

**Energy Conservation and Waste Heat Recovery**  
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**Lecture - 31**  
**Heat exchanger (Contd.)**

Welcome back, we were discussing heat exchangers and particularly, we were trying to do some sort of analysis I have told that we will not derive any formula, but we will just give you the formula. So, that will be some sort of a recapitulation and while I will provide you with the formula for heat exchanger analysis, we will also do some sort of we will give also some sort of explanation to it.

So, we have done the first method which is very basic that is for that is LMTD or log mean temperature difference method the mean temperature difference for either a parallel flow heat exchanger or a counter flow heat exchanger is given by some sort of a logarithmic term. So, that is why; it is called log mean temperature difference or in abbreviation LMTD and if we go to the slide once again.

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**1. F-LMTD**

F-LMTD In the analysis of heat exchangers total heat transfer rate  $Q$  is of primary interest. The log-mean temperature difference (LMTD or  $\Delta T_{lm}$  for counter and parallel flow is defined as

$$LMTD = \Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$$


Total heat transfer between hot and cold fluids in a counter flow arrangement

$$Q = U_m A \Delta T_{lm}$$

Where  $\Delta T_1 = T_{h,i} - T_{c,o}$      $\Delta T_2 = T_{h,o} - T_{c,i}$  (Counter Flow)  
 $\Delta T_1 = T_{h,i} - T_{c,i}$      $\Delta T_2 = T_{h,o} - T_{c,o}$  (Parallel Flow)

LMTD represent maximum temperature potential for heat transfer that can be obtained in counter flow exchanger.

**Special Case:** In case of counterflow with  $(\dot{m}c_p)_h = (\dot{m}c_p)_c$ , the quantity  $\Delta T_{lm}$  is indeterminate. In this case, by applying L' Hospital's rule  $Q = UA(T_h - T_c)$  with  $(T_h - T_c) = \Delta T_1 = \Delta T_2$ .



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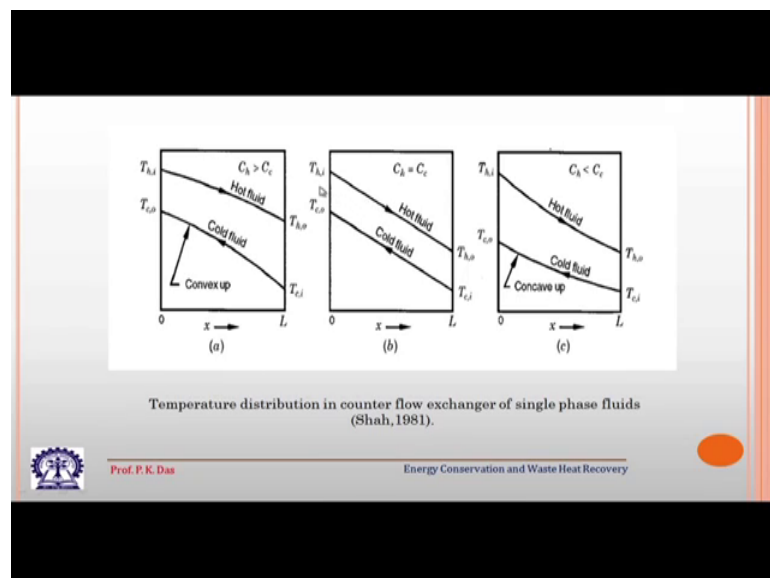
So, the formula is given over here and then once we know; what is LMTD which is the main temp mean temperature difference for the entire heat exchanger. So, we get the  $Q$  that is the rate of heat transfer is equal to into area into delta TLM where delta TLM is given at the top.

Now, here I like to tell you one thing the is the mean overall heat transfer coefficient it is dependent on area sometimes we find the heat exchange area that is different particularly for a tube if you take a circular tube inside the circular tube you have got one heat transfer area and outside the circular tube we have got a different heat transfer area. So, heat transfer area is relevant when you have got convective heat transfer which mostly we are concerned.

