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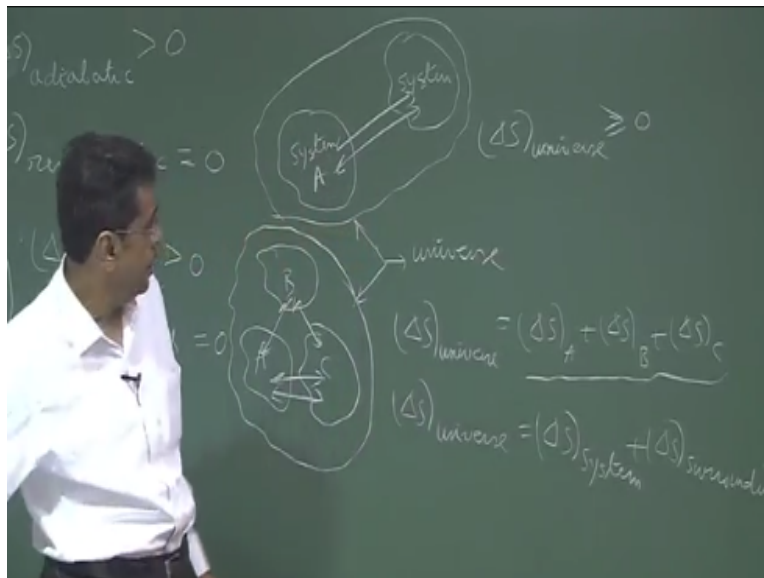
**Course**  
**on**  
**Laws of Thermodynamics**

by  
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**Lecture 16: Entropy Change in Closed Systems (Contd.)**

Good afternoon I welcome you all to this lecture session of the course laws of thermodynamics.

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Last class we have seen that the change of entropy with  $\Delta s$  in an adiabatic process is  $>$  than 0 always because of internal irreversibility if in heat there is no heat transfer if the process is reversible that internal irreversibility is 0 then reversible adiabatic process now question of external irreversibility because the process is adiabatic  $= 0$ .  
These 2 important things we have come to know at the conclusion of the definition of the entropy through Clausius inequality now an isolated system and isolated system which does not interact with this surrounding may be considered as an adiabatic system there may be changes that can take

place within the system, system may not be at equilibrium state and this changes EPT is irreversible we consider the  $\Delta s$  we can consider as isolated system is  $>$  than 0.

If we consider the change inside the isolated system that take place from one part of the system to other part if you consider a single system  $\Delta s$  isolated in case of reversible change = 0 so this two combine we can write  $\Delta s$  of an isolated system of an is  $\geq 0$  means that the change of entropy of an isolated system is  $> 0$  if the changes with the system is reversible or if the change within the system is revisable then equality sing now in this context is very important thing is that if we have two system, system A and system B interact with each other in terms of energy mass everything.

This two system as a whole and if they do not interact with anything else in this surrounding the two system itself consider constitute and isolate system when this is not for two system if we have number if system A, B, C they are mutually interacting with each other can make an isolated system and this type of isolated system in a very loss term is known as universe that means we consider in our universe there are process take place between system to system between we interact with each other there is number of interaction there are number interaction take place.

But universe is such that it does not interact outside the universal entire thing is within the universe so if that limited sense in classical thermodynamics and isolated system which constitutes interacting systems constitute the interacting systems in the surrounding as universe and therefore and they behave like an isolated system obviously I told that constituting this interacting systems and therefore we can write with this that  $\Delta s$  universe is  $\geq 0$ .

That means if we consider an isolated system consisting of interacting systems either two or more number then  $\Delta s$  of this combined system is  $> 0$  when the interactions will be each other when the interactions will be when the interactions will be irreversible when the interactions will be revisable  $\Delta s$  is universe will be equals to 0.

