Fluid Mechanics
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Lecture - 46
Introduction to Laminar Boundary Layer Part – I

Good morning, I welcome you to the session of fluid mechanics. Today, we will be starting a new topic that is introduction to laminar boundary layer; earlier in few discussions or in earlier sessions, while discussing viscous flow, we have recognized that due to the fluid viscosity the velocity at different layers of the fluids are different. This is because of the viscous effect of the fluid; moreover it is because of the frictional interaction between the fluid and the solid. The velocity of the fluid element at the solid surface with respect to the solid surface is 0, when fluid flow passed a solid surface; this is known as no slip condition. If we consider this solid surface is at absolute rest, then we can tell the no slip condition as the velocity of the fluid particular at the solid surface is 0.

So therefore, we see the velocity of the fluid particles are gradually retarding or decreasing to a 0 velocity at the solid surface. So therefore, the velocity gradients are very large at the solid surface. So therefore, if we have a very high flow velocities passed a solid surface so velocities will be retarding near the solid surface and will be 0 at the solid surface. Now it has been found in practice that the, if the fluid flow with a very high velocity or the fluid viscosity is very low. Then what happens? Is that this velocity gradient that is gradient of the velocity in a direction normal to the flow of fluid is limited or arrested within a very narrow region, near the vicinity of the solid surface. So, this gives the preliminary introduction to the concept of boundary layer.

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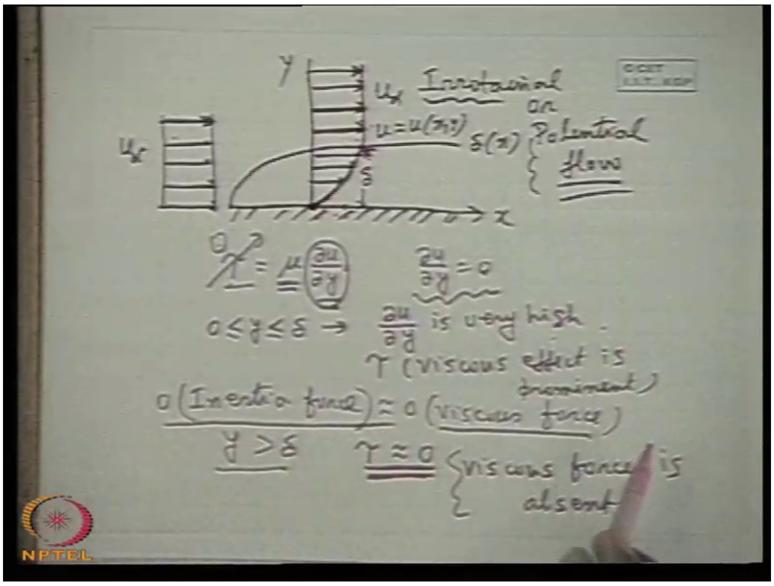

Let us investigate the situation like this, let us consider a the fluid flow over a flat solid surface a simple case, for example, a flat solid surface; what is the situation, let us consider the fluid approaches with a uniform velocity, u infinity, whose magnitude is u infinity? So, when the fluid first flow pass this solid surface, we see that there is a formation of a layer like that, that we just define a layer like that, so that at any section we can describe the velocity profile like this, we can describe the velocity profile like this.

Let me first draw, then I will explain the situation; that means, well that means, at any section, the velocity starting from 0, because of the no slip condition of the fluid. Increases from 0 to the free stream velocity beyond a certain distance, this distance from the (()) this is the delta; this is very small depending up on the flow velocity. So, therefore, we see that within a small distance in a direction perpendicular to the surface plate we consider this is as x and this as y we see in the direction y. Within a small distance from the plate the velocity gradient is confined, which means beyond a small distance delta from the plate in the y direction, fluid approaches almost is original uniform velocity; who which is known as free stream velocity.

So, the velocity profile is almost uniform beyond a certain distance delta. Who, which is a function of the direction x, which increases in the direction x like this. So, this the phenomena based on which therefore, we can write a very interesting thing. The shear stress at any layer in the flow of fluid as we know from our basic knowledge; can be written as mu in to del u del y, if u is the velocity, which is a function of both x and y at any section. So, it is del u del y. del u del y is the velocity gradient, the gradient of velocity in this normal direction; which is nothing, but the shear rate or rate of shear stress. So, shear stress at any layer is simply mu del u del y.

So, therefore, we see the value of shear stress beyond that thickness delta where the flow velocity is almost equal to free stream velocity u infinity. That is uniform del u del y is 0, which means the shear stress is 0; that means, even if there is a viscosity of the fluid because of the velocity gradient being 0. The shear stress vanishes; that means, the influence of shear stress is therefore, arrested within a very small layer or within a very small distance from the plate. So, therefore, we can write that 0 less than equal to y, less than equal to delta; we can see we can tell that del u del y velocity gradient is very high and at the same time the shear stress or the viscous effect, viscous effect is prominent, viscous effect is prominent; which means that within this region the inertia force and the viscous force are of equal order of magnitude, because inertia force is always there because the fluid is accelerated in the direction.

So, inertia force the order like that we can write the order of inertia force is equal to the order of viscous force. Whereas we see that when y is greater than delta beyond a thickness beyond which the velocity attains almost the u infinity velocity. So, tau is almost equal to 0, that is shear stress vanishes because del u del y becomes almost equal to 0 and viscous force is not prominent. So, viscous force is vanishing, viscous force is absent; that means, this part of the flow is an irrotational flow that is an irrotational or potential flow, irrotational or potential, irrotational that is the flow which is executed by an inviscid flow, that is an inviscid fluid.

Therefore we see that even for a viscous fluid beyond a certain distance in the direction normal to the fluid flow from the surface, the fluid velocity becomes almost uniform in the direction normal to the flow. So, that the flow becomes irrotational or potential flow. There because of this gradient being 0, the shear stress vanishes, shear stress vanishes. So, therefore, at this distance; that means, a large distance away from the plate, tau is almost 0 and viscous force is absent.