For radiative heat transfer also it is important, but we are mostly concerned with convective heat transfer and that area depends whether you are considering it from inside of the tube or considering it outside the tube. Now depending on that will also be different. So, is area dependent and here we have got this formula  $Q$  is equal to into a  $\Delta T_m$  and  $\Delta T_1 \Delta T_2$  everything has been given for counter flow and parallel flow; I have explained it there could be a special case in case of counter flow the quantity the heat capacity rate if that becomes equal for both the streams; that means, for the hot stream and the cold stream then.

We have to take the mean temperature difference simply as  $\Delta T$  or  $\Delta T_1$  or  $\Delta T_2$ ; let me explain it or we can explain it going to some sort of a previous slide.

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So, let us go to this slide. So, here you see heat capacity rates are same for both the hot fluid and cold fluid. So, here the temperature difference will be  $\Delta T_{hi} - T_{co}$  here it will be again  $\Delta T$ , we can all them  $\Delta T_1 \Delta T_2$ , but everywhere in the

heat exchanger the temperature difference will be delta T and that should be the mean temperature difference; obviously, that should be the mean temperature difference.

Now, what you have what we have done if you see that based on these 4 temperatures 2 temperatures at the inlet end and 2 temperatures at the outlet end we have done the analysis this is very important and this is what we can do because these 4 temperatures are measurable and known other temperatures are within the heat exchangers the fluid stream temperature at other point are within the heat exchanger a direct measurement is not possible. So, we have to have the performance of the heat exchanger based on these 4 temperatures.

Now, what we get we get a formula by which by the same formula we will have the rate of heat transfer for both parallel flow heat exchanger and counter flow heat exchanger.

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**1. F-LMTD for Multipass and Crossflow Heat Exchangers**

The LMTD shown in previous slide is not applicable for heat transfer analysis of crossflow and multipass flow heat exchangers.


$$\Delta T_{lm,cf} = \frac{(T_{h,i} - T_{c,o}) - (T_{h,o} - T_{c,i})}{\ln[(T_{h,i} - T_{c,o}) / (T_{h,o} - T_{c,i})]}$$

$P = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{c,i}} = \frac{\Delta T_c}{\Delta T_{max}}$  (Heat actually transferred to the heat which would be transferred if the same cold fluid temperature was raised to the hot-fluid inlet temperature. Hence it is temperature effectiveness of heat exchanger.)

$R = \frac{C_c}{C_h} = \frac{T_{h,i} - T_{h,o}}{T_{c,o} - T_{c,i}}$  (Ratio of  $\dot{m}c_p$  of cold fluid to that of hot fluid and is called the heat capacity rate ratio.)


Where  $\Delta T_{lm,cf}$  is the LMTD for counter flow arrangement with the same fluid inlet and outlet temperature. Hence, in crossflow heat is calculated by introducing a non-dimensional number called F.

$$Q = U A F \Delta T_{lm,cf}$$



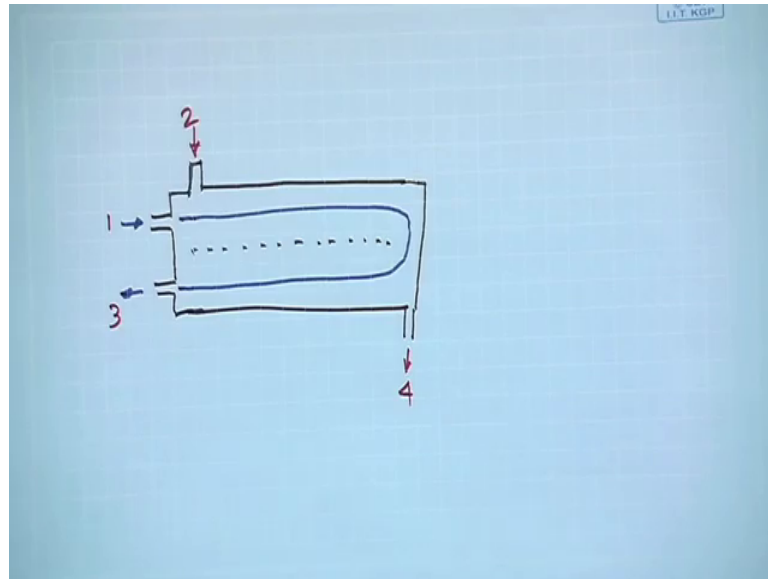
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Now, the question is there are many other different types of heat exchanger; what do we do for them, what we do is like this the LMTD shown in the previous slide is not applicable for heat transfer analysis of cross flow and multi pass flow heat exchangers multi pass flow heat exchangers the thing is that one can think of a multi pass flow heat exchanger like this.

