Fluid Mechanics

Prof. S. K. Som
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 38
Principles of Similarity Part- III

Good morning, I welcome you all to this session of fluid mechanics. Last class we were discussing the application of dimensional analysis that is Buckingham's pi-theorem, in pipe flow problem to recognize the different dimensionless term, that this similarity criteria.

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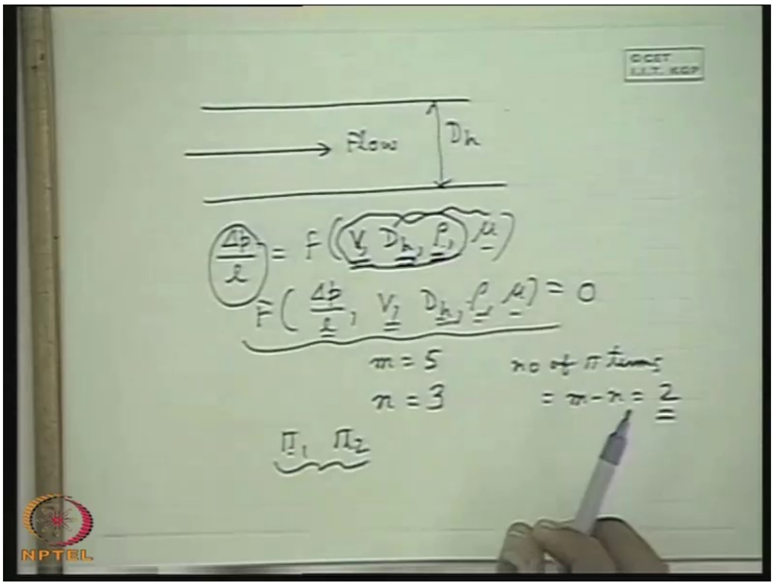

And we arrived at this, if we recall this a pipe flow problem, this is the flow we have found out; that delta p by l per unit length is a function of flow velocity V, the hydraulic diameter D h, which is the pipe diameter in this case D h is the pipe diameter or hydraulic diameter rho and the viscosity mu. Or this was this can be x delta p by l so, delta p by l, V, D h, rho, mu is 0; that is the implicit functional relationship. That these are the variables defining up the problem, where delta p by l is the output parameter dependent variable; which depends upon velocity of flow hydraulic diameter and density and viscosity. So, according to Buckingham's pi-theorem, we see that number of variables m is 5, and number of fundamental dimensions in which these variables can be expressed is 3; that is M L T.

Then the number of pi terms according to Buckingham's pi-theorem is m minus n, that is 2, 5 minus 3 in this case. So, two pi terms will be there pi 1 and pi 2. Now according to this theorem, we will have to choose three repeating variables. Now there are four variables excluding this dependent variable; that means, 1 2 3 4. So, number of choices are there and we have seen that; if we choose V D h and rho, three has the repeating variable to determine the pi terms as per as the procedure. Then we land up with the conventional friction factor and Reynolds number as the two pi 1, pi 2 terms not Reynolds number 1 by Reynolds number. Now, we will show that, if we take any, any other any three; that means, another set, there will be four sets and what will be the pi terms or the dimensionless terms see that.

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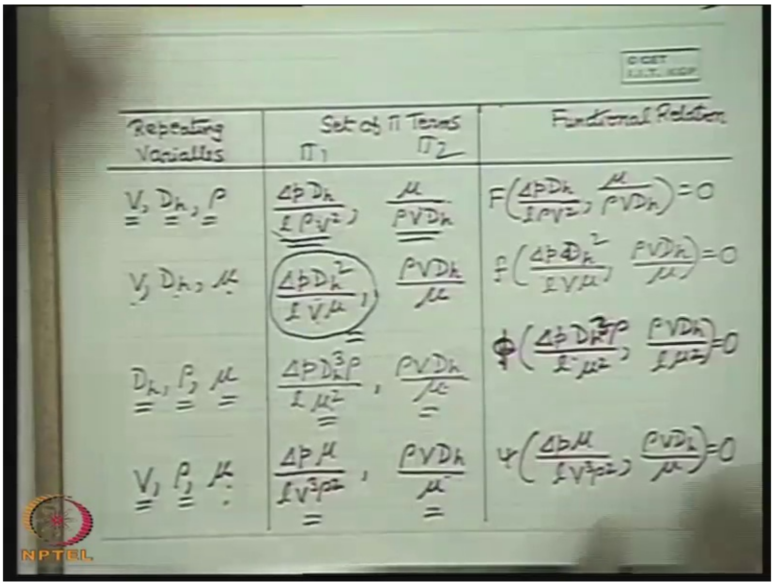

That if we call take repeating variables V D h rho, we land up with this pi 1 and pi 2; that is this is pi 1 and this is pi 2. So, this is the conventional friction factor, this is 1 by Reynolds number and functional relation is function of these and these 0; that means, this is a function of this any way we can take, but if we take V, D h, mu as the repeating variables we get a different dimensionless term. So, this will be just one upon this reverse and this will be a this is also a dimensionless terms, if you calculate the dimension of this it will be dimensionless. So, the functional relationship of function of these and these is 0.

Similarly, if we take these three as the repeating variables, we get these two as pi 1 pi 2.

If we take these three as the repeating variables, we take these two as pi 1 and pi 2 terms. So, that the functional relationship is like this so, this way we take the repeating variables like this. So, if we now the question is that, this way if we take the repeating variables we are landing up with different pi term. Now, the pertinent question comes, how should I know that which pi terms will be we have to take to get the correct result. The answer is like that all the pi terms are physically, mathematically meaningful. Physically also meaningful in maintaining the dynamic similarity criteria and you see these sets of pi terms are not independent, they are interdependent between each other, how? Just you see, it is very simple, that if you compare the pi terms of 1 and 2, if you compare the pi terms of 1 and 2, let this is set I, let this set II.

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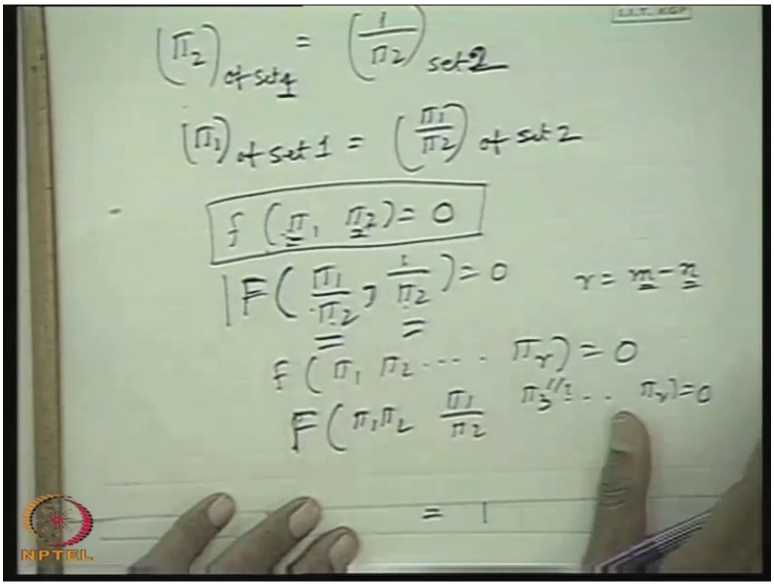

Then you see the pi 2 of set 2, of set 2 is 1 by pi 2 of set 1; that means, if we make the reciprocal it please do not talk in the class if daily I will have to tell that just like a school boy I am sorry this video tape cannot be made for the outsiders I just to the outsiders that if very day just like a kinder garden school I have to tell IIT do not talk do not talk in the class I am sorry I cannot take this class in that case otherwise you leave the class then I will go on teaching for outside students I do not want you if you like to talk you please go out so I cannot take a kinder garden class that each and every day I have to come to the class and tell you please do not talk I am sorry.1 by pi 2 of set 1. So, you see that pi 2 of set 1 is 2, this pi 2 of set 2 is 1 by pi 2 of set 1 or rather I can write that pi 2 of set 1 rather 1 by pi 2 of set 2. Similarly pi 1 of set 1 is nothing but pi 1 or by pi 2 of set 2, of set 2; that means, if I divide this one by this pi term I get this; that means, I can tell, that if I get a functional relationship of pi 1 pi 2 is 0 from set 2; that means, this set.

