Gear and Gear Unit Design: Theory and Practice
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Lecture – 02
Laws of Gearing, Kinematics and Geometry Part – I

Introduction to gear and gear unit design.

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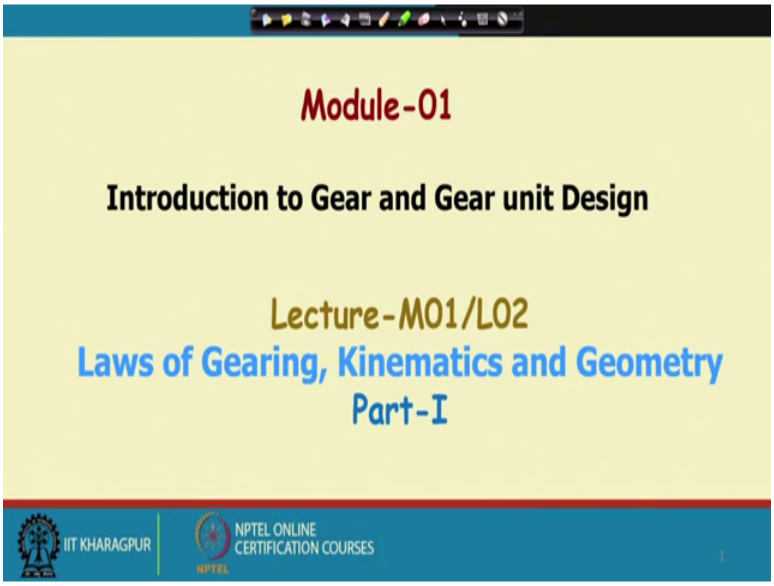

This is module one and the second lecture of module one is on laws of gearing kinematics and geometry part one, in this lecture, I shall cover Fundamentals of gear tooth profiles next gearing laws.

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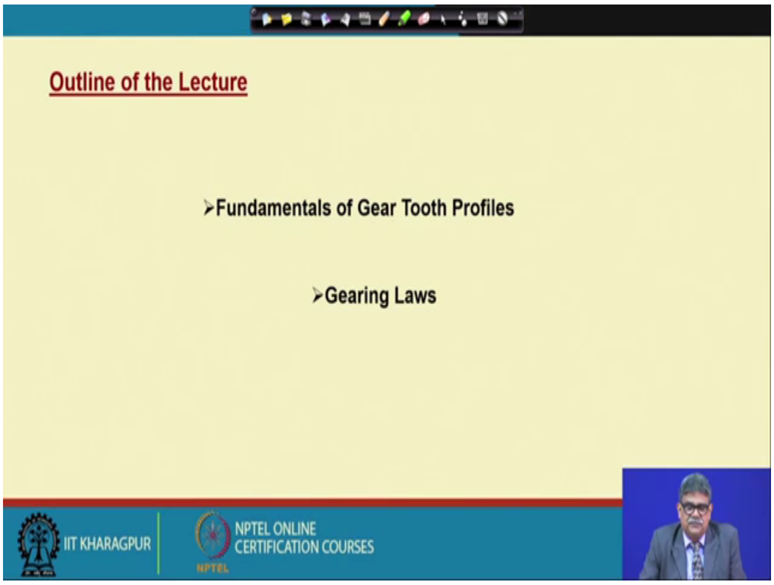

