a A 2 sin theta, now if we just add this part I can tell the force, the net force what F x B already I have written let us write the R x B is the net force with considering the atmospheric pressure force will be m dot V 2 cos theta minus V 1. Now minus p 2 A 2 cos theta plus p a A 2 cos theta, I can write minus A 2 cos theta will be taken common, so p 2 minus p a p 2 minus p a A 2 cos theta plus p 1 A 1 minus p a; that means plus p 1 minus p a into A 1; that means this part I am adding;

that means F x B plus F x a that is the total force on the bend in the x direction. So total y component of force on the bend is F y B that due to the change in momentum that is given by the liquid plus that given by the atmospheric air; that means if I just add in minus m 2 V 2 sine theta, so minus p 2 A 2 sine theta plus that means, I can write minus p 2 minus p a and A 2 sine theta, alright minus rho.

Now this gives a very interesting thing that if you compare this with this and this with this what comparisons we get that instead of p 2 if we substitute p 2 minus p a here also and instead of p 1 if we substitute p 1 minus p a, As if we consider the pressure p 1 here as p 1 minus p a and consider pressure here as p 2 minus p a and then if we find out the forces on the bend from the consideration of the fluid then automatically the forces due to atmospheric pressure on the bend, due to the surrounding air pressure of this surrounding air on the bend is taken care of; which means mathematically that if we prescribe the pressure in terms of the gauge pressure or even if the absolute pressure is given we only take the gauge pressure as the pressures at the 2 sections and find out the forces acting on the bend by the fluid body from the control volume theorem then we get the force component considering the atmospheric pressure force.

(Refer Slide Time: 19:17)



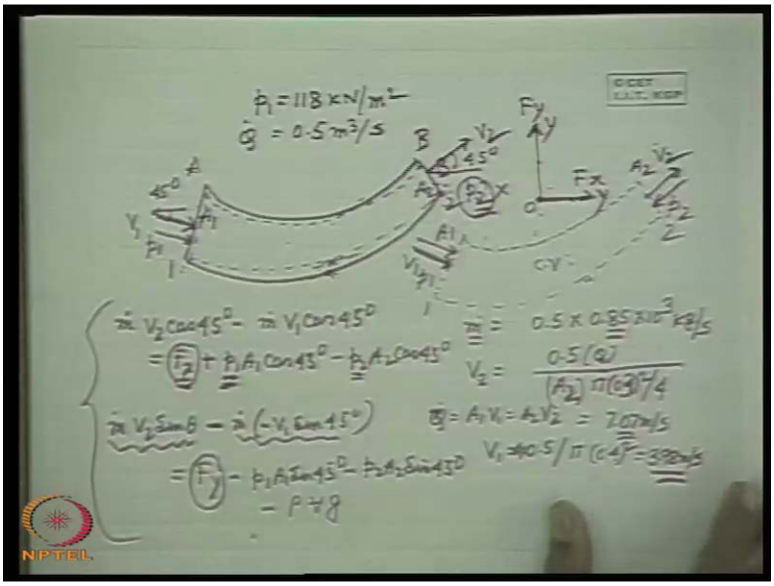
That means p 2 is replaced by p 2 minus p a well and p 1 in the equations which comes automatically from the control volume theorem application of the control volume theorem as p 1 minus p a then this consideration of atmospheric pressure force is taken care of, alright. Now today immediately I want to solve a problem so that things will be more clear to you and how it is applied in the numerical problems.

Let us now concentrate on the problem example, let us go you have understood? This very simple example, Oil of specific gravity; please write a little fast, oil of specific gravity 0.85 flows through a vertical reducing bend at a rate of 0.5 meter cube per second and with a pressure of 118 kilo Newton per meter square; those of you who have purchased my book you can just make a correction in my book, this is a problem exercise problem, that means the problem to be worked out by you, it is giving mega Newton but it will be kilo Newton per meter square this is a mistake, while entering the bend at a that I will show in the figure, at a means the inlet section at a the diameter at A is 0.4 meter and that at B; that means, B is prescribed in the figure that is the outlet section is 0.3 meter.

