Energy Conservation and Waste Heat Recovery
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Lecture - 43
TEG - performance optimization

Good morning, welcome back to the next lecture for energy conservation and waste heat recovery. Today, we will try to wrap up and conclude our discussion on thermoelectric generators.

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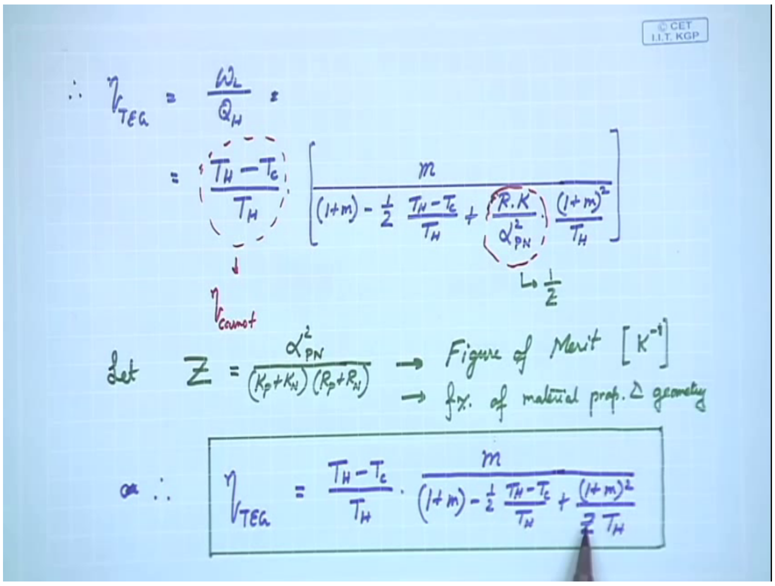

You remember last time what we did was, we tried to do some performance analysis of a thermoelectric module. And we came up with this expression, whether; efficiency of the thermoelectric generator was shown to be a function of a bunch of parameters. It was a function of the temperatures of the two junctions, that is T H and T c it was a function of the properties which is aptly captured by this figure of merit Z, which is see the coefficient divided by the product of electrical resistances of the two junctions of the two elements P and [noise]; as well as the thermal conductances or electrical resistance as well as thermal conductances of the P and N junctions ok.

It also depends on this parameter m which if we recall; is the resistance ratio. It is the ratio of the external load divided by the resistances of the P and N junctions, clear; Also, if you look at it this form is also quite nice, because the first factor that we see here is the

Carnot efficiency, right as we saw. And then the rest of it is the other parameters which is the combination of the resistances, the temperatures as well as the material as and geometric properties.

So, we are going to depend on we are going to do further analysis on this. So, what is this depend on? What does? Now if you look at it what is it that we are going to that we can play around with. Say typically; the heat source we cannot do much that is available to us we have to live with that for example, if the heat source is steam at 150 degree centigrade my T H is 150 degrees I cannot increase it any further. So, I am let us assume that T H and T c is something we do not have control over the T c on the other hand will be mostly ambient temperature or an elevated ambient temperature.

We really cannot bring it down further. So, what can we change we can play around if you look at this expression we can play around with m over here or we can play around with Z. So, let us do that and see what we get.

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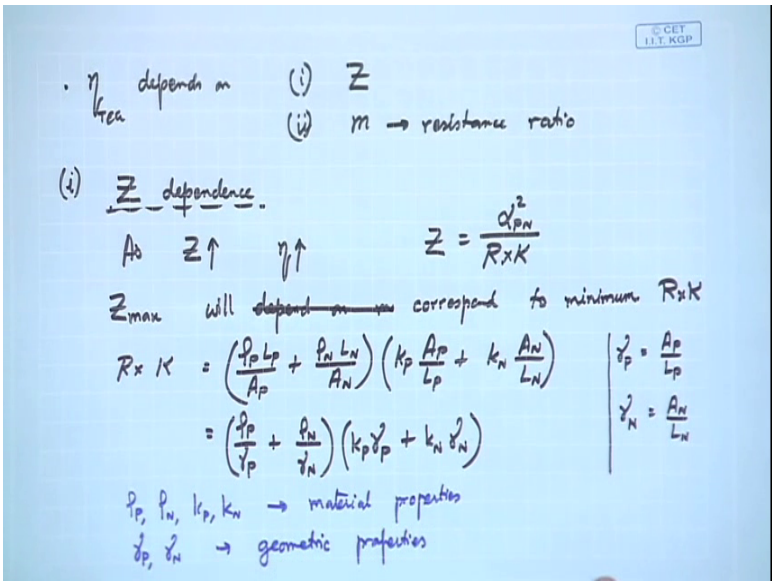