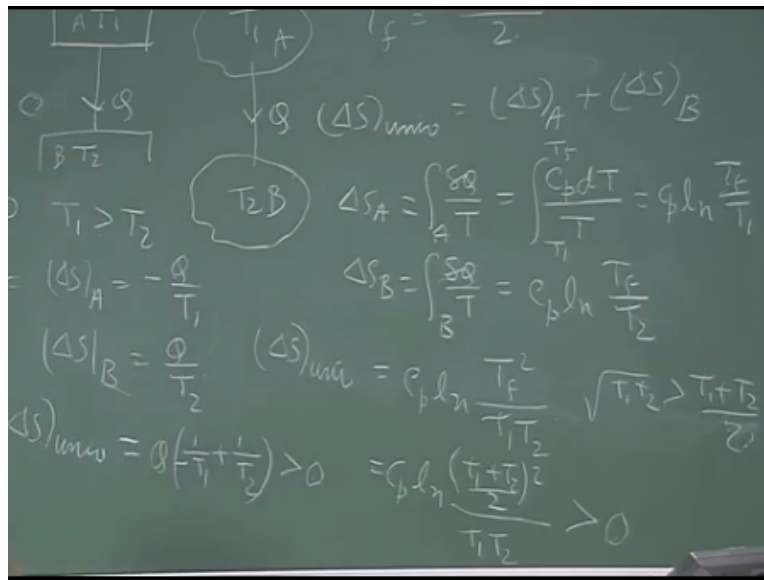
And  $\Delta$  universe that means of this isolated system is nothing but some  $\Delta s$  of all the interacting system  $\Delta s_A + \Delta s_B + \Delta s$  all the interacting system algebraic sum of all the that means sum may be positive sum may be negative in such way that there algebraic sum is always either 0 for a irreversible process or greater than 0 for nay natural process for example 2 bodies exchange heat

I will show you now if the two bodies exchange heat 1 body which rejects it is entropies decreased but one body which gets CD entropies increased but the algebraic sum is greater than 0.

If the process is irreversible and natural process otherwise it is 0, if it is irreversible process of there are two system one is consider that system another is considered that surrounding to that system sometimes we write these expression so  $\Delta S_{universe} = \Delta S_{system} + \Delta S_{surrounding}$  that also can be written  $\Delta S_{universe} = \Delta S_{system} + \Delta S_{surrounding}$ .

Now you see this concept how as it work now let us consider in irreversible heat transfer let us now consider an irreversible heat transfer and irreversible heat transfer.

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$T_1$  and irreversible and natural heat transfer process will take place if two systems are made into contact through that thermal wall, so that heat can transfer in this case where  $T_1$  is  $> T_2$  heat may occur so now entropy change of this system this is system A this is system B there is A is  $-Q / T_1$  why  $\Delta Q / T$  is  $Q / T_1$  okay, and this is  $\delta S_B$  is  $Q / T_2$  and if you consider the changes internally here irreversible so that we can write the  $\Delta Q / d$  is  $\delta S_A$  and since  $T$  remains constant that  $T_1, T_1$  comes out of the integral this becomes  $Q$  and the same  $Q$  is transferred here  $Q_2$ .

So  $\Delta S_{universe}$  will be equal to  $Q \times \left( \frac{1}{T_1} + \frac{1}{T_2} \right)$  this is  $> 0$  because it is  $< T_1, T_1$  is  $> T_2$  that means this is less that means this is always  $> 0$  another example I give it in the two bodies are finite capacity here you have considered thermal reserve here, whose heat capacity is infinitely high and keeps at constant temperature, but if I may contact of two finite bodies  $T_1$  and  $T_2$  and heat transfer takes

place so that T1 gets decreased temperature get decreased as rejects heat and T2 gets increase that it takes and the heat transfer will go and decreasing the temperature.

Difference goes on decreasing and finally the process will naturally stop oh that will I will tell a mean temperature if we consider the heat capacity is that same then you know the mean temperature is nothing but the arithmetic mean by the energy balance  $T_1 + T_2 / 2$  that means heat rejected by this body is equal to like heat gain by this body and if we consider  $T_f$  is the final temperature as we know that it be heat capacities are same  $T_f$  is there arithmetic mean, but now instead of so this a natural process this is a  $T_f$  is  $T_1 + T_2 / 2$  now how do we find out the  $\Delta S$  universe.

