

Gear and Gear Unit Design: Theory and Practice
Prof. Rathindranath Maiti
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 34
Involute Spur Gear Tooth Correction Part – II

Gear and gear you need design module 7. Introduction to Involute Gear Tooth Correction and this is the second lecture on involute spur gear tooth correction.

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Outline of the Lecture

- **Concept of Involute Gear Tooth Correction**
 - **Amount of Profile Shift (Tooth Correction)**
- **Recommendation on Amount of Profile Shift , Minimum No. of teeth at different operating Condition**
- **Characteristics of Corrected Gears.**
- **Effect of amount of Profile shift on Tooth Shape, Under cut etc.**
 - **Forms of Tooth , Under cut etc.**

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Now, in this lecture I shall cover concept of involute gear tooth correction, then amount of profile shift that is the tooth correction, recommendation on amount of profile shift, minimum number of teeth at different operating condition, characteristics of corrected gears, effect of amount of profile shift on tooth safe, undercut etcetera and finally, forms of tooth, undercut etcetera.

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Introduction to Involute Spur Gear Tooth Correction (Contd...):

S & S₀- Plus and S & S₀- Minus Correction (Contd...)

Amount of Correction :

$$\delta = \pm xm$$

S₀ Means no change in Centre distance.

Or, in other words mating gears have '+' (usually pinion) & '-' (usually Gear) correction.

S Correction with change in Centre distance.

$r_p = r = \frac{mZ}{2}$

Root Circle is below Base Circle

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Now, already I have discussed how the gear correction is given? I have also stated that what condition gear corrections are provided to make the gear and pinion of equal strength correction is provided, to make the center distance acquire value the correction is provided, but mostly when the teeth number is small in this pinion where there will be undercut to avoid that undercut teeth correction is essential.

Now, the amount of teeth correction as I told say this is the rack cutter the in blue lines that is the rack cutter. Initially the rack cutter, which exists, was passing through the standard pitch circle of the gear, which is being cut. Now, what we have done we have withdrawn the gear up to in amount by delta. This means that the cutter is not being allowed to go full depth and we are finishing the cutting process there itself so, that is by the pink line pitch line.

Now, when we are finding the contact situation we consider that from that point the height of the rack is one module which is standard. So, in this case as if it is not a cutter it is a basic rack and we can go up to the point, where the corner will touch the intersection of our point of tangency of the line of action and the tooth profile of a teeth.

So, this is the limit now the amount of shift delta is equal to plus minus xm plus is for plus correction; that means, when we are not allowing the cutter to go to the full depth of cut that is called plus correction, this means that in case of external tooth the center distance of the cutter and the gear is being cut will be more by the delta amount for that,

this is plus definitely the cut tooth final cut tooth will have more width and if it is a minus sign then it will give him the extra amount of cut, extra amount of depth, by delta which is x_m . In this case in this figure it is shown as if it is withdrawn, because we are trying to find out how much correction is needed for a teeth which needs the correction where the number is less than critical number.

Now S_0 means no change in center distance again we should keep in mind that this is we are giving the corrections in such a way, that there will be no change in center distance, then we call this is S_0 gear, but actually there is no meaning when it is not meeting with the gear for which it is designed. And if S means there center distance change is possible. This is we need to follow because if we give S_0 correction S_0 gear correction if we mean eat; that means, there is no change in central distance. Therefore, if we give plus correction to the pinion we have to give minus correction to the gear and vice versa.

In case of center distance change, if we if you call S corrections that might be anything pinion is corrected gear is not both gear and pinion corrected both given plus correction both given minus corrections etcetera etcetera. Now, in this case what we have I have shown the dark blue line that is the increasing tooth height this is from the pitch line this height is a F_c into module this is from this point that is addendum factor of the cutter into module addendum factor of the cutter will be 1.25.

So, this means that even if in with known condition the cutter the extra amount, which is clearance among that may go inside the base shuttle at the first point of contact. This will give only route and that that profile is nonfunctioning profile.

