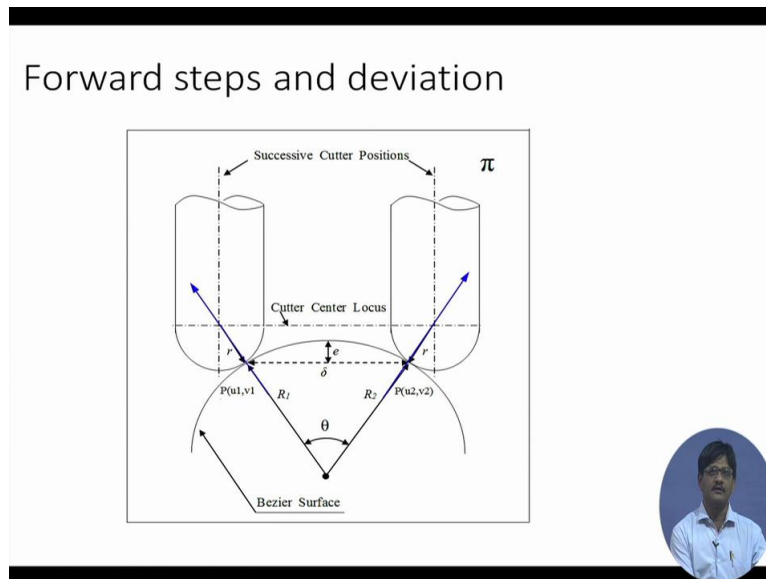


Computer Numerical Control of Machine Tools and Processes
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Lecture 18

Introduction to Computer Control- Role of Computers in Automation

Welcome to the 18th lecture in the open online course “Computer numerical control of machine tools and processes”. So we have discussed curved surface geometry, now we will discuss cutter path generation for curved surfaces, so let us move right into the subject.

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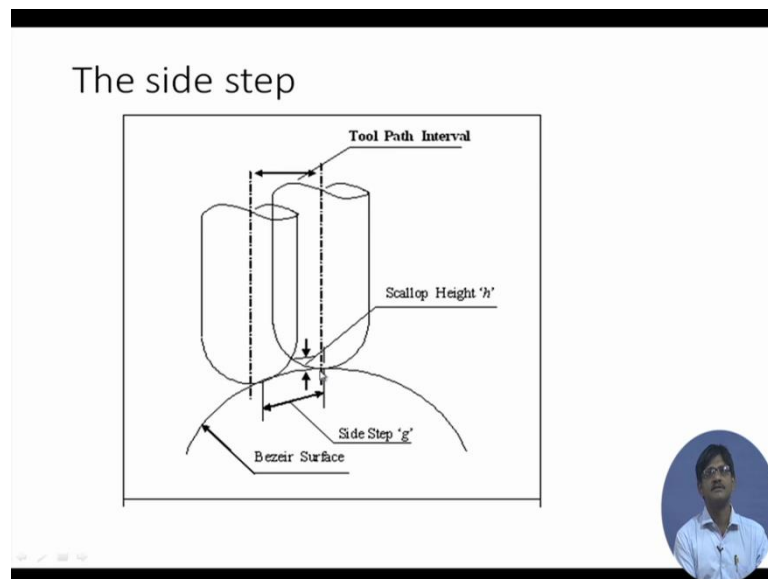


So we will be discussing 1st of all forward steps and side steps which make up the cutter path. If you remember, we had discussed about cutter path on a curved surface and we had also discussed that the cutter path has to be approximated by the cutter by straight-line segments. It moves in small I mean short straight segments to approximate a curve path. However, when we are taking this segmental when we are carrying out this linear segmental motion, the deviation that that the cutter the deviation this cutter path suffers from the design surface okay the curve surface that has to be equated to a particular value which can be tolerated.

If we take a very large value, the surface will appear jagged and might not serve its purpose. If we take E to be very small, then the surface will be appearing smoother and maybe carrying out its required function also satisfactorily, but maybe lot of effort will have to be given for manufacturing it so we have to go for a trade-off and generally we try to make all these segments in such a way that at least what is tolerated that particular dimension is attained here.

So this is one position of the cutter, this is another position of the cutter, it has moved from here to here and if we ignore external gouging at this moment then this is the path, which shows the machine surface, this is the deviation, these are the 2 radii of curvature at these 2 points. And we have to we are at present residing here, we have to somehow calculate the 2nd point at which the cutter arrives.

