

Gear and Gear Unit Design: Theory and Practice
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Lecture - 33
Involute Spur Gear Tooth Correction: Part – I

Module 7 this is on Introduction to Involute Gear Tooth Correction. And in this lecture I shall cover the involute spur gear tooth correction part 1.

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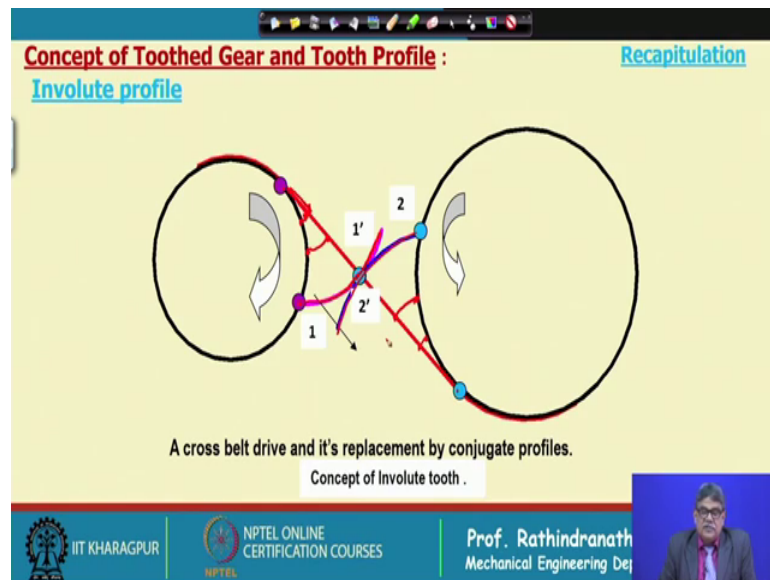
Outline of the Lecture

- Concept of Involute Tooth (Recapitulation)
 - Generation of Involute Tooth
 - Basic Rack and Rack Cutter
 - Concept of Gear Tooth Correction
 - Why Gear Tooth Correction
 - Plus & Minus Correction
 - Amount of Correction (Profile Shift)

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This is introduction to gear tooth correction involute gear tooth correction part 1. And the lecture will have concept of involute tooth will recapitulate, how this involute tooth is generated? Generation of involute tooth basic rack and rack cutter concept of gear tooth correction, why gear tooth correction is required plus and minus correction, amount of correction, that is profile C.

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Now, this already we have seen how the involute teeth profile is generated or we can say that instead of cross belt drives, we can use the involute to the gear drive we can replace the belt by involute tooth, what I have shown here that one small drum and there is another a little bigger drum, which is connected by a belt.

So, we can say that this is one drum and this is another drum this is the belt and let us consider initially it is rotating in these directions. Now, if we consider a point one which has come over here which was initially. So, this we may consider in this drum this point 1, which has come here from this point. And from 2 has come to this point while this drum is rotating actually let us consider first that this point was here.

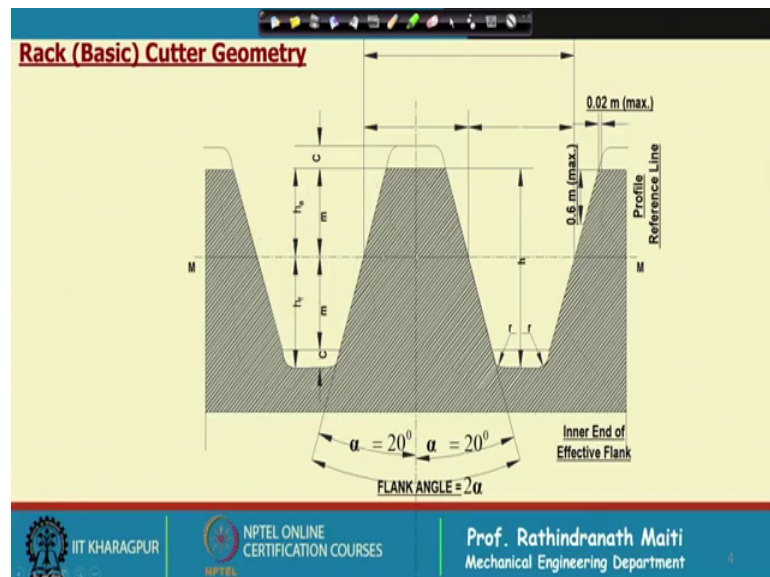
So, while it is moving in this direction this point is coming gradually here. So, if we take a locus of this point next this locus of this point next it has reached here. So, we are getting this profile ok.

