Energy Conservation and Waste Heat Recovery
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Lecture - 42
TEG - performance analysis

Welcome back, good morning and today we are going to continuous part of our course on energy conservation and waste heat recovery. We will continue our discussions on thermoelectric generators, we already have had 2 lectures on this. So, today what we will do is we are going to go into you know analyzing the performance of a thermoelectric generator again thermoelectric generator as we know can directly convert thermal energy that is available to us and in the context of this course we are talking about waste heat recovery. So, let us say we have a source of waste heat or we have a source of thermal energy from which otherwise would have been waste heat and we can harness part of that energy to and convert, it to additional electrical energy, all right.

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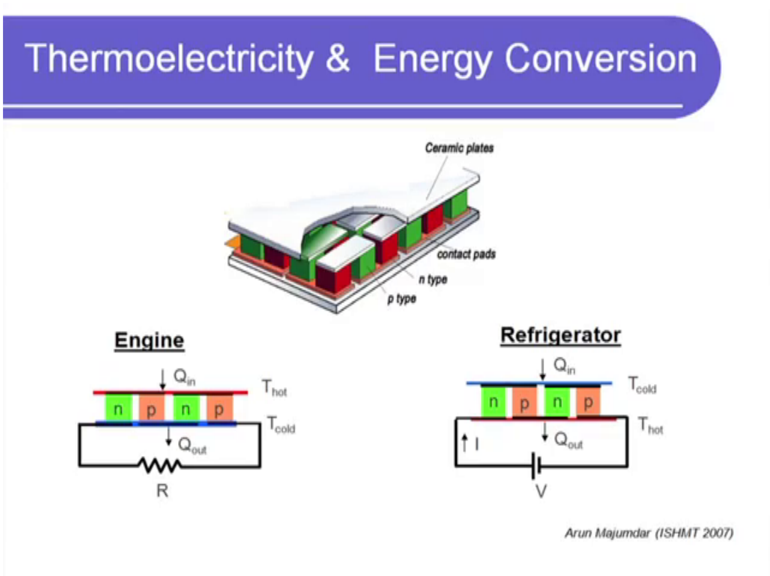

So, before we move on I will just want to show or want to mention another just one extra thing say this is how we saw that a thermoelectric module consists of a series of these np pairs or PN pairs thermoelectric pairs and which are connected in series and then at the top and bottom we have couple of ceramic plates that kind of sandwich them and which these materials.

Now, the thermoelectric module can be used for generation of electricity which is the focus of what we are discussing and this is how it is done as we saw on one we have a series of these N and P types on 1 end we have a heat, it is a source of heat Q N coming in and so, that is at a hotter temperature the other end is at a colder temperature and what we are saying is if we do that we will be able to give rise to an electric current that will flow through a resistor that is outside and that is how we are able to get electrical energy out of it.

Now the reverse is also possible, let us say we have a thermoelectric module and we force a current to flow through that in the manner that is shown over here, then what happens is an electric it gives rise to a temperature gradient across the junctions and as a result of which it is possible to remove heat across at an adverse temperature gradient that is to remove heat or transfer heat from a lower temperature to higher temperature which is what is shown here.

And where do we see this otherwise we see this in a refrigerator you know refrigerators what we see is we are able to in a refrigerator whatever the way; it works is we are able to constantly remove heat from a lower temperature to higher temperature we are removing heat from inside the refrigeration chamber which is at a lower temperature and rejecting it to the outside ambient which is at a higher temperature thermo-electrics can perform the same function in this manner only in this case the electrical energy is the input to the thermoelectric module and the temperature difference is output, but in our focus what we are looking at for energy generation the temperature difference is the input is a cause and the electrical energy that we get out of it is the effect. So, I thought it was is important for us to know the very the other prominent application of thermo-electrics which is as a refrigerator or as a cooler where it is being research and being also used in several applications.

Now let us move on to the thermoelectric generation that is used as an engine which is the left hand cartoon.