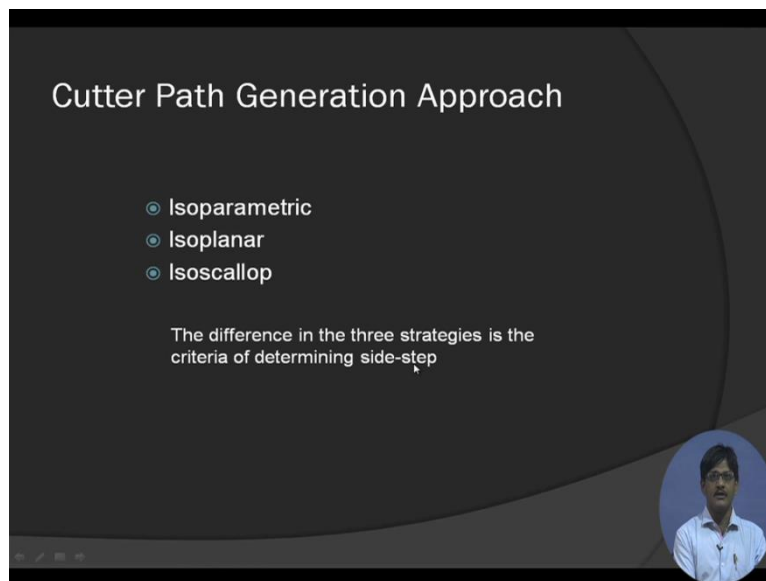
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This shows the sidestep, sidestep means that this is the 1st position of the cutter when it is executing a particular path perpendicular to the plane of the paper and when it comes here, it takes a side shift to start yet another path, in between its leaves behind this uncut material which looks like an upraised triangular portion, it is called scallop and it continues throughout between the 2 cutter paths and creates a characteristic roughness of the surface which is to be avoided at all cost. Since we cannot have 0 roughness because there has to be a shift between 2 cutter paths.

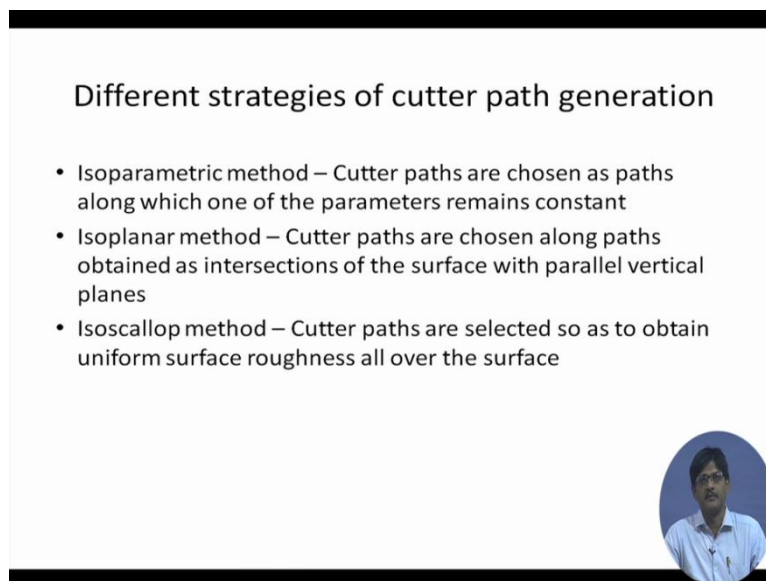
Therefore, here also we define a particular tolerance value within which this cutter surface roughness I mean cutter surface roughness is created by the cutter, this has to be within that tolerance and preferably this roughness should be the same everywhere so that all surface related effects or other surface roughness related effects will be free from all over the surface like say for example, turbulence, et cetera. So this the height of the roughness which is created here it is called scallop height and the distance between the centre lines of the tool, they are called this is called tool path interval and the distance between the cutter contact points is called the side step.

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So there are 3 methods of cutter path generation if we consider the way in which we are defining the side step and forward step these are called Isoparametric, Isoplanar and Isoscallop. What are their differences?