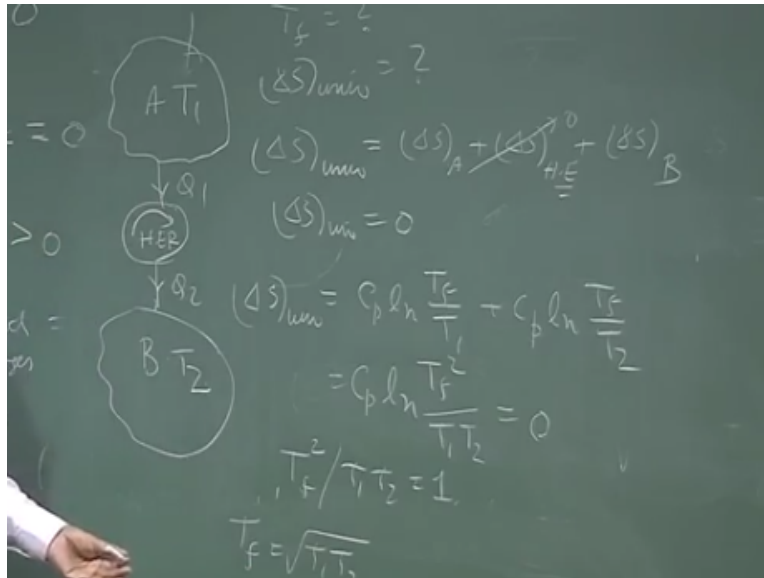
So  $\Delta S$  universe equals to  $\Delta S$  this is body A then this is body B  $\Delta S_A + \Delta S_B$  now what is  $\Delta S$  in this case  $\Delta S_A$  if we consider this is internally reversible, is integral  $\delta Q / T$  for the body and heat is equal to  $C_p \Delta T$  is the heat capacity total heat capacity mass into specific heat  $C_p \times dT$  is the reversible heat transfer /  $T$  and it is from  $T_1$  to  $T_f$  and this becomes is equal to  $C_p \ln T_f / T_1$  see  $T_f$  is  $< T_1$  this is negative obviously if  $\Delta S$  is negative because for A it is reject heat it is cool so therefore we cannot take  $T$  out because  $\Delta Q$  is  $C_p dT$  and  $T$  is changing.

So therefore we have to integrate like this similar way if  $\Delta S$  becomes around  $\Delta Q / T$  this is for A this for B and that becomes is equal to at the similar way  $C_p \ln T_f / T_2$  and this is positive because  $T_f > T_2$ , So  $\Delta S$  universe is equal to  $\Delta S_A + \Delta S_B$ ,  $C_p \ln T_f^2 / T_1 T_2$  and  $T_f^2$  is what?  $C_p \ln (T_1 + T_2)^2 / (T_1 + T_2)^2 / T_1 T_2$  this is always positive we can prove this is because we know that geometric mean root over  $T_1 T_2$  is always greater than the arithmetic mean.

This we have proved now this we can prove that  $(T_1 + T_2)^2$  is always greater than  $T_1 T_2$  that means it is always greater than 0 that means it is greater than 0, so we can prove since the arithmetic mean is greater than the geometric mean that is  $T_1 + T_2 / 2$  is always greater than  $\sqrt{T_1 T_2}$ ,  $(T_1 + T_2)^2 / 4 > T_1 T_2$  or this whole square is greater than  $T_1 T_2$  we can prove simple school level algebra so that entropy change is greater than 0 for a finite word.

Now another very interesting thing comes into picture that if the two finite bodies re attach or connected through a reversible heat engine then what will be its final temperature this is a very interesting problem.

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That if two bodies A at temperature  $T_1$  another body B at temperature  $T_2$  if they are connected through a reversible heat engine sometimes this is a problem that it takes it  $Q_1$  which is going on changing because their finite capacity body, so what will be  $T_A$  and what will be  $\Delta x$  units,  $T_A$  is not the arithmetic mean so you have to find out  $T_A$  because they are not in direct thermal contact, now here to find out  $T_A$  we must know this thing this will act as a clue to find out  $T_A$  since this is the internally irreversibility taken to is 0 for both these things and since this is connected to a reversible heat engine which takes also heat reversibly from this two reserves. So the entire process is reversible this system reversible heat engine reversible this that means  $\Delta$  is universe is  $\Delta$  is A +  $\Delta$  is heat engine +  $\Delta$  is B, now for a heat engine whether it is reversible or irreversible the entropy change is always 0 because entropy is a property and it needs operates on a thermodynamic side 0 and  $\Delta S_A$  and  $\Delta S_B$  they are all operates on a reversible process so that  $\Delta S_A$  and  $\Delta S_B$  summation algebraic summation will become 0 so  $\Delta S$  is universe will be 0.

That means  $\Delta S_A + \Delta S_B$  becomes 0, we take this clue to find out  $T_F$  what is  $\Delta S_A$ , that means  $\Delta$  is universe =  $\Delta S$  it will be the same equation  $L_n T_f \int \Delta Q/T + C_p L_N T_F/T_2$  so that this is  $C_p L_N$ , this expression will remain same earlier one because this comes from the integral  $dq/ \Delta Q/T$  and in this case this becomes equal to 0, that means  $T_F^2/T_1 T_2$  is 1 L and  $T_F^2 T_1, T_2$  0 that means  $T_F$  is geometric mean.