Now, next what we have shown that if we give the minus correction, then it will go it is given by a red line it will minus correction mean means the x_m is equal to delta is equal to minus x_n , but in this case we have given the positive correction. So, again by the pink line the cutter will come to this position. Now, again these red line and the pink line that is shown the height is shown 1 module height is shown 1 module, the extra amount clearance this part is not added to the cutter that is just to understand that that to where it is touching.

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Introduction to Involute Spur Gear Tooth Correction (Contd...):

S & S₀ - Plus and S & S₀ - Minus Correction (Contd...)

This means that mating gears have '+' & '-' corrections not of equal amount.

Or, both may have '+' or '-' corrections.

Or, either of the mating gears has '+' or '-' corrections and other has

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So, with this concept we are now giving plus correction. So, therefore, the tooth thickness will increase, which is shown in this figure the dotted line is the new teeth profile and this shift is delta.

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Introduction to Involute Spur Gear Tooth Correction (Contd...):

S & S₀ - Plus and S & S₀ - Minus Correction (Contd...)

'+' (plus) sign is for the '+' correction i.e., outward shift of cutter.

This means that in case of external tooth the cutter is not fed to its full depth.

In case of internal gear it is vice versa.

However, in both cases the centre distance between the cutter and the gear being cut (or the reference distance if the cutter is basic rack) is larger by an amount of Xm than the standard one.

In case of '-' correction these are reversed.

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Now, the plus sign is the plus correction outward shift of the cutter, this means that in case of external tooth the cutter is not fed to its full depth which already I have mentioned. In case of internal gear it is vice versa this means that if we give the plus correction actually as in case of internal depth is internal gear the teeth is the gap of the

actual number same number of external gear teeth and the gap is the teeth in case of internal here.

Now, what we are doing definitely cutter is inside and we are increasing the centre distance increasing center distance mean projective corrections and for that it is going further it is cutting the internal gear to the further depth. So, that we should keep in mind of course it can be taken in other ways also only we have to remember the sign convention.

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Introduction to Involute Spur Gear Tooth Correction (Contd...):
S & S₀ - Plus and S & S₀ - Minus Correction (Contd...)
Minimum Number of Tooth with Correction (Profile Shift):

Referring to Illustrations:

$$\sin \alpha = \frac{PQ}{PT} = \frac{m - xm}{PT}$$

But, $PT = r_p \sin \alpha$

Therefore, $m(1-x) = \frac{mZ}{2} \sin^2 \alpha$

or, $(1-x) = \frac{Z}{(2/\sin^2 \alpha)}$ \Rightarrow $1-x = \frac{Z}{Z_{\min}}$

Labels in diagram: Pitch Line of Rack Cutter, Pitch Circle, Base Circle, Root Circle is below Base Circle, Tip Addendum, Pitch Circle, Cutter Offset (Away from gear Centre), S/S₀ + (Plus) Correction, Uncorrected / Standard Tooth, Corrected Tooth.

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Now, let us come to the geometry PQ is the portion this is originally base circle this is P is the original pitch point. So, we can write sin alpha is equal to P cube divided by PT that is PQ divided by PT this is the standard pressure angle alpha. Now what is there as we are considering this is the point of tangency with this cutters 1 module height teeth this is we are cannot considering the additional amount clearance amount.

So, for that this amount must be m minus xm this portion because we have shifted by xm and after that this height up to the this blue line, this height is can be defined by module minus xm, because for the standard rack the addendum height should be 1 module and that divided by PT, but again PT must be equal to r p sin alpha whereas, r p is the pitch circle radius of the standard gear being cut.

So, we can write m into $1 - x$ must be equal to r_p is equal to mZ by 2 that is the number of teeth into module divided by 2 we are still considering straight tooth spur gear, and if we put this is by 2 this is becoming $\sin^2 \alpha$ PT we put here or $P \sin \alpha$ and this is another $\sin \alpha$ is coming for that.

