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Course
on
Laws of Thermodynamics

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Lecture 18: Entropy And its Transport

So far we have discussed various phases of entropy transport and we will continue with that and the specific agenda of today's discussion we will to mathematically discuss the various aspects that come with the deception of change in entropy of a system for a control mass system as well as for a flow process so to began with I will try to draw your attention on the following in equality.

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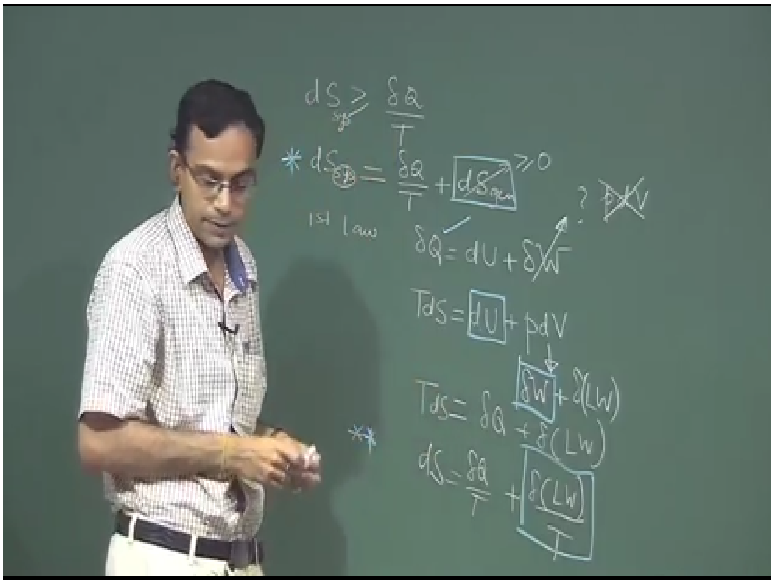

Ds system is ≥ ∂ Q / T so this is something which is I think the most important inequality that we talk about when we discuss about the change in entropy so what is this T something which is very important and we will go through that though various teasing questions and exercises in the subsequent two three modules but this T conceptually is the temperature of the system bounded here cross which this Δ heat transfer is taking this, so it is not any orbiter T it is the temperature of that system bounded here across which the heat transfer is taking place for which you're calculating this ds system.

So now you can also write this or convert this inequality in the form of a equality so you can write ds system is equal to $\partial Q / T$ + entropy generation which is $\geq$ 0m this is for a system now of course the mathematical objective is satisfied here that we all are understand that this terminology is introduced to convert the inequality to 1 equality but what is the physical significant of this in the context of heat and work we have to understand that entropy generation is a general conceptual paradane it should not always be associated with heat transfer.

It could be associated with flow of information in an information system and so but here we are considering the heat transfer as an example, so now we can write so let us do let us do a different exercise let us write the 1st law for a system as ∂Q again we are not considering the change in kinetic energy and potential energy because we are considering that the thermal effects are more important.

So now in place of ∂Q can we write TDS the answer is we can write TDS only for reversible process if it is not reversible process we cannot write ∂Q = TDS now what is ∂W so in place of ∂W we can write the following the first question that let me ask me you that can be write this Pdv, okay so can we write as Pdv what is your answer, so let think about it let us say that we are instead of this we are writing ∂Q = TDS so if we do that then in place of ∂W we can write Pdv with an understanding that we are talking about a simple complex substance.

So that the work is only Pdv work but the Pdv wok is not the work that you can get as output when that you get as output when the process is irreversible so the question is when the process is irreversible what is the work that you get when the process is irreversible the work that you get as an output should be less than Pdv that means because of irreversibility the irreversibility may be associated with fraction free expansion whatever you are losing an opportunity of doing work so you can write Pd vans your actual ∂W + some δLW which is something positive so the last work is 0 when it is reversible process.

So that it will be pdV but if it is not a reversible process then you have last work, so technically speaking you cannot write $LW = pdV$ always so when you write this $TdS = dU + \delta W + \delta LW$ let me show let me try to use here so $dU + \delta W$ this together from here you can write δQ so $TdS = \delta Q + \delta LW$, so that means dS so when we write dS we mean dS system not always we are writing the sub secrete dS system so $dS = \delta Q$ over $T + \delta LW$ over T.