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Let us say this is a multi pass flow heat exchanger let us say hot fluid is entering here hot fluid is going over here and as far as cold fluid is concerned it is entering here it is coming out here and hot fluid is from here to here. So, you see for the top branch of the cold fluid, it is a parallel flow and.

For this branch of the cold fluid it is counter flow arrangement with the hot fluid or hot stream. So, if we have this kind of a heat exchanger which is having 2 pass for the cold fluid then this heat exchanger, we can neither call it a parallel flow heat exchanger nor we call it nor we can call it as a counter flow heat exchanger for this kind of heat exchanger there are many many examples even much complex example cross flow heat exchanger is another one where the we have not derived any formula for cross flow heat exchanger, but how to do then this analysis , but whatever may be the flow arrangement it goes without saying that these 4 temperatures let us say this is 1, 2, 3, 4.

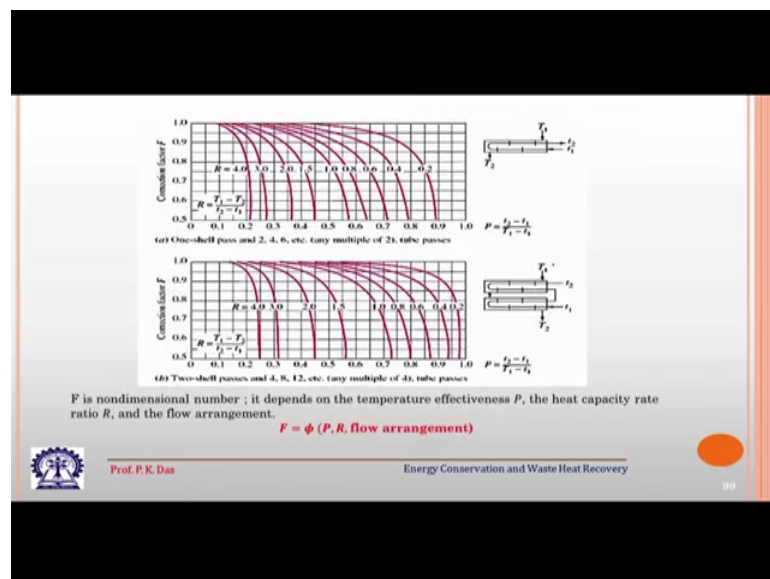
These 4 temperatures are very important and based on these 4 temperatures one can determine the mean temperature for the mean temperature difference of the heat exchanger. So, one method has been suggested it is very which is very useful method that we will use the LMTD formula only, but we will multiply it with some sort of a factor  $F$  and with that we will get the mean temperature difference this  $F$  will depend of course, on some parameters one is this parameter it is called  $P$  and the definition of  $P$  expression

of P has been given heat actually transferred to the fluid which would be transferred if same cold fluid temperature was raised to the hot fluid inlet temperature.

Hence it is temperature effectiveness of heat exchanger this is P and then this also is important this is the ratio of heat capacity rate  $C_c$  for cold fluid and  $C_h$  for hot fluid  $C$ . So,  $C_c$  by  $C_h$  that is R ratio of  $m \cdot C_p$  of cold fluid to that of hot fluid and is called heat capacity rate ratio and then it will also depend not only on these physical parameters it will also depend on the flow arrangement.