So, this will be m by $2 \sin^2 \alpha$ equal to $\sin \alpha$ is there. So, we can write $1 - x$ is equal to x is the factor for teeth correction or shift is equal to number of teeth divided by 2 by $\sin^2 \alpha$. Now we write $1 - x$ is equal to Z by Z_{min} this is if you remember that 2 by $\sin^2 \alpha$ is equal to the minimum number of teeth up to each there will be no undercut ok.

So, that we have defined as a Z_{min} . So, we can write $1 - x$ is equal to Z by Z_{min} mean what is Z that number of teeth we are going to cut ok. So, depending on number of teeth suppose this is 17 . In case of 20 degree teeth suppose we have taken 18 . So, 18 means we will find that x will be some minus value; that means, it can be given further depth of cut. Suppose this is 14 then you will find x is positive; that means, we have to give the positive correction ok.

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Introduction to Involute Spur Gear Tooth Correction (Contd...):

S & S₀ Plus and S & S₀ Minus Correction (Contd...)

Minimum Number of Tooth with Correction (Profile Shift):

$Z_{min} = Z_c$

Which we have already defined earlier as:

$$Z_c = \frac{2a_f}{\sin^2 \alpha}$$

Now for standard gears with addendum factor = 1: $Z_{min} = \frac{2}{\sin^2 \alpha}$

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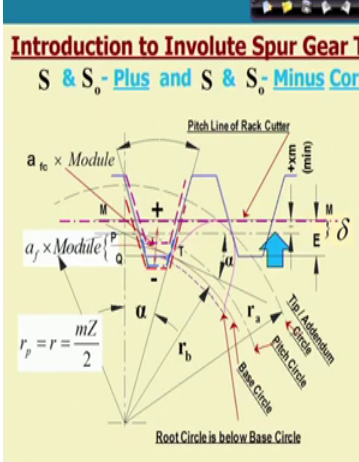
Now, this Z_{min} is equal to Z_c that critical number of teeth which we have already defined, which is given by twice into addendum factor into $\sin^2 \alpha$. Now this addendum factor I have mentioned, because if the cutter is having some different unknown factor, we can go for higher or lower amount of corrections, but for this

purpose during this course we may consider addendum factor is equal to 1. This will be easy to consider and 2 by sin square alpha 20 degree it will be slightly more than 17 and we consider that Z in c is equal to 17.

Now, this is so, we can write itself Z min must be equal to twice by sin square alpha.

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Introduction to Involute Spur Gear Tooth Correction (Contd...) :
S & S₀ - Plus and S & S₀ - Minus Correction (Contd...)



Considering, $1 - x = \frac{Z}{Z_{\min}}$

Or, $x = \frac{Z_{\min} - Z}{Z_{\min}}$

In practice Z can be taken less than that calculated by equation-

$$\frac{2}{\sin^2 \alpha} = Z_{\min}$$

ignoring the minor deviation in tooth profile near the root.

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Now, considering 1 minus x is equal to Z by Z mean we can find out we can calculate the ax is equal to Z min minus Z divided by Z mean; that means, minimum Z, ideally which is equal to the critical number of teeth twice by sin square alpha in case of 20 degree a sorry in case of involute gear with addendum factor is equal to 1.

Now, in practice Z can be taken less than that calculated by the equation of 2 way sin square alpha, that is which is critical, ignoring the minor deviation in tooth profile near the root. Now, one can say that if we give the tooth profile not non involute at the root and if the tooth comes in contact there, then definitely it will highlight the laws of gearing here it can be mentioned that with the tooth deformations already laws are violated in to some extent.

So, if there is minor deviation that may not be harmful and by that process we can go for minimum number of teeth and with that size optimization can be done.

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Introduction to Involute Spur Gear Tooth Correction (Contd...):

S & S₀ - Plus and S & S₀ - Minus Correction (Contd...)

