Fluid Mechanics
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Lecture - 47
Introduction to Laminar Boundary Layer Part- II

Good morning, I welcome you all to this session of fluid mechanics. Last class, we were in the way of deducing the boundary layer equations, which are the equations of motions or the Navier-Stokes equations. For an viscous flow or real flow within the boundary layer and what we had proposed thought that there will be simplification of these equations of motions, within the boundary layer and we are on the, we were on the way of directions of the those simplified forms of the equations, known as the boundary layer equations.

So, we started with the Navier-Stokes equation for an incompressible flow with respect to a Cartesian frame of reference and also a two-dimensional flow, and then made this equations non dimensionalize; that means, we derive the non-dimensional form of those two-dimensional viscous and incompressible flow with respect to Cartesian coordinates and arrived certain, and arrived the non-dimensional forms of those equations.

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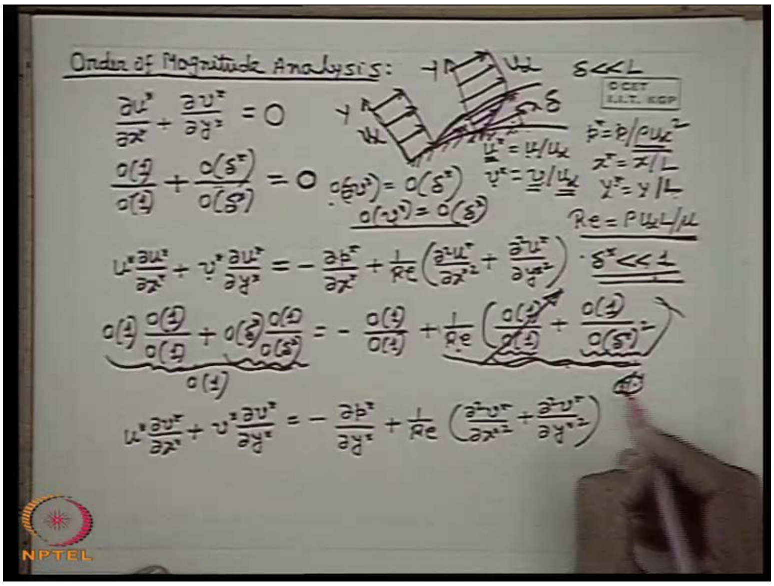

Let us look back what we did earlier. This is the non-dimensional form of the equations, this is the continuity equation, this is the x direction momentum equation and this is the y direction momentum equations, Navier-Stokes equations. Therefore we recall back, the non-dimensional variable where taken as u star the star the super script star was use for non-dimensional quantity. So, non-dimensional velocities u star v star were taken with respect to u infinity, u infinity is the free stream velocity u infinity, which was taken as the reference variable in non dimensionalizing the velocity component.

The pressure was made non-dimensional in this fashion p by rho u infinity square or length coordinates x and y were made non-dimensional as x star and y star, by taking L. Some reference length characteristic length of the problem, it may be the length of the body. So, x by L y by en and we have seen that continuity equation remains as it is; that means, the dimensionless variables are simply substituted by their counterpart, dimensionless counterpart; but in x and y direction momentum equation the things are exactly. So, except with the viscous term, 1 coefficient comes as 1 by Reynolds number; where Reynolds number is defined this fashion rho u infinity L by mu. Now today we will discuss over an very interesting procedure that is order of magnitude analysis.

Now from an order of magnitude analysis, we will find the simplifications of those equations within the boundary layer; now let us find out the order of magnitude analysis. So, first of all we will have to know, what is mean by order? That means for each and every variable dimensionless variable. We can find out the order. For the order we can in a simple manner, we can we can take the maximum value the variable can take; for an example, what should be the order of del u star; that means, the change in the velocity in the x direction . Let the direction of the flow let these be the x direction, let this be the y direction, this be the y direction, this be the x direction. So, the change is this from 0 to u infinity, 0 is the velocity at the surface. So, change is from 0 to u infinity. So, therefore, u star can change from 0 to 1; when u is equal to u infinity u star is 1. So, the order of the del u star that the maximum value the change of u star can take is 1; similarly order of del x star is 1 because x can take the value of L.

So, therefore, change of x takes frame from 0 to takes place from 0 to L. So, the order is also 1. So, the order of the first term of the continuity equation is 1. So, what is the order of the second term? Now, the change in del v star; that means, the change in the order of y component of velocity is first. Not known; that is unknown, before hand before hand we do not know it, because the y component of velocity is generated because of the formation of the boundary layer. So, the liquid is slow down near the valve. So, that the transverse component of velocity is generated, but we know the order of y, order of y; that means, is the delta that is the boundary layer thickness; that means, this thickness delta; because, we are concentrating our attention only within the boundary layer. So, the order of the del v star, what is v star the order of; sorry, del y star. What is y star? y star is y by L. So, y star will be 0 minimum when y is equal to 0, but this change will up to the magnitude of delta; that means, the y star will be maximum value of y star will be delta y L. That means that is delta star. So, therefore, this will be the order of delta star.

Now when the two quantities, this plus this summation of these two quantities make a 0; that means, the order of first term must be equal to the order of the second term. The order of these 2 terms are to be same. So, that they counter to each other with a negative sign. So, that this becomes 0. So, therefore, the order of del v star has to be delta star. So, the consequence of the continuity gives us that the order of delta v star del v star is equal to order of delta star.

