Gear and Gear Unit Design: Theory and Practice

Prof. Rathindranath Maiti
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 04
Involute Toothed Gear Properties & Terminology

So, in this first module introduction to Gear and Gear Unit Design, my next lecture is on Involute Toothed Gear Properties and Terminology.

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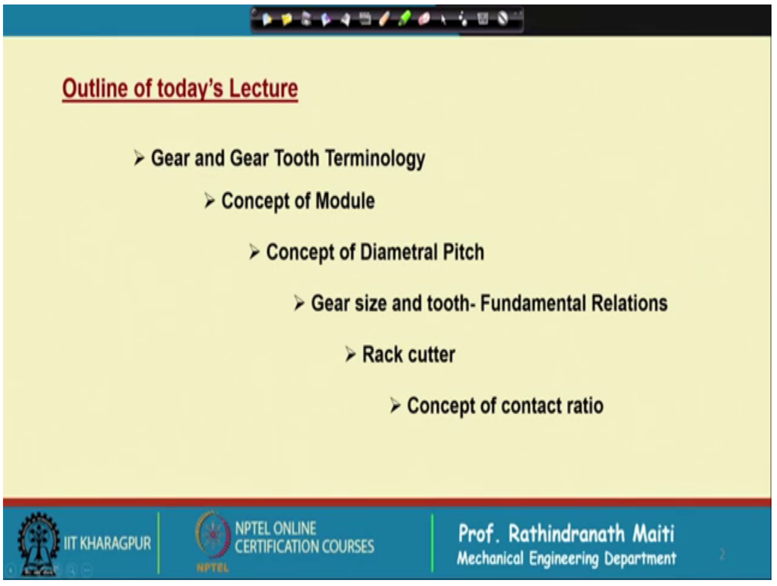

In this lecture, I shall cover gear and gear tooth terminology, concept of module, concept of diametral pitch, gear size and tooth fundamental relations, I shall discuss a little bit about the rack cutter which is the basic cutter for generating the involute gear tip. So, to say for other teeth also each profiles they are having a basic rack cutter. So, I shall discuss a little bit about the rach cutter and then finally, we will have the concept of contact ratio.

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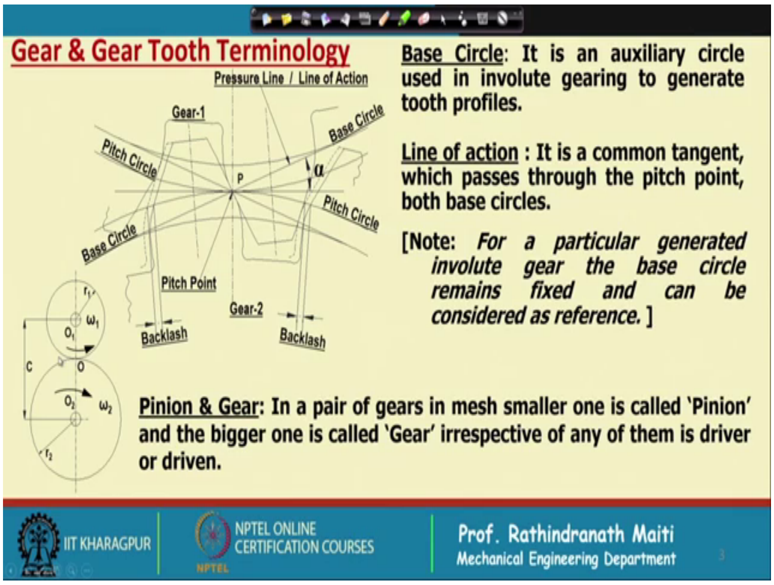

Now, if we consider the involute gear teeth, then as already discussed this is we can consider a cross belt drive and if we take a point on the cross belt drive that generated the involute profile, and on to drum we can set that profile and we can replace the belt. Now the question is that how to maintain the continuity in the motion for that we must have these profiles at a certain interval and when one profile is going out of touch, then another profile should come into contact.

Now, this is for one side motion, if we consider the other side motion; that means, what will happen if you rotate in this 2 drum in the opposite directions. Definitely these set of profile will not work, we have to take another side of profile which you can consider mirror image of these 2 profiles and that we can put on the drum..

This means that if we consider this drum, in that case this type of profile we can generate. Now another pair will come in contact at the moment of when one leaving the contact or even earlier, earlier is the better which will give us the concept of the contact ratio and we will discuss later anyway. It is the tooth proportion is such that at the pitch circle the 2 thickness is equal to gap thickness and it is equal for both the meeting gears whatever may be the teeth number.

If we consider backlash of course, there will be a little thickness will be little less than the exact thickness. Now what is the pitch circle pitch circle is nothing, but a circle that is proportion to the teeth number of the meeting gears on the center lines, if we join 2 centers and if we divide this line in proportion of the tooth numbers that will give us the pitch circle radius.

Now, through that pitch circle which is intersecting the centerlines that point is called P, that point P is called pitch point and the line through that pitch point, touching the 2 drum or which are the base circle is called the line of action or it is also called pitch like pressure line or line of action..