So, I would say that eta TEG, that is, depends on 1 Z and 2 m which is again resistance ratio clear. So, let us take each of them one by one, First one is as we can see for Z dependence. It can be shown quite clearly that as Z goes up my eta also goes up plain and simple; Z increases my efficiency increases.

Now, what is z again? Is alpha squared times R times K. So, what does this mean that my Z max will depend on or will correspond to minimum value of R times K. So, that is important. So, let us do that and see what happens. So, R times K is rho l rho P L P over A P plus rho N L N over an times the thermal conductances which is K P thermal conductivity times A P over L P plus K N A N over L N clear. So, what I will do here is I am going to define a ratio of area to length, just so that it becomes easier to deal with this I do not have to write it all the while; I will say that gamma of P is equal to A P over L P gamma of N is an over L N nothing spectacular this is just to make lives simple here.

So, then what this becomes is this becomes rho P over gamma P plus rho N over gamma N times K P gamma P plus K N gamma N fair and simple right. So, now, see for a given material rho P rho and K are fixed. So, I can write here with a different color, that rho P rho N K P, K N these are all material properties. On the other hand gamma P and gamma N are geometric properties. So, this I can play around let us say, I do not cannot play around with material; I can play around with these for a given set of material.

So, what I will do is?

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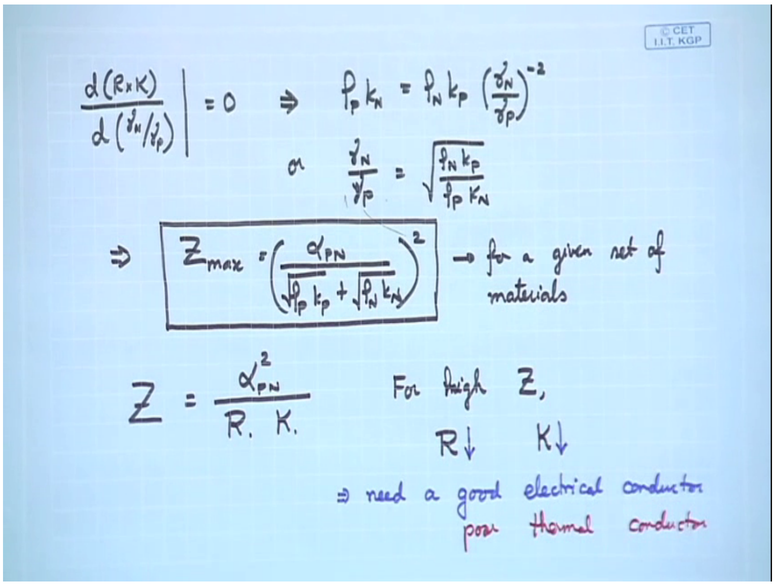

I will do something like d of R times K divided by d of gamma N over gamma P. I will set this to 0 and this will imply that rho P K N is equal to rho N K P times gamma N gamma P to the power minus 2 or simplified further gamma N over gamma P is going to be rho N K P divided by rho P K N. So, this implies that Z max corresponds to alpha P N divided by becomes a very nice expression rho P KP plus root over rho N K N this whole thing whole squared.

So, this is for a given set of materials. So, for a given set of materials, I can play around with the geometric parameters and if I satisfy this equation, I choose the length and area such that this equation this expression is satisfied I get the Z max for that set of materials. Now, the other thing that I would like to point out which is probably very very important; probably the most important when we talk about any research on thermoelectrics is R times K. So, this expression let us look at this once again. If I just look at for a given dimension, in the previous one what did we did we assumed the materials is given and we can play around with the dimensions.