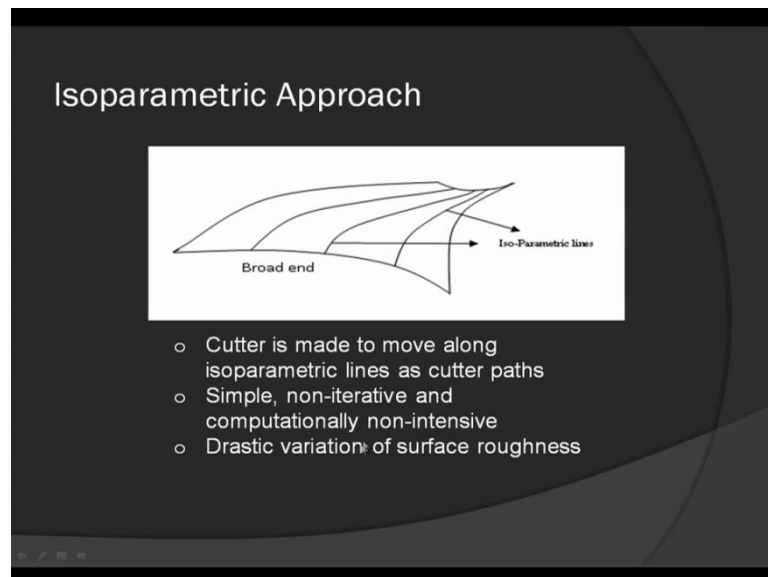
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The Isoparametric method has the cutter paths chosen as the Isoparametric lines themselves, so if the cutter paths are Isoparametric lines on the surface, they are easy to find out they are easy to determine and therefore this process is called known as Computationally non-intensive problem okay, it is not very difficult to find out the cutter paths on the surface. So obviously we would always try to resort to Isoparametric method of cutter path generation if

there are no other problems associated, what can be the other problems associated? The very shape of the curve for example...

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Say this is a particular surface and this in order to cut this surface we are we have to decide whether we will go for Isoparametric method or some other method and see due to some reason we have to cut from this side okay from this side we have to come take the cutter to the other side, not the other way not this way. So if that be so, with that precondition, we will find that if we are resorting to Isoparametric strategy of cutter path generation then essentially these cutter paths will be clustered at one end and they would be widely diverging at the other so that obviously the roughness which will be generated on this at this end of the surface, it will be much less than the roughness which will be created at this end because we can easily make this statement without proof by just observation that if the sidestep is higher, the resulting scallop height will also be higher.

So just for the sake of avoiding computational load, if we go for Isoparametric method here we are going to have a problem, and the problem is this that the roughness will definitely not be uniform and it will shoot up at this end. So this is the advantage of Isoparametric machining non-intensive computationally and this is the disadvantage of Isoparametric machining that surface roughness cannot be controlled all over the surface especially for odd shaped jobs.

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Different strategies of cutter path generation


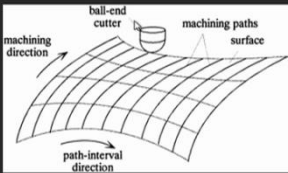
- Isoparametric method – Cutter paths are chosen as paths along which one of the parameters remains constant
- Isoplanar method – Cutter paths are chosen along paths obtained as intersections of the surface with parallel vertical planes
- Isoscallop method – Cutter paths are selected so as to obtain uniform surface roughness all over the surface



Coming back, we have another method which is called Isoplanar. In this one cutter paths are chosen along paths obtained as intersection of the surface with parallel vertical planes. So I have parallel vertical plane is just like that multiple-choice question that we had just like a knife I am going to slice the surface into different slices and those slice interaction curves are going to be my cutter paths, see the figure which will make it clear. And in the Isoscallop method we choose the cutter path specifically such that the surface roughness created by those scallops will be uniform everywhere, so this we have seen already.

