Gear and Gear Unit Design: Theory and Practice

Prof. Rathindranath Maiti
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 10
Crossed Helical Gear – II and Worm Gear

We are continuing with Module 2, design of spur straight and helical bevel and worm gears. Today is the last lecture of this week of this module this is lecture 10.

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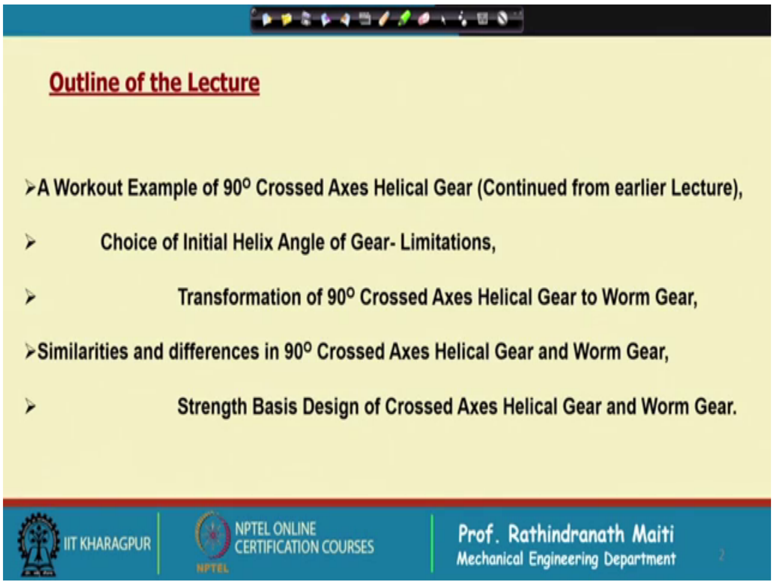

This is continuation of crossed helical gear it is part 2 and one gear. In this lecture I shall cover a work out example of 90 degree Crossed Axes Helical Gear. Choice of initial helix angle of gear and their limitations, Transformation of 90 degree crossed axes helical gear to worm gear, Similarities and differences is 90 degree crossed access helical gear and worm gear, Strength basis design of crossed axes helical gear and to worm gear.

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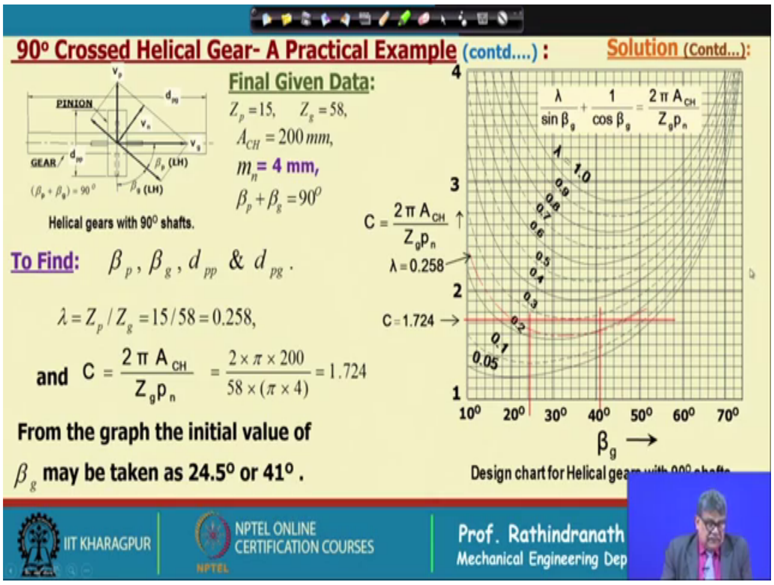

Now, coming to the work out example on 90 degree crossed helical gear, what we initially taken that number of teeth in pinion was 15, number of teeth in gear was 58, center distance specified 200 millimeter, module was 5 and beta p beta g is 90 degree, but we found that with 5 millimeter module the solution was not available. So, we reduced the module to 4 millimeter and then we are trying to find out the solution.

Now, what we have to find that is already discussed, but what we have to find that is the helix angle of pinion helix angle of gear, pitch circle diameter of the pinion and pitch circle diameter of the gear.

Now first we calculate the lambda the reciprocal of transmission ratio that is number of teeth of pinion divided by number of teeth of gear, which is 15 divided by 58 and which is equal to 0.258. And that line as you see it is it is plotted on the graph already developed graph, this graph is the C; here it is shown what is C, which is 2 pi into center distance divided by number of teeth of gear into normal pitch.

That versus that against the helix angle of the gear. Again I mention in crossed helical gear the direction of helix angle will be same for both pinion and gear.

Now, C as calculated the value becomes now with the new module for millimeter is 1.724, which is also this line is showing that line, this is the line for C we have approximately, we have taken 1.74. And the lambda is plotted here that is this red line that is for the value 0.258, that we have extrapolated norm drawn actually with an assumption within guesswork we have extrapolated this.

Now, somehow this figure is not come correctly, because of the perhaps the due to the aspect ratio. Anyway this is the ultimately 2 cross point this graph will be slightly in higher side, because it is a 0.258 it should start somewhere from here. So, it would pass through this point.

And from these 2 point we get 2 initial route 2 initial value for the a a starting the estimation of helix angle that is beta g, Alex angle of gears. From this graph it is 24.5 and 41 degree.

So, first we will try with 24.1.