And if I tell another functional relationship of pi 1 by pi 2 of from set 2 and 1 by pi 2 is equal to 0. I arrive the set results for set 1 because, pi 1 by pi 2 is this pi term pi 1 of the set 1 and 1 by pi 2 is the pi 2 of set 1; that means, from set 2, I can go to set 1, just by some combinations of pi terms. So, this has become one a rule or a corollary, that if we land up with any number of pi terms, a combination of this pi terms will also be a functional relationship, will a valid functional relationship keeping the number of pi terms same.

So, we will see this way some, some sort of combinations between these pi term will be; that means this pi 2 same as this pi 2, but 1 by this pi 2 is equal to this pi 2. We will see some combination with this; we will definitely land up with this. You divide this by this we will get this or we will make some combination between these two we will get this one.

So, therefore, we will see that we will get any of the sets of pi terms, from other sets by some recombination of the existing pi term; that means, if we take any three repeating variables, we will be landing up with the different pi terms, but all the results are equally correct because, this pi terms are interdependent pi term. So, therefore, there is a corollary of the pi theorem, that if there are number of pi terms, pi r number of pi terms which is m minus n, m is the number of physical variables, hence in the fundamental dimension in which they can be expressed. Then this is similar or equivalent to another functional relationship of any combinations like, pi 1 and pi 2, pi 1 by pi 2, pi 3 to the power half any such combinations, but number of pi terms will be same just like this pi 1 pi 2. So, this is equivalent to pi 1 by pi 2 1 by pi 2, but the question comes in all the cases the non-dimensional terms may not be physically meaningful; for example, here the second term here is Reynolds number. So, here it is 1 by Reynolds number; however, this is true, but in this case these three sets, this dimensionless term though they represent the dynamic similarity because, they are interdependent through another pi term, but they do not have physical meaning this have the physical meaning and is equal to the friction factor; that is the only difference.

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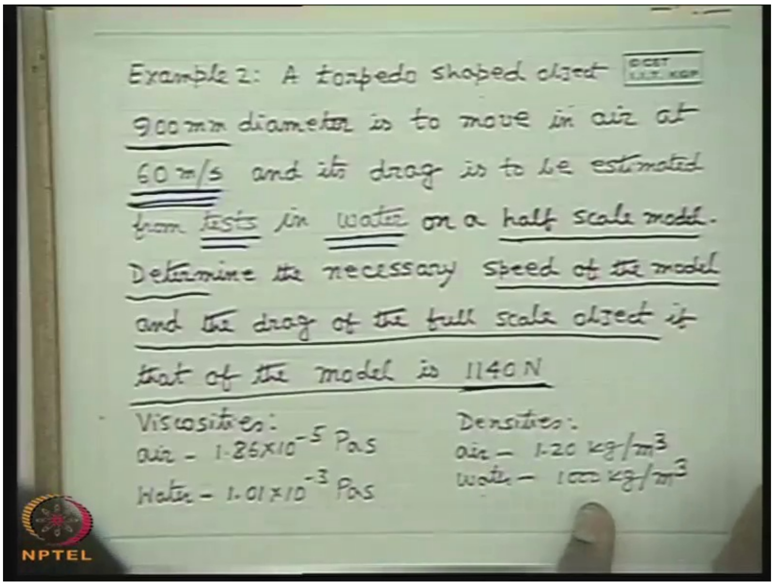

Now, next I like to solve one interesting problem, before I switch over to another problem or another analysis. Example 2: you see that this problem, a torpedo shaped object I am going very fast because, this is very simple of 900 millimeter diameter is to move in air this a problem done in my book, but still for more explanation I just to explain it more I just do it here is to move in air at 60 meter per second. This is the actual condition, that a torpedo shaped object of 900 millimeter actual diameter, is to move in air; this is the velocity of the torpedo is actual condition and its drag is to be estimated. So, under this condition the drag force to be estimated from tests; that means, test in laboratory in water so, test has to be made in water not in air on a half scale model. So, laboratory water is available because, the flow of air is difficult to create, but the flow of water may be easier to be created. So, therefore, it is done on a half scale model, even size will be half, determine the necessary speed of the model. So, under this condition what should be the speed of the model? That means, the speed of the torpedo in the laboratory and the drag of the full scale object, if that of the model is this. That means, what should be the speed of the model we have to first find out and at that speed for a test under water, if the measure drag in the laboratory is 1140 Newton; what should be the drag of the full scale. So, viscosities of air are given, densities of air and water viscosities of air and water are given, densities of air and water are given; these are the properties to be required.

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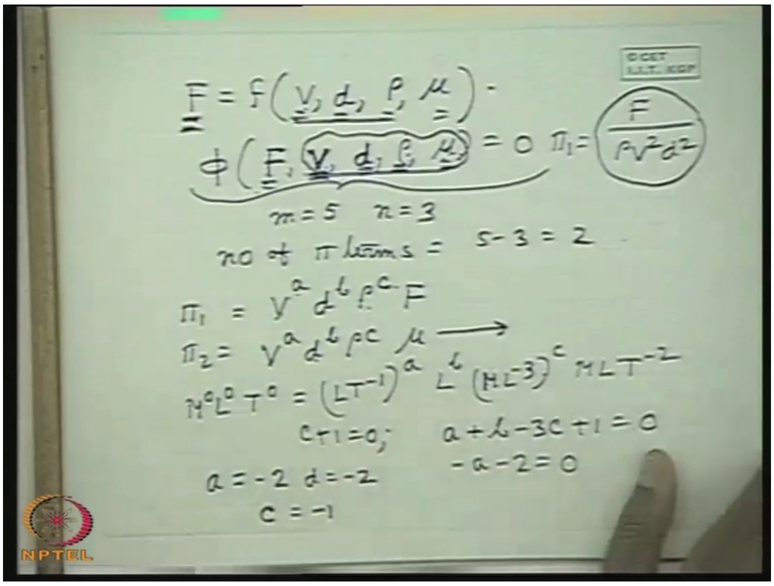

Now, you see this problem has to be first identified, that the drag of the torpedo; that means, drag is F is become, this becomes a function of velocity of the torpedo V, the diameter of the torpedo d; obviously, and what are the other things the velocity the diameter of the torpedo and the density and viscosity.

So, therefore, drag depends upon the velocity, the diameter of the torpedo the density and the viscosity and sorry this is the function of this; or we can define that the entire problem is guided by this variable, this is the output variable first we write the dependent variable is a function of this independent variable. So, in all including the dependent variable; these are the implicit function by which these phenomena can be expressed. That phenomena of drag, while air passes or flow air flows cause the torpedo is like, that the drag force flow velocity, diameter of the torpedo density and viscosity, this functional relationship is equal to 0.