So, in ideal fluid flow or ideal fluid or ideal fluid flow theory can be applied in this region, but in the near vicinity of the plate we are within certain thickness the velocity gradient is very high and prominent and this is maximum at the surface. So, that the shear stress is exerted at the surface and the shear stress in the flow field is prominent within that region. So, in this region the flow is just like a viscous flow which we have discussed earlier where the viscous force inertia force is of equal order of magnitude.

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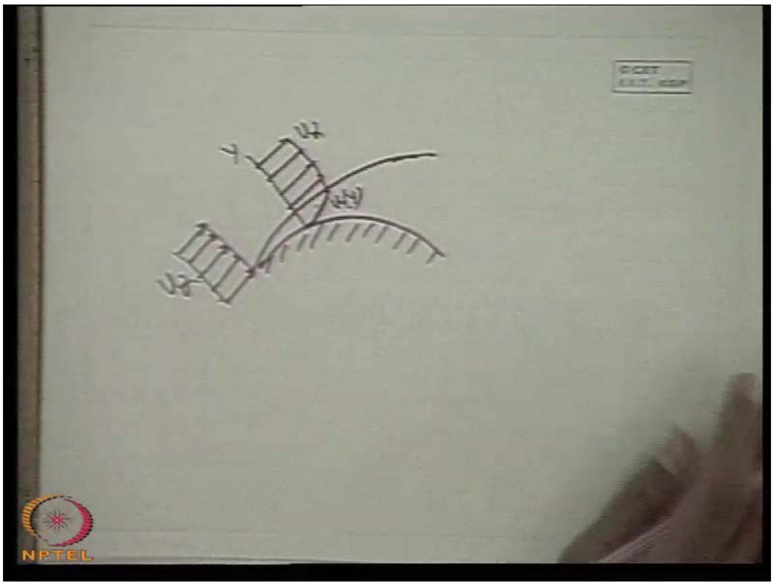

Now this happens for all types of fluid, it is not that for a all types of surface, for a plane surface, we can explain this for a cap surface just like the flow positive cylinder. We can also explain this, like this, that there is a uniform flow which is approaching with a velocity of u infinity; that is uniform, when if first a cap surface there also within a thickness with a velocity is varying. So, this is u infinity. So, this is u. So, this is y; u as a function of y, within this small distance. That is the boundary layer. So, this is the concept.

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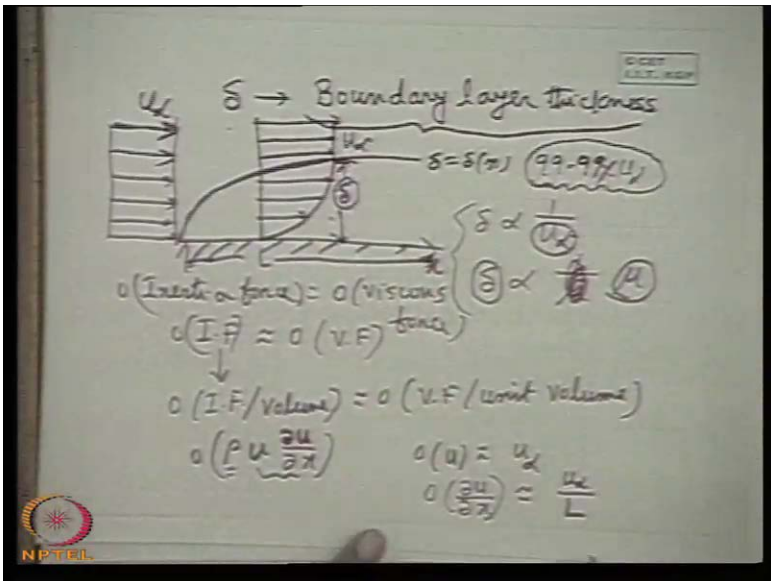

Now the question comes that, what is this boundary? This delta, this thickness is known as boundary layer thickness, boundary layer thickness. Within which the velocity gradient is prominent and the viscous effect is arrested, boundary layer thickness; that is the thickness of the distance from the solid surface, again if we draw this flat surface concept. So, boundary layer this is a function of x goes on increasing.

So, now next question comes that if this is this. In fact, this there is no such physical boundary, it is a hypothetical boundary and another thing we will see afterwards that, this thickness delta if we consider up beyond which the velocity does not exactly becomes u infinity; this is an asymptotic approach towards the free stream velocity. So, that beyond a finite thickness the velocity attains almost; for example, 99.99 percent of u infinity, where we can define the boundary layer thickness; as that thickness prescribing the attainment of certain percent of the of the u infinity. Just like the establishment of flow, we have seen that studies never have matched theoretically, but if we consider the time for which the there is 99 percent of the steady state reach that; that is known as the time for establishment. Here also the thickness beyond which the 99.99 percent of the u infinity is, but for our understanding we consider that as if there is a thickness beyond which it attains exactly u infinity.

Now, the next question comes, that this thickness which we define as the boundary layer thickness; within which the velocity gradient is prominent then the shear stress effect is arrested, what is this value of this thickness and on which it depends. This is the information that this if the flow velocity free stream velocity u infinity or the approach velocity is very high, delta is very low; that means, delta is an inverse function of 1 by u infinity and delta is also an inverse function of viscosity; that means, if viscosity is low sorry it is not an inverse function, direct function. That means fluid with a very high velocity and fluid with a low viscosity gives a lower value of boundary layer thickness then; that means, if the fluid flows with a very high velocity this thickness becomes lower and lower and if the fluid has a very small viscosity the thickness becomes lower and lower. An approximate quantitative analysis we can make from an order of a magnitude analysis.