For an example in case of 20° involute Z_{\min} or Z_C is calculated as 17 (17.097).
 But in practice even without correction it is taken as $Z_{\min p}$, which is as follow:

$Z_{\min p} = 12$ for slow speed application,
 $= 14$ for medium speed application,
 However, it is $= 18$ for high speed application.

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So, there are some recommendations, now for an example in case of 20 degree involute Z_{\min} or Z_C is calculated 17, it is actually 17.097. Now in practice let us consider this nomenclature we put $Z_{\min p}$, that is for practice we can take this is 14 sorry we can take this is 12 for slow speed application. We can take 12 for slow speed application mind it, we normally is supposed to take 17 number of teeth whereas, it is recommended that without correction even if we can go for 12 teeth, if the speed is very slow.

Then it is recommended as 14 if this number can be taken 14 for medium speed application whereas, if we go for high speed then it is better to go for 18 not even 17, we should go for 18. So, that there would be less dynamics this means that the violation of conjugacy can be accepted for slow speed or medium speed.

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Introduction to Involute Spur Gear Tooth Correction (Contd...):
S & S₀ - Plus and S & S₀ - Minus Correction (Contd...)

Therefore, factor x is expressed and used in practice, as:

$$x = \frac{Z_{\min p} - Z}{Z_{\min}}$$

Example: Let Z be 13 (20° pressure angle) and it is of slow speed application.

Then a correction of

$$x = \frac{12 - 13}{17} = -0.0588$$

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Now, let us see what will be the teeth number or correction factor therefore, factor x is now expressed as in practice $Z_{\min p} - Z$ divided by Z_{\min} this Z_{\min} is in case of involute sorry 20 degree will be 17; that means, this Z mean we have to calculate using $Z C$ is equal to twice by sin square alpha whereas, $Z_{\min p}$ will be the number recommended for the practical application. Now, example if we take an example we will understand in better way let Z be 13 we have taken 13 teeth number and 20 degree pressure angle and it is of slow speed application.

Then, the correction according to the formula this is becoming 12 minus 13 by 17 which is minus 0.0588 close to 0.06; that means, we can give this much of minus correction; that means, we can further deepen the cutter and allow the undercut.

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Introduction to Involute Spur Gear Tooth Correction (Contd...):
S & S₀ - Plus and S & S₀ - Minus Correction (Contd...)

'-' sign indicates that more depth of cut can be given although the gear tooth is already has undercut.

If the same gear is to be used for high speed application then minimum correction is required for smooth running.

$$x = \frac{18-13}{17} = +0.294$$

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Now, this minus sign indicates that more depth of cut is given, but how it will work as I told there will be no heart although it is not necessary to give this minus corrections, but it is shown that mind even minus correction can be given. Now let us consider the same 13 number teeth we are using for high speed application, in that case we need to go for a correction of amount 0.294 nearly plus 0.3 the half of the negative correction for slow speed what we found that is numerical value, but no meaning.

So, this is quite interesting that for high speed low speed we can go for different amount of corrections of the teeth, normal practice the in industrial gearboxes 13 tooth number below 17 in case of 20 degree involute is not used. If it is straight tooth spur gear at all, if it is helical gear then we can perhaps go for 15 14 15 because there the amount of correction to calculate the amount of corrections although it will be complicated, but we can roughly say we have to consider then the formative number of teeth which is Z divided by cos cube of hellish angle.

So, that means, for 13 teeth suppose if we take the 15 degree probably it will come close to 17, but there the amount of calculating the correction factor is different because it is in the transverse plane, so, sorry other plane.

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Introduction to Involute Spur Gear Tooth Correction (Contd...):
Characteristics of Corrected Gears:

- Avoidance of undercutting,
- Attainment of a predetermined modified centre distance,
- To increase root and flank strength of tooth,
- Betterment of sliding and contact relations, and
- To shift the beginning of effective profile away from the base circle.

With the gear correction (both types) the base circle does not change.

Therefore, all calculations can still be referred to the base circle.