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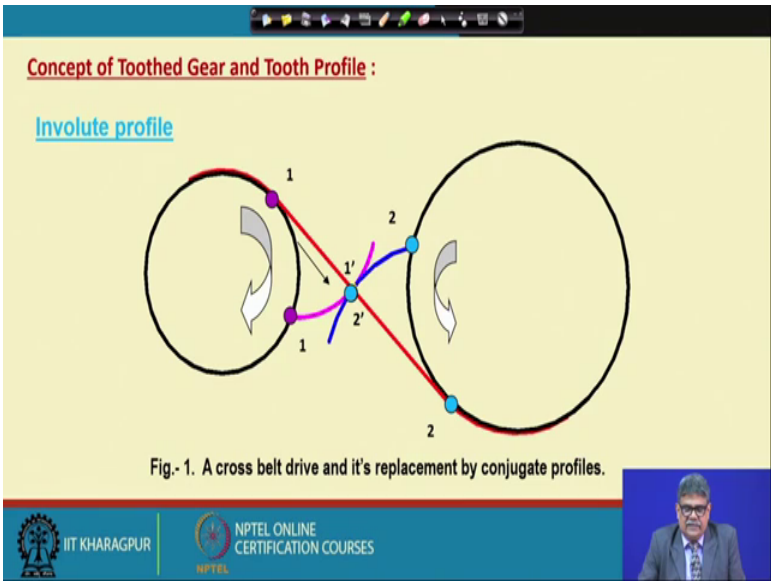

Let us consider the cross belt drive with flat belt, there is a small drum which is usually driving may be driven also and a big drum, it might be equal also size of this drum may be equal also.

However this is the belt cross belt and let us consider a point at the tangent of the belt with this drum, then this drum is rotated in clockwise direction and due to this motion the belt will move in this direction and the other drum is moving in anti-clockwise directions. Now, this point gradually moving towards this clockwise direction on the drum and on the belt, it is coming towards this point.

Now, after certain rotation this point will come here whereas, on the belt this point will come at this point one does. Now, if we consider the each and every point and draw a locus then we will get this profile on this drum. Similarly, if we consider the point 2 in other drum and if we rotate in the opposite direction, then we will get when the point 2 will reach at this point, we will get a profile which is nothing, but the locus of these points as it is travel as it travels.

So, this means that we are getting these 2 profiles on this drum due to this motion now if we remove this belt and allow these 2 profile to touch it each other and then again if we considered the first motion, the motion will be exactly same as it is with this belt, this means that this cross belt drive can be replaced by these 2 drum with these 2 profile.

So, therefore, this is we may consider the first concept of gearing how we can come into tooth gearing for power transmission now as we know on a subtle if a is let us consider this is a thread it is a knowing then these the curve generated on this point is called involute. So, therefore, this must be involute, but the question is that in case of belt drive, we get a continuous motion has belt is a closed loop, but how we can get the continuous motion with these profiles the obvious alternative is that we should have such profile at certain interval and that interval must be uniform and then if we can make this drum with these profiles such that when one is the disengaged at least another one will be engaged, then we will get the tooth gear.

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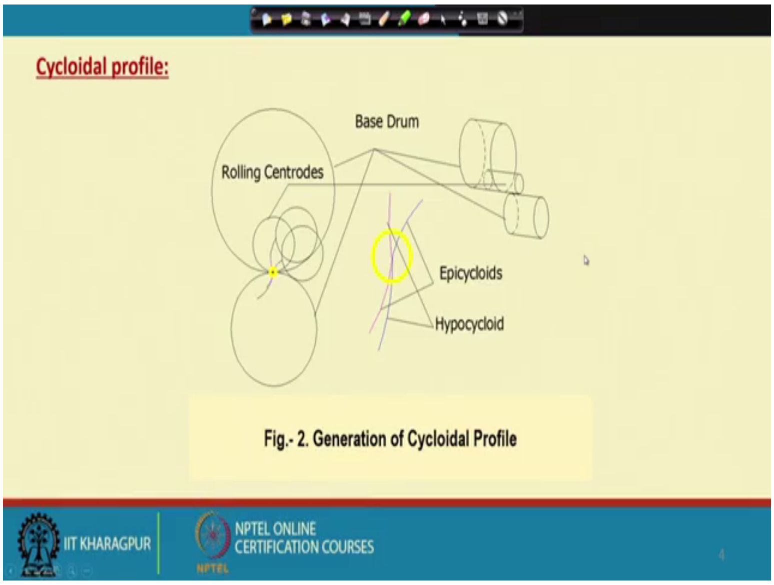

Now, what I have shown that is the generation of involute, but also we can transmit the motion between 2 drums from one to other by using three drum let us consider this is a hollow drum and this is a solid may be hollow also, but what we are doing we are using another cylindrical body on the top of the surface which is touching the inside of this surface, this means that this cylindrical drum and this cylindrical drum, they are touching each other they are they are the outer line will touch each other and this drum is touching inside and this inside of this bigger drum and outside of the smaller drum here.