So, generally what we get we can express by I mean Q the heat transfer by this expression where  $u$  is the overall heat transfer coefficient  $A$  is the area  $F$  is the correction factor this is what we have introduced now which will be a function of P and R and of course, the flow arrangement and then  $\Delta T_{LM}$  counter flow for counter flow heat exchanger whatever  $\Delta T$ , we will  $\Delta T_{LM}$  we will get whatever LMTD, we will get considering counter flow arrangement.

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Then  $F$ ;  $F$  I have told that it depends  $F$  is a number and it is of course, like this in counter flow arrangement we get maximum amount of heat transfer that is most conducive for maximum heat exchange between 2 streams. So, the  $F$  value will be equal to one if we consider counter flow heat exchanger. Now if it is not counter flow heat exchanger then; obviously, the value will be less than one.

So, we will have this kind of curves it has been already calculated and made available for our ease of calculation ease of estimation. So, here you see this is your  $F$  and this is a function of  $P$  and where  $R$  is taken as parameter. So, different curves are for different values of  $R$  and then I have told that it depends on the flow arrangement. So, this is for a particular type of heat exchanger where there are 2 passes like this; this is for another heat exchanger where there are 4 passes like this. So, for each and every heat exchanger arrangement we have got curves like this. So, basically we have to pick up the curve for a specific fluid flow arrangement, then we have to know the value of  $P$  and  $R$  and from this particular curve which we have selected already.

For the heat exchanger we have to get the value of  $F$  then we can use the formula which we have already mentioned and we can get what is the rate of heat transfer. So, you see this is what I was telling  $F$  is a function of  $P$   $R$  and flow arrangement that is what we can see from this particular slide also

So, this is one way of determining the heat transfer rate this is way direct way if the 4 temperatures are given temperature set of the 2 streams at both the inlet and exit. So, then from there we can determine what is our rate of heat transfer. So, this is one typical for one typical kind of analysis this kind of formula which is  $F$  LMTD formula that can be used. So, let us try to understand what kind of suppose we know the 4 temperatures at the beginning we try to design some sort of heat exchanger; we know  $Q$  that is the rate of heat transfer which we have to go for and the 4 temperatures which we can which we like to have. So, these 2 things are fixed and then we have the flow arrangement; that means, whether what kind of tube; tube Diameter, etcetera whether it is counter flow or parallel flow. So, those kind of arrangement and tube diameter; that means, geometry etcetera are the type of geometry that is known. So, you know  $Q$  you know  $\Delta T_{LM}$  then.

Overall heat transfer coefficient that also is known because the tube diameter you have selected and from there you know what is the heat transfer coefficient. So, basically one can calculate  $a$  once you calculate  $a$ ; then probably either the number of tubes or the tube length you can calculate. So, this is a method which is very suitable for design or new design of course, the way I have told it is not as simple as that because  $u$  and  $a$  they could be interrelated even  $\Delta T$  and  $Q$  we cannot separate them out the way I have told that

separately I know Q and delta T. So, there could be some sort of iterative procedure needed, but for design.


If the 4 temperatures are known then this F LMTD method is a very good method unfortunately in many of the cases these 4 temperatures are not known; we know let us say only 2 inlet temperatures and we know some other quantity. So, then the F LMTD method becomes a very roundabout method a TDS method not impossible to apply for those kind of analysis or problem, but it becomes little bit time consuming TDS more number of iterations are to be needed.

So, there should be some other method where the life is little bit simple. So, that method is known as effectiveness NTU method and we are going to have a very brief overview of effectiveness NTU method applicable.

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**ε - NTU Method**

- Applicable when inlet or outlet temperatures of the fluid streams are not known.
- Method is based on fact inlet or exit temperature difference are function of  $\frac{UA}{C_c}$  and  $C_c/C_h$ .
- Dimensionless Form
  - Capacity rate ratio:  $C^* = \frac{C_{min}}{C_{max}}$ , where  $C_{min}$  and  $C_{max}$  are the smaller and larger of the two magnitudes of  $C_h$  and  $C_c$ , respectively, and  $C^* \leq 1$ .
  - Exchanger heat transfer effectiveness: It is the ratio of actual heat transfer rate in heat exchanger to the thermodynamically limited maximum possible heat transfer.