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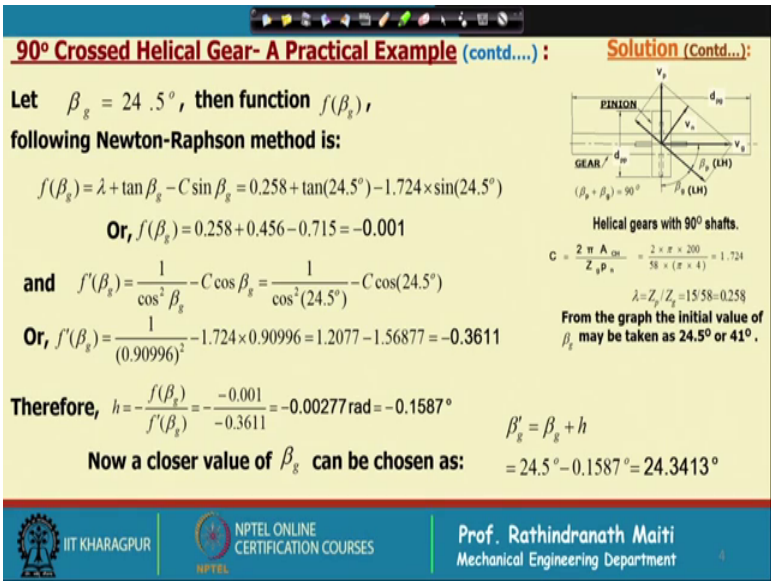

Now, the as already described the Newton-Raphson method for that, now the function of the helix angle of gear is expressed as lambda plus 10 of helix angle of gear minus C into sine of helix angle of gear.

And substituting the values 0.258 has a lambda and angle as 24.5 degree and C as 1.74. We get that f b g that a beta g that is function of helix angle of gear becomes minus point not 1, minus point not 1.

Now, we differentiate f beta g with respect to the angle beta g, which becomes 1 by cos square beta g minus C into cos beta g is equal to 1 by cos square 24.5 degree minus C into cos 24.5 degree. And as calculated it becomes minus point not 3 6 1 1.

Therefore, we can calculate the next increment in the angle which is expressed by minus function of beta g divided by the f dot beta g, and as calculated it is 0.0277 radian or minus 0.1587 degree.

Now, here this minus sign is automatically there and whatever value is coming we can we should take care about this sign. This means that this angle actually it will reduce. So, in the next trial we can we get a closer value closer to the actual reality beta g can be chosen as beta dash g is equal to beta g plus h.

So, beta dash g is nothing, but beta it is a new beta of that is new helix angle of the gear, which now becomes 24.3413 degree.

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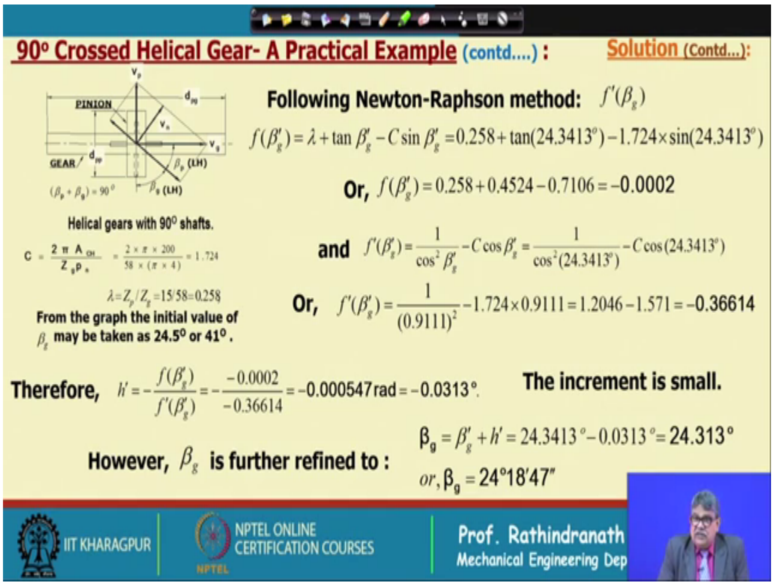

Now with that new value we again write the function of the helix angle of the gear. And substituting this new angle keeping the lambda same, because lambda is not changing. We get that value becomes minus point triple not 2 it is further reduced triple not to and differentiating that function with respect to the new angle and substituting the values. We get beta dash sorry function of beta dash g is equal to minus 0.3 6 6 1 4 minus 0.36614.

Therefore the new incremental value will become again it is minus, but it is very small minus point triple not 5 4 7 radian, which is equal to minus point not 3 1 3 degree. This increment is really small. So, one can terminate the calculation here itself and take this angle.

However, we take this refined value which becomes, then if the now beta is beta dash g plus h dash is equal to 24.313 degree ok, which can be expressed in terms of minutes and seconds 24 degree 18 minutes 47 seconds.

So, this is this value we have accepted, but this can be refined further carrying on the calculation. If we carry out calculation further at one point we see that incremental value is again increasing, we can go up to that and we can find out those values.

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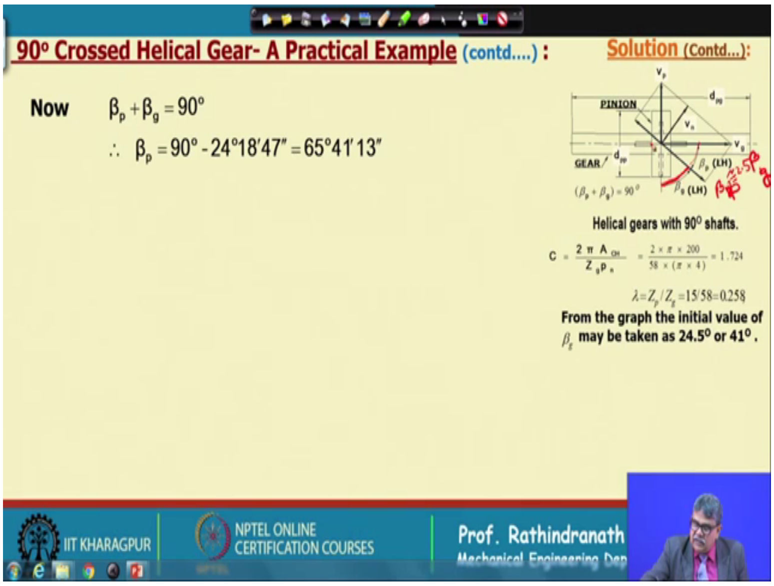

If we look into this figure right side, that this means that this angle this angle is larger than this angle almost 2 and half times. So, beta g is almost equal to 2.5 times of beta p sorry beta p is equal to 2.5 times of beta g this is another way.