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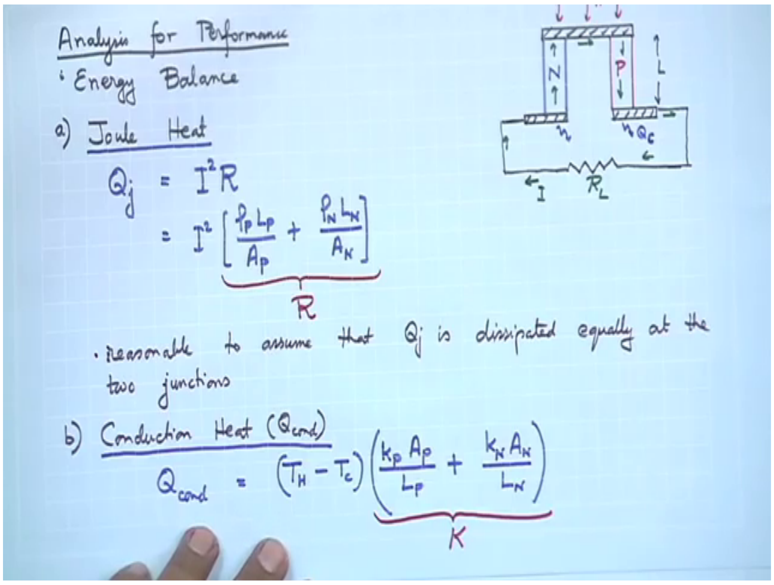

So, let us look into this if we saw last time and again we will take just a single pair of N and P type and try to analyze this configuration. So, let us do a little bit of analysis for performance. So, what we will do is we will do an energy balance to start with. So, let us do an energy balance, but before that lets look at the different source terms the first one we will talk about is joule heat. So, let me say; what is joule heat.

So, joule heat is Qj let us write it as Qj and that is going to be I squared times R, we know that I squared R heating and again, we will write it as I squared and we have what is the R, this is the resistance that is offered by the 2 semiconductors 2 semiconductors one N type and P type. So, R is royal over A. So, we will write it as rho P LP over AP for the P junction and rho N LN over an for the N junction clear. So, many a times in future we may denote this whole thing as capital R. So, I just want to define it right now.

Now it is reasonable to assume. So, let me make that note here or at least state verbally it is reasonable to assume that this joule heat is dissipated equally at the 2 junctions at the hot junction and the cold junction. So, reasonable I would say to assume that Qj is dissipated equally at the 2 junctions. So, if that is the case again once again let us look at this we have defined the outer or the external resistance as RL the current is I the heat that is supplied is Q h and the heat that is rejected the cold junction as QC. So, QH; QC is defined; we know and we are looking at if we operate the thermoelectric module what are the additional energy terms that we are going to come across.

So, the second one we will call it as the conduction heat or Q cond sorry QC is already Q cold. So, why where does this have why does this happen this happens because we know that across any material if we apply a temperature gradient there will be conduction of heat right because there is no perfect insulator heat will be conducted; however, low or; however, high it is. So, we are going to write conduction heat as Q cond is equal to what is it TH minus TC times the thermal conductance what is that Q is ka dt dx . So, therefore, I will write it as KP where K is the thermal conductivity AP over LP plus KN an over LN; these are thermal conductances just like these are electrical resistances . So, from our basic knowledge of heat conduction heat transfer we know that Q is L over K A sorry thermal Q is delta T times K times the cross sectional area divided by the thickness across which the heat is conducted in this case the length.

So, again let me just quickly write down; this junction is this is the length L. So, for N and P, it will be accordingly it will be the length of N and the length of P. So, this again sometimes later we may refer to as K clear. So, we have joule heat we have conduction heat there is a third one which is very important and which is actually coming from the thermoelectric effect.

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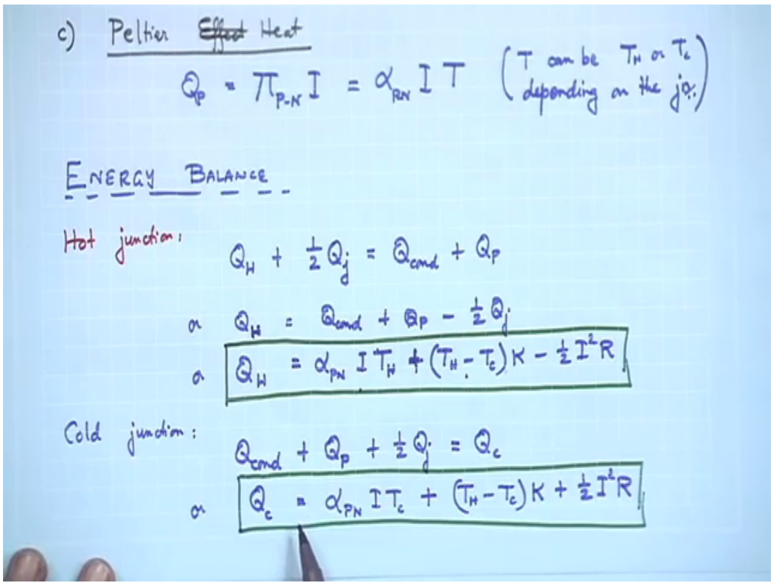

And that is the Peltier heat remember what was the Peltier effect; the Peltier effect stated or pelt sorry Peltier heat the Peltier effect stated that over and above joule heat there is an additional amount of heat which we call the Peltier heat which must be removed or absorbed in order to keep the junctions isothermal.