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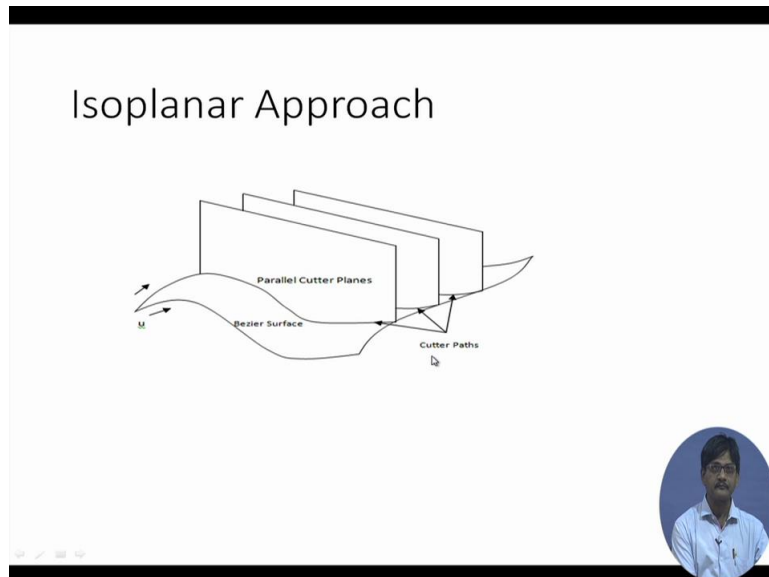
Isoparametric cutter paths



This is a depiction of Isoparametric cutter path like if this is the machining direction, the cutter is moving this way, takes a sidestep here, comes back here and it is moving by zig zag

method and ultimately the machine surface will look like this and this will be the cutter path, scallops will also be of those upraised portions will also be oriented in this direction.

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This is an example of Isoplanar approach, what does it show? It shows those parallel vertical planes which are producing cutter paths by intersection with Bezier surface or for that matter any curved surface. This is the U direction I am sorry the W has got obliterated, this is the W direction that means the 2 parametric directions are shown, the parallel vertical planes are shown and the cutter paths are obtained by the intersection of these planes and the surface. Now one question automatically appears that why are we doing this? What is the specific advantage that we have over say Isoparametric method?

So for that if you kindly remember that in the Isoparametric method we had a surface which was wide it had it had a very wide edge on one side and it was very I mean very narrow on the other side because of which their cutter paths were diverging at one end and clustered at the other, but in this case since these cutter paths are parallel to each other I mean these planes are parallel to each other, their intersection lines with the surface, they are not going to have any divergence that way. That means their distance on the X-Y plane is remaining is going to remain constant all through the cutter path extent, but distance between 2 cutter paths might well be different from the distance between 2 other cutter paths.

But in between 2 successive cutter paths or adjacent cutter paths we are always going to have the same distance, so that divergence which was occurring in Isoparametric method is going to be avoided. But mind you, the roughness is still not going to remain same because

roughness is a function of the side step which is remaining constant here. The what you call it the radius of curvature of the cutter and the radius of the curvature of the surface perpendicular to the direction of cut that is not going to be constant for giving this Isoplanar approach because that is unique to the surface, it depends on the surface not on any other aspect that we are applying.

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Isoplanar cutter paths

- cutter paths are obtained by the intersection lines between a free form surface and a number of parallel vertical planes
- Surface roughness is not uniform

Parallel Vertical planes

Bezier surface

Cutter paths

So Isoplanar method in more detail, it looks like this and the observations that we have made just now, cutter paths are obtained by intersection lines and the surface roughness is still not uniform, but of course the cutter paths are not diverging here.

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Isoplanar forward steps

$$n = (R_u \times R_w) / |R_u \times R_w|$$

$$c = \pm(n \times p) / |n \times p|$$

$$dR = \frac{\partial R}{\partial u} du + \frac{\partial R}{\partial w} dw = \frac{\partial R}{\partial u} \Delta u + \frac{\partial R}{\partial w} \Delta w$$

$$= R^u \Delta u + R^w \Delta w$$

$dR \cdot c = \delta$ and $dR \cdot p = 0$

$$\Delta u (p \cdot R_u) + \Delta w (p \cdot R_w) = 0$$

$$\Delta u (c \cdot R_u) + \Delta w (c \cdot R_w) = \delta$$

eqns (1) & (2)

For some quick calculations involving the forward steps to be taken along this particular cutter path on an Isoplanar cutter path, so how would we calculate the Isoplanar forward step? 1st of all we observe that this particular normal to this to this point say. Yes, having this as a starting point and we are targeting this particular point, I am here and I want to reach this particular point, so for that we are saying that this is the normal at this point and it is obtained by cross product of 2 of the tangents which are the U direction tangent and W direction tangent which we have already understood how to calculate them.

I am afraid, here we have use subscripts for the tangent depiction while we have up till now studied about tangent with superscripts, they are the very same thing but I sorry for the deviation. So it is divided by its magnitude in order to make it a unit vector, so we have n as the unit normal vector at this particular point, where the tool is touching the surface initially. C is the vector which is tangent to this cutter path okay, this is the curve obtained by the intersection of the plane and the surface and C happens to be the tangent to this curve, this intersection curve and the starting point. Now, how to find out C ?