Now, if we consider the same way a point here then if we rotate in the opposite directions it will be here, then when it is coming over here, then like this and gradually we will reach into this. So, this is another profile right. Now, if we remove this belt and allow this to profile to drive the other say suppose this is the driver and this is the driven, then while it is rotating like this from coming from this point to this point this will be pushed as it was in the cross belt. So, this is the basic concept of the involute tooth profile.

Now, to have the smooth gearing action what we need these profiles to be put on the drum at an equal interval. And again these profiles should not interfere with the other should not found with the other while it is rotating, then only we will get the smooth motion as in case of cross belt drive.

Now, these two drums are called basic drum in case of involute gear teeth this is important this is the best circle on which the profile is generated. Physically we cannot see on the gears which is the base circle, except the root circle and tip circle we may not have feeling there, but base circle will be there on which the profile is there ok.

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Now, if we consider a circle of infinite radius, then that will become a straight line, but edge on the circle and involute can be generated. So, what will be the involute profiles on the straight lines that will be a basic rack these are involute on you can say down the straight line. We have earlier I have discussed then after considering the involute profile, if we would like to design a gears we have give some proportion to the spacing of the profiles both forward direction in the opposite directions.

So, there will be a gap in between and it is standardized that had a standard pitch circle, that gap and that tooth thickness will be same that can be used as a gears gear and pinion. And then if we make that base circle is infinity, then we will get this rack and as we see that this profile has been generated on the same line.

So, alternatively we can say if we take a flank and one of the gears opinion as a cutter, then it will definitely generate the other gear keeping the same center distance consider the previous one this to drum. Now small drum we have made with the teeth with a cutter the other one we have kept a solid flank.

Now, we are rotating and allowing to cut the teeth, then definitely it will be able to generate the other profile. Similarly this rack cutter is also if it is used as a cutter that will be able to generate the gear. Now here I have shown that the different dimensions of this rack cutter, but if this basic rack is used for transmitting the motion.

In that case addendum will be one module standard addendum will be 1 module, but in case if we are using that as a cutter then this there will be extra material on the teeth which is the height of which is nothing, but the clearance in the opposite side clearance on the gear side. That means, for the cutter this addendum will be 1.25 for 20 degree standard tip because the addendum we take 1.25 we key 0.25 clearance whereas, at the bottom it might be more although it is see it is written here, but it can be kept more no harm at here. Essentially this should be 0.25 of the module and as there are sharp corners cannot give the better profile at the corner slight corner radius is given ok.

So, with this basic rack cutter we can generate the gear as well as there are pinion cutter also, but as the generated gear whether it is by the rack cutter or whether it is by the hob cutter or pinion cutter basic equivalent gear on the plane of spur gear direction of spur gears statehood spur gear this will be already same only the motion will be different, if we use the pinion cutter motion will be one type if we use the rack cutter motion will be the other type anyway this idea why I have given.

Because, we will need that how using this cutter concept we can give the correction to the teeth here different proportion are given which you can read also say this is inner end of effective flank and this is at at the standard pitch line, it is called pitch line or circle this width of the teeth will be equal to width of the gap. And this is the circular pitch in case of gear here of course, it is the straight pitch which nothing, but e is equal to π into m module and if we consider this angle this angle is the standard pressure angle in this case it is 20 degree.

So, by general a drawing such a rack is not at all difficult and not even manufacturing of such that is difficult for that we do not need any gear cutting machines, we can generate with any machines any milling machines or which can generate this path profile.

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Minimum Number of Teeth in a Gear, Interference and Undercut, & Concept of Gear Tooth Correction :

Recapitulation

Minimum (Critical) Number of Teeth:

Root dia = $Zm - 2(1.25m)$
 Base Circle dia = $Zm \cos \alpha$
 Root dia < Base Circle dia

Labels in diagram: Pitch Line of Rack, Pitch Circle, Addendum Circle, Base Circle, Root Circle, Pitch Line of Rack, Pitch Circle, Addendum Circle, Base Circle, Root Circle, Pitch Line of Rack, Pitch Circle, Addendum Circle, Base Circle, Root Circle.