Now, if we then if we consider a point on this then if we rotate this roller and then if we take a point on this then this point will generate a profile inside as well as outside because this is the same way as I have shown in earlier case if we consider the locus of the point on this drum and while it is rotating on this drum, then it will generate a curve. Now, this curve which has generated on these 2 drums when it is rotating inside it is hypocycloid oh sorry this is rotating outside of a drum that is epicycloids and inside of a drum that is hypocycloid and in that way, we get also a profile which can be fixed on these 2 drums and this can be replaced and we can transmit the motion.

Now, this is another gear another profiles for the gear which is called cycloidal profiles. So, what we have seen that we can use the involute or we can use the cycloid to transmit motion from one drum to other. So, definitely these 2 can be used for gear tooth profile;

however, here question is that is these 2 are the only profiles answer is no there are many many other profiles, which must be conjugate and we can use as a gear.

Actually if we fix any curve on a drum and while we rotate one drum touching other thin or maybe with the cross belt if that attached profile generate another continuous profile, then that must be conjugate and that can be used as gear now the question is that whether shall we use all such profiles or we should be selective or is there any other reason that we have to stick to few of the profiles we will I will come to that.

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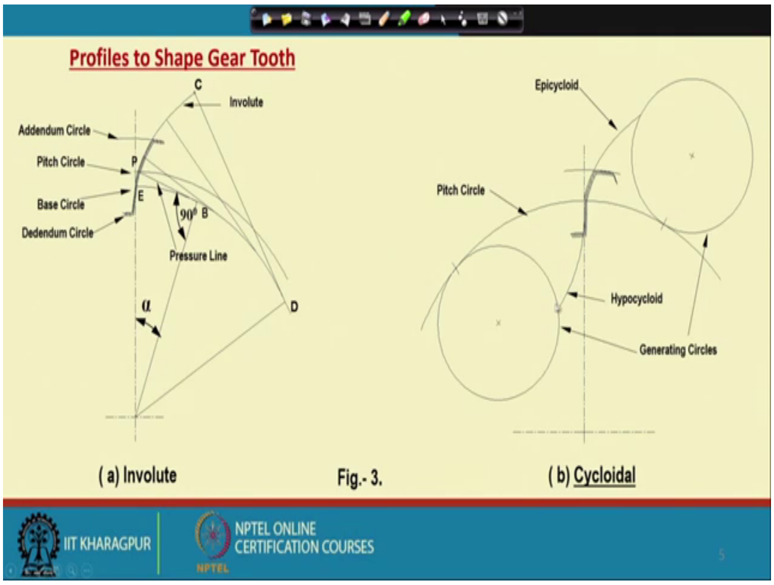

Now, I have shown the how the involute is generated now this involute profile is fixed on the drum on which it is generated and then that gives a gear tooth profile now definitely as I told that we have to use the profile one after another on the drum at a at a fixed distance then that will give the continuous motion..

So, a tooth we will look like this involute tooth will look like this which is generated on this on a drum and if we consider a tooth that should be of a specific size the size should be such that when one pair is out of touch then other pair will be engaged and in that way say this is being unwanted and this profile is being generated and we consider the tip circle which is called addendum circle and this is called pitch point what is pitch point, I will come later and this is called pitch circle.

Pitch circle is a reference circle which will be there when 2 gears are rotating and there the common circle which is touching each other we shall come later on to that, but this is called pitch point it is on this line and this is called base circle on which this profile is generated; however, this tooth may be below the base circle although this portion is not involute then this circle is called either dedendum circle or root circle, but this portion is not used for tooth contact only it is used for the clearance.