Now, therefore, if we apply Buckingham’s pi-theorem, first we see how many variables 1 2 3 4 5; so, m is equal to 5. So, number of fundamental dimension in which these variables can be expressed; there is no temperature, only mass length and time so, 3. So, therefore, number of pi terms, number of pi terms is equal to 5 minus 3 is 2. So, how to determine to 5 terms? First of all we have to consider the variables; that is repeating variables. If we take V d rho, this three as the repeating variables because, the choice is between this four. Again number of choices will be there, but usually we take V d rho;

one with the geometric dimensions, one with the flow characteristics. If flow velocity is there these are the rules, thumb rules flow velocity should be taken another is the rheological properties preferably rho or other mu, better you take rho V d rho. Then the procedure is that first pi term will be unknown integer exponent, V to the power a d to the power b and rho to the power c and one of the remaining variable let F.

And another will be V to the power a d to the power b rho to the power c the other remaining variable mu. So, automatically number of pi terms it will match so, all the remaining variables apart from the repeating variables have to be multiplied with the repeating variable, raise to the power unknown arbitrary indices. So, number of pi terms will automatically match because, there will be two rest, there will be two rest; that means, out of this five parameters, if three are taken as repeating variables. So, two variables will be left so, there will be two pi term.

Now, if I express the dimensional analysis, now I do the dimensional analysis; this m 0 because, pi 1 is a non-dimensional term. So, V is L T to the power minus 1 a, d is L to the power b, rho is L M L to the power minus 3 c and F is M L T to the power minus 2. So, therefore, if we equate then you get, c plus 1 is equal to 0, c plus 1 then with for the L a plus b minus 3 c plus 1 is equal to 0 and for T minus a minus 2 is equal to 0.

So, therefore, if we solve it we will get, a is equal to minus 2, d is equal to minus 2, and c is equal to minus 1. So, if you put it you get, the pi 1 term as F by, F by rho V square d square so, this is one pi 1 term. Similarly if you solve for pi 2, if you solve for pi 2 this if you put this, L by L T to the power minus 1 to the power a L to the power b M L T to the power minus 3 same thing only mu is M L T to the power minus 1.

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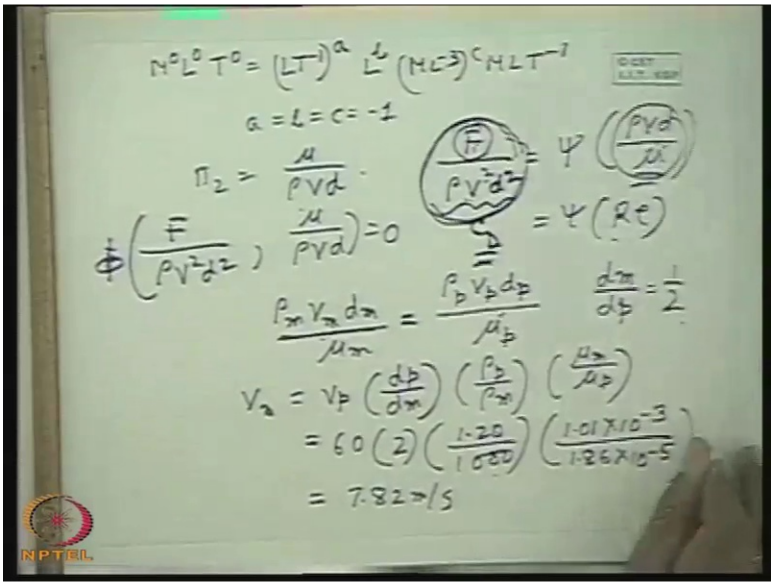

Let us write why unnecessarily. So, for pi 2 if you write it so, pi 2 similarly M to the power 0 L to the power 0 T to the power 0 is equal to L T to the power minus 1 whole to the power a. So, d is L to the power b so, rho is M L to the power minus 3 c M L T to the power minus 1. If we equate it so, you will get, a is equal to b is equal to c is equal to minus 1, you have to get it. And therefore, you get pi 2 is equal to a b c minus 1 means; if you put it here, mu by rho V d so, mu by rho V d.

So, therefore, what is done first, first the pi 1 and pi 2 terms are recognized. So, therefore, the problem is now defined as, F by rho V square d square it is a functional relationship of some phi F by rho, rho V d by mu, mu by rho V d, we can write like that mu by rho V d is equal to 0; or we can write, F by rho V square d square is a function of just I can make a reverse rho V d by mu. That means, this is known as drag coefficient conventional drag coefficient; that is drag force divided by the dynamic head 2 is not there, rho V square times the square of any characteristic dimension. So, that is a function of Reynolds number.

So, therefore, what we do first, we reduce the problem in terms of dimensionless parameter; that means, drag force, velocity, diameter, density, viscosity we club it. So, c d as a function of Reynolds number. Now you see, that for model testing this parameters have to be kept fix so, first criteria for model testing is, that Reynolds number have to be kept fix. So, therefore, Reynolds number for the model, if I give the subscript for the model, must be equal to Reynolds number for the prototype. That means, you are allowed to do the experiment with water, you are allowed to do the experiment with different diameter, but you have to make this combination, that Reynolds number same for dynamic similarity.

So, if you make this so, if you know the diameter ratio from the scale factor, you can find out the model speed. That means, we can write from here, that V m that model speed what is the answer, the V m is equal to V p d p by d m into rho p by rho m into mu m by mu p. So, if you put this value, V p is given that is actual velocity is 60, now scale factor half scale model; that means, laboratory scale is half. That means, d m by d p is half so, this is 2. So, rho p by rho m is the density ratio, that is 1.20 divided by 1000 as it is given density of air density of water and mu m by mu p; that is the viscosity of water model is the water, model fluid is the water and this is the viscosity of air, as it is given that is the prototype and you get this is equal to 7.82 meter per second. That means, we will have to make the experiment under identical Reynolds number.

Now, what we do, we find out the c d from our laboratory and from that c d we code, decode the drag force in actual case. So, therefore, what is meant, that the c d also will be same, if the Reynolds number is same. So, whether it is model test or prototype because, c d is a function of Reynolds number; when Reynolds number assumes a unique value, c d will be assume with the unique value. That means, that means the c d in both the cases will be same or in other words, this dimensionless terms also will be same in both the cases.