So, this value means the helix angle of the in the pinion is taking almost like a thread, but mind it there will be 15 teeth. So, we will find the teeth are something like this on the gear it is from the bottom ok.

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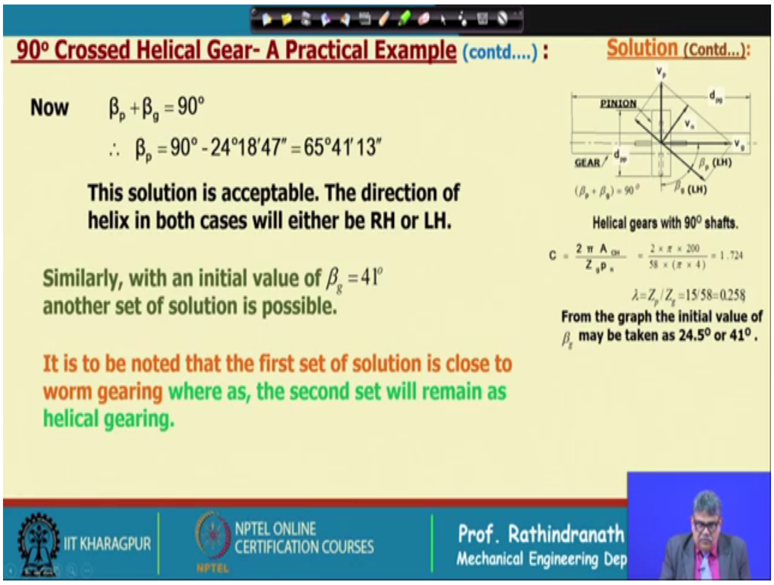

Now, after these calculations, so, we have accepted this solution already the direction of helix in both cases we either be right hand or left hand. In this case the figure it is shown here it is with left hand, it can be taken right and also both right hand.

Similarly, with an initial value of beta g is equal to 41 degree; another set of solution is possible. Now in case of crossed helical gear both are acceptable, what we will find later? That in one case in comparison to one case with the other, we will find in some cases the loads are some loads are in higher side in some in another case the friction is more something like that, but both are acceptable.

Now, it is a required the strength basis designs of these 2 sets of gears and then we can decide on which one we should take. It is to be noted that the first set of solution is close to one gearing. This means that where beta p is very large. So, if let us consider that beta p is 65 degree; that means, from this axes it is coming like this yeah it is coming like this something like this.

So, this means that the thread this is coming like a thread. So, it is close to the solution of the worm gearing.

Next we will come to that that how the this cross axes gearing transform into the worm gearing. Whereas, the second set will remain as helical gear second set, it is very difficult to we cannot come close to the worm gearing, that we will realize later.

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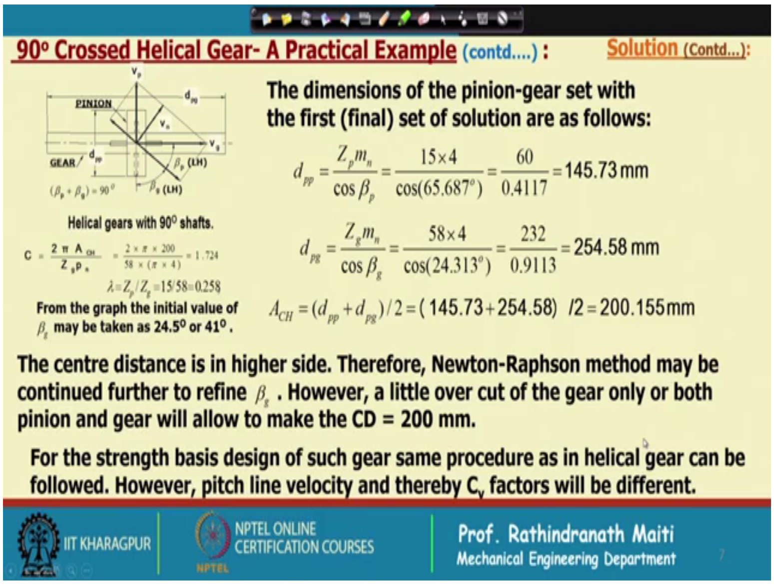

Now, the dimensions of the pinion and gear set with the first set of solution are as follows. Dimensions mean here we have shown only the pitch circle diameter, which can be calculated as Z p into number of teeth into module divided by cause of the helix angle respective helix angle. And in that case we get the pitch circle diameter is equal to 145.73 millimeter and pitch circle diameter of the gear becomes 254.58 millimeter.

Now, center distance can be calculated summing up those diameters divided by 2, which becomes 200.155 millimeter. This means that we wanted to have 200 millimeter, but it has become 200.155 millimeter. So, next the question is that why it has come like this and what we need to do. Now I have tried the several solutions by using the hand calculator, but I found that it never became it did not come below 200 it may be to 200.1413 something like that.

Now, this might be if we calculate this with a help of computer probably we can enter into that zone. However, even if it is not available still the solution is possible either we can increase the center distance, while we are machining we can increase the center distance a little bit. Even if we consider that we need the center distance 200, but 200 millimeter, but the coupling can be used with input machines and output machines such that perhaps 200.2 millimeter center distance can be used.