We can argue that if C is tangent to this curve it is essentially a tangent to the surface which we studied in that MCQ therefore, it is essentially perpendicular to the normal because all tangents at a particular point on a surface will be perpendicular to the normal at that point to the surface, so C is perpendicular to the normal n . Further since C is lying on the plane because essentially if the curve is part of the plane therefore, C has to be part of the plane as well because C is tangent to the curve. So if C is belonging to the plane, lying on the plane therefore, it has to be perpendicular to the normal to the plane as well. And we have designated small p as a normal to the plane, so C is perpendicular to n and C is also perpendicular to p , which means that C can be obtained by cross product of n and p and that is what we have written here.

Unit C vector okay tangent vector to the cutter path must be having this expression that means it can be solved this way if we know p and if we know n . Now, what we have done after this is that any tangent okay any tangent to the cutter path is basically having expression as we have discussed previously sorry let me just look at it this way dR , we are taking a small incremental distance on the surface, small incremental distance on the surface how can we express this? What we have said is that a small incremental distance on the surface can be expressed since it is function of 2 parameters U and W , it can be expressed as partial in the U direction multiplied by du + partial in the W direction multiplied by dw .

This is a very standard expression of the complete differential and it represents a small incremental movement along the surface from the point under consideration to another particular point whose value would be defined by these 2 finite differences Δu and Δw , so I am essentially trying to move from this point to this point and I am saying that this particular distance that I am covering, I can express it in terms of the partials in this manner, partials and finite differences in this manner that is all quite good, acceptable, but where does it lead to?

It leads to this that we are now going to operate this particular movement that we have talked about operated by 2 conditions that we are going to apply on it. What are those conditions? If this happens to represent a movement on the cutter path, then it 1st has to be on the plane that is accepted yes, if it is made to represent the movement on the cutter path then it has to lie on this plane. So that we can that condition we can apply by saying that $dR \cdot p$ must be 0, because if it is lying on the plane it must be perpendicular to p and therefore, $dR \cdot p$ is 0 that is what we have written here.

Second is, we are saying that since we are talking about very small movements on the surface therefore this can well be approximated to lie in the direction of the tangent at the starting point so that we can say that if we take the dot product with the tangent to the cutter path at this point with dR , then essentially it is going to give me the magnitude of dR nothing else because dR is particular vector and the C vector we are assuming to be having absolutely the same direction, dR is so small that it will be it will be in the direction of C only. So having accepted that mentally, we are writing $dR \cdot c = \Delta$. Now what is Δ ? Δ is the length of this segmental distance that the cutter is supposed to cover from the 1st point to the next point.

But we do not know what is going to be the length, so we assume some guess for the moment, now what is going to be this guess value? For the time being we have applied this case and the last Δ that we come up with that we are applying here okay. So we write out these expressions as $dR \cdot p = 0$ here by applying the expression of dR from the partials and the finite differences and we also write out this $dR \cdot c = \Delta$ here I do not know Δ , so I put guess value. So how much do we know this? So we know R_w I mean R_u , we know R_w which are the partials, $\Delta R \Delta U$ and $\Delta R \Delta W$,

We know p if the plane is specified, do we know C ? Yes, it is the cross product of p and n , so practically everything is known except Δu and Δw , which makes them linear

simultaneous that means simple simultaneous equations. Simple simultaneous equations will allow us to find out Delta u and Delta w, which essentially means that from a point U, V, I can solve for point U + Delta u, v + Delta v which is going to have the application of these 2 conditions successfully, what are those 2 conditions? That this particular segmental movement will end up somewhere on the plane itself along the cutter path and this segmental motion will also have a length of Delta, having understood this let us see the next development.

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Forward step – contd...

$$R = \frac{(R_1 + R_2)}{2}$$

$$\delta = 2\sqrt{R^2 - (R - e)^2}$$

eqns (3) & (4)

Now we take a global look at the results that we have obtained. 1st of all, if the 2nd point okay 2nd point we already have in our hand because now we have found out Delta u and Delta v and therefore we have a numerical value of U2 W2 from U1 W1, so we were at U1 W1, we have found out U2 W2 just to make you recapitulate part. These 2 expressions help us in find out U2 W2 because the Delta values are being found out. Once we have this, we say that in that case we find out the radius values at these positions, you must say how do I know radius values? Please remember those formidable expressions I have shown you one point in time of the radius, we will come back to it once again.