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When the inlet and outlet temperature of the fluid streams are not known fluid temperatures are not known.

Method is based on the fact inlet or exit temperature difference are function of these 2 things; that means, one is UA by Cc or it could be C min and another is heat sorry another is heat capacity rate ratio now in dimensionless form we introduce one term called Cs star what is which is known as capacity rate ratio or heat capacity rate ratio; this is C min by C max where C min and C max are the smaller and larger of the 2

magnitudes of  $C_h$   $C_c$  respectively and  $C_c$  star will be less than equal to 1. So, this is one important parameter for our analysis the exchanger heat the exchanger heat transfer effectiveness or this effectiveness epsilon.

It is the ratio of actual heat transfer rate in the heat exchanger to the thermodynamically limited maximum possible rate of heat transfer.

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Actual heat transfer is obtained by using energy given off by hot fluid or energy received by cold fluid,



$$Q = (\dot{m}c_p)_h (T_{h,i} - T_{h,o}) = (\dot{m}c_p)_c (T_{c,o} - T_{c,i})$$

If  $c_h > c_c$ , then  $(T_{h,i} - T_{h,o}) < (T_{c,o} - T_{c,i})$

If  $c_h < c_c$ , then  $(T_{h,i} - T_{h,o}) > (T_{c,o} - T_{c,i})$

The fluid that might undergo maximum temperature difference is the fluid with minimum heat capacity rate  $c_{min}$ . Therefore, the maximum possible heat transfer is expressed as

$$Q_{max} = (\dot{m}c_p)_c (T_{h,i} - T_{c,i}) \text{ if } c_c < c_h \text{ or } Q_{max} = (\dot{m}c_p)_h (T_{h,i} - T_{c,i})$$

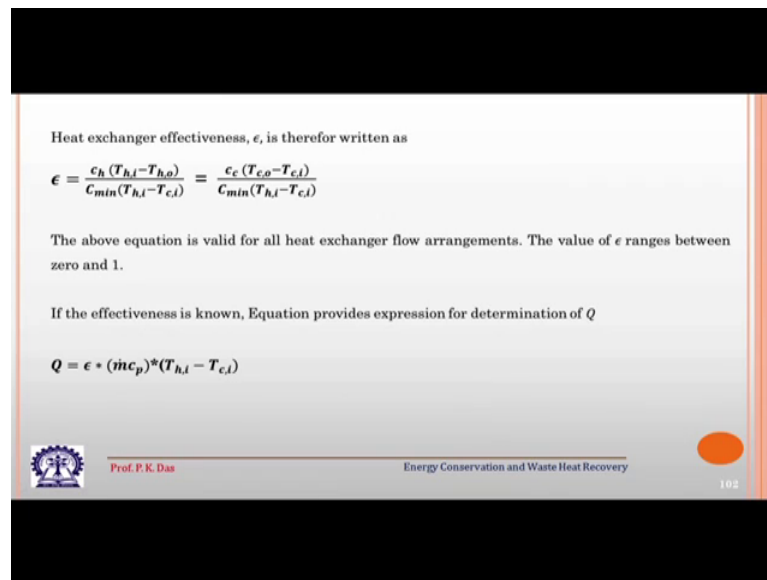
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So, what is that  $Q$  dot actual heat transfer rate can be obtained like this  $Q$  dot is equal to  $mcp_h$ ; that means,  $mcp$  of the hot fluid side multiplied by the temperature change in the hot stream or it is the  $m$  dot  $C_p$  of the cold fluid side multiplied by the temperature change of the cold stream if  $C_h$  is greater than  $C_c$ , then we will get this relationship otherwise we will get this relationship the maximum heat transfer possible, let us see how we can get this the fluid that might undergo maximum temperature difference is the fluid with minimum heat capacity rate  $C$  mean.