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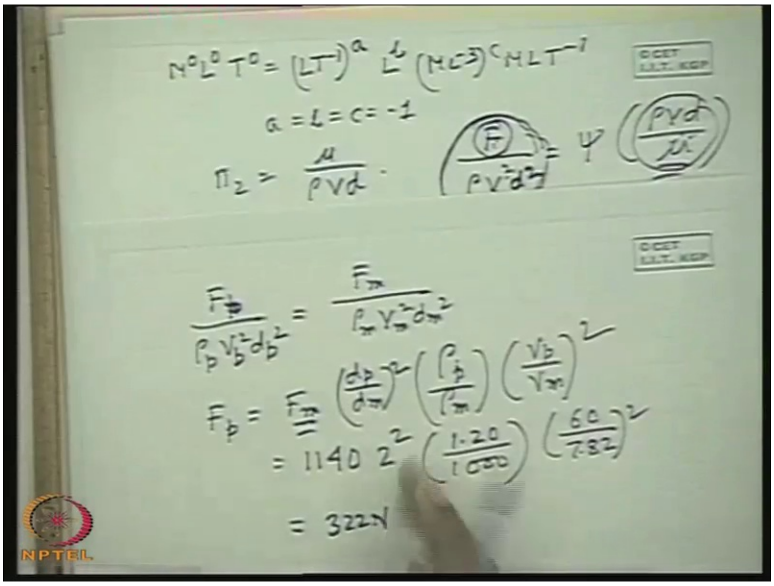

Which means that dimensionless terms, which means that this dimensionless terms F m or rather p first I write F p rho p V p square d p square is equal to F m rho m V m square d m square, it has to be same; from which I can find out F p is F m into d p by d m square into rho p by rho m so, into what is that V p by V m square. So, therefore, if I know the drag force in the model, I can find it, this is the 2 square rho p by rho m is what? rho p is 1.20 and rho m is 1000 and V p by V m, V p is 60 meter per second and V m just now we have evaluated so, therefore, we get this F p as 322. Now you see, this problem the explains two things very beautifully; first of all how to obtain the similarity or dimensionless parameters by the use of Buckingham's pi-theorem for a particular physical problem, then how to apply this non-dimensional terms for this problems between laboratory test and the actual practice; where the problem is same, problem physics is same, but they are under altered conditions.

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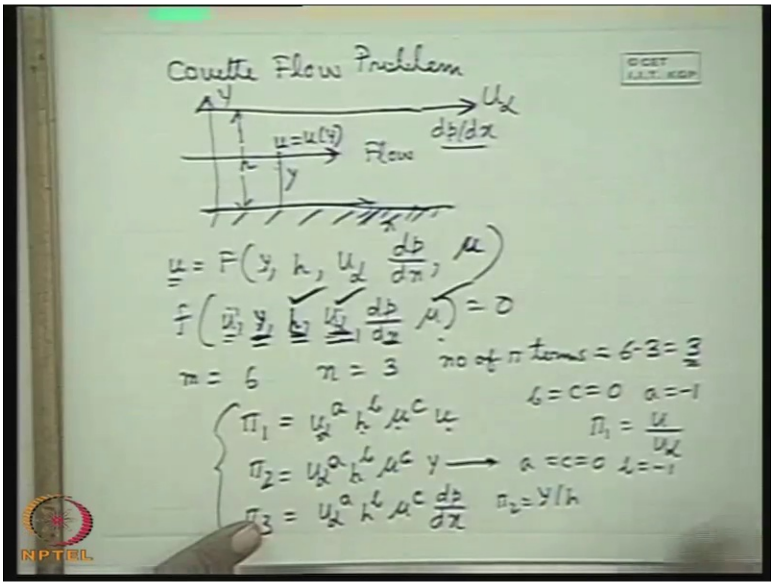

Now, I come to another very interesting situations, that if two variables in a problem (( )) are same in dimension; that means, two length scale or two velocities, then their ratio will automatically be a pi term. Just an example I tell, remember the Couette flow problem, what we have discussed in relation to analytical solution of Navier-Stokes equation; that one plate is fixed another plate is moving with a velocity u infinity and we have found out, we have taken this as the axis and we have found out the velocity distribution with y. And this is also a problem where a d p d x a pressure gradient d p d x is imposed on the problem. So, if I define a problem like that Couette flow problem, that one plate is moving with u infinity velocity, another plate is rest and there is a d p d x pressure gradient; which causes the flow. So, you find u at any location y; that means u as a function of y, this we found out from exact solution of Navier-Stokes equation. And we have seen, that how from the exact solution of Navier, Navier-Stokes equations, we derive the expression in dimensionless form.

Now, let us apply the Buckingham's pi-theorem to obtain the same expression how? You see that. Now here the problem is, that u at any location is a function of the location itself because, u is a function of y another definition, I am sorry that is the plate spacing is h. So, it will be a function of the plate spacing h, it will be a functional of u infinity; that is the velocity of this plate, it is a function of d p d x and it will be a function of the viscosity mu. Because, this is a problem where inertia force is 0 so, rho is not coming into a picture; this is a physics of the problem that you will have to understand, this is of course, difficult at this test. So, it is governed only by the pressure force than viscous force. So, there will be mu only so, y, h, u infinity, d p d x and mu. So, u depends upon this; that means, we can tell, that a functional relationship between u, y, h, u infinity, d p d x and mu defines the problem, this is the implicit functional relationship by which the problem can be described.

So, now if we apply the Bernoulli this dimensional analysis, first we write m is equal to 1 2 3 4 5 6, n is equal to what is n? What is n? 3 because, M L T 3. So, number of pi terms, number of pi terms is equal to 6 minus 3, it is 3 so, there are 3 pi terms. Now, the choice comes with the this, what is that? Choice comes with the repeating variables. So, u is the dependent variable we exclude u, better you take u infinity h and y; u infinity is the flow characteristics and u not y and h because, this will be wrong. So, two repeating variable, should not have the same dimension so, u infinity you take h. So, one with the flow characteristics I have told; another with the geometric characteristics, that better take h because, y is a variable distance better take variable dimension should not be taken, that h and another with the fluid property here rho is not there mu.

So, if you take this 3, then remaining other 3 so, this 3 will define the 3 pi terms, how? You see, pi 1 is equal to, what is pi 1? h u infinity mu. So, you take u infinity first to the power a, h to the power b and mu to the power c any one u, pi 2 u infinity to the power a, h to the power b very simple mu to the power c another one let y; and pi 3 is equal to u infinity to the power a, h to the power b, mu to the power c, d p d x there is no other parameter left. So, automatically it will take care of 3 pi.

Now, if you substitute the dimensional formula of this, L by T, L T to the power minus 1, this L M L to the power minus 1, T to the power minus 1 and this u is L T to the power minus 1; this way if you do you see here, this pi term will tell b is equal to c is equal to 0 and a is equal to minus 1. From this pi term, you will get pi 1 is u by u infinity.

Similarly, for from this pi term, if you work it out you will get the same thing. If you work it out for the exponents a b c, the values of the exponent a b c by equating with the left hand side; M 0, L to the power 0, T to the power 0. Here also you will obtain for this a is equal to c is equal to 0 and b is equal to minus 1; that means, y by h is pi 2.

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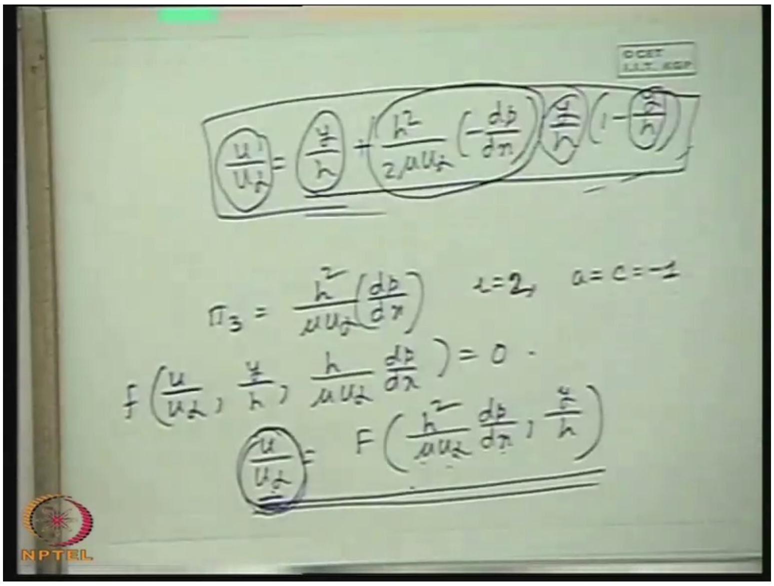

But the pi 3 if you work out, work out you will see pi 3 will be, h by mu u infinity into d p d x pi 3; which means, b is equal to 1, you will get a is equal to c is equal to minus 1. So, that you get this. Therefore then we can tell, that u by u infinity a functional. So, therefore, we know that finally, we replace the relationship as a functional relationship of the pi terms; pi 1, pi 2 and h by mu u infinity d p d x is equal to 0. So, you can write, u by u infinity is a function of h by mu u infinity d p d x and y by h. So, I have solved this problem because of two reasons; one is that, now you can see, that by the exact solutions we have found out u by u infinity as a function of h by mu u infinity d p d x, h square sorry h a square by mu infinity, I am sorry this will be h square, h square probably h square by mu u infinity. I am sorry, this will be h square by mu u u infinity I am sorry.