. So, we are going to denote that as QP Q Peltier and from Peltier effect, what do we know of that we know that that is going to be equal to pi P to N times the current that flows through it, right and again this can be shown to be equal to alpha P N times I times T and this T can be TH or TC depending on the junction . So, with these definitions what we will do next is we will actually do the energy balance. So, let us go for the energy balance the first starting with the hot junction.

So, hot junction we can write the energy balance as QH clear plus QH is again remember it is the heat input. So, that is what is coming in what else is the input there we said that half of the joule heat is dissipated over there. So, Qj half of Qj and what is going out; what is going out of the hot junction let me bring back the schematic. So, QH is an input Qj is an input what is going out of here the conduction heat will be from the hot junction to the cold junction. So, I will write it as Q conduction and what else this is the hot junction. So, therefore, the Peltier heat has to be constantly removed in order to keep it constant in order to keep the temperature at TH. So, QP.

So, let us rearrange this I will go a little fast now here. So, this will be Q conde Q conduction plus Q Peltier minus half of Qj and we can write it as alpha PN I TH plus TH minus TC times K minus half I squared R remember K plus R we have already defined I am sorry, I kind of swept these 2. So, this is actually Q P and this is actually Q conduction sorry this is actually Q conduction this is actually Q p. So, this is equal to this; this is equal to this, but I hope you understand. So, we have got an expression for the QH which is the heat input as a function of these different parameters. So, let me box that because we are going to use it later. So, this is going to be an important expression.

Second one will be the cold junction . So, similarly in the cold junction what can we do what is coming to the cold junction is Q cond the conduction the heat that is conducted away from the hot junction is coming to the cold junction. So, that is an input again the Peltier effect Peltier heat has to come in because the heat has to be absorbed in order to maintain it in the temperature plus half Qj as before and what is going out is QC, all right.

So, this again we can write it as QC will come out as alpha PN I TC plus TH minus TC times K plus half I squared R if we do the mathematics we will get this. So, this is my other important expression. So, I have got an expression for the heat that is coming to the hot junction and the heat that is being dissipated in the cold junction in terms of various parameters in terms of temperatures in terms of the current that is flowing in terms of the material properties like the conductivity and resistivity as well as the seebeck coefficient clear.

So, therefore, what do I have to do I have to find out what is the amount of electrical energy or electrical work that I can get out of this thermoelectric module and what is that going to be that is going to be the current times this resistance sorry; I squared R that is current square times RL.

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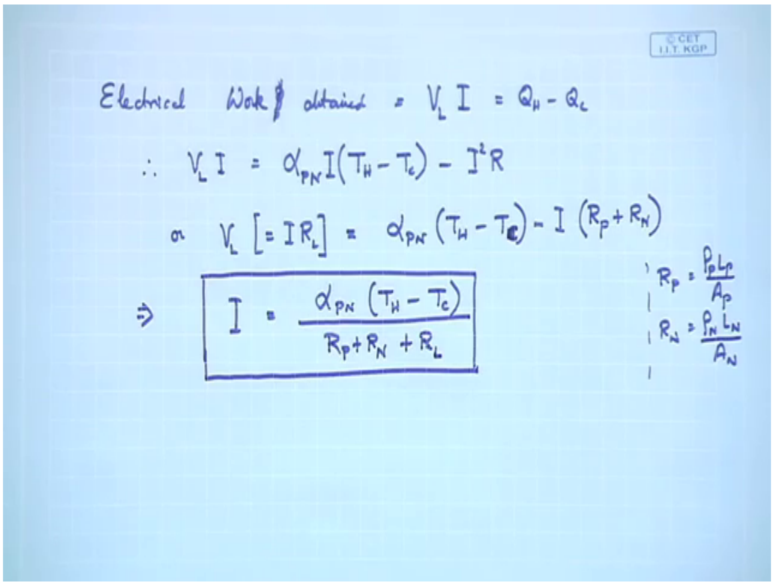

So, let us write that down and say therefore, the electrical energy or work obtained is going to be VL times I, this is a voltage across the load times I and what is that going to be Q h minus QC first law of thermodynamics therefore, V L which is the voltage across the load times I, if you now use both the expressions that we derived before it will beautifully come lot of things will cancel out and it will come out to be as alpha times the temperature difference TH minus TC minus I squared R clear. So, therefore, sorry alpha P N times I times TH minus TC, clear.