Also, the circular pitch (at standard pitch circle) remains same, although the thickness of the corrected tooth at standard pitch circle is different from its standard value which is $\pi m/2$.

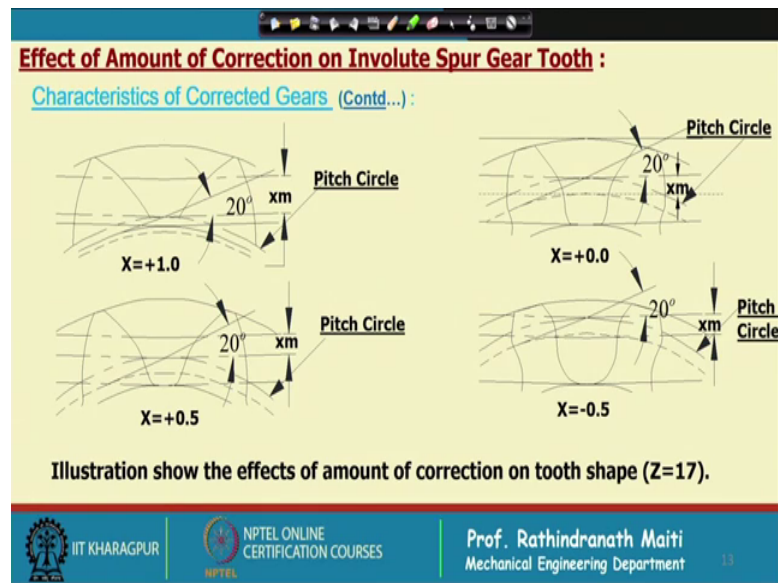
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Now, if we consider the characteristics what are the characteristics of corrected gear? First of all this is evidence of undercutting this is the first character of the corrected gear corrected means in this case profile shift not the profile correction, attainment of predetermined modified centre distance.

So, suppose the center distance is not multiple of 5 or multiple 10 we would like to bring to that value. So, we can provide the corrections to increase route and flank strength of teeth betterment of sliding and contact relations, because if we go for the positive corrections. In that case contact will be at higher the if we consider the cylindrical part of the profile the it will have higher the radius of curvature. So, definitely this will be better, than to shift the beginning of a effective profile away from the base circle you see this contact very close to the base circle is not desired, because there the gear where that it is touching at the root it is having practically 0 radius of curvature.

So, from contacts this point of view it is not good with the gear correction both types; that means, if we go for profile shift or profile modified we should remember the base circle does not change it will it will never change. Therefore, all calculations can still be referred to the base circle also this circular pitch at standard pitch circle remains same. Although the thickness of the corrected tooth at standard pitch circle is different from it is standard value which is πm by 2 the tooth thickness at of standard gear at pitch circle is πm by 2.

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So, that will remain same. Now, let us observe what happens if the correction is given? So, effective amount of correction on involute spur gear teeth. So, we have taken a teeth here it is the tooth number is 17, we have taken 17 number of teeth and then we have provided different amount of correction. Now, as it is told the 17 number of teeth actually do not need any corrections although for high speed it is better to go for at least 18, straight tooth spur gear or in case of roughly, we can estimate that in case of helical gear the formative number of teeth should be 18 at least.