That means this is a bend which is reducing bend; that means bend is converging it is changes in diameter and the internal volume of the bend is 0.1 meter cube that is the internal volume of the bend, neglecting friction find the force on the bend in magnitude and direction, very important is the neglecting friction find the force on the bend in magnitude and direction. Second I m telling that oil of specific gravity 0.85 flows through a vertical reducing bend, vertical means the bend is in a vertical plane, reducing the what is redundant because it is always clear for that is diameter is reduced or reducing bend this is a terminology for the bend; so the practical terminology expanding bend, reducing bend, so that is why it is.

So in a vertical means which is very important for our problem here from the physical point of view it is in a vertical plane, at a rate of this is the flow rate pressure at some section A; that is the inlet section is given the diameter at A is given, diameter at B is given. This is the inlet, this is the outlet that I will show in the figure and the internal volume of the bend is 0.1 meter cube; obviously the internal volume is required when it is a vertical plane I you know that the weight has to be considered or the fluid within the bend. So neglecting friction find the force on the bend in magnitude and direction. How to solve this problem now let us see the problem, our let us the problem is like this the figure is like this.

(Refer Slide Time: 21:55)



So now some more data which is not given which should be reducing, it is given this is A and this is B and the perpendicular that axis rather along which the velocity V 1 is coming this makes an angle 45 degree. Similarly here also V 2, let this section is one this section this is given in the figure this is required 45 degree. So pressure p 1, this is in a vertical plane and this is p 2. So p 1 is specified as 118 kilo Newton per meter square, p 2 is not given, Q is given Q dot that is the volume flow rate 0.5 meter cube per second; what else is given? This diameter is given 0.4 meter that means I know A 1, I know A 2 alright. So now we have to find out the forces, internal volume is given, now what we will do now we take a control volume obviously; that means, let us define the control volume like this say again the same thing in the same drawing I could have done that means this is the control volume. So therefore, here this is the control volume.

Let us consider x y in this direction, o x y and let us consider the forces acting on these at F x and F y on the control volume, this o can be here or here because it is fixed control volume so therefore o origin can be anywhere outside the fixed frame of reference. Then this is V 1, this is V 2, p 1 p 2 all I am giving in terms of 1 2, A 1 A 2, p 1 is also acting like this, p 2 acting like this. Now you tell me what I will write in F x direction momentum? The same thing that what I will write that m dot into now first of all if I take this V 2 cos theta that is theta is 45; that means V 2 cos 45 minus the influx what is influx? Influx is m dot now V 1 cos 45 is in the same direction you see that V 1. So this will be minus V 1 cos 45, so minus V 1 cos 45 same positive direction. So this is the momentum a flux is equal to what we can write is equal to first you write F x plus p 1 A 1 cos p 1 A 1 cos 45 minus here remember this p 1 and p 2 are in gauge pressures very good. So that if I take gauge pressure the effect of ambient pressure is taken care of, so there is no other force acting on this control volume. Then y direction p 2 A 2 very good cos 45, then what is y direction force? Please tell me.

So now we can find out F x from here, then y direction please tell y direction, you tell me m dot; now this is the positive direction though A flux is in the positive direction V 2 sin theta. What is influx minus influx m dot, please; is it V 1 sine 40 is it V 1 sin 45 only? Right minus, Minus V 1 sin 45, very good that is why it will be positive because a flux momentum minus the momentum influx. So momentum influx equation is that minus because net x, x a flux of momentum, but momentum itself as a sin because of the sin of the velocity vector relative to the chosen positive direction sin of the coordinate axis here V 1 sin 45 is in the negative y direction. So that this momentum influx will be negative and this will be added.