And order of del v star means the order of v star. Why? This is because, the velocities is 0 in the flow field at the solid boundary. So, any change in the velocity, the order of any change in the velocity will be the order of velocity itself; because, the velocity changes from a value 0. So, therefore, the order of delta v star will be equal to the order of v star and that become equal to the order of the delta star. So, for a very small value of the boundary we have thickness delta because with respect to its length L.

We can tell that delta star is very small than one this is the basic concept of the boundary layer, which tells us that the order of the velocity component in y direction is very small as compared to its x direction component in the free stream. That means viscous is very less than once this delta star is very less than 1; which means the y component of velocity very very small, is very very small as compared to the x component of velocity as free stream. So, this is the consequence of the, another magnitude analysis for the continuity equation.

Now let us find out the order of magnitude analysis for the x direction momentum equation, let us see that first term in the left hand side, that is the acceleration term u star is the order of 1. We know, because of u star can approach the value 1, when u approaches the value u infinity. So, delta u star is in the similar order of u star, u star and delta u star are of the same order, has the same order since u has used a value 0 in the field. What is the value of order of delta x star? We have earlier, it is 1. Now, the order of v star as already been found to the order of delta star alright. So, order of del u star is 1 and order of del y star is delta star. So, you see the order of these quantities is 1, order of this quantity is also 1, order of delta star by order of delta star.

So, we see that as a whole the left hand side is of the order 1, let us see the order of right hand side quantities; now order of delta p star is the order of p star p by rho u infinity square, this is the dynamic head which is equal in the order of the pressure in the free stream. So, therefore, the p star may approach rho u infinity square. So, that the order of p star is 1. So, order of p star is 1, delta p star is also 1, order of delta x star is 1. So, order of the first quantity is 1.

Let us write 1 by r e, Reynolds number. The order of Reynolds number is not known; this depends up on the problem and we know for a flow viscous flow if the Reynolds number is very large, then only the boundary layer thickness is very small.

For another magnitude analysis we already have shown, that the boundary layer thickness; non dimensional. That is delta by l with respect to any characteristic length of the problem is proportional to 1 by root over R e. This also will be discussed here will be found out here now. Let us find out the order of del square u star del x square; del square u star mean the order of u star, that is equal to order of1 and del x star square also order of 1. What is the order of this quantity, del square u star is the order of 1 because the numerator is the order of u star. Now delta y star square. So, it will be the order of y star square.

What will be the order of y star delta y star square? Delta y star means the change in the y coordinate which is delta 0 to delta within the boundary layer. So, change in y star will be delta by L change in delta star. So, therefore, y star square will be the order of delta star square. So, therefore, del y square start will be order of delta y star square. Now you see that the order of this term is one, but the order of this term is very high, because if we consider that delta star is very less than 1. So, order of this term is much more than the order of this term. So, therefore, we can say that the order of the terms within the brackets is dominated by this term only or in other words we can we can neglect the this term since the order of magnitude is very less as compared to this since delta star is very less than 1.

Now, we see that if this term the viscous term as a whole to be to exist within the boundary layer because the physical picture is there within the boundary layer viscous force is of equal order of magnitude of the inertia force then the entire order of the entire term has to be in the order of 1. And if it has to be so, then the order of 1 by sorry, rather we can write this way.

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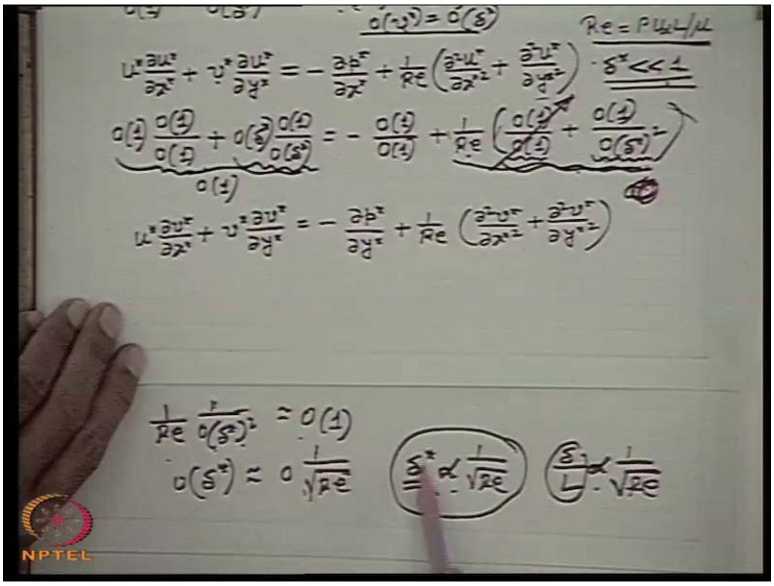

That case we can write the order of 1 by R e, 1 by R e into order of delta square has to be 1; otherwise what will happen, this term will not be equal to the term will be equal in order to the inertia term. So, therefore, of this term to be 1, 1 by R e order of 1 by delta square has to be the order of 1, but this term we can neglect because the order of the terms within the bracket will be dominated by this term only.