Now we know in this region, inertia force, inertia force, if the order of inertia force is equal to the order of viscous force. All right; order of viscous force. Let us right inertia force by I F is equal to order of viscous force, order of inertia force into order of viscous force. Now what is the order of inertia force? So, order inertia force we know per unit volume. Let us consider the order of inertia force per unit volume we can write, is equal to order of viscous force per unit volume.

Now if you recollect our earlier discussion in viscous flow, we know that inertia force per unit volume can be written as, the order can be written as; rho u del u del x. why? Because, it is rho times the acceleration and acceleration if it is x; let us consider a steady states acceleration you know that u del u del x, it is in the order of u del u del x, that is the convictive acceleration. Now, if this is. So, this inertia force rho u del u del x. So, in this case what is the order of u? The order of u is u infinity is the free stream velocity and order of del u del x, this again I will explain in detail; when you will be deriving the boundary layer equation is equal to u infinity by L, is any characteristic length L is any characteristic length of the problem. For example let this is the length of the plate total length of the plate. So, therefore, u del u del x is u infinity square by L.

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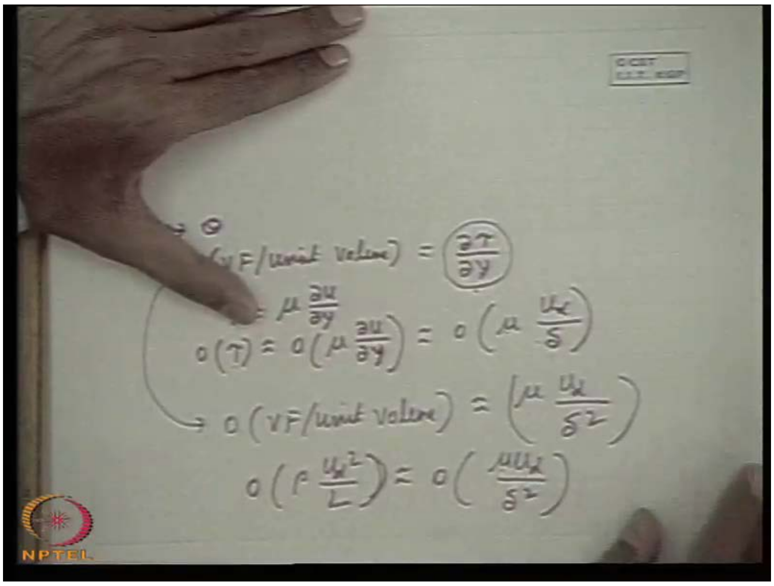

So, therefore, the order of the inertia force per unit volume the order of this becomes equal to the order of becomes equal, to order of rho u infinity square by L. In terms of the input parameters of the problem; that means characterizing parameter of the problem free stream velocity u infinity and the characteristic length L. So, it becomes in the order of rho infinity square L, now what is the order of viscous force per unit volume? Let us write. So, therefore, the order of viscous force per unit volume, what is that? You know that viscous force per unit volume is equal to del tau del y; this can be proved that if we consider a fluid element here. If we consider a fluid element here where this is a shear stress here is tau and this we have discussed earlier in the viscous flow theory and if the shear stress here, is the shear stress is tau plus delta of del y d y; then we can write that the next shear force in this element per unit volume if we consider d y in this direction if we consider d x this length and unit length in perpendicular direction. Then we can show simply that shear viscous force per unit volume of del tau plus del y.

Again what is tau? Tau is equal to mu del u del y. So, therefore, the order of tau the order of shear stress is equal to the order of mu del u del y. Now, what is the order of mu del u del y, is the order of mu del u is the change in velocity that is the order of u infinity and del y is the change in the length in the y direction. Now we are considering only in the boundary we are reach here; where the extent is in the y direction is up to delta boundary layer thickness. So, therefore, the order of delta y is the order of delta because beyond which the viscous force is not prominent therefore, the effect of viscous force is 0. So, therefore, here it will be the order of delta. So, therefore, the order of shear stress is mu y u infinity by delta. So, the order of del tau del y another del y is there.

So, therefore, the order of viscous force per unit volume will be the order of del tau del y; that means, tau the order is this therefore, the order will be mu u infinity del y is again delta, delta square. So, this will be the order of viscous force per unit volume. So, therefore, if we make now you see the order of inertia force per unit volume is rho u infinity square by L. So, we write the order of rho u infinity square by L, will be equal to the order mu u infinity by delta square. This will give the physical picture that within the boundary layer inertia force and viscous force are of equal order. So, inertia force and viscous force are of equal order, which gives us that the order of rho u infinity square by L is equal to the order of mu u infinity by delta square.

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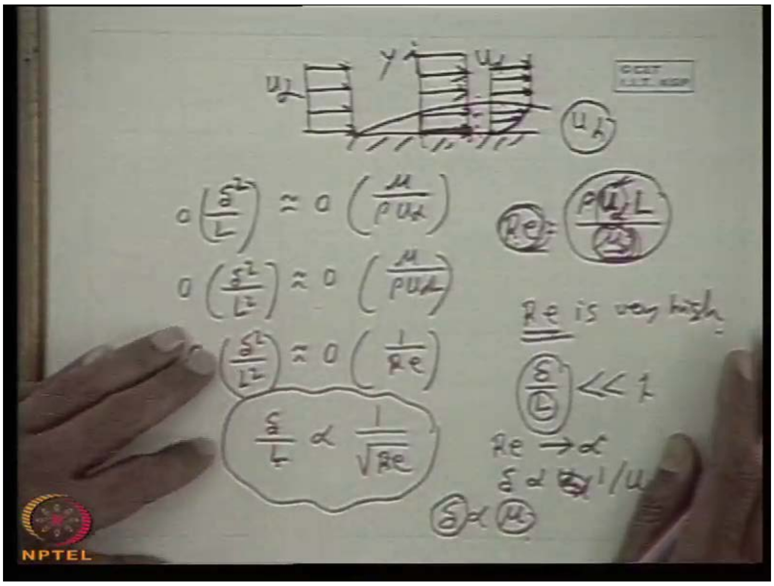

Now if we solve this, we can get if you take this we get delta square by L, the order of delta square by L is equal to order of mu by rho u infinity or simply we can write the order of delta square by L square is equal to order of mu rho by u infinity.