So, therefore, what is this? So, therefore, we can also write as VL which by the way is I times RL is equal to alpha times a temperature difference minus I because one of the currents cancel out and let me write it down as R P plus RN, right. So, R P as we know we have defined before RP is rho P LP over AP and similarly RN is rho N LN over an clear. So, therefore, what does this give this gives I which is the current that is flowing through it is going to be alpha PN sorry TH minus TC I am sorry TH minus TC divided by RP plus RN plus external resistance RL, clear.

So, what does the current flow depend on this is an expression that nicely captures all the independent variables on which the current flow will depend on it will depend on the material properties because the seebeck coefficient alpha PN is a material property, it will depend on the temperature difference. So, higher the temperature difference higher is the current that we are going to get it is also going to depend on the dimensions as well as the electrical resistances of the P and N junctions of the P and N materials right. So, the electrical resistances the material part comes from resistivity the geometry part forms from the length and cross sectional area and finally, last, but not the list it is also going to depend on what is the external load that I am putting N, clear.

So, the current depends on all these parameters it depends again once again on the material properties in terms of seebeck coefficient as well as the electrical resistivities of the P and N junctions it depends on the geometrical properties and the geometric dimensions rather like the length and cross sectional area of the P and N junctions or the P and N elements, I am sorry and finally, sorry it also depends on the temperature difference of course, very very important and finally, the external load clear.

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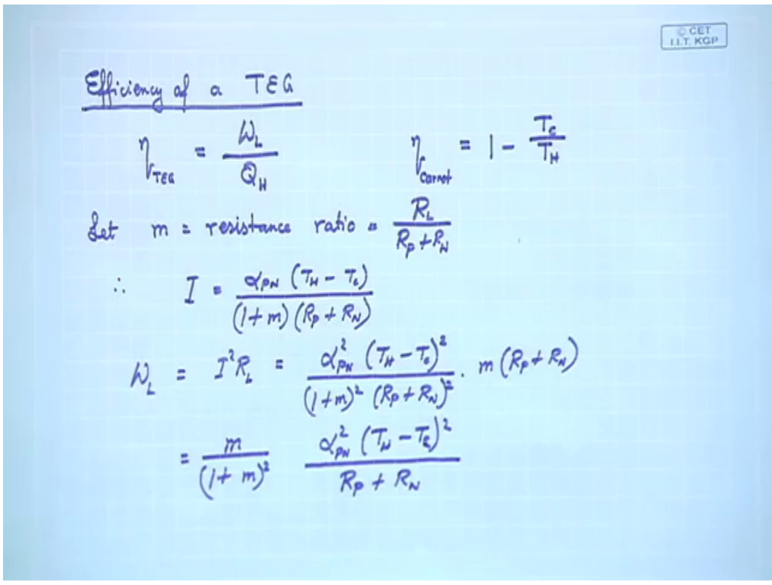

So, next what we will do is we will continue our analysis and say how do we calculate the efficiency of TEG; how do we calculate this. So, let us write that eta which is normally what is denoted or what is used to denote efficiency what will be the eta of TEG the basic definition of efficiency is the work output which is WL divided by the energy input which is QH; clear here we will also write one more thing that eta Carnot for an for a thermoelectric device is going to be one minus TC over TH we know this right. So, this is going to be the Carnot efficiency and the thermoelectric generator efficiency will be definitely much lower than this, but it at this point let us also keep this in mind and later on we are going to use it.

. So, what I am going to do next is this will be a mathematical exercise let m is equal to resistance ratio and defined as R L over RP plus RN. So, which is the external resistance divided by the thermal intrinsic electrical resistances of the thermoelectric elements. So, therefore, I can write I which is a current is going to be alpha PN TH minus TC divided by one plus m into RP plus RN therefore, the external work WL is going to be I squared RL and you can show it to be as alpha squared PN TH minus TC whole squared divided by one plus m squared RP plus RN squared times RL is what m times RP plus RN m times RP plus RN.

So, lot of things will cancel out and finally, what we will have is m divided by one plus m whole squared times alpha squared TH minus TC whole squared this does not cancel unfortunately divided by RP plus RN clear. So, next what we will do is we will move on therefore.