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Now, first of all although I have discussed what should be the minimum number of teeth. If we look into this figure then where is the base circle? Now this there is the base circle on which the profile is generated, but root may be below the base circles actually if we consider the root circle this must be equal to 4 straight tooth spur gear Z into module minus 2 into 1 point sorry radius means root circle dia $Z m$ root dia is equal to Z into module minus twice into 1.25 into module ok.

And base circle \cos of the sharing in this case $\cos \alpha$, now if we equate if you consider this 2 at 1 point we can find if we go on decreasing the teeth number at 1 point we will find root dia will be less than the base circle dia, but there is no harm as long as the no gearing action is coming below the base circle ok.

So, this concept we should have in in case have a pressure standard pressure angle say 20 degree this number for which the base circle will come below teeth number is above a critical number which I have discussed earlier, but here again I shall discussed this issue. So, first of all we will consider what is the minimum critical number of teeth.

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Minimum Number of Teeth in a Gear, Interference and Undercut, & Concept of Gear Tooth Correction :

$r_p = r = \frac{mZ}{2}$

Root Circle is below Base Circle

Recapitulation

Minimum (Critical) Number of Teeth:

$$PQ = a_f \times m = r - r_b \cos \alpha$$

$$= r(1 - \cos^2 \alpha) = r \sin^2 \alpha$$

or, $a_f \times m = r \sin^2 \alpha = \frac{Z_c \times m}{2} \sin^2 \alpha$

$$Z_c = \frac{2a_f}{\sin^2 \alpha}$$

Where, $a_f \times \text{Module}$ (a_f being a factor) is the addendum (active) height of the rack tooth.

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Now, to find out that what we shall consider say let us consider this is the pitch line of a basic rack, but not the cutter this has this is not the cutter basic rack. So, if we allow this to meet with a gear the point where it can meet that is first meeting that might be the tangent of line of action with the base circle of the waiting gear.

So, now, if we consider that say PQ is the addendum factor into module in case of standard we can consider this is just one module. So, we can write now PQ is a f into module must be equal to r here r is the pitch circle radius of this gear meeting here and minus r_b this one is the r_b this radius into $\cos \alpha$ minus this distance Q to sale this is oh and this is r we have considered. Therefore, equating we get PQ must be equal to r minus 1 minus $\cos^2 \alpha$ minus r that is equal to minus sorry that is equal to r sine square alpha.

Now, this gives us a condition that addendum factor into module is equal to r sine square alpha and r is equal to now we consider that it is the minimum or critical number of teeth, for which this teeth of the basic rack is meeting first meeting at the best circle intersection of the base circle and the involute and the tangents of line of actions; that means, this point ok.

So, from there we can have that equate this Z_c can be written equal to 2 into a alpha divided by sine square alpha. And as the addendum factor stand standard addendum factor is one this can be as well written 2 by sine square alpha e for alpha is equal to 20

degree standard pressure angle 20 degree this Z_c becomes 17 point not something it is slightly more than 17 slightly more than 17 and safely, we can take the teeth number is equal to 17 minimum number of state tooth spur gear with 20 degree.

Now, if it is less than that that in that case with addendum is equal to 1 this contact point, this contact point will come below the base circle and which will not be involved and definitely the conjugacy will not be maintained. Here is a question can you take the teeth number less than the Z_c for a system mind it that route might be below that base circle because the route is non-acting portion of the gear it is not taking active part.

So, route can be given say for example, when it is moving in this direction as you see this had come much below the edge this rack is moving in this direction. So, this will come up to this point. So, this route must be here and so, but it will not affect the gearing action as it is below the base circle below the base circle ok. Now the question is that once we make it below this number in case of 20 degree it is 17 below 17 in that case definitely this cutter we read this point will be below the base circle and there will be non-conjugate actions as well as there will be a undercut like this; that means, then tooth will become like this.

So, definitely we cannot use a tooth like these either we will get a conjugate action below this point what can be done? One easy method of correcting the tooth is that we can shift the profile in the upward directions, with same cutter and probably for which this root will come become same like this, but then this affective portion of this one will not come below the base circle.

So, that is the concept of gear tooth corrections, which we will see more in the next slides. Now in this case as I have already described a is the addendum factor suppose, if you take that an addendum factor slightly less in that case we will have the less number of teeth, but for the we will consider always on the standard one.