So, order of delta square by L square is equal to the order of; what is this if we define the Reynolds number as we have seen the definition of Reynolds number is rho sorry here one L will be there we have divided by L, rho times the characteristic velocity here in this case this is the free stream velocity and a characteristic length L divided by mu. So, therefore, if we define the Reynolds number based on the characteristic velocity as the free stream velocity, free stream velocity as the characteristic velocity. Here you must know one thing forever, that the characteristic velocity defining the Reynolds number in case of flow pass solid body is the free stream velocity. This length varies depending up on the problem in this case, this is the length of the or the length in the direction of flow; this is the viscosity, this is the density rheological property. So, this way we define the Reynolds number our typical Reynolds number. So, this 1 by Reynolds number.

So, here we see that delta by L is proportional, this means the order are same; that means the proportional root over by Reynolds number, which is a very important conclusion. So, therefore, we see in terms of dimensionless term we can tell, that when Reynolds number is very high; that means, if a flow takes place with a very high Reynolds number, then the boundary layer thickness is very small compared to any characteristic length of the problem; that means, the ratio of delta y by L is very very less than equal to 1. So, as Reynolds number goes on increasing this becomes very low. So, in a very limiting case when Reynolds number is infinity tending to infinity; that means, in finite velocity. So, this becomes 0.

Again we see that high Reynolds number means either high velocity of flow; that means, when the velocity of flow is high delta by L is very small. So, in terms of dimensionless parameter the boundary layer thickness becomes small; that means, delta is proportional to u infinity; if you put this value. So, we can see that delta is inversely proportional 1 by u infinity; that means, if u infinity attains an infinite value. So, delta is almost 0. Again from this if we put here dimensionless term from here we can see that delta is directly proportional to mu; that means, when mu is 0. For an ideal flow what is Reynolds number infinity? Though Reynolds number is defined for an inviscid fluid, but if you want to define it for an inviscid fluid it is infinity. That means delta by L is 0; which means, when mu is 0, delta is 0. Therefore, for an inviscid fluid there is no boundary layer thickness; that means, there is no retardation of the fluid.

So, if the fluid approaches with a uniform velocity u infinity. So, at any section the fluid velocity will be like that u infinity uniform and near at the surface also fluid flows with the velocity u infinity; that means, fluid will slip from the surface. So, mostly no slip condition is much prevailing in case of an inviscid fluid. So, therefore, there is no velocity gradient in a direction, perpendicular to the flow direction the flow is uniform in that direction same; velocity also at the solid surface the fluid velocity is the same, as that in other places. So, that u infinity is constant; that means, delta is 0 for an inviscid fluid. So, therefore, if the fluid viscosity is very small or fluid velocity is very high or in one word if the fluid flow is very high than Reynolds number. So, the region near the plate very small region delta region becomes thinner and thinner. So, there is a thin region near the solid surface within which the velocity gradient; that means, this one will be prominent and the shear stress will be affected.

Nevertheless one thing you have to remember that there will be some thin region because from here you can see the mu can never become 0 or infinity can never become infinity. So, that delta can never become 0. That means whatever high velocity the fluid can posses, whatever low viscosity the fluid can have, there must be a thin region adjacent to the solid surface within which the velocity gradient and the consequent effect of this viscosity is arrested. Beyond which the fluid can be treated as an inviscid fluid that the fluid flow is irrotational or potential flow. So, therefore, we see the boundary layer device the flow in to two regimes; one is the viscous flow regimes, another is the irrotational or potential flow regimes.

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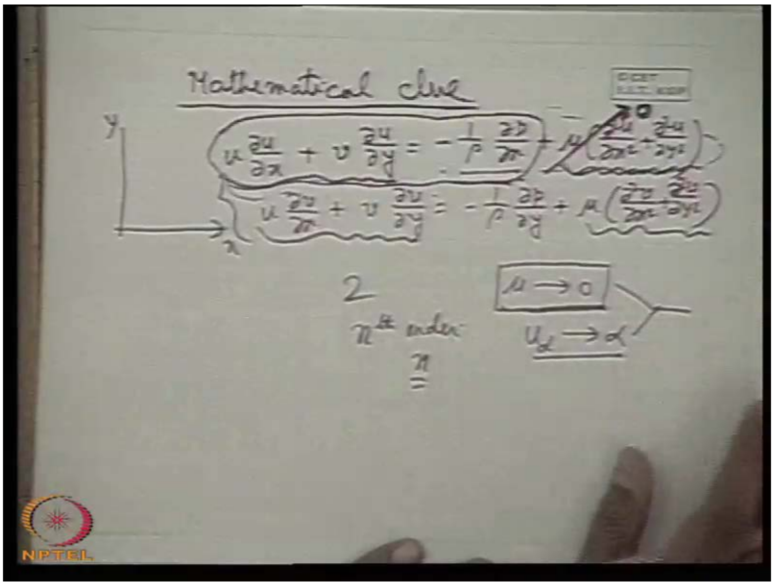

Now, I will give some mathematical clue to this problem, which first came in to the mind of prandtl, who gave the revolutionary concept of boundary layer that mathematical clue, a very interesting mathematical clue. Now you see, if you remember the viscous flow equation of motions which are known as Navier-Stokes equation for a 2 dimensional Cartesian frame of reference if you write these equations.

Let us recollect this equation, u del u del x with x y co-ordinate with Cartesian x y co-ordinate x direction plus v del u del y is called to minus 1 up on rho del u del x plus mu del square u del x square plus del square u del u del y square. Similarly, we have at y direction equation u del v del x equation of motion plus v del v del y is equal to minus 1 upon rho let us considered 2 dimensional equation for an incompressible steady flow del square del v del x square plus del square v del y square.