So, I am sorry, this will be h square by mu u infinity. So, therefore, this pi 3 terms where I have explained b that b is equal 2 h square by, but the difference is that the most interesting thing and informative thing is that by the exact solution of Navier-Stokes equation, we did use the relationship; if you remember u by u infinity is y by h plus h square by 2 mu u infinity into minus d p d x into y by h into 1 minus y by h.

Now, here the problem is that, here we cannot express this explicit functional relationship we can only recognize that, this is the dimensionless dependent variable, this is one non-dimensionless independent variable; it is another dimensionless independent. So, only the dimensionless variables both dependent and independent we can fix, but by

Buckingham's pi-theorem or dimensional analysis we can never find out the actual explicit relationships. For example the earlier problem, earlier problem also the drag force is a function of Reynolds number, what should be the explicit form of the functional relationship, is very difficult to be established by dimensional analysis. This cannot be done; this has to done either by a theoretical analysis or by doing experiments from the empirical points by regression analysis. Otherwise the explicit relationship is not possible, but in comparing the results from model to the prototype; it is not necessary to have an explicit relationship, if we know the pertinent dimensionless terms governing the problem, it is All right.

So, therefore, one of the conclusion is that, if two terms having same dimension their ratio is automatically pi term; that means, in defining a problem, if I see there are two variables of same dimension y h. So, y by h will be a pi term, similarly two variables u here u and u infinity are of the same dimension, u by u infinity is a pi term.

So, now I conclude the Buckingham’s pi-theorem by telling this, what are the rules of Buckingham’s pi-theorem? First identify the problems by its physical variables, governing the problem; that you will have to detect sometimes it is told and in actual case, it will not be told for the purpose of the examination. It may be described that is a problem is described by these variables, physical variables.

Then you can you have to find out let m is the number, if physical variables describing the problem; then you identify the number of fundamental dimension, in which the physical variables can be expressed let this is be n. So, then m minus n will be the number of pi terms or the dimensionless term describing the problem, now to find out the dimensionless terms you will have to fix the number of repeating variables, the number of repeating variables will be n. So, there will be m minus n terms left.

So, choice of repeating variables will be like, that one should be flow parameter, another should be geometrical parameter and another should be property of the fluid. Then this three repeating variables so, pi terms is set by multiplying; the three repeating variables raised to arbitrary unknown indices and the one of the rest dimensional variables. So, this way, m minus n pi terms will be formed and then the values of a b c will be found out by equating the indices of the fundamental dimension equated, for the individual pi term.

Corollaries are like this, that if these two variables of a particular problem are of same dimension their ratio will be automatically a pi term. Another thing is that, if you land up with certain pi terms by choosing certain repeating variables, you can make a combination of the pi terms keeping the number of pi terms same to describe the same problem. All right, you may see that you have not landed up with the conventional terms; then you see some combinations will give you the conventional terms, then you tell that this into this divided by this is also equivalent to that particular functional relationship; keeping the number of pi terms fix.

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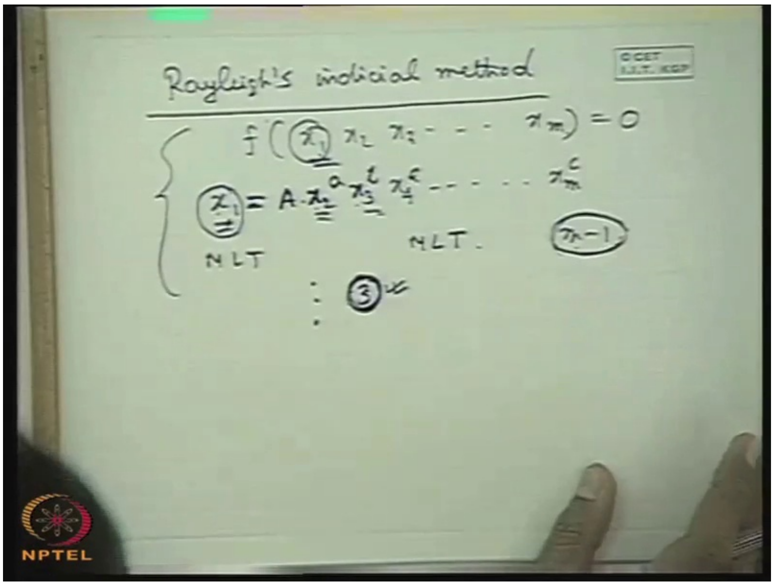

Now, I will tell you what is Rayleigh's indicial method, now next is the Rayleigh's indicial method. So, there is another method known as Rayleigh's indicial method in determining the, in determining the problem, in determining the pi terms dimensional analysis; that means, in determining the pertinent pi terms for a problem. So, just like Buckingham's pi-theorem, Rayleigh's indicial method I will describe; this is also based on the dimensional homogeneity of the physical variables involved in a problem.

So, what Rayleigh's indicial method does it recognizes for example, a problem is defined as x 1, x 2, x 3, dot dot dot x m number of variables. So, what it does? It first recognizes the dependent variable or output parameter and expresses this, as a power law type of relationship some constant, A x 2 to power a power law type of dependency or power law type of dependence, with the dependent variables.

That means with the independent variables sorry; that means, the in Rayleigh's method we first recognize the dependent variable and express the dependent variable, as a power law product of the indices product of the independent variable raise to unknown indices with a constant multiplying with a constant. Then this unknown indices what happened we have got, if we equate the M L and T in from right hand and left hand side, we have got three constraining equation, three constraining equation. That means, that if we have a number of unknowns for example, here if m is this so, m minus 1 unknowns will be there, a b c like that because, dependent variables is here. So, m minus 1 independent variables, if total variables are m. So, m minus 1 unknowns, but we have got 3 constraining equations so, these 3 equations are eliminated. So, m minus 1 unknown, they are expressed in terms of 3 variables; less than three variables. So, three variables are eliminated for m minus 1 unknown, in finding out the grouping of the pi terms. So, this is the key point of the Rayleigh's indicial method or the central role of the Rayleigh's indicial method.