We employ basically those expressions, if you say that no I could not understand it and I do not want to use it, fine in that case what we can do is we can draw normals, normal at least has been found out very easily as the cross product of the 2 partials and therefore I can draw the normal, so geometrically I can draw the normals and let them intersect at some point and I will say that these approximate values of R 1 and R 2 at these points. You may point out another problem, these normals are not lying on the same plane that they are not coplanar, in

that case we are having the plane the cutting that vertical plane with which we were intersecting, project those normals onto the vertical plane so that they are not coplanar and find out their point of intersection so that you get R_1 and R_2 .

Now are these going to be different? They can well be different because R_1 and R_2 , they represent the curvatures of the surface at these 2 points and at these 2 points they can have different values of curvature. So having found out R_1 and R_2 we take the mean value $\frac{R_1 + R_2}{2}$ and do a calculation, what is this calculation? This calculation we bring in the value of the tolerance into the calculation, we take this particular triangle which is right angle having hypotenuse = $\frac{R_1 + R_2}{2}$ that is R_{mean} R is this hypotenuse, this side is $R - E$, E value is the we have equated it to the deviation and it will be equal to the form tolerance say 50 microns.


So it will be available with us as a number, so we have $R^2 - (R - E)^2$ equal to this particular distance and what is this distance? It is nothing but $\frac{\Delta}{2}$, so what do we have is known, we have found out R in this that round about manner geometrically for example, we know the value of E so that this right hand side can be completely worked out. So we can get the value of Δ by putting in a value of R and putting in a value of E , but we already know Δ but this Δ is coming from a slightly different consideration that is the application of tolerance value, so it might well not match with this Δ I mean the Δ that we have worked out from the surface consideration.

If it does not match, we take this new Δ , go back to the expression and go back to the calculations, again find out the next point and that way find out another value of R and go on repeating this cycle as an iteration. So after several iterations if we observe that whatever Δ we put in at the beginning, it is coming out as the same value from this calculation we accept that particular Δ and move onto the next particular point and this method is called Marching and this way we can find out the forward steps one after the other.

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Steps in Isoplanar forward steps determination


1. Get a point on the edge of a surface $R(u,w)$. This is the start point of a cutter path. This is the intersection point of the edge of the surface and the vertical plane that you have started with.
2. Define a guess value of δ to go to next point on the intersection curve. Generally this guess equals final δ of last forward step.
3. Apply SPI algo (surface plane intersection algorithm) to get the next point on the surface on the cutter path – that is, get Δu and Δw
4. Refine by CPI (curve plane intersection algo)
5. Once you get u_1, w_1 and u_2, w_2 – you can find the normals at the two points. Take their projections on the vertical intersecting plane.



So let us have a quick look at the way in which we are moving. Get a point on the edge of the surface so we have to start from the edge, this is the start point of the cutter path and this is the intersection of the edge of the surface and the vertical plane that you have started with and this is found out by something called curve plane intersection algorithm, which I had not included here. So after that we define a guess value of Delta to go to the next point on the intersection curve and generally this guess equals the final Delta of the last forward step. Apply the surface plane intersection algorithm to get to the next point on the surface on the cutter path that is obtained Delta u and Delta w.

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6. Producing them backwards – get the radii at the two points. Get their mean = R
7. With the values of e (say 50 microns) and radius R find value of δ (from equations (3) and (4)). *If the guess value of δ matches this value – select δ as path length. Go to 2. Else*
8. Run SPI again with changed value of δ (that is – go to 3)
9. Continue



Refine by curve plane International golf, once again I have not discussed it. Once you get U_1 , W_1 and U_2 , W_2 , you can find out the normals at the 2 points, take their projections on the vertical intersecting plane, producing them backwards, get radii at the 2 points, get their mean equal to R with the values of say form tolerance 50 microns and the radius R . Find the value of Δ and affect the guess value of the Δ matches this value, it is good, we can start the next point then otherwise, you again go back find out Δ you and Δ w afresh with this new value of Δ and iterate until and unless you get a convergence. And this way the forward steps in Isoplanar method can be found out, thank you very much.