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Minimum Number of Teeth in a Gear, Interference and Undercut, & Concept of Gear Tooth Correction : Recapitulation

Minimum (Critical) Number of Teeth:

$$Z_c = \frac{2a_f}{\sin^2 \alpha}$$

Where, $a_f \times Module$ (a_f being a factor) is the addendum height of the tooth.

and, $a_{fc} \times Module$ (a_{fc} being the cutter addendum factor) is the addendum height (active) of the rack cutter.

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Let us have a look into the more details about the tooth corrections what we are doing? Now, we are considering that the cutter is having this is instead of rack it is having the cutter; cutter means it is addendum it is teeth is equal to normal the addendum of the gear teeth.

So, therefore, though we have considered a_{fc} ; a_{fc} for standard it will be 1.25 into module then our cutter is here. So, this is the cutter now with this cutter if we cut this say 17 teeth we are cutting the 17 teeth with this cutter with the rack cutter, then our teeth the teeth will be something like this, because we have used this teeth ok. This teeth we have used and we have got the profile like this it will work it will be allowed, but when the teeth number is less what we can do we can this was the initial pitch line we can move the teeth in upward directions that is say we have come over here.

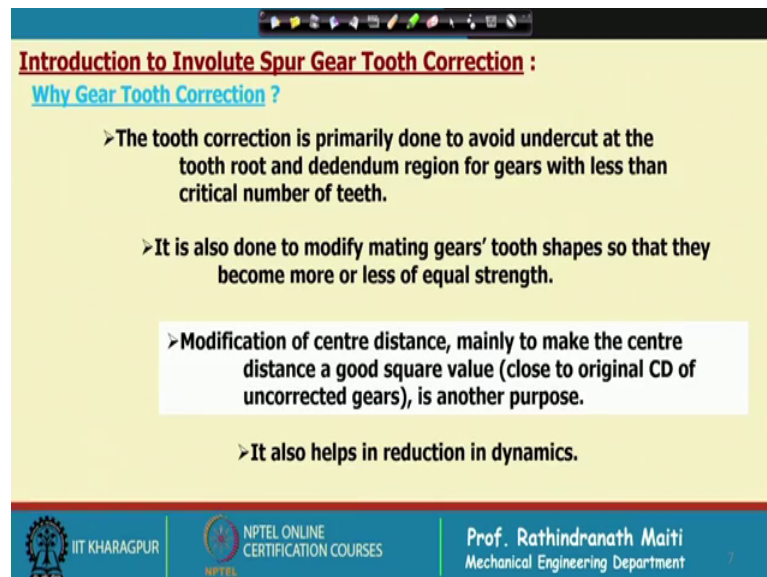
And, now we are generating the same with the same cutter we are using this say earlier when the pitch line was here with this cutter it could be at the bottom much bottom, but now as it has been shaped we will get still an effective profile, which can be used for conjugate actions.

Now, this similarly this cutter also can be instead of withdrawing it can be fed inside say for example, large gears it is much more teeth number is much more than 17 in case of 20 degree then definitely we can make the tooth thin and teeth will not till go under the

base circle or it will not generate the profile which is and which is not in conjugate action.

So, this is minus correction and this is called the plus correction, plus correction means that we are withdrawing the rack cutter; that means, with external teeth the center distance is increasing. If it is a pinion cutter then the center distance between the pinion cutter and the rack cutter is increasing sorry the pinion and the pinion cutter it is increasing. So, we call plus correction and if it is going inside that is it is making the tooth thin we call minus corrections.

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Introduction to Involute Spur Gear Tooth Correction :
Why Gear Tooth Correction ?

- The tooth correction is primarily done to avoid undercut at the tooth root and dedendum region for gears with less than critical number of teeth.
- It is also done to modify mating gears' tooth shapes so that they become more or less of equal strength.
- Modification of centre distance, mainly to make the centre distance a good square value (close to original CD of uncorrected gears), is another purpose.
- It also helps in reduction in dynamics.

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Now, why we need gear tooth correction the tooth correction is primarily done to avoid undercut at the tooth root and addendum vision for gears with less than critical number of teeth say; that means, if we in case of α is equal to 20 degree if we take in the teeth number less than 17. So, what we can do we can give the plus correction; that means, tooth we can make a thicker and we can have a with the undercut.