If we equate this we see that delta star the order of delta star becomes equal to the order of 1 by root over R e or simply delta star is proportional 2. This already we have shown earlier, already we have shown earlier that the order of delta y L, delta star is delta by L is proportional to 1 by R e or delta by l is proportional to 1 by R e. So, therefore, if R e is very high delta by L will be very low; which means the boundary layer thickness will be much smaller compared to any characteristic length of the problem.

So, therefore, this is the conclusion from the order of magnitude analysis in the x direction for the x direction momentum equations, for which we get the order of delta star is proportional to the, is equal to the order of 1 by root over R e or delta star is proportional to 1 by root over R e. So, we can tell that r e is proportional to order of delta star square or 1 by R e is proportional to sorry; 1 by r e is proportional to the order of delta star square. So, that this cancels and makes the term in the order of 1.

Now, at the same time if we make a make an order of magnitude analysis for the y direction equation we will arrive then, to an interesting result. What is this? Let us see, the order of this is 1; now it is known everything is known, order of delta v star is delta star, order of this is equal to 1 plus order of v star is delta star, order of del v star is delta star, order of del y star is delta, order of delta star. Now we see the order of this term is delta star order of this term is delta star; that means, as a whole we can say the order of the left hand term; that means, the inertia force is of the delta star. That means order of this term is delta star; that means, the inertia force is of very less order of magnitude which physically implies that the inertia of force in the transverse direction or in the y direction is very small.

Now let us equate the order in the right hand side. Now, in doing; so, we make the order del p star, del y star to be unknown beforehand because we do not know the order of del p star del y star which means we do not know the way the pressure will be varying within the boundary layer. We have to find out this that, what is the order of variation of the pressure within the boundary layer. So, therefore, this will be made unknown and other terms order of magnitude is known to us for example, del square v star; that means, it is in the order of del v star or v star which is equal to the order of delta star. The order of x star square is order of 1, order of v star is order of delta star and order of y star square is delta y star square is order of delta star square. So, here also we see this star the order of this star means 1 by delta star, whereas order of this term is delta star.

Now, when delta star is very less than equal to 1, then what happens? This term small while this term is high; so, the order of this term is much higher than the order of this term by a factor of 1 by delta star square. So, that the order of this term can be neglected with comparison to the order of this term or we can say in other hand that the order terms within the bracket will be dominated by the order of this term only; that means, del square v star del y star square. Similar way the order of the terms within the brackets for x direction momentum equations were dominated only by the order of this term; del square u star del y star square not by this term del square u star del x star square the order of this term is small in comparison to the order of this term.

So, therefore, we see that the order of this term is in the or the order of delta. So, order of this term is in the order of 1 by R e to 1 by delta star. We know from the earlier analysis that the order of 1 by R e is equal to the order of delta square. So, that this term order becomes the in the order of delta. So, therefore, if we replace this here then we see the delta star square 1 by delta star. So, order of this star the viscous term the right hand side term the order of this term is also the order of delta. So, therefore, this term is also in the order of delta star the inertia term is also in the order of delta star. So, therefore, there is no other way out or go that the order of del p is del y square will be also in the order of delta.

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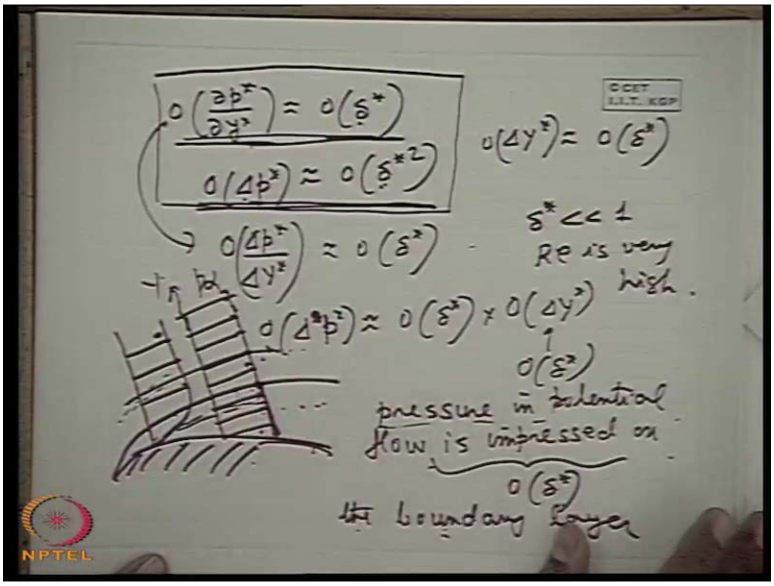

That means, the term the order of this term del p star del y square will be equal to del star. That means order of delta star; this gives a very good conclusion I just tell you before that I write. So, therefore, the order of the pressure difference within the boundary layer, will be equal to the order of delta star square while the del y star the order of del y star is is also equal to the order of delta star. So, therefore, delta p del p del y means if we write we can write the order of del p star by del y star this is equal to order of delta star. So, order of delta p star delta p star is equal to order of delta star into order of del y star; which is also equal to the order of delta star.

So, that the order of delta p star is equal to the order of delta star square. This is the most useful conclusion of a boundary layer. What is that? Let me show you that; for example, if there is a boundary layer over a curved surface in general. So, if this is the boundary layer, then this shows that within the boundary layer this is the velocity profile within the boundary layer the pressure difference is negligible because delta star is very less than 1. R e is very high is, very high. So, that the order of delta star is very small; that means, that pressure gradient within the boundary layer is very small therefore, the pressure difference is still smaller; that means, there is no pressure difference within the boundary layer.