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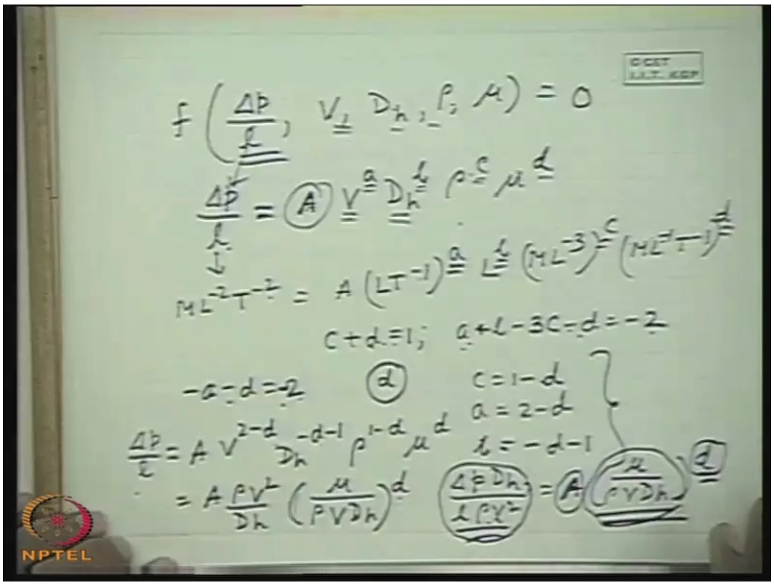

Let us describe the Rayleigh's indicial method for a pipe flow problem, then only it will be clear. The, what is pipe flow problem? Pipe flow problem we know, that delta p by l, function of delta p by the pipe flow problem we just define like this; D h rho mu 0. So, the first problem which I solved, that this is the defined describe by this, 1 2 3 4 5 variables. Now Raleigh's indicial method does like this, they first identify the dependent variable and they it starts with that let delta p by l becomes a function of the independent variable of this nature; V to the power a D h to the power b rho to the power c mu to the power d.

Let us express or let us assume that delta p by l becomes this type of relationship, bears this type of relationship with the independent variables. Then what it does in the next step? That equates the fundamental dimensions; what is the fundamental dimension? M L to the power minus 2, T to the power minus 2, A L T to the power minus 1 whole to the power a, L to the power b, M L to the power minus 3 whole to the power c, M L to the power minus 1 T to the power minus 1 whole to the power d.

Now, if we equate the M L T, you will get, c plus d is equal to 1. Equating this a plus b minus 3 c minus d is equal to minus 2; that is L a plus b minus 3 c and T for T minus a minus d is equal 2. So, here the difference is that, if we express like this and equate the M L T dimensions, we will be always getting three constraining equation. But here we see, a b c d four unknowns are there so, what we can do three we can eliminate; that means, we can express all the unknown in terms of 1.

Let us consider, that we will express all the unknown in terms of d. So, what it comes? d c is equal to immediately 1 minus d, a is equal to 2 minus d and what? Please there is any mistake what is please tell quick, minus a minus d is equal to minus 2; this is All right? minus a minus d is equal to minus 2, this two are All right. Now c is equal to 1 minus d, a is equal to what is a? a is equal to 2 minus d and if you put c and d you will get, b is equal to, well you will get, b is equal to minus d minus 1.

So, everything is put in terms of d, then I put here, delta p by l is equal to A V to the power a; that means, V to the power 2 minus d, D h to the power minus d minus 1, rho to the power c that is, rho to the power 1 minus d. So, everything now is expressed in terms of d. Now this can be written like that, if we take rho V; that means, this is separate rho V square and what is that? D h minus 1 by D h, then we can club V to the power minus d, D h to the power minus d, rho to the minus d mu; that means, mu by rho V D h to the power d.

So, therefore, it is done or you can write if you multiply this, delta p D h by l rho V square is A into mu by rho V D h to the power d; that means, I can recognize the dimensionless terms, but this A d values I do not know, I do not know the explicit form. This has to be found only from experiments or from theory, but I know that delta p D h l by rho V square is one variable in dimensionless form; which is the dependent variable and independent variable is mu by rho V D h, this is the way the Raleigh's indicial method does. That means, it identifies the dependent variable and express the dependent variable, as a product form of the independent variable raise to the power unknown integer indices, multiplied with a constant.

This is a style, this you can make one, this does not play a role in this process; then equate M L T from both the sides. You get three constraining equations number of unknowns may be more, but you can eliminate three; for example, here number of unknowns are four. So, if you eliminate three, you will be expressing in terms of only one and you can club it. If the number of unknowns are five we would have eliminated three; that means, five unknowns could have been expressed, in terms of only two unknowns and you will see, that two unknowns are such that you can club the grouping for the pi terms automatically. So, this is clear, this is Raleigh's indicial method.

Now, another so, these two methods we have discussed dimensional analysis by Buckingham, Buckingham's pi-theorem; another is the Raleigh's indicial method, but most interesting or I shall tell the most easy method is the Buckingham pi-theorem; Raleigh's indicial method sometimes gives us problem because, if there are more number of pi terms. So, a problem is governed by large number of variables, then what happens, sometimes in eliminating three from the three constraining equations with a large number of unknowns may be difficult job and there is chance of performing mistakes. So, it is better to proceed with the Buckingham pi-theorem to recognize the dimensionless terms defining a problem.

So, therefore, again I repeat the dimensionless pi term dimensionless Buckingham's pi-theorem, first you recognize the number of variables of the system; then you recognize the number of fundamental dimensions expressing, that variables defining the problem. And then the number of pi terms will be m minus n, here very interesting question comes, none of you have asked this; sir what happens, if m is equal n or m is equal to less than m, if m is less than equal to n. That means, a problem is described by number of variables; which is either equal to or less than the number of fundamental dimensions, then m minus n 0 or m minus n less than 0. So, there is no pi term; that means, no process is described by that. When m is equal m less than n is an impossible situation and m is equal to n means, this represents a equilibrium state of a system. So, a process has to be described, m has to be more than n so, m can not be less than n. So, therefore, m minus n will be the pertinent dimensionless terms defining the problem.

Then you choose the repeating variables judiciously number is n and these repeating variables must contain; one flow parameter, another geometric characteristics, another what is another one? One flow, one geometric, another is liquid property or fluid property. Then you go for finding out the different pi term, if two variables have the same dimensions their ratio is automatically a pi term.

So, this is all in discussing the similarity principle and one must know that, why we are finding out the dimensionless term; because, these are the terms representing the criteria of similarity. So, if one has to make experiments in the laboratory to predict the performance for the actual systems operating under different set of conditions; yes. He can do the experiments under alter sets, but his dimensionless parameter, the dimensionless parameters between the two systems operating under different conditions have to be kept fixed, to maintain the criteria of similarity.

And moreover the experiments that have to be performed is reduce the number of experiments because, the influence of one variable will show the influence of other variables in a group; in a particular group. These are the two advantages of making use of the dimension analysis and through the Buckingham pi-theorem and Raleigh’s indicial method in developing this functional relationship between the dimensionless terms of a particular physical phenomenon.

Thank you.

Good morning. I welcome you all to this session of fluid mechanics. Today we will start a new chapter that is flow of ideal fluids. We will describe here a brief discussion on the flow of ideal fluids. Now what is an ideal fluid? We have already recognized the definition an ideal fluid is a fluid, which has know viscosity and therefore, flow of such fluids known as inviscid fluids which has know viscosity, is known as ideal fluids. In addition to this non viscous nature of the fluid, that means 0 viscosity, we will impose here two other properties of the fluid that incompressible that density remains constant and at the same time irrotational, the rotation in the flow of fluid is 0.