So now compare this * with ** okay compare the * with ** okay so we can write if you compare the * with the ** we can clearly write that this entropy generation is nothing but this LW/T that means entropy generation is always associated with the last opportunity of doing one that can be taken as a physical interpretation of entropy generation in a system where you are trying to use some heat transfer to convert that into work so if you are using some heat transfer to convert that into work there is a maximum potential of that heat to be converted to work, but you may not be able to utilize that maximum potential.

Because of entropy generation or irreversibility in the system, so physically you may have last opportunity of doing work for last work, so this is the first physical interpretation that I want to give and then using the same inequality let me try to give you a different prospect let us give an example.

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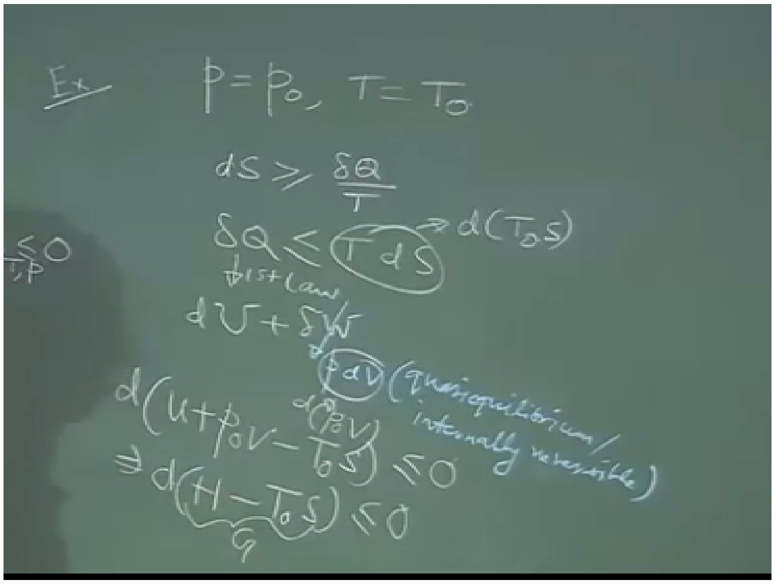

Ex $P = P_0,\ T = T_0$

$dS \geq \frac{\delta Q}{T}$

$\delta Q \leq \boxed{TdS} \rightarrow d(T_0 S)$

first law

$dU + \delta W$

$\boxed{pdV}$ (Quasiequilibrium/ internally reversible)

$d(P_0V)$

$d(U + P_0V - T_0S) \leq 0$

$\Rightarrow d\underbrace{(H - T_0S)}_{G} \leq 0$

≤ 0

T,P

Where you have process with P = P_0 and T = T_0 that is pressure and temperature they are kept as constant but still there is a change within the system how is it possible, remember that for system to be in equilibrium it has to be mechanically equilibrium thermal equilibrium and phrase and chemical equilibrium, so these two indicate mechanical and thermal equilibrium but it does not guarantee in phase and chemical equilibrium, so they are put still be a change within the system even if you keep.

These two same and let us consider certain such a change so we can write again for such a system dS > = δQ over T and again this T is absolute temperature by this time you know very well that in when you are considering something in the context of second law that T is absolute temperature, so now you can write δQ < = TdS if you apply the first law in place of δQ you can write dU + δW neglecting changes in kinetic energy and potential energy when can you write this δW as pdV.

When it is a internally reversible process or pass equilibrium process so pass equilibrium or internally reversible these two terms are equivalent or internally reversible, so now this is a process where T is kept as constant so you can write TdS or rather sorry I have written δU by mistake, so dU d of U + now this pdV you can write d of P0V because P0 is a constant you can take it within the difference here, so d of U + P0V similarly this you can write as d(TOS) so u+ P0V – TOS <= 0 this means d of what is u+ pv, U + PV is the enthalpy H so d(H) with reference to pressure P0 – T0S <= 0, so here we come up with another mathematical function which is a combination o properties this we call as Gibb's function or Gibb's free energy of the system.

So DG so that means we can say that any spontaneous process will take place if at a constant temperature and pressure the Gibb's function or Gibb's free energy is decreasing so a spontaneous process cannot take place if the Gibb's free energy is increasing and it comes to equilibrium when it is a minimum that there is no further change in the Gibb's free energy.

So why I am telling you this because in this particular course we do not have a lot of scope of discussing about the Gibb's function and its interpretation in the context of phase and chemical equilibrium but I want to give you a very basic elementary physical picture on the Gibb's function its physical interpretation so you can see that these G therefore is the driving force for a chemical change to take place or driving force for a phase change to take place.