So in case of direct thermal conduct of two finite bodies of same capacity is arithmetic mean while it is connected by a reversible heat engine internally reversible it is of the system at 0 then the mean temperature is root over  $T_1, T_2$  is the geometric mean okay. Now after this I will solve

few problem okay I think the concept is clear to you therefore you see before I close this I tell you that it is now concluded that for all natural processes in this universe which is considered as an isolated system takes place in such a way.

That the  $\Delta$  is of the universe is great than 0 and this is the direction of constant I told at the beginning of this class that there is directional constant for any process, any natural process occurs in such a way that it makes a permanent indentation to the surrounding, that means entropy change of the process plus the entropy change of the interacting systems are surrounding algebraic sum of this two mass be greater than 0.

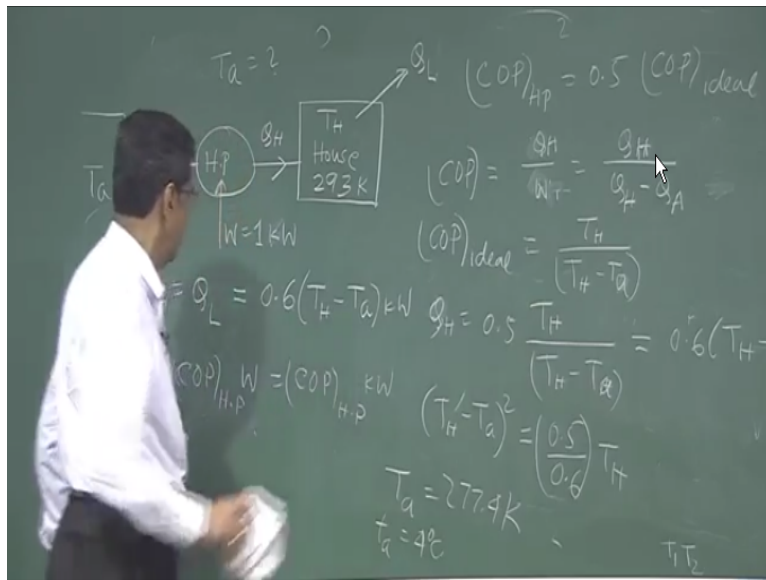
If the processes are natural, that means if you consider an entropy band of in an universe the entropy is monotonically increasing that means entropy change of the universe always increases for example, I am teaching to you this is a process in this process I increase the entropy of the universe, so there is an increasing the entropy of the universe, entropy is generated in the universe so this is the directional constant of any natural process, okay in general. Now I will solve some problem, let u see this problem.  
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**Problem 5:** A heat pump has a coefficient of performance that is 50% of the theoretical maximum. It maintains a house at 20°C, which leaks energy of 0.6 kW per degree temperature difference to the ambient. For a maximum of 1.0 kW power input, find the minimum outside temperature for which the heat pump is a sufficient heat source.

A heat pump has a coefficient of performance that is 50% of the theoretical maximum, it maintains a house at 20°C which leaks energy of 0.6 kW per degree of temperature difference to the ambient, that means temperature difference from the house and the ambient. For a maximum

of 1.0kW power input to the heat pump find the minimum outside temperature for which the heat pump is a sufficient heat source, okay. Now let us come to the board and see what this tells.

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Now there is a house this is the house and there is a heat pump let the temperature of the house is  $T_H$  which is  $20^\circ\text{C}$  it means  $293\text{K}$  actually it is  $273.15$  you have to add because the thermodynamic scale of temperature is define, which has defined, which has been defined as I told but the scale is like that  $0^\circ\text{C}$  corresponds to  $273.15\text{K}$  but you can neglect this  $0.15$   $293\text{K}$  and heat from gives heat  $Q_H$  to this house and it takes heat from this surroundings, this is the surrounding temperature  $T_a$  which we have to find out, so this takes heat from the surrounding let this is  $Q_A$  we have to find out  $T_a$ .

And it takes work as  $1\text{kW}$ , let us work with the  $1\text{kW}$  heat and find out what is the value of  $T_H$  and that will ne  $T_a$  and that will be the minimum value, why I will tell you afterward. This house

leaks energy which is  $Q_L$  is the heat at which the heat leaks from the house to the surrounding. Now to maintain the house at 293K  $Q_H$  will be automatically by common sense this much less than 293K it is the heating the house in knowing tank in Western countries.