So, this is basically purpose it is also done to modify mating gears tooth shapes. So, that they become more or less of equal strength usually in gear design what we have seen that pinion for the same material definitely pinion will be weaker, because it will have less amount of formative number of teeth the magnitude of formative firm factor you can say firm factor magnitude of firm factor of less than gears. So, this means that pinion is weaker of course, by using different material we can make them of equal strength, but

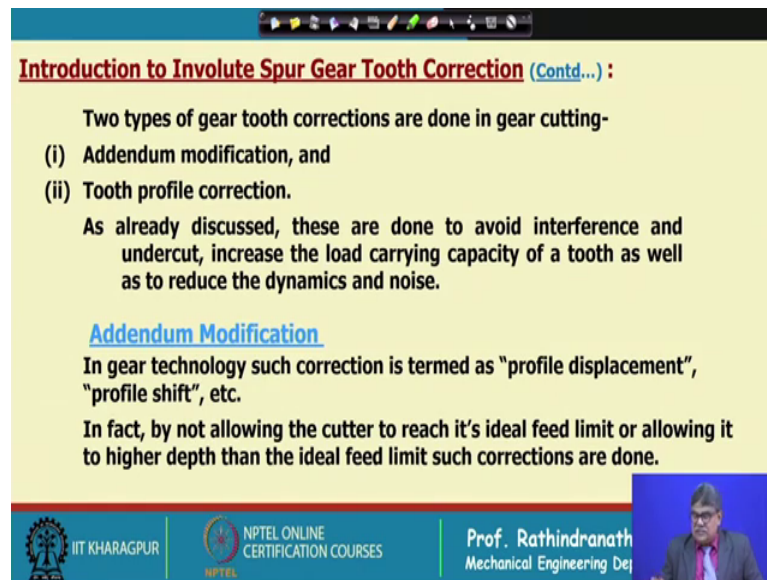
what we can do even there is different material still just to bring the stress level equal we can modify the gears.

So, that is another purpose which is done keeping in mind that gear set will have more life if it is not corrected. Modification of center distance mainly to make the center distance a good square value, close to original cd of under uncorrected gears is another purpose; that means, we can give small corrections say for example, we have taken a gear set and module even the helical angles such that, it is not multiple of 5 the center distance is not multiple of 5 or multiple of 10 and we would like to make it multiple of 5 and 10 say center distance has become 103 millimeter.

We would like to make it 100, then possibly we can give negative correction to both pinion and gear maybe more to the gear less to opinion or only 2 gear to bring the center distance 100 millimeter or else we can increase it by increase it up to 100 5 millimeter in that case probably we will strain them that pinion teeth will give positive correction to the pinion.

So, it is possible to make the center distance x square value, which is preferred usually just introducing correction. It also helps in reduction in dynamics if one is one teeth is weaker than other in that case one will have more deflection than the other and then when the load is released the it will have more motion than the other. So, dynamics will definitely will be higher, if then if they are of equal strength. So, this is another effect of the gear tooth corrections.

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Introduction to Involute Spur Gear Tooth Correction (Contd...):

Two types of gear tooth corrections are done in gear cutting-

- (i) Addendum modification, and
- (ii) Tooth profile correction.

As already discussed, these are done to avoid interference and undercut, increase the load carrying capacity of a tooth as well as to reduce the dynamics and noise.

Addendum Modification

In gear technology such correction is termed as "profile displacement", "profile shift", etc.

In fact, by not allowing the cutter to reach its ideal feed limit or allowing it to higher depth than the ideal feed limit such corrections are done.

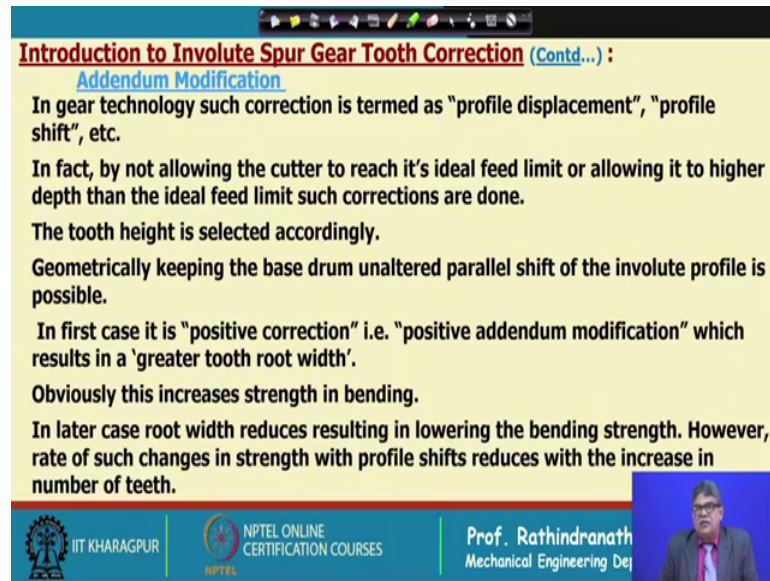
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Now, two types of gear tooth corrections are done in gear cutting one is the addendum modification. And other is tooth profile correction.