Similarly, so, this is called pressure line or line of action which is which passes through this speech point now this if we this is the involute and this angle is 90 degree and this angle is alpha is called the pressure angle, if we consider another gear is meeting then this line will touch the base circle of the other gear and we would call that this is the line of action and this would be the pressure angle.

In case of cycloidal tooth, these 2 are generating circle these 2 circles are generating circle as I have described in the in earlier slide. So, that we will give us this type of profile which in open I may look like an involute profile, but there will be obvious difference also now this in this case in case of cycloidal tooth this circle is called this circle itself is the pitch circle as well as base circle whereas, it is not called base circle, it is called describing circle whereas, in case of involute teeth, this is called base circle that is the basic drum on which this involute has been generated.

And this is epicycloids and this is hypocycloid.

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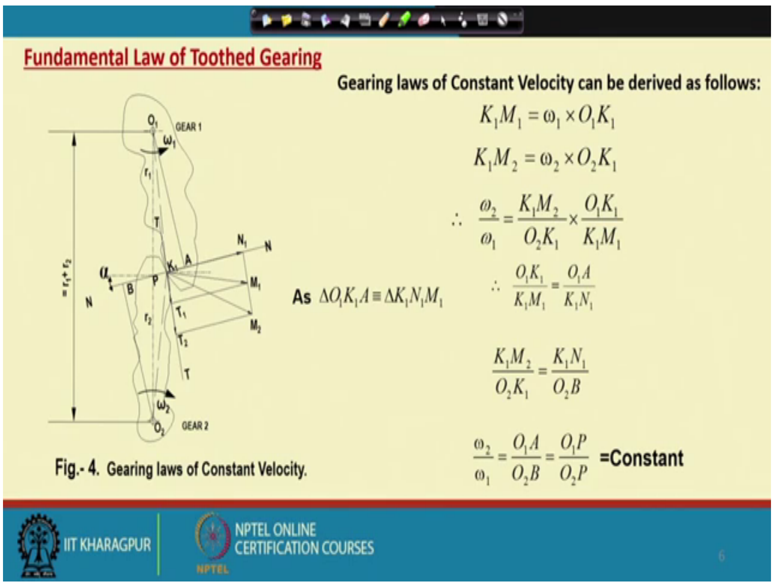

So, I have shown only 2 profiles involute and cycloidal as I told that other profiles are there, but we are not we will consider only the involute profile why for that; let us look into this gearing law. Now, what is the basic purpose we have transmitting power from one to other power means the speed and the torque when the speed increases torque will reduce and usually in most of the cases, we will consider, there should have the constant reduction or constant increase in speed or torque usually the prime mover through which the drive gears is connected that is of constant speed and output will be also constant speed this means that the ratio of this gear ratio of this basic drum riff of the describing circle should be a constant.

Now, we shall examine that through a gearing let us consider these two are gear profiles these are 2 gears gear 1 and gear 2 and then this profile has come to a contact point which is K 1 and then if we consider the gear one is rotating at omega 1 speed, then we can draw a vector the velocity vector which is which can be written K 1, M 1 is equal to omega 1 into 1 into K 1, here I would like to mention that this angle is 90 degree because as we are presenting in this way, this is 90 degree although we have, this is not proportionate, but considered this is 90 degree.

Similarly, K 1, M 2 for the rotation of the gear 2 will be omega 2 into O 2, K 1 and this angle is also 90 degree ok. Now, if we write this equation and consider divide one by other second one by the first one, then omega 2 omega 1 is equal to K 1, M 2 by O 2, K 1 into O 1, K 1 into K 1, M 1, this can be easily derived now.

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Now, has this if we consider the triangle, then O 1, K 1 a is similar to K 1, N 1, M 1 because this angle is alpha angle and as this has rotated by this this amount then this angle will must be equal to this angle. So, there these 2 are similar triangle and due to that we one we can write O 1, K 1 is by K 1, M 1 is equal to O 1, a by K 1, N 1.