Now this  $Q_H$  will be such to maintain the house at 293K must equal  $Q_L$  that means the heat which it leaks must be giving by  $Q_H$  a steady state when the house as attend 293K that means  $Q_H = Q_L$  and  $Q_L$  is it is what 0.6kw  $Q_w$  what is this  $Q_L$  is 0.6 KW per degree of temperature difference that means  $0.6 \times T_h - T_a$  that much KW, because by definition 0.6kw per degree difference  $T_h - T_a$   $Q_L$  this is  $Q_H$ . Now we have to express  $Q_H$  now in terms of this temperature  $T_a$  and  $T_h$  so that we can find out the  $T$ , minimum temperature  $T$ .

How you will find out now this  $Q_H$  you can be written as the coefficient of performance of the heat pump in to  $w$  because coefficient of performance is defined as  $Q_H / w$  and  $w = 1$  so therefore it is simply this cop of the heat pump. That much KW why because  $w$  is 1kw now we have to express cop in terms of temperature and it is written in the problem the heat pump as a coefficient of performance which is 50% of the ideal one that means cop of heat pump is 0.5 of the cop of the ideal.

Now ideal cop we can write cop is usually can be written  $w/Q_H$  or I am sorry extremely  $Q_H / w$  or  $Q_H / (T_h - T_a)$  and for a ideal cop this is the basic definition then they are proportional to their temperature, so therefore cop will be 0.5 of this so therefore we can write  $Q_H = 0.5$  of this that means  $Q_H = \text{cop of heat pump } w$  is one so  $Q_H = 0.5 (T_h - T_a) / (T_h - T_a)$  very simple problem  $T_a$  small am I have given small a sorry.

$T_h - T_a$  and that must be equal to  $0.6 (T_h - T_a)$  okay so therefore it must be equal to  $0.5 (T_h - T_a)$ ,  $T_h - T_a$  must =  $0.6 (T_h - T_a)$  because this is  $Q_H = q_a$  so therefore we get  $(T_h - T_a)^2 = 0.5 / 0.6 \times T_h$  this is very simple  $0.5 / 0.6 (T_h - T_a)$  is  $T_h - T_a$  and if you solving  $T_h$  is 293 K you get the value of  $T_a$  the negative value you just in ignore which will give a value of  $T_a$  higher than  $T_h$  so we take the positive value the answer is 277.4 K that means  $T_a$  is in the usual centigrade for Celsius scale is  $4^\circ$  Celsius so this is simply this problem okay I think you have understood this problem well so how I have done this  $Q_H$  is equal to  $Q_L$  is by definition is  $Q_H$  is  $\text{COP} \times W$  is 1 kilo watt so they are simply it is COP kilo watt.



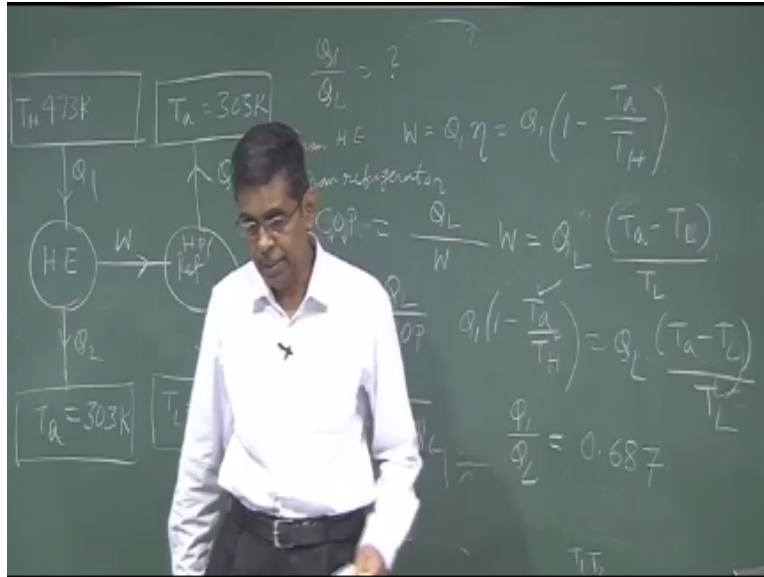
So then right COP of the hit pump is .50% of this is ideal we cannot right in terms of temperature for non ideal hit pump but it is .5 of the ideal hit pump COP so therefore  $Q_H$  is equal to COP is equal to this 60% okay.