First of all addendum modification means; that we increase or decrease the addendum as well as we shift the profile and put profile file corrections again we should consider the profile itself is slightly main non involute and it is made in such a way after deformations, it will become close to involute, but that is not possible because that will be load dependent deflection will be load dependent. So, profile corrections are usually given to give the better life and that is not an easy task. Normally profile shifting and addendum truncation or adding to the addendum is the general way of correction.

This is done simply why withdrawing the cutter or not allowing the cutter to cut into full depth or allowing the cutter to cut into more depth. So, that type of corrections are widely used and next onwards we shall consider only the profile shifting addendum modification and profile shifting. So, this part we have already discussed and now addendum modification is that in gear technology such correction is done as profile displacement or profile shift, which I have mentioned. In fact, by not allowing the cutter to reach its ideal feed limit or allowing it to higher depth, then the ideal feed limit such corrections are done.

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Introduction to Involute Spur Gear Tooth Correction (Contd...):
Addendum Modification

In gear technology such correction is termed as "profile displacement", "profile shift", etc.

In fact, by not allowing the cutter to reach its ideal feed limit or allowing it to higher depth than the ideal feed limit such corrections are done.

The tooth height is selected accordingly.

Geometrically keeping the base drum unaltered parallel shift of the involute profile is possible.

In first case it is "positive correction" i.e. "positive addendum modification" which results in a 'greater tooth root width'.

Obviously this increases strength in bending.

In later case root width reduces resulting in lowering the bending strength. However, rate of such changes in strength with profile shifts reduces with the increase in number of teeth.

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In gear technology already I have mentioned this the tooth height is selected accordingly.

So, if necessary we can truncate the addendum or we can add that in them, geometrically keeping the base drum unaltered parallel shift of the profile input profile is possible. And first case it is positive correction that is positive addendum modification, which results in greater tooth root width. Obviously, these increase strained in bending in later case root width reduces resulting in lowering the bending strength; however, rate of such changes in strength with profile shape reduces with the increase in number of teeth.

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Introduction to Involute Spur Gear Tooth Correction (Contd...):
Addendum Modification (Contd...)

Such a correction is sometime called as 'profile modification'.
However, correction or modification of the geometric shape of the profile is put into a separate category which is termed as '*tooth profile correction*'.

In a mating gear pair such a correction can be done without changing or with a minor change in centre distance (to make the centre distance a roundup figure which is usually multiple of 5 mm in metric system) the centre distance.

It is sometimes called as '+' '-' (plus-minus) correction. '+' correction is given to the pinion and '-' correction is given to the gear. With the changes in root widths Lewis form factors of both pinion tooth and gear tooth are altered.

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Such a correction is sometimes called as 'profile modifications', but in true sense there should not have any confusion profile correction is that we are altering the involute profile to some other profile to have the better gearing action. However, the correction or modification in geometric shape of the profile is put into separate category which is done as tooth profile corrections, which I mentioned innovating gear pair such a correction can be done without changing or with a minor change in center distance to make the center distance around a figure, which is usually multiple of 5 millimeter in metric system.

It is sometimes called as pass plus minus correction without calling is profile ship to involve plus minus corrections plus correction is given to the pinion and minus correction is given to the gear that is usual, but as I told that to make the center distance lower the minus correction can be given both of course, looking into the strength of the teeth with the change in route width Lewis form factor of both pinion tooth and gear tooth are altered.

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Introduction to Involute Spur Gear Tooth Correction (Contd...):
Addendum Modification (Contd...)

Thus by selecting a suitable '+' '-' correction we can reduce the difference in $S_d Y$ for pinion and gear in Lewis formula, as follows, making the pinion and gear teeth of almost equal strength.

$$m = \sqrt[3]{\left(\frac{2T \cos \beta}{Z\psi}\right) \frac{1}{S_d Y}}$$

It optimizes not only the gear sizes but also increases the life of a gear set.