So, therefore, we will mainly discuss the flow of an ideal fluid with incompressibility, ideal incompressible; that means inviscid, incompressible and irrotational flow. So, let us recall few of the corollaries of the incompressibility and irrotationality in a flow field.

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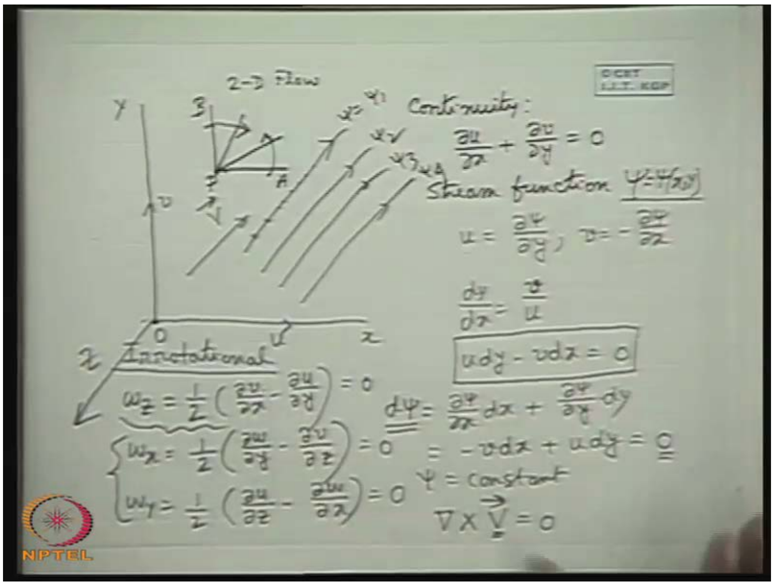

Let us concentrate our analysis on a two-dimensional frame of reference. Let us take a two-dimensional, x and y in a Cartesian coordinate system. We consider a two-dimensional flow in a Cartesian coordinate system described by x and y axis.

Now, in this case we know that a incompressible flow, for an incompressible flow the equation of continuity, the equation of continuity conservation of mass, for an incompressible flow is del u del x plus del v del y is 0, where u and v is the velocity in the x direction and y direction; that means, velocity vector v has the component u and v in along x and y axis respectively.

Now, as a consequence of this continuity equation we define a function, stream function, we have discussed in details about this earlier stream function psi which is a function of space coordinates x y. In such a way that the velocity components are defined like this u is del psi del y and v is minus del psi del x; one is positive another is negative. So that, the psi function that is the stream function automatically satisfies the continuity. You see that if we substitute u and v. From this definition we will get that this automatically becomes 0; that means, stream function automatically satisfies the continuity, this we have already recognized earlier.

Now, you see that we know that for a stream line this stream function psi remains constant along the line. Now, if were call the definition of a stream line, stream line definition we know that the tangent to it at any point represents the velocity vector. And the equation of a stream line is that d y d x in a two-dimensional frame is equal to v by u because, the slope of this stream line gives the direction of the velocity vector. So, for a two-dimensional case, it is like this which gives the u d y minus v d x is equal to 0. This is nothing, but the equation of a stream line.

Now, let us see that if we expand psi in a differential form, we can write d psi, psi being a function of both x and y; we can write del psi del x d x plus del psi del y d y. Now, del psi del x by definition is v minus v and del psi del y by definition is u d y. Since the equation of a stream line; that means, along a stream line u d y minus v d x is 0 so, d psi is 0. So, along a stream line d psi is 0; that means, psi is equal to constant.

So, therefore, we know that this stream lines for example, if this be the stream lines that any instant in a steady state, this stream line remains same at all instant of time. So, psi is equal to constant; that is psi 1, let this is psi 2, let this is psi 3; that means, we can ascribe a single value of psi for each and every point along a stream line. So, this we have discussed earlier.

Now, if the flow is irrotational, if the flow is irrotational, now in a two irrotational flow means we have discussed earlier that is a flow where rotation is 0; so, rotation has got three components along the three axis, coordinate axis; now in a two-dimensional flow there is only one component of rotation; that is the rotation about the z axis. How do you define rotation? We defined it earlier the rotation is the arithmetic average of the angular velocity of two linear segments which were initially perpendicular to each other; that means, if we take a point p and a p and b p are the two linear segments mutually perpendicular to each other. Then, because of the flow field this tendency of rotation of these linear elements in the opposite direction; for example, if we take the arithmetic average of the angular velocities of these two linear elements, we get the rotational component that we have already derived earlier. This with respect to z axis; that means, an axis perpendicular to the plane of this figure; that means, at p the axis is perpendicular to the parallel to a z axis here which can be represented like this. It will be half del v del x minus del u del y.

Now, we can recognize the other component of the rotation; that means, x; that means, in y z plane, rotation in y z plane, this is rotation in x y plane. That is rotation about z axis; rotation in y z plane that is about x axis, rotation like this. It will be y and z; that means, del w del y minus del v del z. Rotation in about x axis that is omega x del w del y minus del v del z. Similarly, with respect to y it will be del u del z minus del, y it is; that means, del w del x; del u del z minus del w; so, three rotational component. And for a irrotational flow all this three components become independently 0. For a two-dimensional case we do not come across this, defined in an x y plane; so, rotation is only this; so, this becomes 0. So, this three together is written in the form if you recollect is 0 where V were is the velocity vector which has got three distinct component u v and w.

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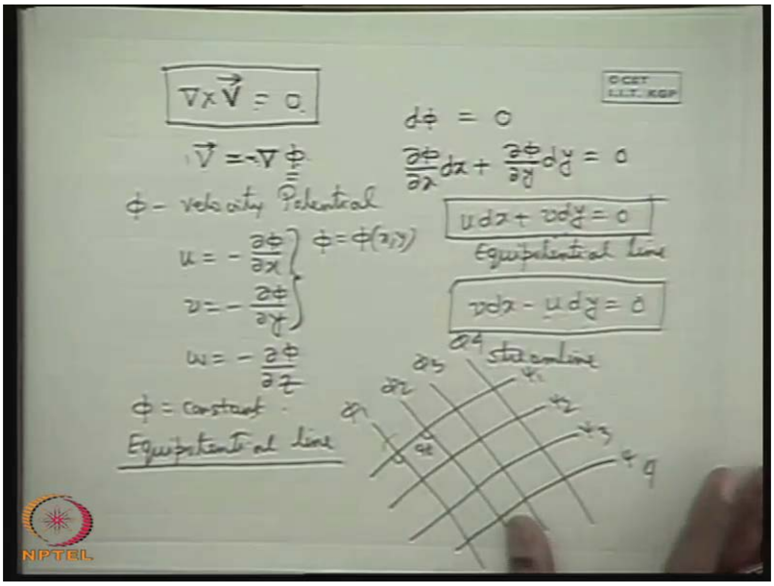

Now, for an irrotational flow del v del x minus del u del y is 0 or we can tell that curl of velocity vector is 0. Now, a consequence of this mathematical relationship is that this vector V can be expressed as a gradient of a scalar function phi. So, this is the mathematical theorem that the curl of any vector is 0; that vector can be obtained or can be expressed as a gradient of a scalar function. So, this scalar function phi is known as the potential function. If this vector is the velocity vector in case of irrotational flow curl of this velocity vector is 0. Then velocity vector can be obtained as a gradient of the scalar function phi which is known as velocity potential function, velocity potential function or simply velocity potential.