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**Problem 7:** We wish to produce refrigeration at  $-30^\circ\text{C}$ . A reservoir is available at  $200^\circ\text{C}$ , and the ambient temperature is  $30^\circ\text{C}$ . Thus, work can be done by a cyclic heat engine operating between the  $200^\circ\text{C}$  reservoir and the ambient. This work is used to drive the refrigerator. Determine the ratio of the heat transferred from the  $200^\circ\text{C}$  reservoir to the heat transferred from the  $-30^\circ\text{C}$  reservoir, assuming all processes are reversible.

Now come to the next problem, now we come to the next problem now this is the same problem we wish to produce refrigeration at  $-30^\circ\text{C}$  a reservoir is available at  $200^\circ\text{C}$  and the ambient temperature is  $30^\circ\text{C}$  a reservoir is a  $200^\circ\text{C}$  and the ambient temperature is  $30^\circ\text{C}$  thus work can be done by a cyclic heat engine operating between  $200^\circ\text{C}$  reservoir and the ambient this work is used to drive the refrigerator determine the ratio of the heat transferred from the  $200^\circ\text{C}$  reservoir to the het transferred from  $-30^\circ\text{C}$  reservoir assuming all processes are reversible.

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Now this is like this we have ambient we have a hot source at  $200^{\circ}\text{C}$  that means  $473\text{K}$  this is  $T_H$  hot source then we operate the heat engine with the ambient the heating takes heat  $Q_1$  and rejects it  $Q_2$  to an ambient temperature  $T_H$  which is how much  $T_H$  and ambient  $30^{\circ}\text{C}$   $T_a$  which is equals to  $30^{\circ}$  that means  $303\text{K}$   $30+273$   $303\text{K}$ .

Now this heat engine drives a heat pump refrigerator here you write refrigerator, refrigerator which takes a same thing heat pump the same device heat pump or refrigerator takes heat from – cool temperature  $T_L$  low temperature which equals to  $-30^{\circ}\text{C}$  that means this is what is the  $-30^{\circ}\text{C}$  okay.

Let me see the  $-30^{\circ}\text{C}$  that means it is  $243\text{K}$  it takes  $Q_L$  and it gives to the ambient temperature, ambient temperature in this case here see it Cit pumps heat from the low temperature and gives to the ambient temperature which is  $303\text{K}$  and this heat you consider as  $Q_H$  which it gives here so  $Q_1$   $Q_2$  are the heat taken by the heat engine giving by heat engine this is by refrigerator given by the refrigerator to this were it access the heat part.

Now our problem is that we have to find out  $Q_1/Q_L$  what is  $Q_1/Q_L$ . So it is very simple  $W$  is the connecting it, that means we connect  $W$  with  $Q_1$  and then we connect  $W$  with  $Q_L$  and equate it and then we get the result, because the same  $W$  driving the refrigeration, we have temperature this from where the refrigeration has to be made and we have an ambient where the refrigerator will reject.

But do we have a source, so that with this as source and the ambient as your seeing a heat engine can operate at that heat engine can drive the refrigerator that means the same  $W$ . So from the heat engine, we get  $W$  is  $Q_1 \times \eta$  and here all processes are reversible. It is told in the problem that assumes all processes as reversible, so  $\eta$  is what?  $Q_1 \times 1 - T_A / T_H$ .  $W$  is  $Q_1 \times 1 - T_A / T_H$ , from here  $W$  how can I find out?  $W$  here COP is defined as from refrigerator, from refrigerator COP is what? COP is  $Q_L / W$ .

So  $Q_L = W = Q_L / \text{COP}$  okay and at the same time COP since this is a reversible heat pump COP is  $Q_L / (Q_H - Q_L)$  that is  $T_L / (T_A - T_L)$  because this is reversible heat pump or refrigerator, so we can express this in terms of the temperatures  $T_L / (T_A - T_L)$  okay. So now  $W =$  what from the refrigeration from the refrigerator? We get  $W = Q_L / \text{COP}$  and COP is that, so  $W = Q_L \times 1 / \text{COP}$ , that means  $T_A - T_H / (T_A - T_L) / T_L$  that is this temperature.

So these two are equal  $W$ , so therefore  $Q_1 \times 1 - T_A / T_H$  that is the efficiency  $Q_L \times T_A - T_L / T_L$  here we get everything, so we get  $Q_1 / Q_L$  if you take this side divided by this, we know  $T_A$  303 we know  $T_H$  473 and we know  $T_L$  is 243. So you know all the temperatures, so therefore we can get the ratio  $Q_1 / Q_L$  which is  $= 0.687$ , okay today up to this thank you.