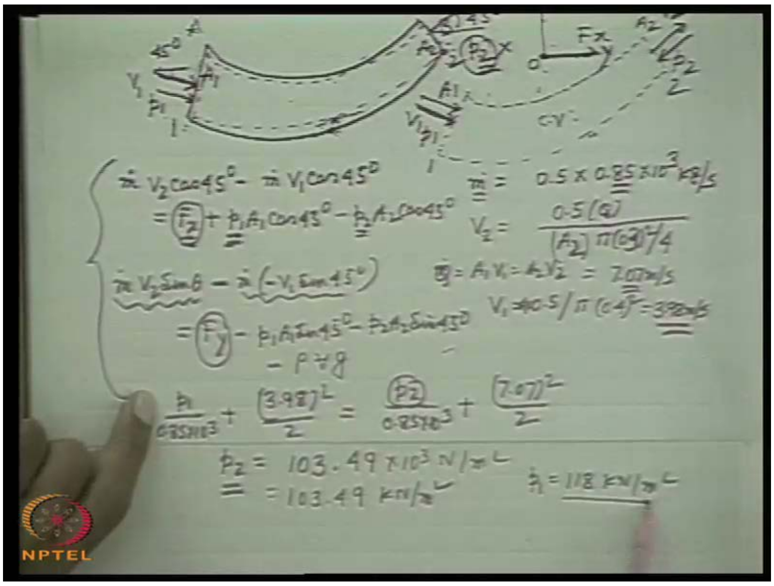
So that net momentum in a flux in y direction, positive y direction as chosen here will be the plus; this is the only difference is equal to then F y as we have already chosen let F y is the force acting on the control volume. Next part is minus p 1 A 1 sin 45 alright, minus p 2 A 2 sin 45. So as per as the momentum theorems, theorem is constant there is no problem with us. Now we will have to, please, yes very good; what is that? Minus rho just we write like that in the general nomenclature. Now thing has to be found out, what is m dot; m dot is 0.5 meter cube per second into rho, rho is 0.5 into 10 to the power 3 , specific gravity 10 to the per 3 meter cube per kg. So this becomes equal to kg per second, so m dot. How to find out V 2 and V 1? Please tell me how to find out V 2.

Good Q by; that means, this is Q by A 1 cubicles Q is a 10, sorry Q is A 2 V 2 because m dot or simply Q dot if I write rho is constant is A 1 V 1 is A 2 V 2 continuity; that means, if we equate here this is the A 1 into V 1, if we equate here A 2 into V 2. So what is A 2 in bracket, A 2 is pi what is d; d 1 is 0.4 meter, so 0.4 whole square by 4 so it is very simple, you can find out V 2. If you want the values then I can tell you this is being done, but I think that you can do it any time well values are yes, the values at V 2 it is given as A B; in the problem it is given A B, so better I am not changing it 1 means A and 2 means B. So B v is; that means V 2 is 7.07 meter per second. So diameter is more so velocity will be, similar way V 1 is 0.5 by pi into sorry this will be 0.3.

So, this will be 0.4 whole square, here 4 will come pi the square by 4. So this will be 3.98 metre per second, so we know Q, so we know V 1 V 2. So what else we have to know; p 1 p 2, how to find out p 1 p 2? here is the concept how to find out p 1 p 2, how to find out p 1 p 2 please tell me, Bernoulli's equation and that is why this what is not redundant neglecting friction because the momentum theorem goes with friction also. Because we have consider the F x and F y is the force acting there lies the concept on the control volume, and this force can take care of any reaction force interaction between the solid surface and the liquid. It can have a ratio force that it can ratio frictional force, so the combination of which; that means all sorts of interaction between the solid and the fluid layer is taken care of by. So no restriction has been made here where we have considered in general the force is acting on the control volume F x and F y by the interaction with the solid surface.

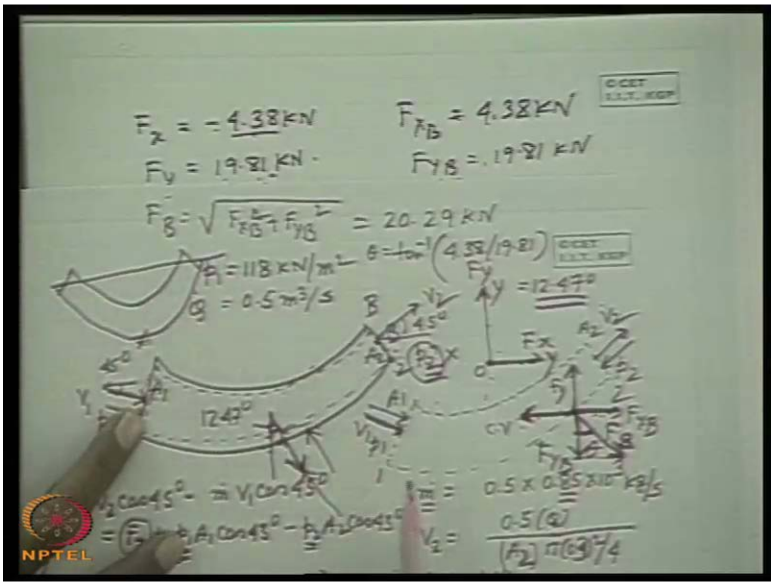
In that we have not taken that solid surface and this fluid is playing with a frictional system; that means, ideal fluid. So therefore, here there is no restrictions but when a problem is given neglecting friction these what is not redundant it has got some meaning. So the meaning comes immediately, becomes immediately apprehend when we want to find out p 2 because p 1 is given; if p 2 was given we were not bothered and again if what stored frictionless then we could have been dilemma we have to show that the Bernoulli's equation is valid if it is not tallied then we could write that the what frictionless is wrong in the problem.