Now, let us say the dimensions are fine I can play around and I do this, but what else can I do to make this even higher. What are the kind of materials that we should have in order to maximize this performance of the thermoelectric? If you look at this; however, you see that there is an anomaly. You have three terms alpha you have a thermal resistance and electrical conductance. So, what do I have to do I have to maximize alpha I have to minimize electrical resistance, but I have to minimize thermal conductance also. So, for high value of zeta or for high value of Z my R needs to be low and my thermal conductance also needs to be low.

So, in other words; I need a good electrical conductor and a poor thermal conductor. Now, unfortunately the two of them it is very difficult to satisfy both of them. It is not easy to have a high a very good electrical conductor which is simultaneously a poor thermal conductor, because most of them go hand in hand. For metals for example, both are governed by the movement of electrons. So, how can you have a good electrical conductor and a bad thermal conductor. Similarly in nonmetals also you have thermal conductivity you have components because of electron movement and you also have component because of lattice vibrations which is also known as phonons.

So, as a result you know for the almost last 60, 70 years scientists have been trying to material scientists mostly have been trying to come up with materials where we can maximize Z T or the value of Z, but unfortunately for almost 50 years from 1950s to early 2000 it was it remained constant. Lots of you know formulations, lots of new materials developed, but finally, when you put it; put it into practical use we are all around the same around the same value of Z t equals to one ok.

So, just keep in mind Z t, because it becomes Non-dimensional that value is often used. So, we were stuck at Z t equals to one for all for a long time for a given set of dimensions. And then what happened was, because of advances in nanotechnology the almost two groups parallelly came up with material which was again based on bismuth telluride super lattices. They were able to grow these lattices in the nanoscale and grow it in dimensions such that the electrons could pass through because their mean free path of vibration is lower.

So, it will let the electrons pass through and therefore, thermal; electrical conductivity is not impacted, but; however, it will arrest the movement of phonons and thereby impacting the thermal conductivity and arresting thermal conduction. So, this is just a side note from the research story I am telling you that again to what happened was it is normally very difficult to have a material which is a poor thermal conductor and a good electrical conductor, but around 10 to 15 years back two research groups parallelly came up with the super lattice structures at the nanoscale such that what happened was it would let electrons flow through, because electrons have lower mean free path of vibration.

But, it would arrest the movement of phonons and thereby it would arrest the lattice vibrations lattice movements and thereby impact the thermal conductivity. So, as a result we got materials which was a good conductor of electricity, but a very poor conductor of heat. And, then as a result what happens was z t value which was stuck at one for a long time at least in the lab scale was shown to go to almost 2.5 both of them big jump and this happened, because we were able to go these structures we were able to grow these structures at the nanoscale at dimensions which would arrest phonons and let electrons pass through.

So, just a little side story on that on the research on materials part which also helps us to increase or maximize the value of Z or zee. So, again coming back to what we were looking at we had we tried to maximize the value of Z by playing around with the geometric parameters which is within our control and we said that the maximum value of Z is this for a given set of materials and this comes from satisfying this relation gamma

N over gamma P is square root of rho N K P over rho P K N alright; what was the second thing that Z depend on?

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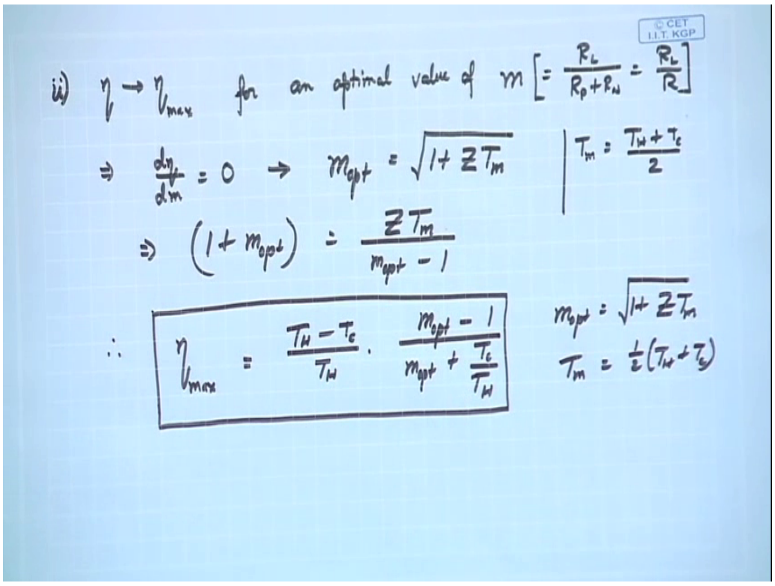

It depend on the value of m.