So, these are the terms responsible for the viscous flow, if you make this become 0 for an inviscid fluid; when mu is equal to 0. So, this equation becomes the Euler's equation, this is the inertia force that is the mass times the acceleration it is written per unit mass basis. So, it only the acceleration part. So, convictive acceleration in x direction, convictive acceleration in y direction for a steady flow and this is the pressure force. So, this part is the Euler's equation this is the addition for the viscous time or the viscous flow. This we have already seen in the viscous flow theory. Now you see these equations are second order differential equations, now one thing you see that when the viscosity is very small very very small for example, mu tending to 0.

Then what happens if you neglect this term. Then an approximate equation can be developed which is the Euler's equation, but if we solve these equation for the entire flow field for a limiting condition of viscosity tending to 0 or u infinity tending to infinity this two are same. When we will non-dimensionalize these equation you will see that, but here there is no chance of seeing this two thing same. So, therefore, we consider here when mu tends to 0, for that limiting condition if we approximate this equation by just dropping this term, then it will be erroneous mathematically to solve this equation for the entire flow region. Do you know why? This is a mathematical contradiction or this is the mathematical constant that you cannot approximate any differential equation by dropping one term or a number of terms which reduces the order of the equation. Now you see the order of the equation that is, this Navier-Stokes equation is 2 second order and the order of the equation is contributed by this term. This is the second order term. So, therefore, if we make this term 0; that means, the equation is transformed from second order to first order. Now you see the order of a differential equation for it is solution requires the same number of boundary condition That means if you have to solve a differential equation of second order, we require two boundary condition if we solve a differential equation of n th order, n th order we require n number of boundary conditions.

So, therefore, when we suppress the order of a differential equation, then we require a less number of boundary condition for this complete solution; that means, few boundary conditions are become redundant. So, actual physical situation is such, even if mu is very small there should be a term like that which gives a, second order characteristics of the equation, whenever we make an approximation by suppression this term; that means, we suppress the order of magnitude we reduce by 1. So, one of the boundary conditions will be missing. So, that one physical information is missing.

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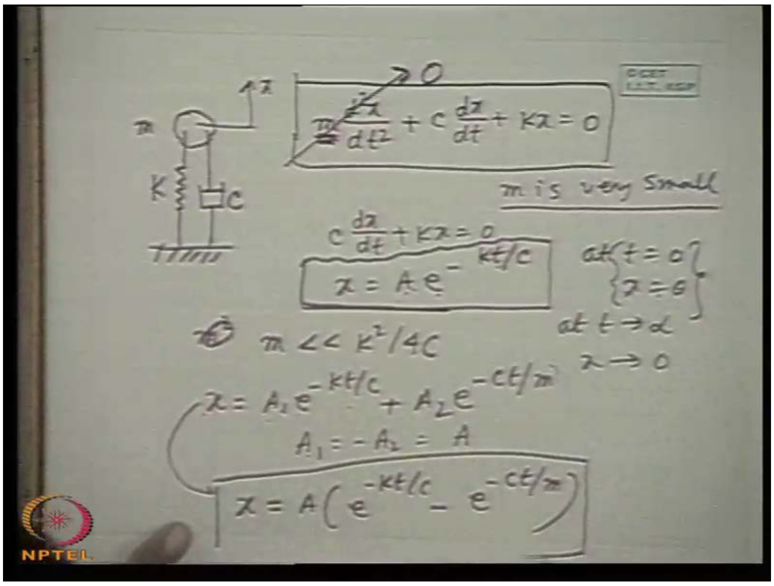

I will give you a very interesting example in this aspect. Let us consider a vibration of spring system which you can very well appreciate of mass in. And let us consider the typical vibrational system like this with a spring of constant K, spring constant K and a damper and a spring mass, vibration of a spring mass system with a damper whose damping coefficient is c.

So, the differential coefficient for this if we consider in this direction x is n d square x; that means, if we give a small excitation at t is equal to 0 and leave it. So, then we can find out the vibration or the displacement of this mass by this equation, by this solution of this equation which probably you know very well c d x d t plus k x is 0. This is well known equation from the theory of vibration, simple case. Now if you consider a point mass; that means, mass is very very small. So, that mass is m is very very small, is very small. So, now at the very first attempt we will be tempted to make this term 0 because very small mass; that means, this term should be 0. So, why not make this term 0 and approximate this equation. In that case what is happening a second order equation is being reduced to first order equation.

So, therefore, what is that equation d x d t plus k x is 0. What is the solution of this equation? Solution of this equation is x is equal to A very simple e to the power minus k t by c. So, this is the solution of this equation. Where you see, one thing is clear; that we can we have only one boundary condition. What boundary? We have at only one boundary condition that therefore, we cannot use the boundary condition at t is equal to 0 x is equal to 0. For example this boundary condition we cannot solve because that t is equal to 0 x is equal to finite otherwise a will be 0. So, we cannot solve this boundary condition rather we can use the boundary condition at t tends to infinity. What is that x tends to 0 this is automatically satisfied by this. So, this is the simplified equation, but what happens if we solve the entire equation; that means, second order equation with the idea that m is very small.

That means mathematically if you consider, m square very very less than; that means, m rather sorry m less than, very very less than k square by 4 C; then if we solve it then you will get a complete solution of this; we will give, e to the power minus k t by c plus another constant A 1 is a constant, e to the power minus c t by mc t by m. This is the complete solution; now in this solution if we can use this equation, t is equal to 0 boundary constant which we could not use here. So, if you use this you will get A 1 is equal to minus A 2 and let this is equal to A, the same boundary condition.

Why? Because when m is exactly 0, then this is what m is exactly 0, this will not come in to picture, this is 0. So, this is x is equal to A 1 e to the power minus k t by c. If we compare these two equations, A 1 will be A. So, A 1 is minus A 2 A. So, therefore, the complete solution is x is equal to A into e to the power minus k t by c minus e to the power minus c t by e. Now if we compare these two equations, now you see the interesting picture.