Similarly, if we consider the other circle then K 1, M 2 divided by O 2, K 1 must be equal to K 1, N 1 is O 2 B, this means that if I consider omega 2 by omega 1, then O 1 A O 1 A divided by O 2 B and again from this similar triangle property of this O 1 a P and O 2 B P we can write this must be equal to O 1 P by O 2 P, but these 2 are nothing, but the center distance and they these 2 gears is always have a common circle which is touching at the point P which is nothing, but the pitch circle and therefore, this ratio is always constant.

Therefore if omega 1 is uniform omega 2 will be uniform with a constant ratio. So, this is the basic laws of gearing now here interestingly we can look into this as this velocity vector is having a common projection K 1, N 1 and this this one also has a common projects and K 1, N 1 that is why the velocity remain constant this line velocity velocity along the line of action remain constant and this ratio remain constant.

But if we consider their component along the K 1 directions one is K 1,T 1 and another is K 2, T 2; obviously, these 2 vectors will be same equal at the point P, but other points there will be difference this means that while this tooth contact consider this is the involute profile this is giving a constant velocity along the line of action, but they are having a rubbing action on each other except the pitch point that due to this rubbing on this teeth there will be some losses where which can also be calculated.

So, for gearing this should be maintained; now it is it can be shown with the involute teeth this is easily maintained, it also can be mentioned here in case of involute teeth, we have this line this line of action is a straight line which is nothing, but the cross belt, you can say that like a cross belt, but in case of cycloidal tooth that path is cut, but still with that card and if we draw the geometry the same thing can be proved for cycloidal tooth also.

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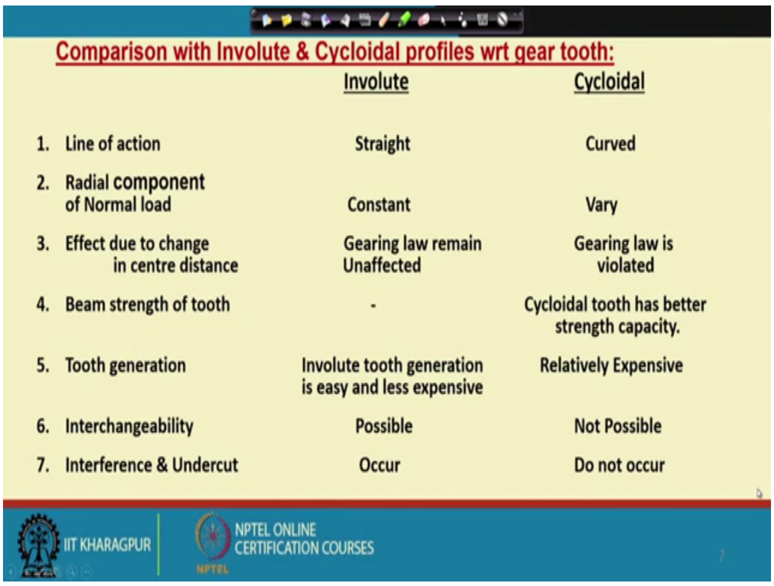

Now if we compare the tooth profile of involute and cycloidal then in that case what we find the line of action is straight in case of involute where is in cycloidal, it is curved the radial component of normal node, if we consider the normal load that definitely we will act through the line of action that will be radial component will be constant in case of involute, but at as the path of contact is curved in case of cycloidal that will vary what does it mean that in case in case of involute tooth the radial load which is going to thus act will always remain constant for a constant torque whereas, in case of cycloidal tooth for a constant torque that will vary.

Now, affected due to change in center distance if you change the center distance in case of involute teeth what we find that there is to base circle this base circle is being driven by say like a cross belt that is there is a common tangent; that means, if we even if we move this 2 point, but still the same involute can we will touch each other and therefore, gearing laws will remain half unaffected whereas, in case of cycloidal teeth the gearing law will be violated because as long as if they are moved then the true describing circle they are away from each other and this at that portion the gearing law will be violated .