That means the one draws the pressure distribution, you will see that in the potential region; that means, beyond the boundary layer pressure is uniform in this transverse direction why; let this is be infinity. So, the same pressure is impressed within the boundary layer. That means there is no pressure variation within the boundary layer. So, therefore, the pressure may vary in the direction of flow in this direction depending up on the curvature of the body, but the pressure at the adjoining boundary layer with the potential flow remains same throughout the boundary layer. That means the variation of pressure within the boundary will be guided by the variation of pressure in the free stream or the potential flow. So, the pressure in the potential flow is totally impressed on the boundary layer.

So, this is a very important conclusion that pressure in potential flow, pressure in potential or inviscid flow is impressed on the boundary layer, impressed on the boundary layer. So, therefore, to know the pressure distribution within the boundary layer, one has to only solve the ideal flow equations or the equations for potential flow to know the pressure distribution, outside the boundary layer and the same pressure distribution will hold good within the boundary layer. Because the pressure at the adjoining boundary layer and the free stream or the potential flow will be same as that in the boundary layer, within the boundary layer there will not be any be any pressure gradient or pressure difference within the boundary layer will be very very small. That means the pressure in potential flow is impressed on the boundary layer; this is the most useful consequence of the order of magnitude analysis of the y direction equations of motions.

And now you can probably recognize the fact that how useful is the analysis of ideal flow theory or inviscid flow theory. No fluid has got 0 viscosity, but still we assume an hypothetical fluid with a 0 viscosity and analyze the ideal flow theory. So, that from the ideal flow theory the pressure distribution which we get is same as that in the, boundary layer.

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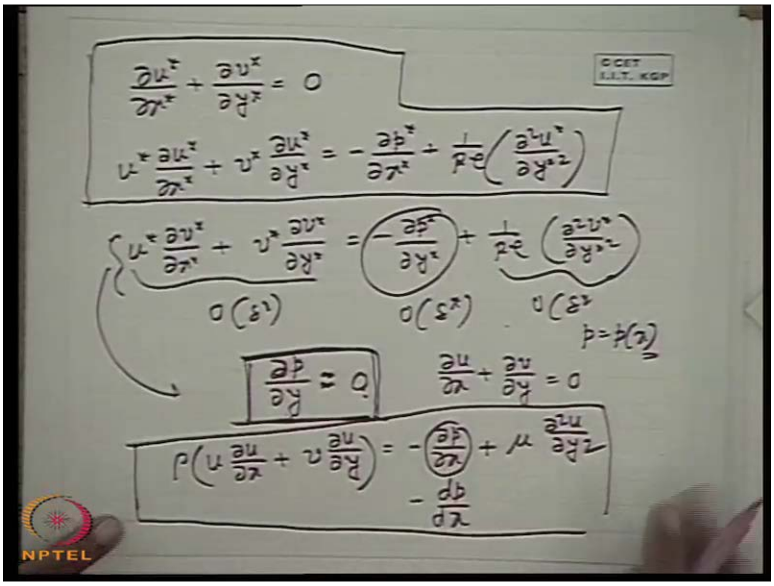

Now if we write the non-dimensional form of the equation of the order of magnitude. We get the simplification like that, there is no simplification in the continuity equation, but there is a simplification in the h direction equation; del u star del x star this term is retained, now the order of these 2 terms are same. So, that both the terms are retained, is equal to minus del v star del x star plus 1 by R e. Now here we have already seen the order of this term is less than the order of this term. So, therefore, we write only this term del star u star square del y star square.

Similarly for the y direction equation, all the order is same neglected. If we see we should not write, I write it and then only we will. So, what is the consequence of y direction momentum equation that, the order of analysis magnitude analysis that; this is the order of delta star. So, this is in the order of delta star. So, this is the order of delta star; that means, the y direction equation can be written as del p del y is almost 0, as very small order del p del y. So, this is the consequence of the order of magnitude analysis of the approximation of the boundary layer in y direction.

So, y direction gives the this result del p del y is 0. So, therefore, we are left only with two equations 1 is the equation of continuity and another is the equation of motion for x direction. Now, if we write the this in the dimensional counterpart. So, what we will get we will get the continuity we will get del u del x plus del v del y is equal to 0 and for this we get u del u del x if you recall plus v del u del y is equal to minus or rho if you write here minus del p del x plus mu into del square u and del y square. Since del p del y is 0; that means, the pressure gradient within the boundary layer is negligible this can be replaced as minus d p d x, because p is a functions of x only.