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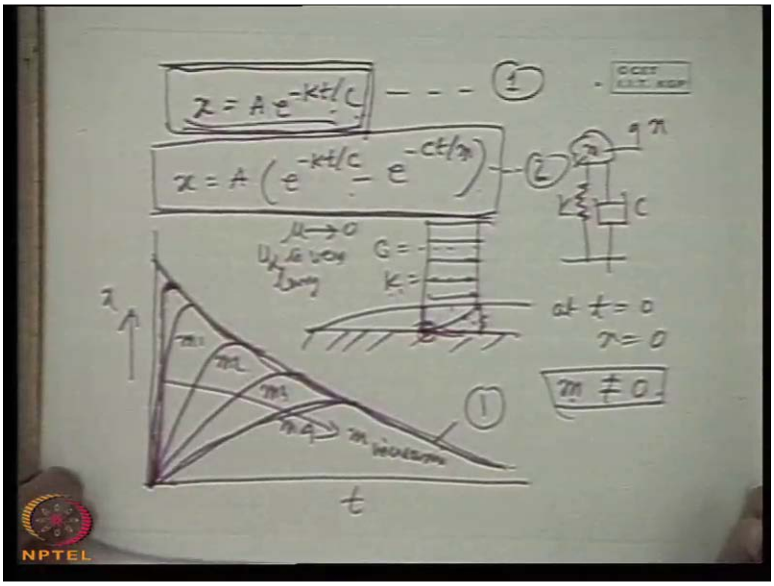

Now, I write again these 2 equations, 1 is discarding the first the second order term that means solution of this; that means, x is equal to A e to the power minus k t by c, this is one simplified equation and another is that by solving the full equation for small values of n that we have developed just now.

X is equal to A into e to the power minus k t by c e to the power c t by m. Now if we compare these 2 equations in terms of a figure you see, that if we now draw x versus t. Now let us here also see the condition, that the physical system. See this is k. So, this is x. Now if we draw these graph at t is equal to 0, this gives A, the maximum and they need; that means, this is equation 1, let this is equation 2. So, this is equation 1. And we are solving it for a given value of c and k, that means we are plotting the curve for giving the value of c and giving the value of k; now for the same value of c and k, if you plot this graph for different values of n what would you get; that means, just if we plot these graphs for the same values of c and k. For who which we are drawing the equation 1. So, if you find out the equation 2 for different values of n we can see that, there will be a family of curves with different values of m, m 1, m 2 different values of m. And in this direction m is increasing, m is increasing in this direction.

So, therefore, we see if m is very small the curve is like that, what does it mean? This equation we are able to solve at t is equal to 0, x is equal to 0 and in practice what will happen? That if, we give a small excitation at t is equal to 0 and leave it this mass will first attempt to a very high displacement and then it will damp out, but this attainment of this displacement from the initial position t is equal to 0; this duration will be smaller and smaller as the m is increasing. But nevertheless this part will be there. So, that we can see for a small m the larger portion of its response coincides with this equation; whereas, when we go for high m it is not so, but we can argue that whatever may be the small mass. If it is not exactly becomes 0, for a non 0 mass because m can never become 0; even for a point mass it has got some value. We see this portion of the curve; that means, this attainment from 0 displacement to a maximum displacement which may take place for a very small duration of time is absent by this solution

So, from here we can understand that for an ideal flow whatever may be the viscosity, viscosity may be very very small close to 0 or u infinity is very large nevertheless, we have a very small thickness within which the viscous effect is prominent. So, that the boundary condition for the no slip is not satisfied if we approximate the equation to the ideal situation; that means, we are unable to detect this one. So, here the maximum part of the response coincides with this equation here also.

The maximum part of the flow field coincides with the ideal flow theory, nevertheless there will be smaller and smaller, but some non 0 value of the thickness delta within which the viscous effect is prominent. This is because, in the approximated equation for potential flow or ideal fluid flow this no slip condition boundary condition we cannot satisfy the similar case; here we cannot satisfy the initial boundary condition. So that, this portion of curve will never show this small portion; even for very small mass where the displacement from 0 is attained here. So, this is the mathematical clue which came into to the mind of prandtl that we cannot suppress the order of equation; that means, when the flow velocity is very high or viscosity is very low.

We cannot approximate the full equation of motion for a viscous flow that naviers stock equation by dropping entirely the viscous terms and hence by reducing the order of the equation. So, that one of the boundary conditions will be missing in the solution of this equation. And that is missing of that boundary condition will not reveal the actual flow picture very adjacent to the solid surface. So, that thickness may be smaller and smaller, but at the solid surface the flow has to be 0, but the solution of ideal fluid does not give the 0 velocity at the solid surface. So, this is besides the physical and mathematical clue for the concept of the boundary layer.

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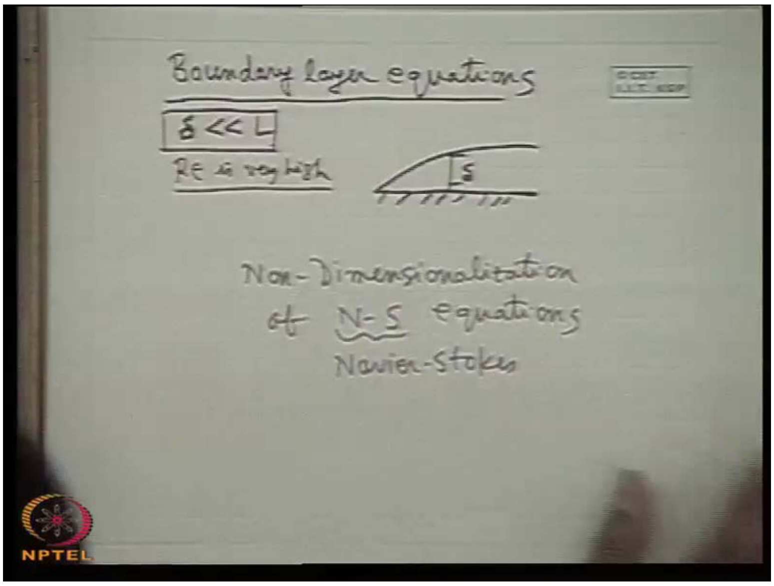

Now I will come to the development of boundary layer equations. Now boundary layer equations, what is boundary layer equations? Boundary layer equations are the equations of the motions within the boundary layer. Now with this idea that the boundary layer thickness is very very small as compared to any characteristics length.