And we have already discussed that for a phase or chemical equilibrium the chemical potential concept with the picture and that chemical potential is linked with the partial moral Gibb's function. Now in the two examples that we have consider in this today's lecture so far we have considered the cases of irreversible situations but let us consider an example where we consider reversible adiabatic process of an ideal gas.

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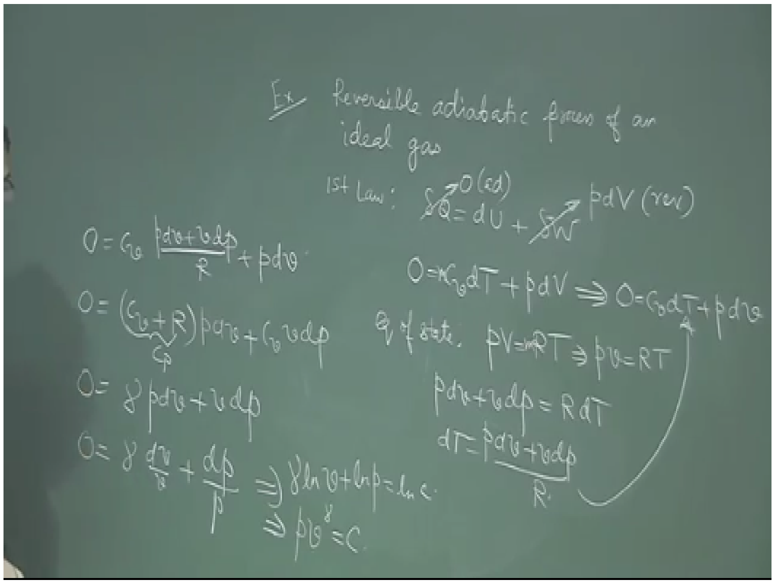

So let us apply the first law and second law for this, so let us apply the first law. δQ = DU + δW now because it is reversible you can write this PDV Ideal gas we consider it to be a simple compressible substance with the pressure volume temperature changes are more important than other changes and because it is adiabatic this is 0 so you can say so this PDV is for reversible and this is for adiabatic.

So we have used both the considerations of reversible and adiabatic now for an ideal gas DU is CVDT this we already discussed in one of our previous lectures and we know for an ideal gas the equation of state is PV = RT so let us apply it for a unit mass so if you so this will be M CVDT if it is total mass so if you apply per unit mass it will be CV0 = CVDT + P x d of specific volume, right.

So for an ideal gas you can write P x specific volume = RT so if you take the differential of both sides you have PDV + VDP is RDT. So you can write DT is PDV + VDP/R, let us substitute this here so if you substitute this here then you get 0 = CV PDV + VDP /R + PDV. So you can write 0 = CV + R PDV + CV VDP, CV + R is CP and if you divide both sides by CV then you have CP/CV = γ. So 0 = γ PDV + VDP so you can divide both sides by PV and what you get is 0 = γ dv/v + dp/p so you can divide both sides by pv and what you get is 0=γdv/v+dp+p so if you integrate now you will get γlnv+lnp=lnc or this means pv^{γ}=constant, so this shows that for a reversible adiabatic process of an ideal gas the pressure volume will essentially shows that pv^{γ}=constant.

With this back ground now let me give you a teaser, so let me relax the condition that it is a reversible process, this is the next example it is an adiabatic process of an ideal gas but need not be reversible that is the next example, so what happens here.

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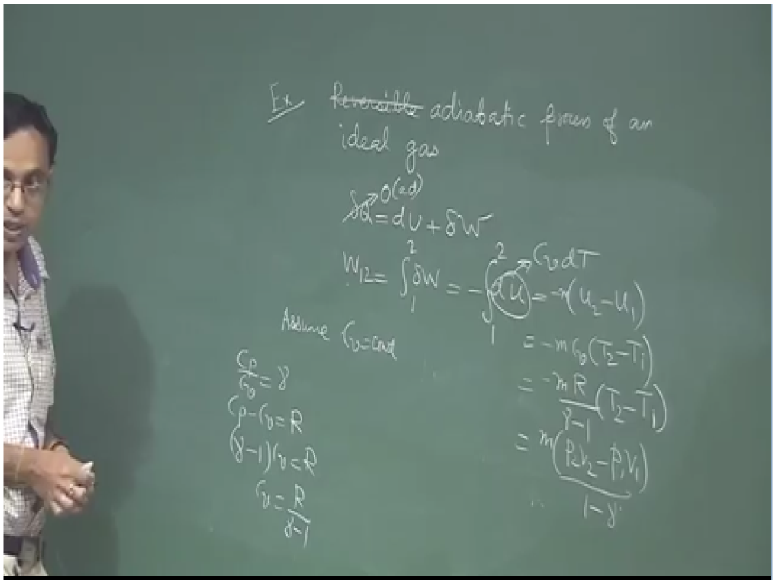