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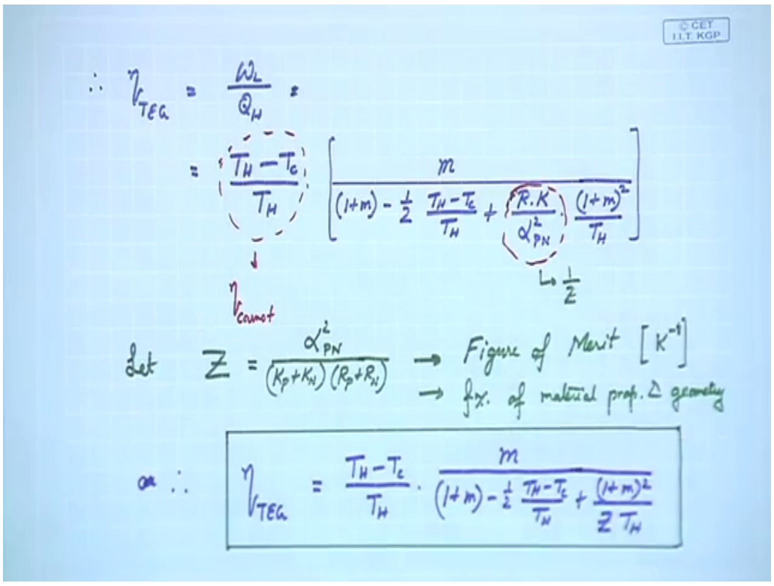

The efficiency of the thermoelectric generator is going to be WL over QH which is going to be what we just found the expression for WL and divided by QH for which we had derived an expression before. So, this is my expression for WL and this was my expression for QH except that over here I am going to just use some of these m definitions. So, I will put R as R P plus R N and so on.

if you do all that and I will leave it to you as an exercise you will finally, end up getting an expression in this manner TH minus TC over TH times this will be a long expression the denominator the numerator is going to be simple, it is just an m the denominator will look something like this one plus m minus half TH minus TC over TH plus R times K which is again RP plus RN times KP plus KN divided by alpha square PN times one plus m whole squared divided by TH this is how it will look.

Now, lets spend some time looking at this expression TH minus TC over TH what was this if you recall this is Carnot efficiency this times I have a factor which encompasses a lot of properties it encompasses the temperature difference as well as absolute temperature the hot junction it includes the material and geometric properties through the resistances and conductances and it also includes what is the external load and the resistance of that external load that we as that we have put through this term m clear. So, therefore, the efficiency depends on the material properties the materials that we are using it depends on the temperature difference as well as the absolute temperature and it also depends on the external load that we have put in. So, in order to get the maximum efficiency we need to try and optimize each of these .

So, one last thing that we will do over here is this term is very important I will just circle this term. So, I am going to define this let define this by a term Z or z. So, Z is actually alpha squared PN divided by KP plus KN times RP plus RN. So, this is one over Z actually. So, this becomes one over z. So, this is a very very important figure of merit when we talk about thermoelectric if you go and tells anybody that I am working with thermoelectric the first thing that they will ask you is what is the ZT value and the Z.

Typically the out of that ZT is this parameter T of course, is the typically either the temperature of the hot junction or the mean temperature of the 2 the Z as you will see has dimensions of Kelvin to the power minus one that is why ZT is important to make it non dimensionless and what does it depend it is a function of material properties and geometry and more importantly material properties because geometry you can tailor or you can use.

So, therefore, what we will end this lecture is by this expression therefore, I would say the def the efficiency of a thermoelectric generator is given by the Carnot efficiency TH minus TC over TH times m divided by one plus m minus half TH minus TC over TH plus one plus m whole squared divided by Z TH one of the most important expressions in thermoelectric generators is this expression. So, as we see the efficiency depends on a variety of parameters that we just saw and here we have defined something very very important which is the figure of merit Z having dimensions of one over Kelvin.

So, today we started with doing some analysis and we are trying to and what we did was we came up with an expression, but by doing energy balance we came up with an expression for the efficiency of a thermoelectric generator as a function of its material properties as a function of which dimensions as well as a function of its circuit properties; in the sense that what is the resistance of the load that I have put externally. So, in the next class, we what we will do is we will complete this exercise and we will try to wrap up thermoelectric generators. So, again hopefully you have learned something new today and we will continue with this learning in the next class.

Thank you very much.