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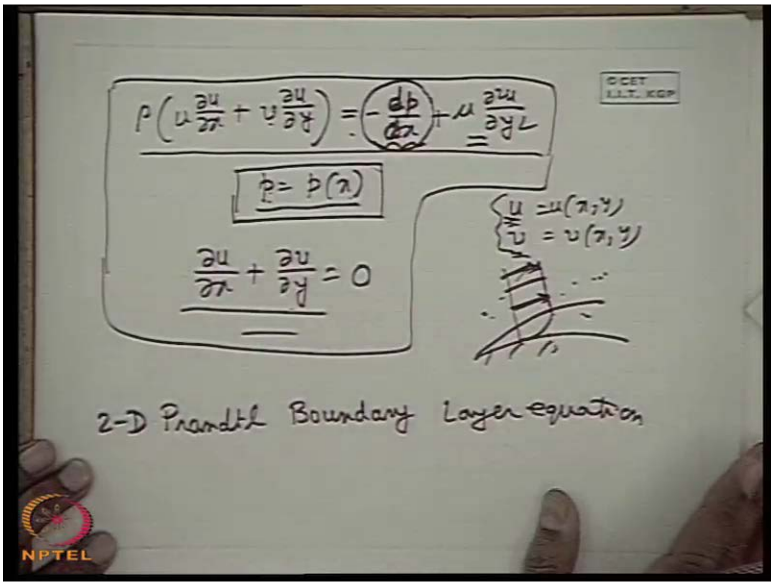

So, these, these equation is known as Prandtl boundary layer equation. This is known Prandtl, Prandtl the person who first discovered this revolutionary concept of boundary layer; Prandtl boundary layer equation, Prandtl boundary layer equations of course, two dimensional Prandtl boundary layer equations. So, this is the two dimensional Prandtl boundary layer equation. So, again we write the boundary layer equation two dimensional again because, this is very important u del u del x; that means, both the terms in the inertia of forced or the acceleration per unit volume or mass as you see here rho is multiplied per unit volume remains the pressure gradient is of the same order of magnitude oh sorry this is now d p d x plus mu l square del u square del y square. So, this equation if you write this, so, it suggests automatically that p is a function of x only; that means, the y direction equation have already given fullness that p is a function of x only.

So, therefore, this is the equation along with the equation continuity del u del x plus del y which means, that we have only 2 unknowns u and v as a function of x and y to be solved and this can be solved from these two equations. Because, the pressure is not unknown within the boundary layer because, pressure is solved as a function of x from the solution of the ideal flow beyond the boundary layer; that means, this is the surface. So, pressure equation will be solved by solving the Euler’s equation, or the Bernoulli’s equation in the potential flow region. So, the d p d x is already known for the boundary layer equations the pressure is solved in the form of a function p with x from the solution of Euler’s equation or Bernoulli’s equation reading the potential flow.

Then the only unknowns remain is the u and v component and this can be solved by the boundary layer equation; that is the equation of motion in the x direction within the boundary layer, along with the continuity equation. All right, I think we will understood this philosophy of boundary layer equations where, by the order of magnitude analysis we can make it simplification of the equation of motions, and the viscous term can be written as only mu del square u del y square. And also you can we have proved the most useful significance of the boundary layer that, the pressure in the potential core is same as that in the boundary layer; that means, the pressure is impressed within the boundary layer. So, the pressure as a function of flow direction is found out from the solution of the potential flow equation that is, the Euler’s equation or the Bernoulli’s equation. And then the boundary layer equations are solved for the few velocity quantities only, for a two dimensional case there are two velocity component. So, we have two equations one boundary layer equation in one x direction and another one is the continuity equation.

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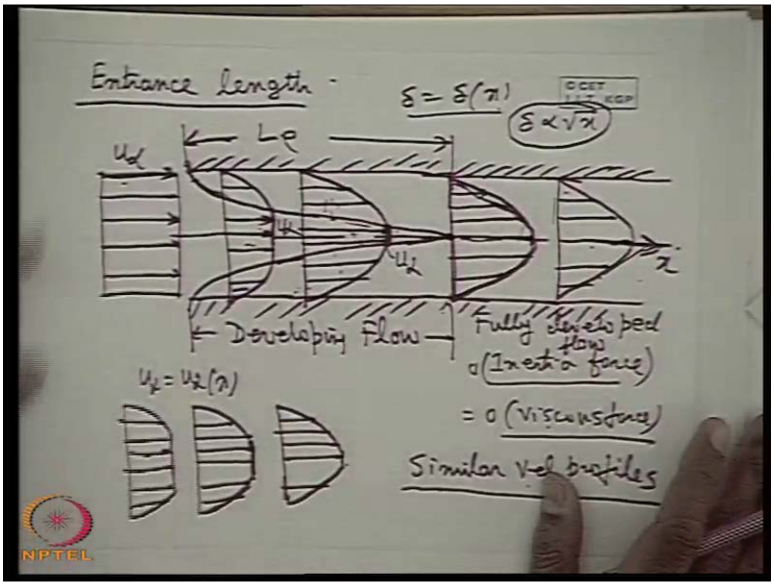

Well now we will come to a most useful concept of boundary layer separation, but before that I will test entrance length concept, entrance length concept of entrance length. Now, we see this so far we have discussed for flow over a surface, now what happens when the flow approaches in a close duct, for example in a duct is the circular duct or it may be a duct with any cross section. If the flow approaches with an uniform velocity through a duct what happens, let us see what happens, if the flow approaches to a duct.

Let us consider the flow approaches with a uniform velocity like, this with a uniform velocity as the flow touches these two surfaces what happens if we take this as the axis of the flow the flow will be symmetric about this axis. Since the flow condition is symmetric well then what happens, the boundary layer will start growing from this surface both the surfaces. So, boundary layer will start growing and it will meet the axis at some point, so, what happens in this region if we draw the velocity profile which will be symmetrical about these axes. We will see the velocity will very only within the boundary that, steeply and then it will, what it will do? It will just be constant throughout this; the same picture will be there. That means this part is the potential flow, and this part will be the viscous flow.