Ex ~~Reversible~~ adiabatic process of an ideal gas

$\cancel{\delta Q} \xrightarrow{} 0 \text{ (ad)}$

$\delta Q = dU + \delta W$

$W_{12} = \int_1^2 \delta W = -\int_1^2 \boxed{dU} = -m(u_2 - u_1)$

$dU \rightarrow C_v dT$

Assume $C_v = \text{const}$

$= -m C_v (T_2 - T_1)$

$= -\frac{mR}{\gamma - 1}(T_2 - T_1)$

$= m\frac{(p_2 v_2 - p_1 v_1)}{1 - \gamma}$

$\frac{C_p}{C_v} = \gamma$

$C_p - C_v = R$

$(\gamma - 1)C_v = R$

$C_v = \frac{R}{\gamma - 1}$

So δQ=du+W by the way for the reversible adiabatic process of an ideal gas if you have pv^{γ}= constant the work done we have already seen that is integral pdv so p2v2-p1v2/1-γ in the chapter of heat and watt we have derived this, we can example of a reversible polytrophic process we have air we have use the index n here just use γ instead of n. Now we will find out the work done here, so it is a adiabatic therefore δQ is 0 it is not reversible.

So we cannot write δW=pdv so W12=∫δW from state 1 to state 2 is equal to –du integral of this, so –(u2-u1). So as an example, assume so du is what so this is per unit mass so m.du so m.u2-u1 so assume Cv=constant, so this if you assume Cv=constant this is then becomes mCv(T2-T1). So now you know that you can write Cp/Cv=γ and Cp-Cv=R so from this you can write Cv in terms of R Cp you can write as γCv so γ-1 Cv=R so Cv=R/γ-1.

So –mR/ or minus you can absorb okay, -mR/γ-1 so you can write R(Tspv) so this you can write (p2v2-p1v1)/1-γ this is something which is bound to create a confusion within you that with the reversible adiabatic process whatever answer we got as work done the same answer we have got as work done when we relax the requirement of reversible process, so the work is becoming same for reversible and irreversible process so where is the fallacy.

Actually the work done p2v2-p1v1/1-γ is true for a reversible process even if Cp and Cv are functions of temperature, but for an irreversible process we show that this is true only if Cp, Cv are constants otherwise this integration cannot be done in this way. So this is true only for calorically prefect gas, but not for all ideal gases. Calorically prefect means only when Cp, Cv is constant but if Cp, Cv is not constant then we cannot use this, okay. The next agenda will be to discuss about.

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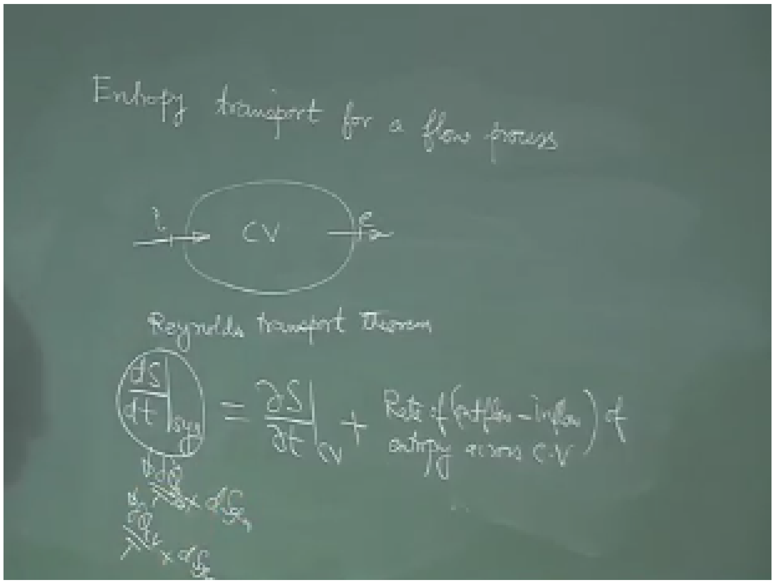

Entropy transport for a flow process, so let us say that there is a control volume there is some inflow boundary there is some outflow boundary, so we can write the entropy transport across the control volume in terms of the entropy transport of a control mass system, why we intend to write it in this way is because we already know how to describe the entropy transport trough across a control mass system. So we want to use that to extend it to the entropy transport for a control volume so for control mass assister we have already read the ds = ∂q/ T + d of entropy generation.