So, eta will tend to eta max for an optimal value of m; m again let us repeat let us recall R L over R P plus R N or R L over capital R clear. So, let us do that this will be a simple mathematical exercise d eta dm equals to 0 will give us m optimum is equal to 1 plus Z or Z T m, where T m is actually the mean temperature T H plus T C by 2 it can be shown you can do it as a as an exercise. You take the value of the efficiency which we derived here and just differentiate with respect to m, all these are constants and you will get this expression.

So, what does it mean? This means that you do this you square the two terms etcetera; all that and I would say that 1 plus m opt will be equal to Z T m divided by m opt minus 1. So, therefore, eta max turns out to be T H minus T C over T C T H, which is a Carnot efficiency times, why did I do this? I did this to get something which is a very simple expression and this is that expression this is eta max and here let me write m opt 1 plus Z T M and T M is half of T H plus T C, is already defined here clear. So, this is how I can maximize the efficiency with respect to the resistance ratio m by choosing the external resistance such that that the ratio satisfies this equation.

So, the question may be asked is I mean can I do both? I try to maximize the efficiency by maximizing the value of Z and I try to maximize the efficiency by maximizing or by come coming up with the optimal value of the resistance ratio. And the answer is yes; you know what we will do is we will first play around with the geometry of the thermoelectric elements such that this Z is optimized and we get that Z max value and then in order to play around with m and satisfy this relation, we will play we will alter the external resistance value R L ok.

So, we will only play around with R L and that is how we are going to get this value of m optimum. So, it is possible clear alright. So, this was for maximum efficiency alright, but; however, what we also need to do is see efficiency is definitely very important, because if I have an amount of thermal energy available to me how much of it can I extract is definitely important, but also equally important probably depending on the application again is, what is the maximum work output which may not correspond to maximum efficiency that is for a given set of elements and resistances and all that what is the maximum work that I can get out.

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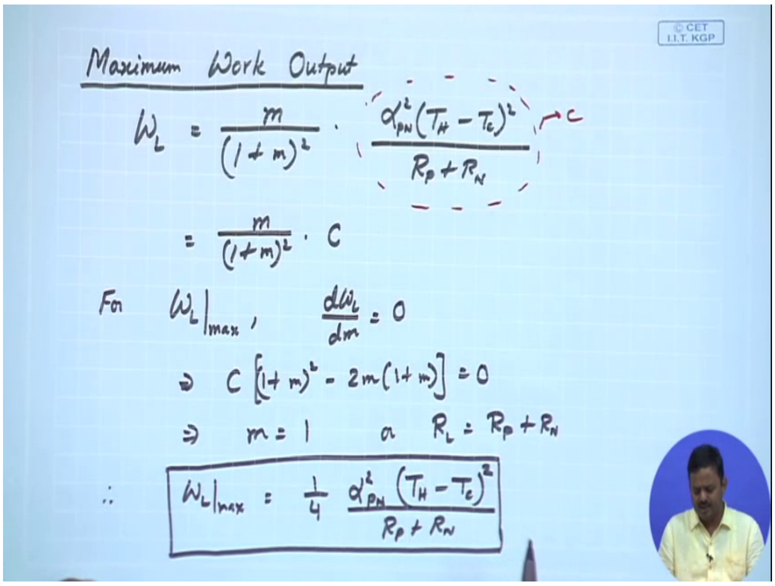

So, to do that for maximum work output, what I will do is? I will go to this value of W L which I recall is m into 1 plus m whole square times alpha squared T H minus T C whole squared alpha P N divided by R P plus R N. So, let me define this number by some constant let us this equal to C again just to all I am trying to do is simplify things a little bit. So, m 1 plus m whole squared times C. So, for W L max what happens, d W L dm is equal to 0. So, which implies C into 1 plus m whole square sorry minus 2 m 1 plus m equal to 0 which implies m is equal to 1 or R L is equal to R P plus R N. This gives you the maximum output work output clear.