(Refer Slide Time: 31:21)



So in frictionless it is given p 2 is absent; that means we can find out p 2 by writing the Bernoulli's equation formula in 2 sections. So the writing Bernoulli's equation is in these 2 sections p 1 and p 2 then we can find out that p 1 you can write the Bernoulli's equation; p 1 by rho, rho is 0.85 into 10 to the power 3 plus V 1 square by 2, what is V 1? 3.98 whole square by 2 is equal to according to Bernoulli's equation this body this 2 points the elevations are the same then we can write p 2 by 0.85 into 10 to the power 3 plus V 2, what is V 2? 7.07 whole square V 2 square, so this p 2 is found out and this p 2 is if you find out you will get here it is given as p b because a and b higher started with 1 and 2; however, this into 10 to the power 3 Newton per meter square or simple 103.49 kilo Newton per meter square.

(Refer Slide Time: 32:42)



So problem is over now if we put this value p 2, p 1 is known as 118, p 1 is known given by the problem. So here is the tricks that you take the frictional effect and find out this 118 kilo Newton per meter square you get the value of F x everything is known except F x and F y.

And if you do so, you will find a value of F x which is equal to; I tell you the value of F x will come minus 4.38 kilo Newton and the value of F y if we calculate it will come 19.81 kilo Newton. Therefore we can say that if you place this figure with this value; that means the force on the control volume is acting in this direction F x and this direction F y, F y come in positive that means the forces acting in this direction or this magnitude and the force in the x direction is acting with this direction with this magnitude; that means now I show by the direction with this diagram.

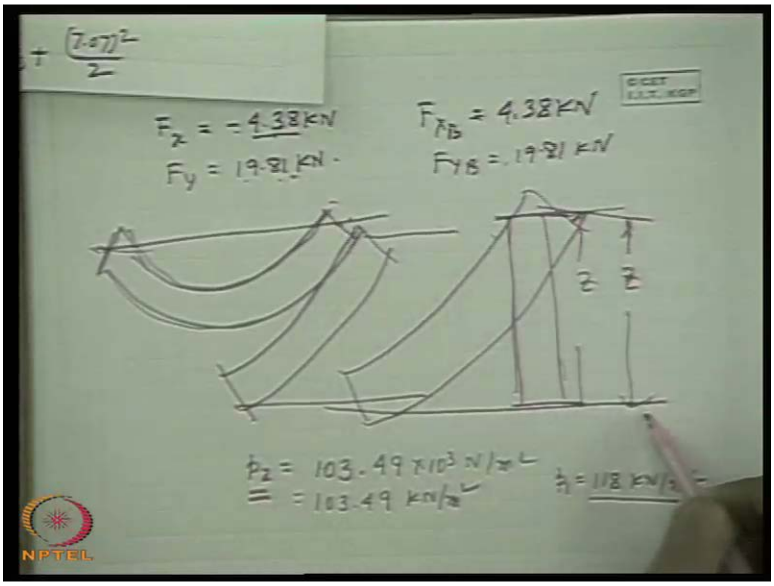
So therefore forces on the bend will be acting, this is the forces on the bend F y B and F x B. So therefore F x B will be now I am writing the same magnitude only kilo Newton and F y B is 19.81 kilo Newton. I am not giving any sign because I am showing it this is the F x B and this is, because it is just the reverse opposite because this is acting on the control volume so this is acting on the bend. So the resultant will be acting somewhere like that in that direction.

So if this is the resultant force F B, so F B is found out as vector addition of these; that means F x B square plus F y B square. So this 4.3 square 19.81 square under root which gives you a value of 20.19 kilo Newton and if we prescribe the angle with the vertical we can write theta is equal to tan inverse horizontal angle, with the vertical means tan inverse the perpendicular will be the horizontal component 4.38 by 9 its simple school level vector problem and this becomes equal to 2l.4 square, obviously the vertical forces heavy so it will be more towards the vertical.