Now, a negative sign is deliberately used to make the things similar to all phenomenological equations, where any flux is proportional to the negative gradient of the potential function. So, therefore, a negative sign is implied. So for that, if you just expand this; then, we can write for three-dimensional cases with respect to a Cartesian coordinate system; simply we can write u is equal to minus del phi del x, v is equal to minus del phi del y and w is equal to minus del phi del z. So, for a two-dimensional case we can deal with this two; u is equal to minus del phi del x, v is equal to minus del phi del y. And the curve or the line along which phi remains constant, is known as equipotential line, is known as equipotential line, equipotential line.

Now, let us find out the equation of equipotential line. Equation of equipotential line means phi is equal to constant; this is the equation, but we try to derive it in terms of u and v; what to do? You write it in a differential form that d phi is 0. Now what is d phi? d phi is, because phi is a function of x and y. This is the scalar function point coordinate x and y in case of two-dimensional system. In three-dimensional flow it is a function of x y and z. So, it is del phi del x d x plus del phi del y d y and that must be equal to 0 along a equipotential line.

So, del phi del x is minus u, del phi del y is minus v. So, therefore, we see u d x plus v d y is 0. So, this is the equation of equipotential line, equation of equipotential line, equation of equipotential line. Similarly, equation of stream line is v d x minus u d y is 0. This is the equation of stream line, this is the equation of stream line. So, immediate inspection of this two equation said that, they are mutually orthogonal; they are perpendicular to each other, u d x plus v d y is 0 and it is v d x minus u d y is 0. The slope of this line at any point then, slope of this line at any point are perpendicular to each other.

So, therefore, we can say that if this be the stream lines, this be the constant stream lines psi 1, psi 2, psi 3, psi 4. Then the orthogonal families of curves will be the constant; five lines. So, they will be, this corners will be 90 degree. So, they mutually form a orthogonal sets of trajectories. So, this is constant psi line; these are the constant five lines.

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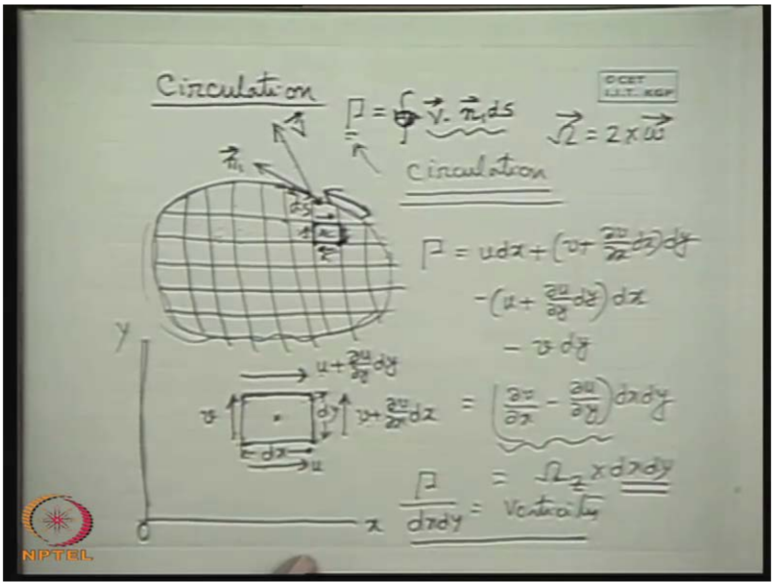

Now, we come to the concept of circulation; what is meant by, circulation, what is meant by circulation? Let us consider in a flow field, a closed-loop, a closed-loop and bounded by a closed contour. Now, if we designate some small elemental length as d s where, the velocity vector is v bar; then, the contour integral of this v dot let us define a vector, normal unit vector along the tangent to this; that means, along the tangent to the curve at this point; that means, along the direction of this linear element d s and unit vector n 1 we define, then n 1 d s; which means that, the elemental d s is given a vector concept by multiplying it with a unit vector along the elemental line; that means, along the tangent to that line.

Then, this quantity if we make a contour integral over this closed contour; then we define this as the circulation gamma, which is the symbol. So, gamma is the circulation, circulation. It is simply means that, the length a linear elemental length multiplied by the component of the velocity along that length. So, velocity vector may have in any direction component of the velocity vector along that length and if we sum up all such quantities product of the velocity component, along a linear, along a infinite small elemental length and product of the length times the velocity component along that direction and sum it up over a close contour. Then this is known as circulation around that close contour or close circuit. This is known as circulation.

Now, we can show that the circulation for in a any close contour can be found out by the circulation of small grids or small contours, generated within that big contour, close contour; that means, for example, if this close contour is now divided into a number of small circuits, this big circuit is divided in number of small circuits and you see that, if we have the velocity component like this, now what happens? Before that I must say that when we make this integral, we take certain direction for this integral along the close contour; if we make in this direction usually taken as positive conventionally. So, this is given a direction like that a contour integral or a close contour integral along a close contour is given with a sign. So, this is positive along this sign; that means, conventionally taken the anti-clockwise direction or rather this way you take an anti-clock wise direction. This is not anti-clockwise; so, this is an anti-clockwise direction. Let us consider the anti-clockwise direction; that means, this direction anti-clockwise direction as a positive.

Now, you see therefore, for a small circle the, therefore the circulation will be defined the velocity vector multiplied by this length; velocity vector multiplied by this length; this velocity vector multiplied by this length; this velocity vector multiplied by this length. Now you consider the common edges; that means, this edge the contribution of the velocity in v at this edge in the circulation for this grid, is in one direction and for this grid is in other direction. So, for all this common surface it is like that it contributes in the calculation of circulation for one grid, one close circuit, in one direction and another adjacent close circuit in another direction. So, that they ultimately nullify each other or cancel each other and we are left only with these contributions along the outer most boundary. So, that ultimately if we sum up the circulations of these infinites small circuits we get the circulation around the close circle.

Now let us find out the circulation in case of x y coordinate. Let us see a simple case consider a circuit, we can choose a circuit a rectangular circuit like this, of length d x and length d y. What will be the circulation? Let us consider at this plane the velocity is u and at this plane the velocity will be changed for a distance d y will be u plus del u del y d y. Similarly, let us consider the velocity at this plane will be v; positive direction. So, velocity at this plane will be changed for a distance d x as v plus del v del x d x.

Now, let us find out the circulation with the anti-clockwise direction as the positive direction. Now, what is this? This will be u for this length, this elemental length u into d x and this is, this has got an anti-clockwise rotation about any point within the circuit. This is the way the direction for the rotation has to be taken care of.

Similarly, for this element it is the velocity along this element is v plus del v del x d x. u does not have any component for this which is perpendicular to this linear element. So, therefore, it is the velocity component along the linear element times d y; this gives also in the same directional notation. So, plus v plus del v del x d x times d y. Then for this, it is an opposite direction, but the magnitude is this velocity component times the length d x; that means, minus u plus del u del y d y, this is the velocity component times d x, minus this one in the same direction; that means, minus v times this length d y.

So, if you make it simple. So, it will become ultimately del v del x minus del u del y into d x d y. This is because, see v d y v d y cancels u d x minus u d x cancels, but this is our twice the rotation and this is you know is the vorticity about the z axis. We earlier defined vorticity is the two times the rotation vector; if you represent vorticity as a vector then it is two times the rotation vector. So, this is the vorticity. So, vorticity times d x d y. What is d x d y? d x d y is the area of this contour. So, we can tell that circulation divided by this area is the vorticity, this is vorticity. So, this is a very important theorem from which we can write that vorticity.