Therefore the maximum possible heat transfer is expressed as  $Q_{max}$  is equal to  $m$  dot  $C_p$   $c$  that is  $T_{hi}$  minus  $T_{ci}$  if  $c_c$  is less than  $C_h$  or  $Q_{max}$  is equal to  $m$  dot  $C_p$   $h$  multiplied by  $T_{hi}$  minus  $T_{ci}$ . So, this will be the maximum amount of heat transfer.



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Heat exchanger effectiveness,  $\epsilon$ , is therefore written as

$$\epsilon = \frac{C_h(T_{h,i} - T_{h,o})}{C_{min}(T_{h,i} - T_{c,i})} = \frac{C_c(T_{c,o} - T_{c,i})}{C_{min}(T_{h,i} - T_{c,i})}$$

The above equation is valid for all heat exchanger flow arrangements. The value of  $\epsilon$  ranges between zero and 1.

If the effectiveness is known, Equation provides expression for determination of  $Q$

$$Q = \epsilon * (mC_p) * (T_{h,i} - T_{c,i})$$

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So, effectiveness we can defined like this depending on our whether we like to use  $C_h$  or  $C_c$  the effectiveness can be used like this. So, you see this is the maximum temperature difference possible without violating thermodynamics or without violating second law. So, this is the definition of effectiveness the above equation is valid for all heat exchanger flow element the value of epsilon ranges between 0 and one if the effectiveness is known equation provides the expression for determine in a determination of  $Q$ .

So, suppose I know the effectiveness of the heat exchanger then  $Q$  can be determined simply by  $m \cdot C_p$  multiplied by  $T_{hi}$  minus  $T_{ci}$ ; generally inlet temperatures are known. So, you see the advantage of using this method generally inlet temperatures are known or even as the basis of the design the inlet temperatures are known. So, if I know the inlet temperatures and then if I have already decided what could be the effectiveness of the heat exchanger I am going to design. So, my effectiveness that is also known then I will get the  $Q$  value or vice versa suppose the inlet temperatures are known and I know that how much heat I have to extract or I have to supply then using this particular formula I can know the effectiveness of my heat exchanger.

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➤ Number of transfer unit: It depicts the nondimensional heat transfer size of the heat exchanger.

$$NTU = \frac{AU}{c_{min}} = \frac{1}{c_{min}} \int U dA$$


For counter flow following expression is obtained :

$$\epsilon = \frac{1 - \exp[-NTU(1 - C/c_{max})]}{1 - (c_{min}/c_{max}) \exp[-NTU(1 - c_{min}/c_{max})]}$$

If  $c_c < c_h$  ( $c_c = c_{min}$ ,  $c_h = c_{max}$ ), the result will be the same.

In case of parallel flow, a similar analysis may be applied to obtain following expression:

$$\epsilon = \frac{1 - \exp[-NTU(1 + c_{min}/c_{max})]}{1 + (c_{min}/c_{max})}$$

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Now we come to the next parameter next parameter is NTU the full form of NTU is the number of transfer unit it gives some idea regarding the size of the heat exchanger. So, NTU is defined as AU divided by c mean where A is the area, U is the overall heat transfer coefficient and C mean is the minimum of either of Ch and Cc. So, once we have got NTU as I have told that it is indicative of the size of the heat exchanger it is also indicating of the residence time of the fluid in the heat exchanger. So, basically it gives idea regarding the transfer whether the rate of heat transfer will be very large or very small something like that. So, relative it is a relative kind of a thing for same kind of heat exchanger if we have 2 values of NTU the higher value of NTU gives.

That the size of the heat exchanger is more or inside the fluid stays for a longer time like this and then what is important the effectiveness and NTU they are uniquely related for counter flow the following expression is obtained this is between effectiveness and NTU and it will be the same if Cc is less than Ch the result will be the same in case of parallel flow a similar analysis can be applied to obtain the following expression this expression we will get for parallel flow. So, you see these expressions are little bit complex, but there is a unique relationship between NTU and effectiveness.