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The selecting a suitable plus minus correction we can reduce the difference in $S_d Y$ here in this case, you can see this we can give same for both gear and pinion.

Because in a stage they are transmitting the same load contact load will be same, but here will be the difference because due to the material this may be different $S_d Y$ we have considered the allowable strength considering all factors and Y is the form factor, that will be different that will definitely be less in case of pinion. Now what we can do by modifying keeping this material same by modifying the tooth we can increase or decrease the Y and then in such a relations we can make the T by because the torque in the other shaft is increasing T by this part T by Z into this part more or less same.

So, this module will become exactly equal whether we design the gear or design the pinion. So, this is just to give an hints, but in spite of that in design it is not possible that we will make always the strength is equal only we can decrease or increase a we can increase decrease the differences, in optimization not the gear sizes, but also increases the life of the gear set.

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Introduction to Involute Spur Gear Tooth Correction (Contd...):
S & S₀ - Plus and S & S₀ - Minus Correction
S₀ is '+' '-' S correction is without change in centre distance.

S / S₀ '+' (Plus) Correction

This means that in S₀ correction, if a gear is given '+' correction (to the pinion) then the mating gear is to be given '-' correction by same amount to keep the centre distance unaltered.

'+' correction in (Both External & Internal) Toothed Gear means cutter offset with more CD

(a) shows '+' correction in a tooth .

The diagram shows two gear profiles. The left profile is labeled 'Uncorrected / Standard Tooth' and the right profile is labeled 'Corrected Tooth'. The corrected tooth is thicker at the base. The diagram labels the 'Pitch Circle', 'Base Circle', and 'Cutter Offset (Away from gear Centre)'. A distance delta is indicated between the pitch circles of the two gears.

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Now, there is a it is mentioned as well that s correction in s 0 correction S correction plus minus is 0 correction plus minus, S plus minus correction means center distance will not be same center distance will be varied and plus minus correction plus as I told that in external.

In case of external gear the center distance will be more tooth will be become thicker and minus correction is the opposite whereas, it might be S 0 plus minus correction; that means, S 0 means there is no change in center distance. And as you see in this figure in this figure it is shown there is a plus correction is given and the original profile, which was this one has now become like this after the correction and then the cutter has been withdrawn by the amount of delta.

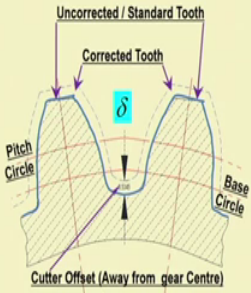
So, plus correction in both external and tooth and gear means cutter offset with more CD centre distance is increasing and minus is opposite.

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Introduction to Involute Spur Gear Tooth Correction (Contd...):

S & S₀ - Plus and S & S₀ - Minus Correction (Contd...)

S / S₀ '+' (Plus) Correction **Correction factor 'X':**



The amount S & S₀ '+' (plus) and S & S₀ '-' (minus) correction is defined by:

$$\delta = \pm X m$$

Where,
 δ is the amount of profile shift or correction factor,
 m is the module, and
 X is the profile shift or correction factor.

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Now, if we consider a correction factor x , then for first plus minus corrections we can write δ is equal to plus minus $2x$ into module. So, that means, we have to mention what is the correction factor plus x or minus x plus means the tooth thickness will be increased and minus means the tooth thickness will be decreased.

So, ignore this value it is it has no meaning. Where δ is the amount of profile shift or correction factor m is the module and x is the profile shift or correction factor.

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1. G. M. Maitra, Handbook of Gear Design. (2nd. edn., 1989 ISBN 0-07-460237-3), McGraw-Hill pub. Co. Ltd. India.
2. Extract from DIN 3960

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Now, for this portion although there is a I have mentioned about the reference books, but for this portion I have followed other books also din standard, but Maitras book will be helpful this will be available this is available in Indian market and this is cheaper than any other available book on gear corrections.

So, that is why I have mentioned this one. So, in this lecture we have basically learned how what is the concept of gear tooth correction in state tooth's spur gear. And in the next lecture I shall give more details about the tooth corrections their limitations and by tooth corrections, what are the changes and how to calculate to thickness etcetera.

Thank you.