Alternatively we can keeping all other data unaltered we can simply give a little bit more cut to both gear and pinion or only to the gear, to make the center distance 200 millimeter; both are acceptable. So, this is the a solution is shown how it is done? The same note I have written here, which I have already discussed. For this strength basis design, now the question is that for the strength basis design of such gear same procedure as in helical gear can be followed the all the data say the lubrication factor and other factors with factor everything we can consider accordingly.

However, pitch line velocity and thereby the C v factors will be different, for these 2 gears that we need to calculate and we a need to calculate the C v accordingly.

Another thing I would like to mention here itself as you see that pitch line velocity of the gear and pitch line velocity of the pinion, these are put in such a way and their components along the line of contact, along the line of contact this gives a sense to us that there will be relative motion at the contact point there will be relative motions; that means, there will be friction.

However, this will not if we just try to visualize this will not be more even if we can calculate how much it is it will not be very high, but still it will be there than ordinary helical gears it will be more than ordinary helical gears.

Now, the we will discuss about the one gear. So, so far what I have shown that is the in cross helical gear how to calculate their helix angle other dimensions and in access then designing though we can follow the same procedure as in case of helical gear, because the usually material will be steel or whatever it might be we can follow the same procedure so, no problem at such.

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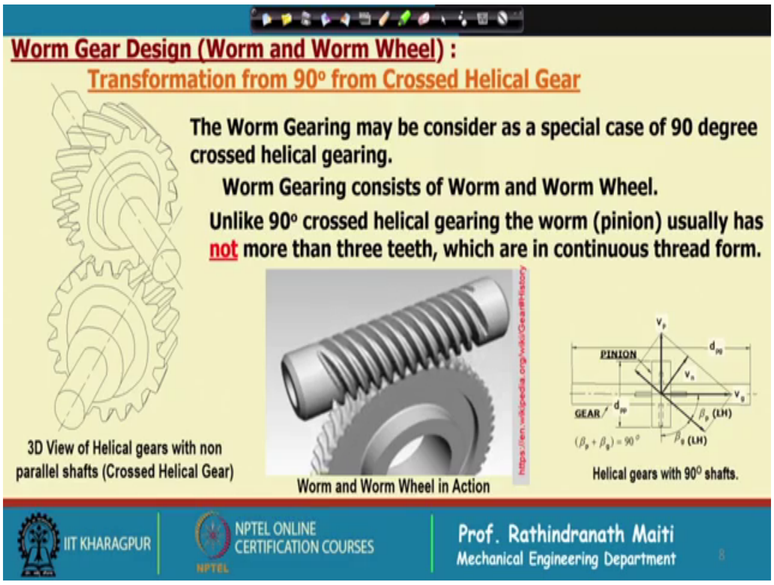

Now, we are saying that same crossed helical gear can be transformed into worm gear, how the worm gear even may be considered as a special case of 90 degree crossed helical gearing. Now this figure this is from an internet I have given this reference I have been tempted to show this figure. As you see this is the worm this one is the worm this one is the worm and this is the worm wheel.

Now, as you find in the worm apparently the it is in the form of thread it is in the form of thread and this means that the angle of beta p is such that it has become a (Refer Time: 20:02) on this shaft.

So, one gearing consists of worm and worm wheel which is shown here and next unlike 90 degree crossed helical gear the worm pinion usually has not more than 3 teeth it might be 4 teeth also, which are in continuous thread form it is like that it is it will be continuous thread (Refer Time: 20:32) apparently it will not look like a teeth.

And here in the left hand side I have shown a 3 d view of crossed helical gear, you can you can looking into these 2 figure, you can find out what is the difference between 1 gearing and 90 degree crossed little gearing both are transmitting the power in 90 degree direction non parallel shafts.

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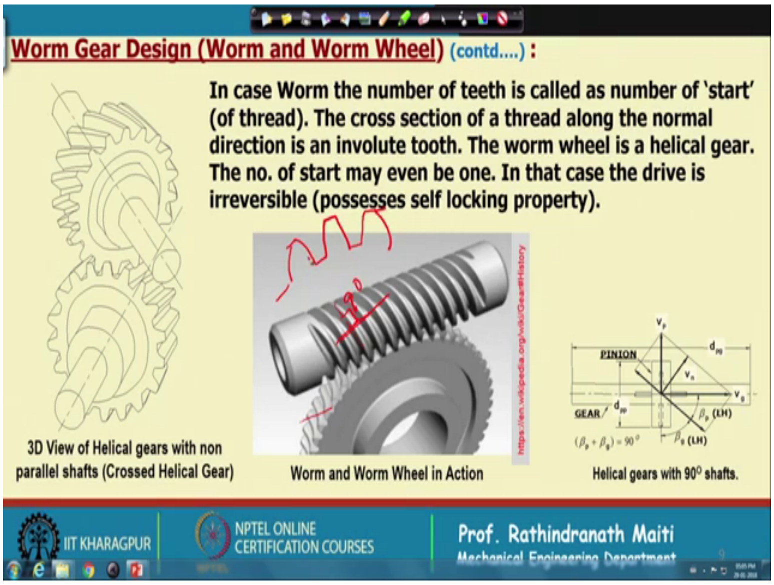

Now, in case of worm the number of teeth is called as number of start of thread you can say it is a thread the cross section of a thread along the normal direction is an involute teeth. This means that, if we cut this in the normal directions that is if I take a pitch line this if we make this is 90 degree. Then this section will look like it is it is involute this is involute and in case of in case of this helical gear also it will look like this.

The worm wheel is a helical gear the number of start may even be 1 number of starting worm may even be one in that case the drive is irreversible possesses self-locking property. This means that in that case suppose it is the single start, in that case if you rotate the worm out flow will be from worm wheel, but if you try to rotate the worm wheel you cannot rotate the worm.