For example in a flat plate, for example, this distance is very small compared to any distance in this direction, because of this characteristic feature and the delta is very small if Reynolds number is very high, when Reynolds number is very high. So, for therefore, for very high Reynolds number flow, when the delta is very small these are known as boundary layer flow. The equation of the motion gets simplified within this region, even the full viscous force the full form of the momentum equation for viscous flow is simplified and that is a simplified form of the equation of motion, written within the boundary layer with the boundary layer simplifications are known as boundary layer equations. So, before that I will first start with the non-dimensional, non-dimensionalization of the equation of motion, non-dimensionalization of Navier-Stokes equations, this you probably recollect that is the equation of motions for viscous flow; that means Navier-Stokes equations.

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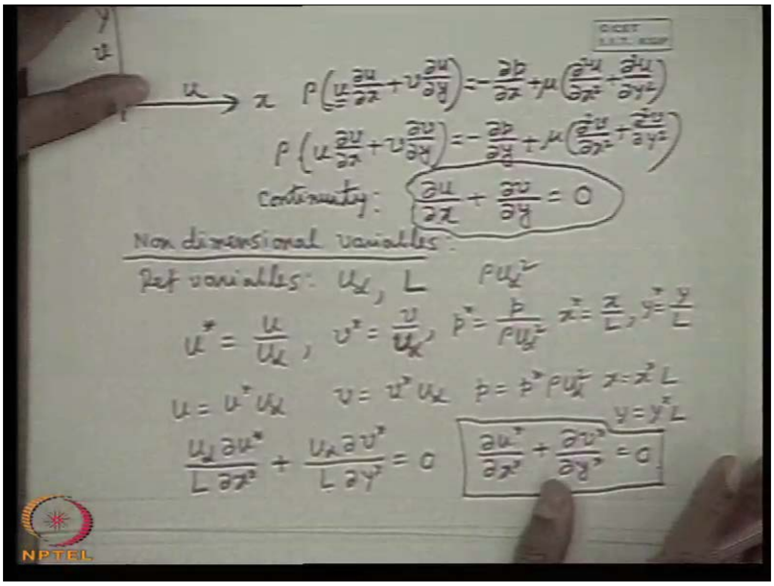

So, let us do that let us consider the 2 dimensional x y coordinate system, let us take x y. Let us consider a 2 dimensional x y coordinate system and a steady incompressible flow. So, what is the Navier-Stokes equation, if we write rho u del u del x in x direction plus v del u del y is equal to minus del p del x, if you recollect mu del square u del x square plus del square u del y square; how to write x direction momentum equation.

What is y direction momentum equation, y direction momentum equation is that means, y direction u is the velocity in the x direction v is the velocity y direction. u del v del x plus v del v del y well is equal to minus del p del y plus mu del square v del x square plus del square v del y square and if you write the equation of continuity along with the equation of motion continuity; that means, these are the equations of motion Naviers-stokes equation, you know for an incompressible flow in a Cartesian coordinate system. So, steady flow and incompressible flow. So, when the flow is incompressible this is the equation; del u del x plus del v del y is the 0.

Now if we write this equation in terms of non-dimensional variables. So, how do you define the non-dimensionable variables, non-dimensionable variables; we have seen that earlier in experimental techniques, we follow the Buckingham’s pi theorem to find out 1 dimensional variable, but in theory the non-dimensional variables are found like that; let us find out the reference, variable with which the non-dimensionalization is done. The reference variables we take for non-dimensionalizing the velocity component as u infinity the reference variable we take in non-dimensionalizing the length quantity as any characteristic length L; which is the length of the plate in direction of flow, any characteristic length of the plate or length of the problem.

Now the non-dimensionalizing the P we take the rho u infinity square as the normal, as the reference variables and now we define the non-dimensional variables as we take star as an superscript u star is therefore, u by u infinity; v star is therefore, is equal to v by v infinity sorry u infinity, u infinity; p star is therefore, is equal to p by rho u infinity square; x star is therefore, x by L and y star is therefore, y star by L.

From which we can write u is equal to u star into u infinity, v is equal to v star into u infinity, p is equal to p star in to rho u infinity square, x is equal to x star into L, y is equal to y star into L. Now a simple mathematical way we can change, these we can replace this dimensional variable in terms of non dimensional variables. How we can do it? It is very simple, if we start this equation; for example, then we see first let us see that this first we do with del u del x plus del v del y is equal to 0 this equation. What is del u? Del u is del u star u infinity.

So, u infinity del u star, I do this one divided by del x. What is x, x star L; that means, L del x star plus del v del y, what is del v? v star, del v star u infinity. So, v star is again the counter part of v; that means, non dimensional variable del v star. Then what is del y? del y is del y star L, u infinity by L comes common. So, that the final equation is del u star del x star plus del v star del y star; that means, the continuity equation remains as it is in its for only the dimensional variables are changed or substituted in terms of dimensionless variables. But it is not so, for the case of Naviers -Stokes equation; why not? Let us see now Naviers-Stokes equation for example, I think you can see it with very well, these are the reference variables I think you can see. Now for the Naviers-Stokes equations, if we write the equations. I think well this, I think you can see this, I think you can see this; now Naviers-Stokes equation if we write, rho it is shown.