So that means we know what is ds of with respect to system so now we want to apply that to ds with respect to control volume so what w e want to do is something like this let us say that we apply the Reynolds transport theorem, so I am straight away writing it for taking the property as s it could be any general extensive property so ds dt for with respect to the system is ∂s / ∂t with respect to the control volume plus rate of outflow minus inflow of entropy across the control volume.

So instead of entropy we could use any property but as an example we are using entropy as a property now what is ds dt of the system this is ∂q/ T + ds gen this ∂q / T is the ∂q system but in the derivation of Reynolds transport theorem what is considered is that there is a control mass system which is go inside with this control volume in the limit as that time ∂t that is under consideration is 10 x 0. So when the control mass system is almost coincident with this control volume that means the δq of the system will be same as δq control volume, so this will become δq cv / t + dsj.

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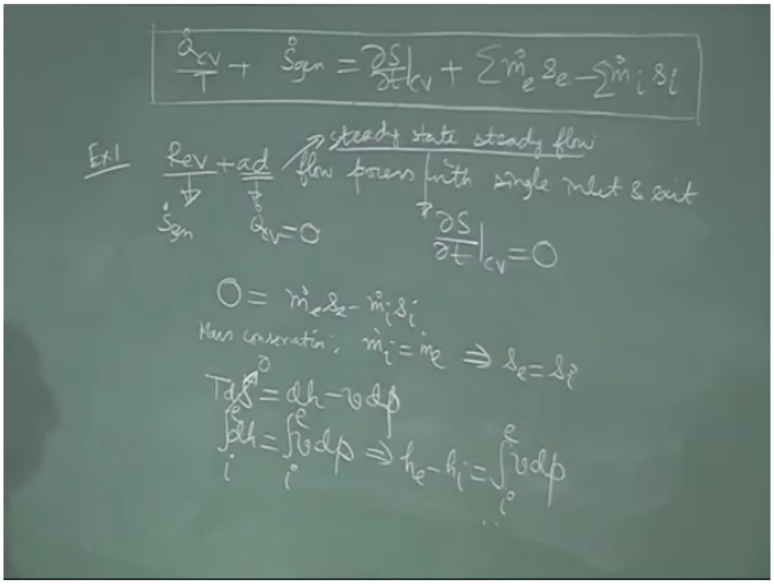

$$\boxed{\frac{\dot{Q}_{cv}}{T} + \dot{S}_{gen} = \frac{\partial S}{\partial t}\Big|_{cv} + \sum \dot{m}_e s_e - \sum \dot{m}_i s_i}$$

Ex1 Rev + ad flow process with single inlet & exit

steady state steady flow

$\dot{S}_{gen}$ $\dot{Q}_{cv} = 0$ $\frac{\partial S}{\partial t}\Big|_{cv} = 0$

$$0 = \dot{m}_e s_e - \dot{m}_i s_i$$

Mass conservation; $\dot{m}_i = \dot{m}_e \Rightarrow s_e = s_i$

$$T\,\overset{0}{\cancel{ds}} = dh - v\,dp$$

$$\int_i^e dh = \int_i^e v\,dp \Rightarrow h_e - h_i = \int_i^e v\,dp$$

Therefore we can write δqcv / t + Ds gen = ∂s ∂t cv + rate of outflow – inflow of entropy is Σ m° e se – Σ m°I si this gives what is m° is the mass flow rate this is waste on the assumption that across the inflow and the outflow boundaries this is inflow boundary this sis out flow boundary there could be many inflow and many outflow boundaries across each boundary the property is uniform so that the total property you can just simply write mass flow rate times the entropy that particular section so this can be thought of as an average specific entropy over the section across which the transport is taking place.

So this is so you can write so you can write this equation or this particular equation as an equation that commands the entropy transport during a flow process, so let me give you one example we will talk about two examples first example reversible and adiabatic flow process with single inlet and exit, so because it is adiabatic you have so one particular in this expression one particular thing which we have missed is that the right hand side is expressed on the rate equation.