What happens we if it is lower than this then R L value is low? So, I squared R i squared R L is going to be not the maximum value; if you have m greater than 1 and so, which means the external resistance is greater than these then what happens the total current the I value goes down right and therefore, I squared R L again will not be necessarily be maximum. So, the i squared R L reaches a maximum point when this is satisfied; that may not be equal to the or may not or that is not necessarily maximum efficiency. So, therefore, let me write down W L max turns out to be one-fourth alpha squared P N, T H minus T C whole squared divided by R P plus R N very important relation.

And, similarly you can put the same; put this value into the definition of efficiency and get the corresponding efficiency. As well and as you will see that that efficiency is not going to be the same as the eta max that we got in the previous exercise; alright. So, that is about the analysis what we did was we did a lot of analysis on a thermoelectric module and we got expressions for efficiency, then we try to see what does efficiency depend on and what all can we do with the within what is in our control to maximize either the efficiency or the work output and we saw that the two are not necessarily the same, actually the maximum work output does not correspond to maximum efficiency ok.

So, depending on what you want? If you want to have the maximum electrical output, if you have a lot of energy available to you and you want to have the maximum amount of work output, then you will go over this. On the other hand, if you want to use the energy very prudently, then maximum efficiency we want to convert most of the output that is a most of the waste heat that is available to us and convert it to the useful electrical energy, then we will do something else which was we will maximize the efficiency.

So, what we are going to do next is? We will wrap it up with a couple of configurations. How can we use? The TEGs the thermoelectric generators in a combined cycle power plant let us say.

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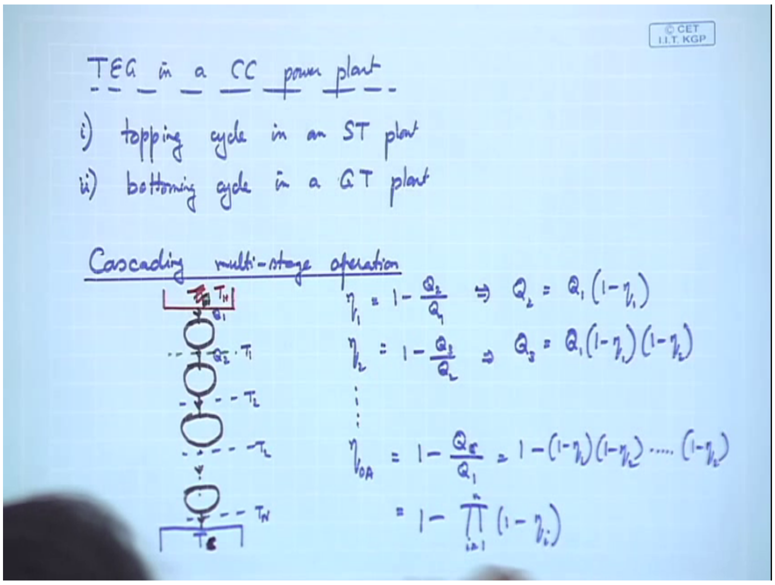

So, TEG in a combined cycle power plant can be used in two ways. These are just passing remarks; it can be used as a topping cycle in an S T plant in a steam turbine plant or as a bottoming cycle in a gas turbine plant. So, what it means is? In the first case, when I am using as a topping cycle, then the what happens is I am I am using the waste heat or I am using the thermal energy source first as an input to the thermoelectric generator and generating electricity at a low efficiency probably, and then using the heat that is rejected that Q C that we saw in the previous analysis using that as the heat input to the steam turbine plant ok.

And as a bottoming cycle in the gas turbine plant is we use the reject heat the exhaust heat or the heat contained in the exhaust gases from a gas turbine and use it as a heat source for the steam turbine. We use it as a heat source for the thermoelectric generation. Finally, what I will do is? I will end up with something called a cascading multi-stage operation. What that means is? If I have a large temperature difference available to me, what I can do is? I can utilize instead of using a simple a single thermoelectric generator what I will do is? I will have T 1 and let us say I have T 2 at the bottom, what I can do is? I can have several TEGs in this manner.