If we know one we can determine the other one in many of the cases what we have got we have got curves for effectiveness or NTU there are a results which are expressed graphically in the form of curves. So, then it becomes easier of course, nowadays with

the help of digital computing calculation of this kind of relationships or tackling this kind of relationship is child's play. So, it is no not at all difficult, but you see I have given just like before I have given the relationship for parallel flow and counter flow there are other kind there could be other kind of flow arrangement there could be other kind of flow arrangement and for them what are we going to do.

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**SPECIAL CASE:**

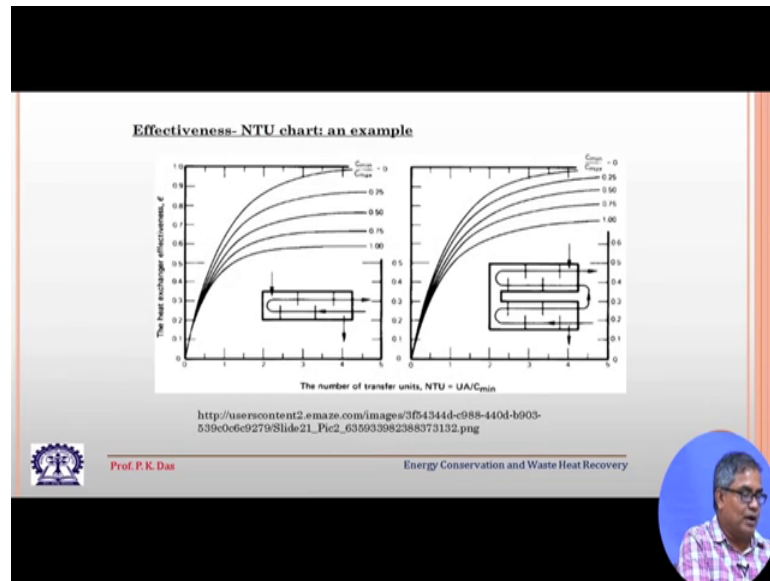
- $C_{min}/C_{max}$  equal to 1
  - $\epsilon = \frac{NTU}{NTU+1}$  ( Counter Flow ) ,  $\epsilon = \frac{1}{2}(1 - e^{-2NTU})$  ( Parallel Flow ).
- $C_{min}/C_{max}$  equal to 0 as in boiler and condensers for parallel flow and counter flow , equation becomes
  - $\epsilon = 1 - \exp(-NTU)$
  - $\epsilon = \phi(NTU, c^*, \text{flow arrangement})$

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Let us discuss some sort of special cases one special case is that the heat capacity rate are equal for both the fluids then see mean by C max is equal to 1 and effectiveness is given by NTU divided by NTU plus one for counter flow and effectiveness is this value for parallel flow see mean by c max is equal to 0. This is another this is another special case where we will have one of the fluid stream is changing its face if some fluid stream changes its face, then the temperature difference between the inlet and outlet is equal to 0 then for finite heat transfer then for finite heat transfer the C value has to be very large, this if there is some finite amount of heat transfer between the inlet point to the outlet point and from there we get that C min by C max is equal to 0 as in case of boiler and condenser.

So, we will have this kind of the equation becomes epsilon is equal to 1 minus exponential of minus NTU in general epsilon is a function of NTU C star and flow arrangement again just for as we have got for F different kind of card.

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So, for this kind of relationship also we can have different type of cars the 2 cases which we have shown for LMTD method the same 2 cases here it is shown. So, this is the flow arrangement for a particular flow arrangement we will get a typical cut. So, this is  $c$  mean by  $c$  max and this side we have NTU and as a function of NTU we will have the effectiveness of the heat exchanger.

So, effectiveness of the heat exchanger that is a function of NTU that is dependent on the flow arrangement and also dependent on the  $c$  star value that is  $c$  minimum by  $c$  maximum this is for another case. So, there are host of curves like this. So, using these curves we can employ the effectiveness NTU method and we can do calculation now depending on the depending one type of problem, we have to adopt either the F LMTD method or the effectiveness NTU method in many cases the iterations required for F LMTD method can be avoided by taking a effectiveness NTU method we will pick up certain examples to clarify these points and then we will see how the heat exchangers are very relevant for waste heat recovery.

Thank you.