And that type of transmission is required in some places say for an example a bell conveyor drum is being driven by the worm gearing and the well conveyed is inclined.

So, if it is stopped still it cannot rotate in the opposite direction. So, material will be at it position this is just an example.

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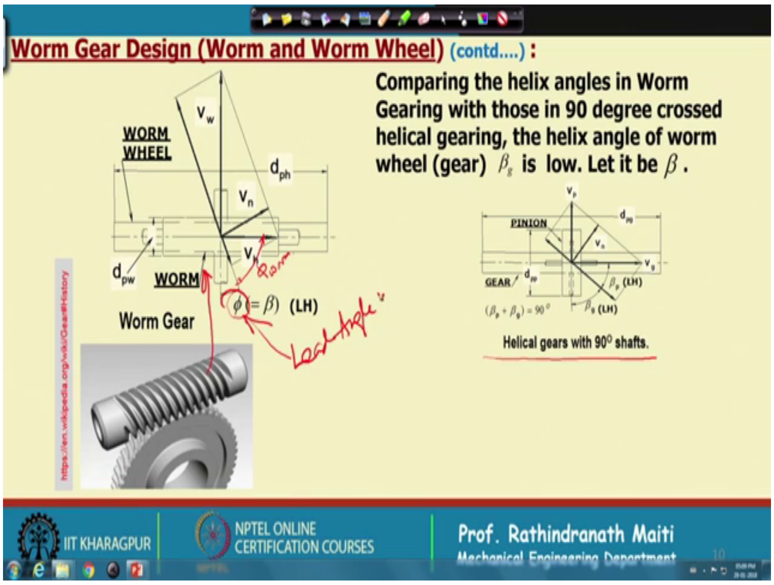

Now comparing the helix angle in 1 gearing with those in 90 degree crossed helical gear. The helix angle of worm wheel beta g is low and let it be beta angle. Now here I have drawn 2 figures, the right hand side this is a crossed helical gear.

So, this is a crossed helical gear with 90 degrees shafts. So, we have shown the line of contact as we have seen that it is possible to make the beta g and beta p both very close to 45 degree it is possible. In the another solution is there the beta p is usually large and beta g will be small and that will be close to worm gear.

In coming to this worm gearing feature as we find that this pinion this has become like a cylinder with a thread, which is shown here it has become something like this I have drawn it here.

And if we think of the helix angle then on the worm gear the helix angle is this much and here in this case if we make it will be coil something like this and no actually it is in the opposite directions the coil will be something like this. And comparing with this cross helical gear, this will be the helix angle of worm, but we do not mention that is the helix angle of the worm rather. We consider this angle and what we call as a lead angle or pitch angle.

In case of single start we should call pitch or lead both will be acceptable, but in case of multiple start 2 3 4 we this angle is equal to lead angle; that means, the thread is coiled with that much of angle and other threads side by side all are having the equal angle, but they are apart by a distance which is equal to pitch.

So, I think this is clear that what are what are will be the angles.

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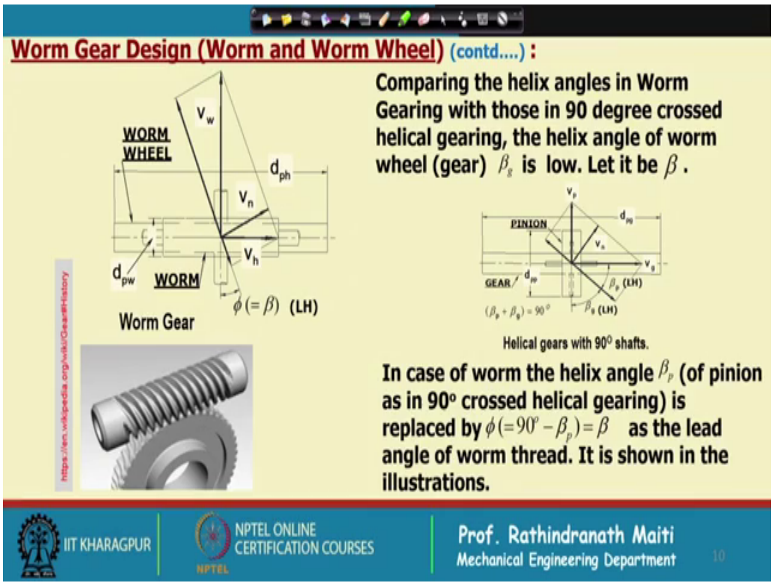

Now in case of worm the helix angle beta p of the pinion is 90 degree crossed helical gearing is replaced by phi, which is 90 degree minus beta p is equal to beta that is the helix angle of gear.

So, now we do not call as beta we call the phi is the helix angle of the worm wheel and phi is the lead angle of the worm, it is shown in the illustration.

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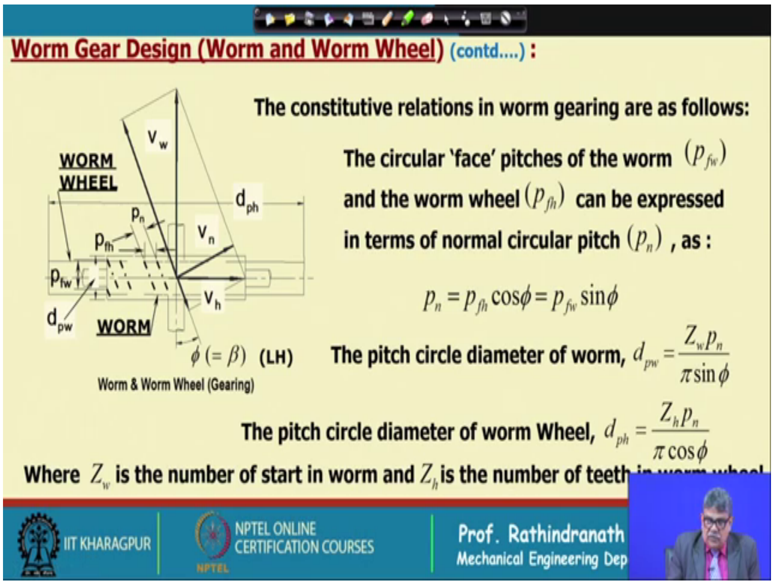

The constitutive relations in worm gearing are as follows the circular or face speech of the worm P f w this is shown here. So, say this is one teeth this is another teeth are like this way or that way, when it is coming to this face? Then distance between 2 teeth the pitch distance is given by designated by P f w that is face pitch of worm.