And since the boundary layer is growing, I have told earlier that the boundary layer is a function of the flow direction, let this direction is x. So, boundary layer is the direct function of x is proportional to x, root x actually or x it depends upon the type of flow it is directly a function of x; that means, it grows. Then what happens, the potential core is getting reduced; that means, at this section the velocity profile will be like this, but at the same time the potential core velocity this part is getting increased, this will be accelerated, this part will be accelerated. And here we will have a velocity variation like this is for a laminar flow, I am showing you the parabolic profile we get that we have already seen the solution of Poiseuille Flow.

But here I am interested in telling you, the concept that this part of the flow is known as developing flow, developing flow; that means, the flow is being developed; where at any section there is a boundary layer flow, boundary layer zone and there is a potential zone. Now see that since the volume fluid remains constant, the average velocity at any section remains constant. So, as the boundary layer grows most more of the fluid is being retarded within the boundary layer, so, that the core potential core velocity is getting accelerated, so, that this u infinity and u infinity is not same. So, u infinity is a function of x and it is increasing with the direction of x with the concept, that the average velocity will remain same as that of the u infinity because, the cross sectional area remains same.

If flow rate remains same, average velocity remains same. So, when the two boundary layers smudges on the axis, we get a profile of this structure for a laminar flow, and here what happens the entire region is the boundary layer; which means the entire flow region. In the entire flow region, inertia force and this viscous force, inertia force and the order of inertia force is equal to the order of viscous force; as it happens in case of boundary layer force.

So, therefore, this region of flow is known as fully developed, fully developed flow. So, therefore, this is developed flow up to which, the development of the boundary layer takes place, and the diminishing of the potential core, and where this is smudged. And fully developed flow in those regions, where inertia of force and viscous force are of the same order of magnitude, and we gain this similar velocity profile. So, velocity profiles are of similar in nature; that means, the nature the profile velocity profile that is the profile means, the distribution in y direction the transverse direction; that means, the direction transverse to the direction of flow remains same along the direction of flow. That means, if we map the velocity of profiles in different sections along the direction of the flow, we will get the same type of the distribution that is known as, similarity similar velocity profiles.

So, in the fully developed flow where, inertia of force and viscous force, will be of the same order of magnitude; when the boundary layer from the two surfaces smudge on the axis the velocity profiles; that means, the distribution in the transverse direction becomes same along the direction of the flow. Whereas, here it is not so, if we see the velocity profile, here velocity profile look like this a more extended potential flow region, where somewhere here we may get a relatively less extended potential flow, because this is the potential region. So, this is the potential region, but what we get we get an accelerated potential flow this magnitude is smaller than this magnitude. So, the profiles are not same here after some distance where it is almost going to merge we get a very small, very small so, that the velocity profiles are going to be changed; that means, this is the velocity profile this is the velocity profile. So, that this is going to be changed. So, when the fully developed flow occurs from the nature of the velocity profile variation velocity profile we see that the maximum is at the center.

So, afterwards in the fully developed flow, the velocity profile becomes similar. So, this length after which the flow becomes, fully developed from the developing region is known as the entrance length. This is usually expressed as a function of the diameter, entrance length to the diameter ratio; it is the hydraulic diameter, in case of a duct and in case of a duct to circular cross section, it is the diameter of the duct which is equal to the hydraulic diameter; which is a direct function of the Reynolds number of the flow. So, it depends upon the regime of the flow laminar or turbulent and also the type of the duct. So, this is known as the entrance length after which the flow develops from a developing one where fully developed one.

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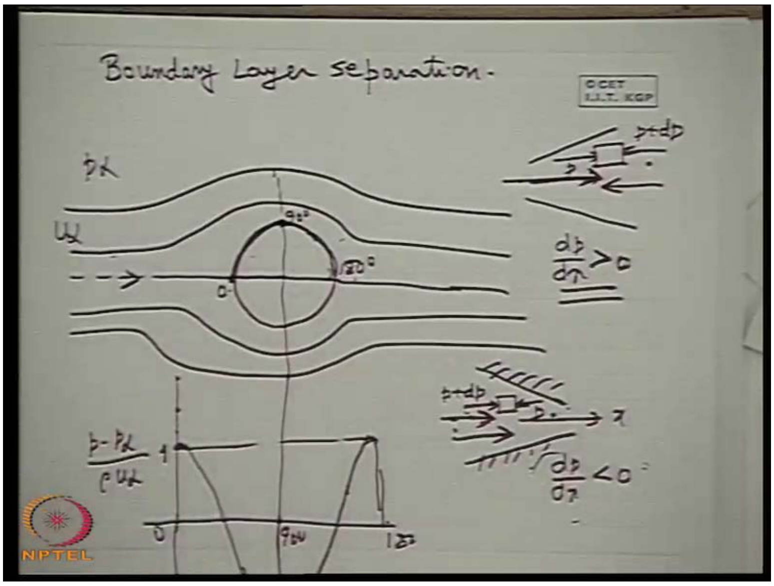

Now we will discuss another very useful concept which is the boundary layer separation. Let us boundary layer separation. Now, let us consider the case that the boundary layer flow, the flow where in passed a cylinder flow, passed a circular cylinder which we have already recognized. Let us consider an ideal flow. If you remember the ideal flow situation that the stream lines were like this. So, one thing we have noticed earlier that the pressure distribution is like this, the pressure if we consider the flow direction like this with u infinity.