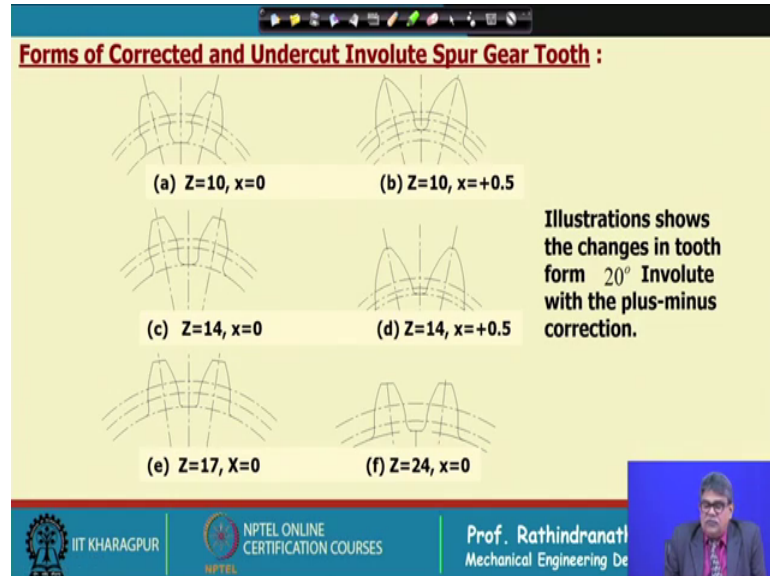
However, we have shown here considering a teeth number 17 and 20 degree involute that is the minimum number of teeth if we give plus one correction then tooth will become like this one thing to be observed the tip is almost is a it is pointed.

So, we will discuss later there is certainly limit we cannot go for any amount of correction for a number of teeth. Now, if it is given no corrections then tooth profile is something like this. And if it is given plus naught 5 it is as you see say shape wise, it is better than you can see it has more widen tooth as well as tip is also is having substance amount.

So, probably on 17 teeth we can go for point 5 corrections and to have better form of teeth. However, if it is given minus naught 0.5 then as you see that the root has become there is undercut this is undercut say this is undercut and as you see that this portion is not useful also the root has become very weak.

So, definitely we would not go for minus 0.5 correction in case of 17 teeth. So, this is just to show what will be the form of teeth.

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Now, again what we have taken we have considered different number of teeth and we have given different amount of correction there just to have a what the forms become after correction. Say if we take 10 number of teeth with 20 degree involute and without correction without correction. So, this is the safe clearly there is an undercut and this section is weak ok. Now if we provide a point 5 corrections which can be calculated we can take say depending on the speed we can calculate, how much correction we can give say let us consider it is for high speed then we will put 18 minus 10 divided by 17.

So, it is almost close to 0.5. So, if we give 0.5 correction then this will be the safe and it is you can say this base circle is here whereas, this root has become above the base circle and this edge this is called truck wide truck wide is having the proper safe route is not that weak as well as considerable teeth is there also. Of course, this with the positive corrections teeth is also increased to keep the height same as 1 plus 1.25 module. So, it can be increased we can truncate off of course.

But this is definitely acceptable. Now, let us consider another set of teeth which we have taken Z is equal to 14 and there is no correction now apparently this tooth can be used of course, not for the high speed, but for medium speed definitely Z 14 and or slow speed no correction can be used ok. But still if we give the correction of 0.5 the tooth form is

very good tooth form is better than the same amount of correction with 10 teeth and also teeth is their substance teeth is there root is stronger.

So, definitely this will be a good choice to give 0.5 correction if we go for 17 number of teeth in that case this is; obviously, acceptable, but if we give correction definitely there will be a improvement root improvement if we go for 20 teeth then what we find that this teeth is far better than 17 14 etcetera without any corrections and definitely that is accepted.

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Bibliography

1. G. M. Maitra, Handbook of Gear Design. (2nd. edn., 1989 ISBN 0-07-460237-3), McGraw-Hill pub co ltd India.
2. Extract from DIN 3960

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So, this for this lecture I have followed different sources, but mostly I have followed G.M Maitras book.

So, I acknowledge very much that he had written the gear correction in very concise form we have I have also consulted dean and Indian standard. So, this is just an extract to give you an idea what is the correction how to calculate the amount of corrections etcetera, but this subject this area is you will need to study more to where the correction is given where not and how it is calculated particularly in the manufacturing, manufacturing of course, the manufacturer they recommend you if you set automatically we will get that amount of corrections.

But there are many many things which we need to follow if we go for helical gear we will gear and other gears, but this is only the basic idea that how the amount of correction is calculated.

Thank you.