Now, if we consider the beam strength in case of cycloidal tooth has better strength capacity that is from the geometry it can be proved there will be from same more or less same size of teeth the cycloidal tooth will have better strength whereas, in case of involute tooth the strength will be less now tooth generation involute tooth generation in is easy and less expensive and definitely cycloidal tooth generation will be relatively expensive.

Interchangeability in this case as we find that the common tangent is generating the involute tooth profile then definitely we can change the size of the drum; that means, there will be interchangeability whereas, in this case the 2 touching circle generating the cycloidal profile and we cannot interchange. So, these gears cycloidal gears has to be manufactured in peer, but still I would say that cycloidal truth are used where for a special purpose and then that has a specific purpose and some certain advantage as well.

Now, interference and undercut this I will discuss later what is called interference and undercut that occur in case of involute teeth whereas, in case of cycloidal teeth this will not occur.

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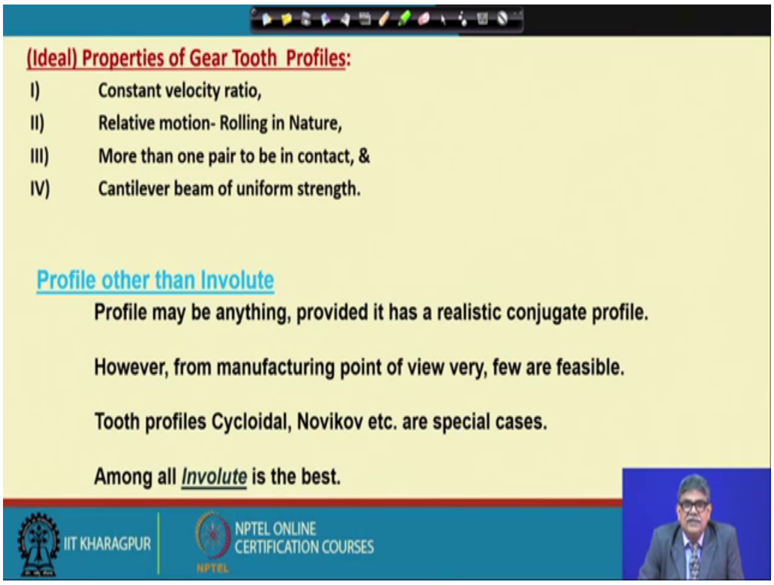

Now, ideal properties of gear tooth profiles should be like that constant velocity ratio relative motion and rolling in nature as much as possible although there will be slide which I have discussed more than one pair to be in contact and cantilever beam of uniform strength that should be ideal gear tooth profile.

Now, profile other than involute as I have discussed profile may be anything provided it has a realistic conjugate profile; however, from manufacturing point of view very few are feasible tooth profile cycloidal Novikov etcetera are special cases, Novikov is another type of profile which is also used for gears; however, it is found that among all involute is the best and our gear design what we will learn now onwards that is mostly on involute gears.

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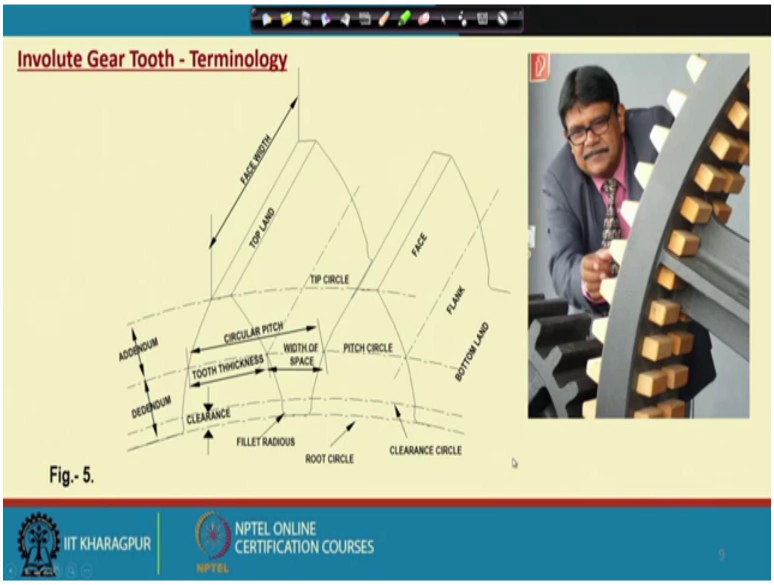