If this is 0; this is theta if we measure in this direction; this is 90 degree and this is 180 degree, the flow. The pressure distribution was like this. If you recollect the pressure distribution v minus p infinity divided by rho u square. So, therefore, pressure was like this; the velocity here is 0. So, pressure was maximum. Then what happened, if we, sorry, we can up to take like this, the pressure here is maximum at one. For example, then pressure at some point goes to 0 and then it becomes negative. So, pressure in this half pressure decreases. So, pressure decreases and reaches minimum here which is minus 3, if you recollect this and goes on increasing and at 180 degree. So, this is 90 degree; this is 0 degree; this is 180 degree. The pressure reaches the same value because of the in visit nature of the flow.

So, therefore, we can understand one thing from this pressure distribution equations that the, this half of the flow due to the curvature the fluid is accelerating that is the pressure is decreasing; sort of a nozzle flow type. This creates the flow, this creates a flow through a constriction while these are of the cylinder provides an advance pressure gradient; that means, pressure is going to increase therefore, the fluid against an advance pressure gradient.

There were situations like that similar to these in case of close dark that if, a fluid flows a real fluid in a converging dark. What happens? The pressure gradient d p d x; that means the, this is the direction is less than 0; that means, pressure in the downstream is lower than that in the upstream. This is known as favorable pressure gradient. That is the flow is accelerating why? It is called favorable pressure gradient that if we consider a fluid element, we will see the pressure here is more than the pressure here. So, therefore, the for example, pressure here is p plus d p pressure here is p. The net pressure here is acting in this direction of fluid motion that is, the accelerating flow or the negative pressure gradient. That is why it is called favorable pressure gradient.

But, in case of flow through a diffuser, that means, for example, this is similar to the flow in this part, but the flow in this part that is a flow through diffuser what happens? Here the pressure gradient is positive; that means, the pressure is increasing in the direction of flow; that means, pressure at downstream is more than that in the upstream then; that means, if you take a fluid element the pressure here is p, then the pressure here will be p plus d p. So, therefore, the net pressure flow is acting in the direction opposite to the flow. So, therefore, this positive pressure gradient flow situation will call is at advance pressure gradient which is being imposed by this part of the cylinder.

Now, the question comes that when the fluid flows it flows with a favorable pressure gradient, it flows with advance pressure gradient we have seen; that means, definitely fluid does not flow by virtue of the energy gradient. Otherwise, the advance pressure gradient flow could not have existed. Actually fluid flows from higher energy to lower energy; that means, by virtue of the energy gradient. So, therefore, fluid element becomes evil to flow against an advance pressure gradient, that is positive pressure gradient that means, fluid element can surround the adverse specially, then that means, the pressure downstream is more than that of the upstream, because of it is additional kinetic energy; that is why the flow against an advance pressure gradient is possible, but what happens due to the formation of the boundary layer, the velocity of the fluid particles is retarded or is lost or is slowed down near the valve.

In fact, we know that at the valve velocity of the fluid particularly 0, if the velocity of the valve is 0. So, therefore, for the particles fluid particles very near to the valv, their kinetic energy is consume, because of the friction and they become unable to overcome or surround the advance pressure and what happens? Because of these pressure forces, which is acting on this fluid element they want to flow or they exactly flows in a opposite direction to the valve. So, this happens only in case of an advance pressure gradient flow, because of favorable pressure gradient flow; even if the fluid loses its kinetic energy near the valve, the advance pressure gradient just I have discussed is giving a force, net pressure force in the direction of the fluid flow and pushes them in the direction of the main valve flow, but in case of an advance pressure gradient, when the fluid particles near the valve loses lose their kinetic energy.

They being acted on the advance pressure gradient force, that is a pressure force in the direction; opposite to the fluid flow they train to flow in the opposite direction, this creates a flow reversal or the back flow and that is the concept of boundary layer separation. So, therefore, a back flow; that means, the within the boundary layer what happens? So, therefore, in this case we can. So, therefore, we can say that, if the boundary layer goes like that near 90 degree, what happens? The fluid particles move in the opposite direction.

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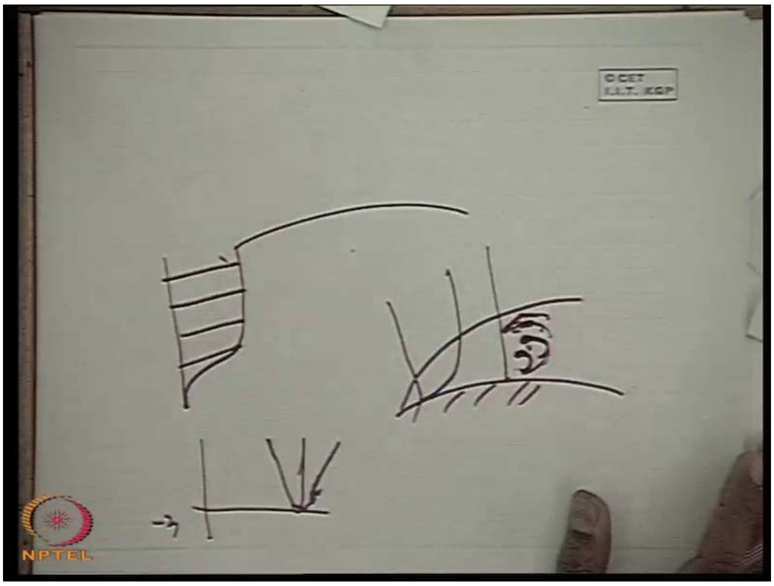