Similarly for the worm wheel this is designated by P f h the face pitch of worm wheel. And if we take their normal directions then in the normal direction model that will be the P n normal pitch and that will be equal for both the worm and worm wheel.

So, P n can now be written as P f h cos of phi and is equal to P f w sine of phi, this geometry can be realized from this figure. The pitch circle diameter of worm d p w is number of start into normal pitch divided by pi sine alpha sorry sine phi, and the pitch circle diameter of worm wheel d p h will be number of teeth of the worm wheel into normal pitch divided by pi to cos of phi. Where Z w is the number of start in worm and Z is the number of teeth of the worm wheel.

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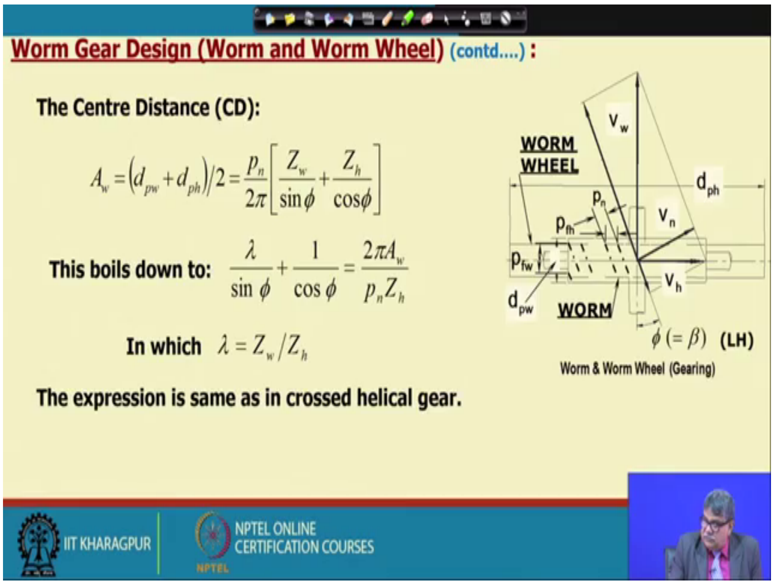

So, adding these 2 diameter and dividing by 2 we get the relation for the center distance, which is shown here and this again we can write in the form what we used for the crossed helical gear and it becomes lambda by sine phi plus one by cos phi is equal to twice pi into center distance, divided by normal pitch into number of teeth of worm wheel. In which lambda is number of start of the worm divided by number of teeth of the worm wheel.

The expression is same as in crossed helical gear already told now; we can use the same chart to get the solution.

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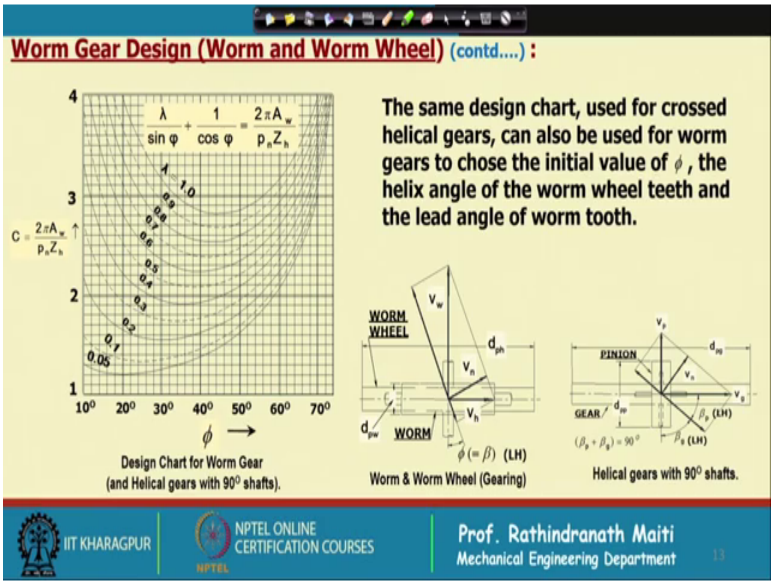

The same design chart used for crossed helical gears can also be used for worm gears to choose the initial value of the angle phi, the helix angle of the worm wheel teeth and the lead angle of the worm teeth.