So, what happens is there are intermediate temperatures that I am going to use, the first TEG operates between; let us just the source T H and T 0 and let us say T H, I am sorry; let me write it down again T H and this is T C. And, then I have T 1, I have T 2, I have T

3, I have T n and so on. So, then what happens is? The efficiencies will be different. So, eta 1 is going to be 1 minus Q 2 by Q 1. So, I have Q 1 coming in over here and then Q 2 over here and so on which implies Q 2 is going to be Q 1, 1 minus eta 1.

Similarly, eta 2 is going to be 1 minus Q 3 by Q 2 which means Q 3 is going to be Q 1 into 2 minus eta 2 times 1 minus eta 2. And, similarly we will keep on going and finally, what we will have is the eta overall is going to be 1 minus Q C over Q 1 and this is going to be 1 minus 1 minus eta 1, 1 minus eta 2 dot dot dot dot dot dot 1 minus eta N or in other words 1 minus the product of I going from 1 to N 1 minus eta i right

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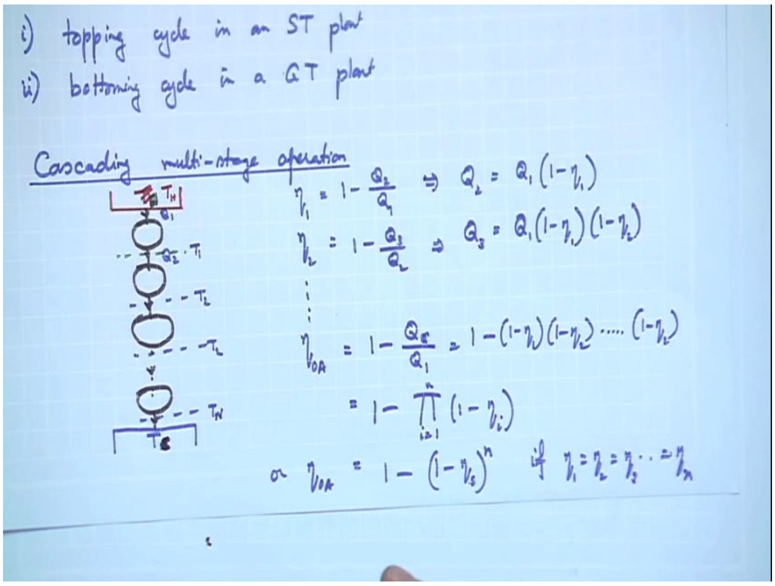

And, in case also with is equal to or eta over all; can be equal to 1 minus 1 minus eta s to the power n if eta 1 equals 2 eta 2 equals 2 eta 3 eta n ok.

Now, keep in mind it does not always mean that if you keep increasing n my overall efficiency increases it comes it this actually this expression gives you that impression that if you keep on increasing the number of cycles in and on or the sorry the number of stages my overall efficiency will go up, but that is not the case, because every time what happens is where as you increase the number of stages I am in I am reducing the temperature drop across each of these thermoelectric generators. So, therefore, my eta s so this the individual efficiencies of each of the thermoelectric generators will keep going down al1right.

So, what it means is if I keep on increasing n my eta s will also change or rather reduce. So, therefore, it does not necessarily mean that if I keep on increasing indefinitely the number of stages I will keep on increasing I will keep on getting higher and higher efficiencies alright. So, it is limited by the temperature band alright. So, that kind of brings us to the end of thermoelectric generation, and we have studied a lot of things. We started with understanding what thermoelectric generation is? What are it is potential applications? Where it is used today by for waste heat recovery? And then what we did was we went into the analysis of the thermoelectric module trying to see, what is the how do we calculate efficiency? How do we do energy balance and then how do we maximize the efficiency and what all does efficiency depend on.

And finally, we wrapped up by at small discussion on cascading multistage operation where for a given temperature difference we can use several thermoelectric generators in stages and how do we calculate efficiencies for that. So, that was the first type of direct conversion device which directly converts thermal energy to electrical energy, using solid state device no moving parts no working fluids and I hope you learnt something new through this discussion and what we will do is in the next lecture we are going to move on to another type of direct conversion device till then.

Thank you very much.