This I can show you here, let this is the slope, when the this is the boundary layer. So, if the advance pressure gradient is there. So, fluid particle actually flows, the fluid particle flows in the opposite direction and if we draw the boundary, the velocity profile, the velocity profile will be like this. So, del u del y will be 0 at this section that, now I will discuss. So, therefore, the boundary, what happens? The fluid particle moves in the opposite direction and creates a back flow or a flow reversal, which cartels the pressure energy of the fluid and pressure is going to be reduced.

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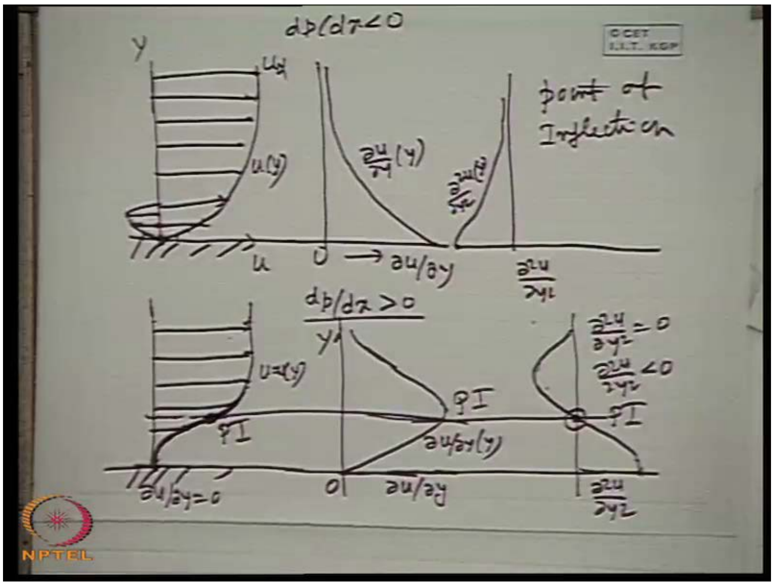

So, now we analyze the boundary layer separation from a mathematical point of view, mathematical point of view; let us consider a flow like this, let us consider a boundary layer flow like this, where the velocity profile is this a favorable pressure gradient flow; let us consider d p d x is less than 0. So, in this case we see this is the u infinity, the velocity profile tends to increase from 0 to u infinity, in this case this is u, this is y, this is u as a function of y. Now, if we draw the slope the variation of the del u del y, it is maximum here and it is going to be 0. That means del u del y profile is like this. So, this direction is y, this direction; now, del u del y this is 0. So, this is del u del y at y; that means, del u del y from a very high value, at the valve gradually approaches to 0. So, what is the del square u del y square we get del square u del y square value, we get del square u del y square.

It is negative at this point and then getting 0. So, del square u del y square is approaching. So, this is del square u del y square as a function of y, from a negative value to at 0. So, there is no point of inflection within the boundary layer. But what happens, when the flow takes place with an advance pressure gradient. So, the boundary layer grows and the velocity profile within the boundary layer is like this. So, if you draw the velocity profile at this point del u del y is 0. So, del u del y is 0 here. So, this is the. Why? del u del y is 0, this is because, we know that at the boundary layer separation point of fluid, which is flowing in this direction flows in the opposite direction; that means, when the boundary layer separation takes place the velocity profile will be like this.

So, just at the juncture of the boundary layer profile the velocity, this curvature this becomes 0; the slope changes from one direction to other direction, the sign of the slope changes from positive to negative. So, therefore, del u del y is 0 at the juncture of the boundary layer separation so, velocity profile gets a shape like this. This is a function of y now, if you draw the gradient del u del y (()), you see initially this part the velocity gradient, the curvature is positive and the velocity gradient increases del u del y increases from a value 0. So, del u del y increases.

So, after this point let this point the again the curve becomes flattened; that means, the slope gets on decreasing, then the curvature changes; the curvature is positive, again the curvature is negative; that means, this part the del u del y decreases. So, del u del y from 0 increases and then decreases and then going to 0. That means this point in del u del y is maximum. So, del u del y as a function of y. So, this is del u del y from 0, this is y and if we draw now, the del square u del y square curve, then we see that initially it is positive, it is positive then it goes on decreasing and then at this point.

This is 0 where the curve becomes flat, where del u del y is decreasing; which means del square u del y square is negative and one thing you have to understand, that far apart from the boundary layer in the adjoining main stream, the del square u del y square has to be 0 from a negative value; that means, del square u del y square must be negative. Why? This is because, the velocity approaches is maximum value and the velocity gradient is going to be decreased from an high value towards 0; like this. So, that the curvature must approach 0 from a negative value. So, therefore, the curvature in this case must cross to the 0 point, this is known as point of inflex. So, this is known as point of inflection. So, therefore, for a boundary layer separation case, the velocity profile must have a point of inflection; well I think today up to this we will discuss again in the next class.

Thank you.