So angle is 12.470; that means, the resultant force will be, that means there is a force acting like this whose angle is 12.470. It is more towards vertical because vertical forces heavy 19.81 as compared to F a. So, this is the force acting on the bend. Now if I have to find out what is the force required to support the bend it will be like this; that means, it is parallel to this. So this is precisely the problem please any query, yes.

Vertical tubes such as (( )) consider the potential energy? I am telling you are not listening to, I told that when I have considered the Bernoulli’s equation the potential head difference between these 2 is very small, there is no considerations that there is a if there is a bend tube like this. So this and this plane may come into the same horizontal plane.

(Refer Slide Time: 35:51)



So it depends upon if there is a vertical tube like this, a vertical pipe like that so there is a potential. If the potential difference is given then you take it or otherwise neglect it. So if bend is like this in a vertical plane there has to be, you cannot tell always there will be a potential the bend may be like this.

So, that the potential head between the outlet and inlet cannot come into picture, negligible depending upon the bend. So if it is not given in the problem we will assume the bend is such and the figure is given such that it appears to be there is no potential head difference, we will write that, I have told that when applying the Bernoulli’s equation we have neglecting the potential. If there is a vertical tube or a bend like this in a vertical plane definitely a potential difference will arise if the figure is like that and it has to be told in the problem, if the problem does not tell it and it is a numerical problem then you have to assume the differential potential head is negligible.

Definitely potential head is there and when I have written the Bernoulli’s equation I have told that I neglect the potential head if potential elevation or elevation difference between these 2, potential head will come; potential head does not mean an absolute potential head that is the reason that the arbitrary datum of potential energy has got no value. This may be potential heavy, high potential it compared to an arbitrary datum, you can make arbitrary datum anywhere and here there is a high potential head. It is the difference which is coming into the picture, if here I will Z 1 here it will be Z 2 g Z 1 it will be g Z

2. So Z 2 minus Z 1 is the important, differential potential head. So difference in potential head we are neglecting here and in this figure it shows that probably this is very small alright.

Yes sir.

p 1 p 2 as gauge pressures

Yes these are very see, these are very ((trivelling)) things what you are asking, it is just conception is clear. If there is a problem nothing is written you assume that we assumed the pressure to be gauge pressure and find it; or you take the pressure a intelligent student will do like this, but a less intelligent student will take that we assumed the pressure as the absolute pressure then do that and then at the end you will by know atmospheric pressure higher to take consideration otherwise I will not get marks again atmospheric pressure you will ultimately loss us time so you have to do that. But if it is not given it is clever attitude to take that when the pressure is not mentioned whether absolute or grid I take this is as gauge pressure. So immediately the calculation will till the net force acting on the bend consideration of the atmospheric pressure, these are relatively useful problems, so problem will give that you have understood.

So you are asking the question I understand as if sir what I will do in the examination or this is not given that is not given. So if you have understand the problem very well you can detect that problem is not expressively well posed, you can mention that and examiner will be very happy; sorry the pressure in the problem is not expressible written either gauge usually when pressure is quoted at this kilo newton per meter square gauge it is not written. So therefore, pressure has to be written clear fully that absolute or gauge. In many applications we do not write it, very difficult to understand because convention changes people think that they people will know that it is gauge pressure.

So you have to categorically write it, just I am giving an example in this count that in my life time I have seen only 1 doctor in my life who after seeing my pressure wrote at this millimetre of mercuric gauge, I was so surprised to see that I told doctor this is for the first time I am seeing a medical practitioner even they going through all the courses in a school level first year MBBS even they do not write this millimetre, even the millimetre they do not write, they write the numeric, know that is not there for, know this is not necessary people know this should be the pressure, this is the numeric if it goes beyond that some 90 plus age is the thumb to left. But only 1 doctor in my life time, only 1 doctor I saw he wrote that 130 millimetre of age gauge systolic and diastolic 90 millimetre of mercury gauge, I was surprised. So expressed it, I thought that it should not be a medical doctor it should be a person of science and engineering. So do not tell it to your relatives about doctors, but if the examination problem is not expressively well put you will assume concepts, conceptually you are alright.

Thank you.