Now, if we consider the involute tooth terminology because we should know something more about the laws as well as the gears geometry and kinematics, but before that it is better to know; what are the different terminology used in gears here, I have drawn 2 involute tooth profiles and then this is the phase width of the tooth, this is sometimes it is called tooth width sometimes, it is called the gear width this width is from one side of the gear to other side that is called face width.

Now, this portion is called top land and this circle is called tip circle or as I have mentioned earlier addendum circle, this surface is called face and this person is called flank the below this pitch circle. Now, this portion is called bottom land, this circle is called pitch circle. Now, definition of each circle is that if we consider the involute tooth then to be circle a we will a line can be drawn through the base circle which will touch 2 base circles.

Now, that will intersect the line joining the centers of these 2 gears that point is called pitch point and the circle through these pitch point is called pitch circles. So, pitch circle physically there is no existence when the gear miss then we will have the pitch circle ok, but in standard gears we take the standard cutter and we when we with the standard gear when we make the standard tooth then the proportion is such that this becomes the circular pitch and this gap equal to this gear width it is not proportionate looking proportionate here, but we have to remember for standard gears on pitch circle this arc thickness of the teeth is equal to arc thickness of the gap.

So, this is called 2 thickness this is called width of space these 2 are equal for the standard cut gears now this portion is called addendum we should remember this height is called addendum and this portion is called dedendum and this is the base circle no this point is called at the root this is called fillet radius this is also called a root and this circle is called root circle or sometimes called addendum circle root circle or dedendum circle and this is addendum circle or tip circle and then this clearance this is called clearance circle.

Now, this clearance circle not necessarily the base circle rather if we consider the tip circle of the a meeting gear that will pass through this point that will touch here. So, this portion is called clearance circle sorry clearance. Now, this clearance this if we consider now base circle usually base circle will be below that clearance circle.

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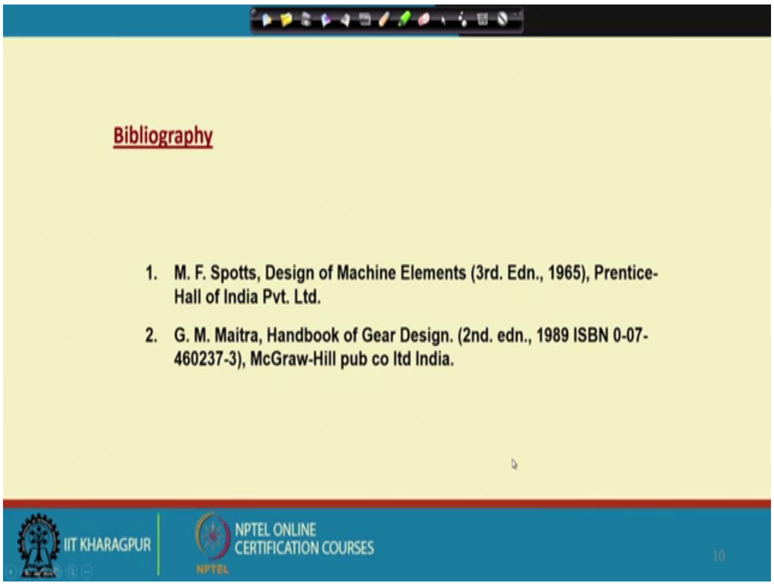

Now, for this portion which I have delivered you can follow a book the design of machine elements by sport now this I have followed the third edition, but it is available the later additions are now available now there is another very good book by G M Maitra handbook of gears and there many informations or gears are there apart from that I there are many other books which some of them, I have mentioned in my introductory lecture.

Thank you.