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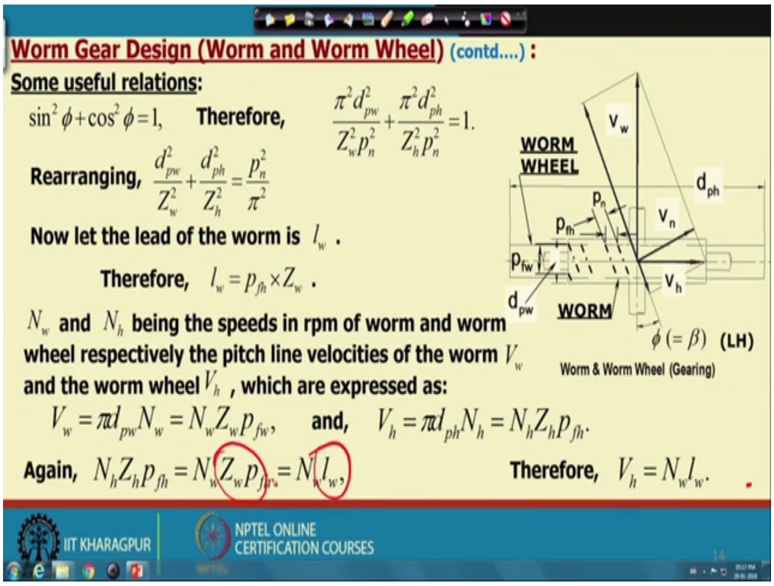

Now, some useful relations are as follows, which is as the sine square phi and cos square phi is equal to 1 therefore, we can write the pi d p w square by Z w square by P n square plus pi square n d d square P h divided by Z Z h square into P n square is equal to 1, which can be rearranged to pitch circle diameter of the worm square divided by number of teeth or number of start of the worm square, plus pitch circle diameter of the worm wheel square divided by teeth number of the worm wheel square is equal to circular pitch normal circulars pitch square divided by pi square.

Now, let the lead of the worm is l w clearly, that lead l w is equal to that face pitch into Z w. This means that. So, this is the face pitch and what is the number of start if we add, if we if we add that that number of face pitch that will give the lead; that means, if you open that one will become the lead of this sorry the lead in this directions.

And then N w plus N h being the speeds in rpm of worm and worm wheel respectively, the pitch line velocities of the worm and the worm wheel which are expressed as pi d p w, N w that is the V w which is shown here.

Say this is V w as we see this magnitude of this one and the V h that is the pitch line velocity of the worm which is shown here, that become N h Z h into P f h. From the figure itself with this angle which some realistic value of this angle helix angle of the worm wheel worm wheel teeth and or the lead angle of the worm, but we find that V w is larger and definitely there will be high friction.

Again if we write that speed in rpm of the wheel into number of teeth of the wheel into face speech of the wheel is equal to number of start of this gear into N w, number of start into P f h. We can replace these by these 2, but keeping this P f h same and then we get as sorry you see this part is nothing, but the lead.

So, we get this relation and we can write V h is equal to N w that number of start of the teeth into l w that Vv h means the pitch line velocity of the gear is nothing, but the axial velocity of the thread, if we look into the thread what velocity it is preceding that will be the velocity of the gears.

So, this relation validates that all the equations, we have right we have written.

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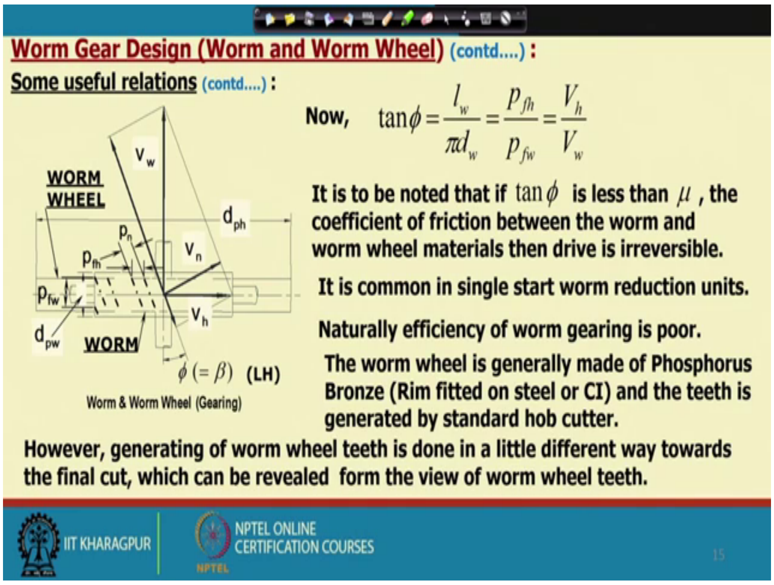

Now, again clearly tan of this lead angle must be equal to lead divided by pitch circle periphery of the pitch circle of the worm. So, if we write this equation that becomes pitch line velocity of the one divided by pitch line velocity sorry pitch line velocity the worm wheel divided by pitch line velocity of the worm.

Now, this value is definitely less than 1, which we find from the figure and if it is a single start then this value becomes much low. And from the relations easily we can we can write that if tan phi is less than mu the coefficient of friction between the worm and worm wheel materials then the drive is irreversible. So, that is the case of self locking which I have already described.

It is common in single start worm reduction units and it is desired. Naturally efficiency of worm gearing is very is poor, the worm wheel is generally made of phosphorous bronze the rim fitted on the steel or cast iron; that means, this worm wheel need the teeth portion is cut on a rim, you which is not steel it is usually phosphorous bronze whereas, very common the worm will be of alloy steel and the um teeth of the wheel can be cut by standard hob cutter.

However, generating of worm wheel teeth is done in a little different way towards the final cut which can be revealed from the view of worm wheel.