So left hand side should also be expressed as a rate equation so instead of ds this it will be s dot gen that is the rate of entropy generation and this will be Q.CV/t so the important thing that we have made important consideration we have made is that right hand side we have expressed on the rate equation in a transport theorem so that left hand side should also be expressed on the rate equation.

The expression for the second law that we have used S2-S1=Q/t+ entropy generation that is not the rate that is the total change so we divide that by time Δt and take the Kelvin as Δt is used so Q.CV IS 0 reversible means entropy generation is here let us consider that in addition to this reversible adiabatic steady state steady flow process this is very important.

A reversible adiabatic steady state steady flow process so if you considering this steady state steady flow process what is essentially means that there is no change of property within the control volume so steady state steady flow process there is no change in the property within the control volume and single in let and single outlet means this £ will be replaced by just you can write 0 is equal to m.e Se- m.i Si.

And the mass conservation is m.e is equal to m.i so you can write Se is equal to Si so this is true for reversible and adiabatic process so this kind of process is also called as isotropic process because the entropy remains constant throughout the process now let us applied the Tds relationship you can apply the Tds relationship for any process so you can write Tds equal to D_H-Tdp so now you apply this there is no change in the entropy during the process, so this is 0 so D_h is equal to Tdp so i to e this is I to e so He-Hi is equal to integral of Tdp from i to e

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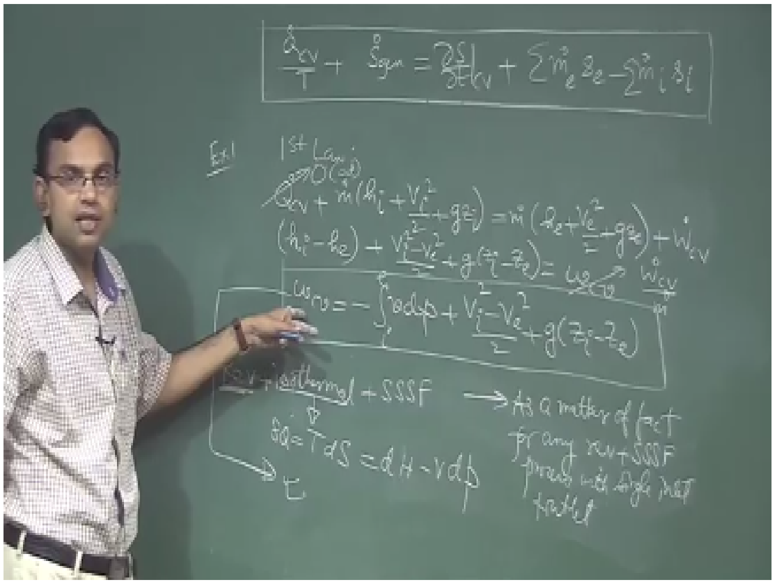

Now let us apply the first law so Qcv+m.*Hi+Vi^2/2+gzi+m.*He+Ve$_2$/2+gze+wcv what is w.$_{cv}$ is w.$_{cv}$ / m dot okay. Now look at this equation hi – he = - integral of Vdp so you can write w.$_{cv}$ integral of Vdp from I to e + Vi$^{2\ -}$ Ve^2/ 2 + gzi – ze, this is true for reversible adiabatic process. If you consider the isothermal process reversible + isothermal + steady state steady flow, so for the isothermal process so we are replacing the adiabatic process by isothermal process, so isothermal process because it is reversible isothermal process you can write Q. = or ΔQ = Tds.

And Tds you can write as dh – vdp, so this you can always write okay, so in place of the heat transfer now you could, now you cannot write 0 but you can write Tds in place of Tds you can write dh – Vdp. So if you write dh – Vdp then you will see that the work done expression comes out to be the same. You just substitute this ΔQ as Tds ad dh – vdp and put it in the expression for the first law. So instead of it will come out to be m dot x h2 – h1 – m dot integral x vdp, where small v is the specific volume.

So you will see that you will get the same expression for the work done, so we are consider two limiting cases one reversible adiabatic and one is reversible infact any reversible process it does not need to be isothermal because here we are not consider any where the t is constant, we have just used Tds this t itself could be a variable therefore we have not committed the t is the constant so reversible isothermal is a special case but it could be any general process.

So for any reversible process, so as a matter of fact for any reversible steady state steady flow process with single inlet and outlet, this is the expression that we get for the work done per unit mass, it is very important expression and we will take up with this expression and go further for our next lecture thank you very much.