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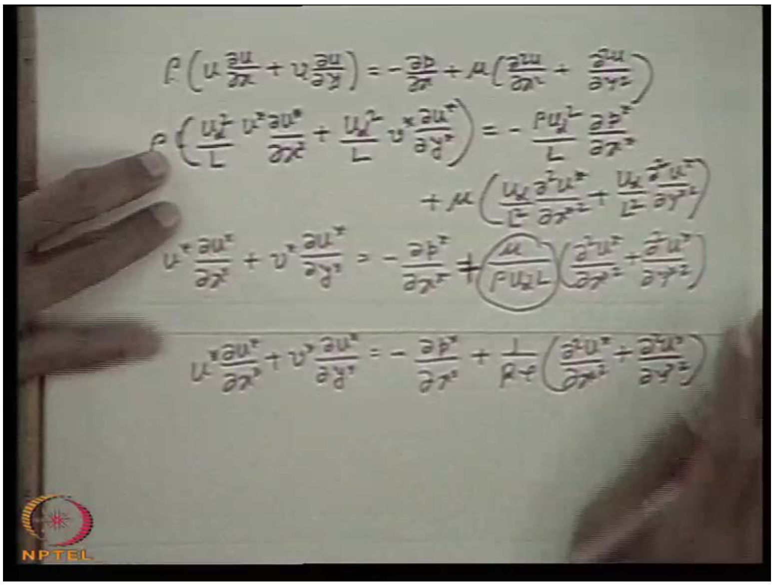

So, Navier-Stokes equations we can write now rho u x direction u del u del x plus v del u del y is equal to minus del p del x plus mu del square u del x square plus del square u del y square. All right, now you see u is u infinity into u star, del u is u infinity into del u star.

So, therefore, we can write u in here u infinity square u star del u star, similarly for del x we get x star L. So, u infinity square L comes common del x star, similarly if I replace v as v star u infinity and del u is del u star u infinity and del y is del y star L. So, similarly we will get the same u infinity square L as common and then v star del u star del y star. Then what is the right hand term minus del p del x. What is p? p star rho u infinity square. So, del p means del p star rho u infinity square; and del x is del x star L. So, therefore, here minus what we get? Rho u infinity square by sorry rho u infinity square and correct L into del p star del x plus mu, what is del square u? u is this. So, del u is del u star u infinity del square u is del square u star del u infinity.

So, therefore, it is simply u infinity del square u star, what is del x square? x is x star L. So, del x square is del x star square, but here you will get L square. So, this is the order of x square; so u infinity by L square. Similarly, if you do that you will get u infinity by L square into corresponding del square u star del y star square. So, therefore, now if we look to this we will see that if you now see this here u infinity square rho u infinity square L comes as common.

So, therefore, we can write u star del u star del x star plus v star del u star del y star is equal to; if you divide this rho u infinity square L rho u infinity cancels. Simply minus so, up to this the form is same only that the dimensional variables are substituted by the non-dimensional variables. Now, here what is happening? mu u infinity L square divided by rho L by u infinity square, you get mu by rho u infinity L; then this term is del square u star del x star square plus del square u star del y star square.

This is nothing, but one by Reynolds number. So, we can write therefore, u star del u star del x star plus v star del u star del y star is equal to minus del p star del x star plus sorry here is the plus term, 1 by r e into del square u star del x star square plus square u star.

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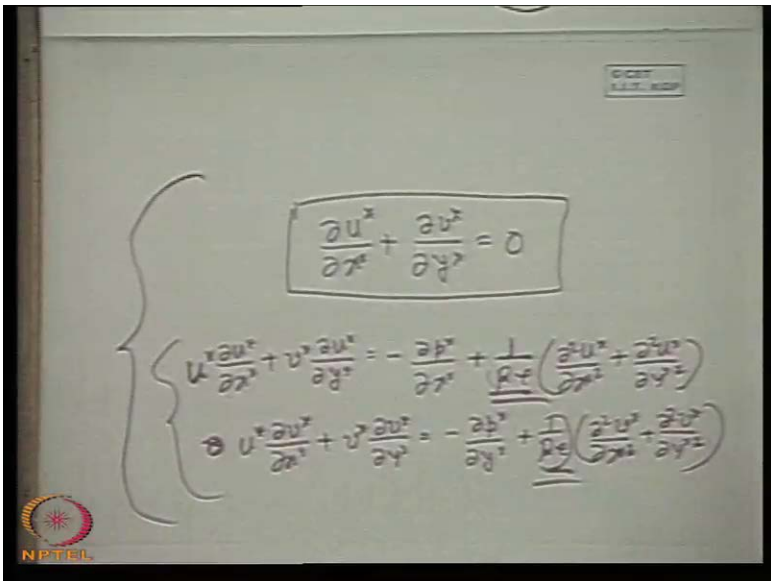

That means we see the non-dimensional form of this equation, x direction momentum equation, is same form if we substitute the variables dimensional variables as non-dimensional variables except there is one coefficient comes with the viscous term instead of mu 1 by r e non-dimensional number; Reynolds number. So, all the terms are now dimensionless. Similar way if you non dimensionalize the y component of velocities, with the same reference variables, you will get the u star, the same way of you do it del v star del x star plus v star del v star del y star is equal to minus del p star del y star plus 1 upon R e; del square v star del x star square plus del square v star del. So, 1 by R e will come in this. So, these are the two Navier-Stokes equation in dimensionless form in the x and y direction.

These are the, but the continuity equation remains in its form, without any coefficient coming with any of the term; that means, they are dimensional equation and non-dimensional equation are exactly in the same form with the variables are substituted by the non-dimensional counter parts. Whereas x and y direction momentum equations are x and y direction Navier-Stokes equation 1 by R e is coming as the coefficient in the viscous term. So, these are the non-dimensional versions of the Navier-Stokes equations along with the equation of continuity. In the next class, thank you I will discuss the dimension the order of magnitude analysis and we will arrive at the boundary layer equation.

Thank you.