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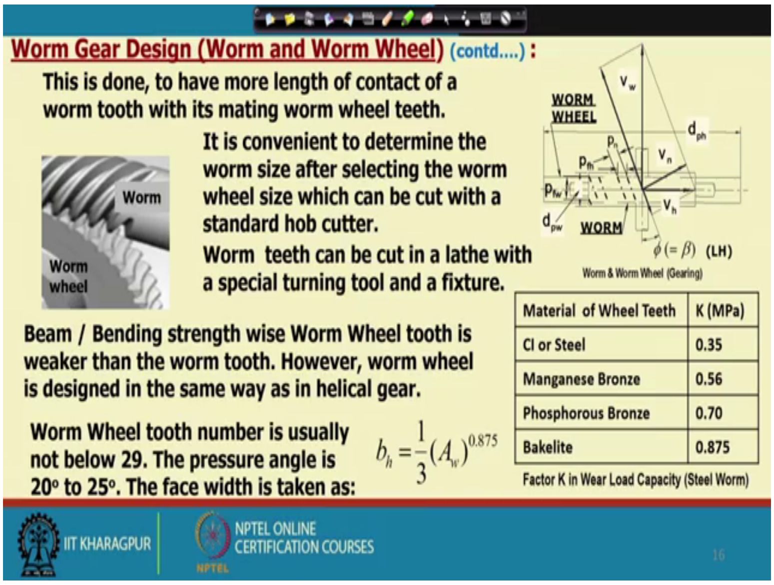

If, we look into this worm and worm wheel then what we find here that this if we cut as you as I told in the normal directions, this is a normal involute teeth. Here also if we cut in this normal directions this will also look like an involute tooth, but if we look into this as if the hob cutter has been deepened on the width of this care. Say this means that actually, if we will find that teeth on the worm wheel it has cut something like this.

So, the hob is cutting like this hob is cutting like this. So, unlike the standard helical gear this in final cut we have to depend this cutter here the manufacturing procedure, the this is maintained and I am not an expert in this directions, but there is a difference between this and this, but still we can cut this worm wheel by a standard hob cutter. Then this dictates us that first we should it is convenient to determine the worms size after selecting the worm wheel size, which can be cut with a standard hob cutter.

So, if we want to design first of all we will think of number of teeth center distance etcetera, but for from cutting point of view first of all the one gear worm wheel sorry worm wheel is selected the angle etcetera. So, that we can cut with a hob cutter, then the worm is cut usually in lead it can be cut in a lead with a special cutter and a fixture.

Now, Beam and Bending strength wise worm wheel teeth worm wheel tooth is weaker than the worm tooth this is because the worm tooth is steel whereas, this is from some other material usually it is phosphorus bronze in most of the cases which I have mentioned ok.

Now the worm wheel tooth number is usually not below 29, the pressure angle is in between 20 to 25 degree say, but we always look for the standard cutter. So, 20 degree is a standard cutter or a 20 degree pressure angle. So, we will consider that in some cases 25 degree is also available in between that 22.5 may also be available, but 20 degree is most common.

Now, the face width this means that the diameter of the worm is almost close to the width of the wheel. There is no meaning that we can take extra-large width or neither it is a meaningful, it will be of less width. And that as a I have shown the worm wheel teeth is made in such a way we get more contact with the worm teeth.

However there is a thumb rule the width of the wheel can be taken as 1 third of center distance to the power 0.875 this is thumb rule, but while we are finalizing the design we can select the actual width. Now again in this case also these we shall we have to design the worm wheel, because the worm is stronger than worm wheel. And the material it might be steel also cast iron, but most common is the phosphorus bronze whereas, manganese bronze is there and also backlight.

Now, these are to have better coefficient of friction. In phosphorus bronze, which can from the strength point of view and from the friction point of view, phosphorus bronze is the best among these lot Bakelite is for light load whereas, if you use for high load steel, but their friction will be more, manganese bronze is in between that.

Now for that that K value in wear load factor that is the wear load capacity of the gear, which we need to check from the dynamic load point of view this K value are given here. Now, apart from that it is customary to estimate the, what would be the efficiency worm gear maybe as low as 70 percent in percent 70 percent.

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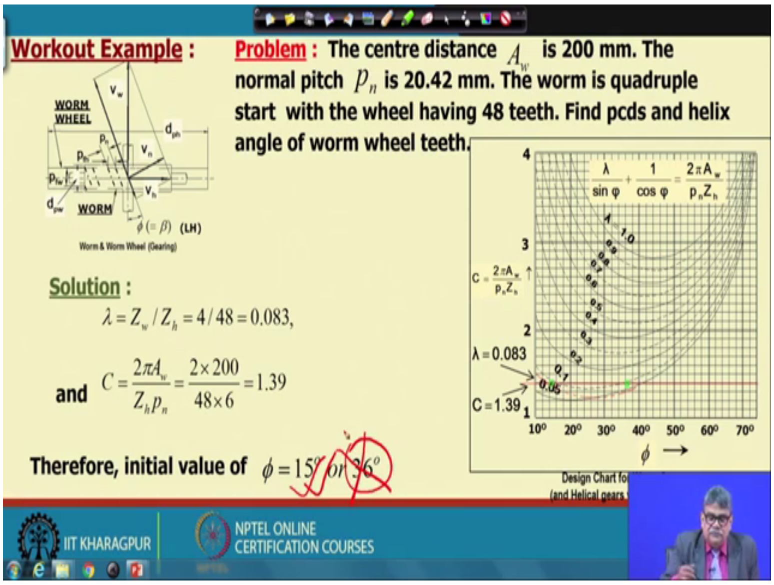

Now, so, if we just we look into a problem a problem is that 200 millimeter center distance and circular pitch is 20.4542, which means the module is 6.5 and the worm tooth is 48 and whereas, number of start is 4, for that we have calculated lambda is point not 83; that means, reduction ratio is 12.

And C is 1.39, but actual case the reduction ratio may be 30 40 is very usual in one gearing.

Now, for these value what we find that 2 values use for the phi that a lead angle 15 degree and 36 degree using the same graph, but as already discussed 36 6 degree will not be acceptable for the one gearing that means this is not acceptable. We will accept only this one and then we will find the solution, this further can be discussed in form of tutorial also.

So, thank you for listening this is the end of the second week lecture and there are many things which can need to be discussed that will be discussed through the tutorial. It will be informed through the tutorial, but from the next week we will take a practical design of helical gear box.

Thank you.