This line of action we can consider the active dividing on the direction of rotation say for example, if gear one is rotating in this directions. So, gear 2 will rotate in these directions and if this is the diver, then contact point is being pushed from this point to the far end on the line of action.

So; that means, in case of the given rotation and this as a driver this will be the line of action or pressure line. And obviously, if we rotate in the opposite directions then the contact will come over here and this will become the active line of actions. Now in this figure in this figure the base circle pitch circle and the pitch point and the backlash is shown. So, it is an, if the definition of the base circle is it is an auxiliary shuttle used in involute gearing to generate tooth profile.

Now, line of action or pressure line it is a common tangent, which passes through the pitch point both base circles passes through the pitch point and it is common tangent to both pitch circles. For a particular generated involute gear the base circle, remains, fixed, and can be considered as a reference. Now if this is very important. We can change the center distance a little bit and as discussed with the change in center the distance still the same gear would work only the amount of backlash will increase or decrease.

So, once we engage the gear at a different center distance than the standard center distance, then definitely the base circle will move and therefore, the line of action also will be different from the standard line of action. Even if we give the pitch point same the working pitch circle will be increased and the pressure angle will be increased in case of the center distance is increased; that means, say for a standard pressure angle of 20 degree alpha, the working pressure angle which is may be considered a F W or F O will become slightly more if we increase the centre distance.

But one thing important still the base circle remain same on which the profile was generated. So, for a standard system; that means, if we consider the cutter is of 20 degree and when the teeth number is selected for that particular gear one thing is fixed, which is base circle that we should remember? This will help in calculating the other dimensions of the gear. Now we often call pinion and gear what is pinion and what is gear? In a pair of gear in mesh, the smaller one will be called as pinion and the bigger one will be called as gear irrespective of any of them is diver of or driven.

Normally force we use the gear for speed reduction and increasing the torque in that case driver is mostly pinion and we call that pinion and gear and there is a misconception that sometimes we call that which is the driver that is the pinion, but it is not true the pinion is the smaller gear. And in case of if the both gear size is same then instead of calling them as pinion better to call way both of them are gear.

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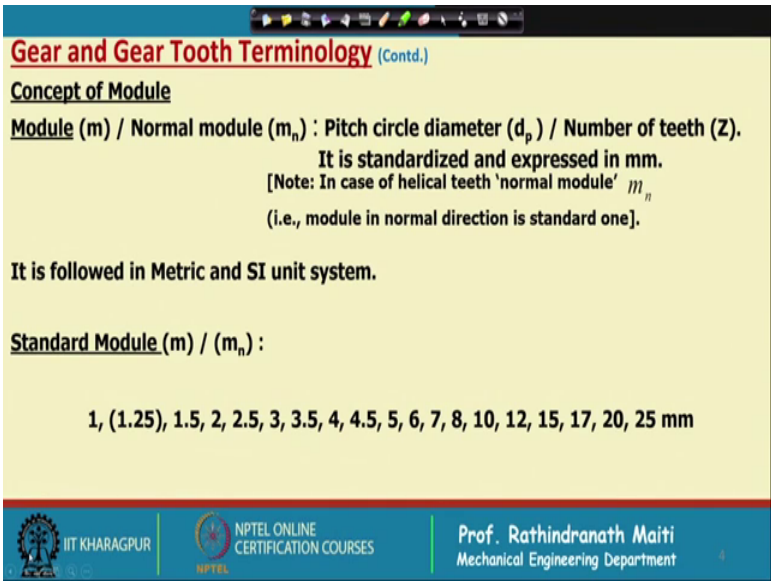

Now, we shall learn a little bit about the other terminology. Now first of all the module already I have discussed, but the module is this expressed in millimeter the module into the number of teeth will give you the pitch circle diameter of the gear. Now as I told the pitch circle again, when the gears are meeting, if the center distance are different then definitely there will be change in pitch circle. Then what we should call the pitch circle of a gear?

If it is cut the standard then we can call that the standard pitch circle will nothing, but the standard module we have considered into number of teeth. Otherwise we have to always define the pitch circle when the gears are meeting, but for the fundamental calculations basic calculations first of all we will we shall consider. The it was a standard gear and then it fit is corrected or modified that, we shall consider and for that we should call standard pitch circle diameter is equal to number of teeth into the module.

Now, again module in case of straight tooth spur gear we simply call it module, but full term should be normal module and in case of helical gear teeth we should call always the normal module, when we shall discuss more about the helical gear I will show that what is the difference between the module normal module and the face module. Now this is followed in metric and S I units.

Now, standard modules are 1, 1.25, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 6 etcetera up to 25 and the dimension is millimeter. So, this is expressed in millimeter now 1.2 5 although it is a standard, but rarely it is used and that is why it has, because it has been kept in a parenthesis and also it is what mentioning that in case of industrial gears normally modules are not less than 2.5.

So, Module below 2.5 are for small and precision gears which usually used for motion transmission mostly not for the power transmission.

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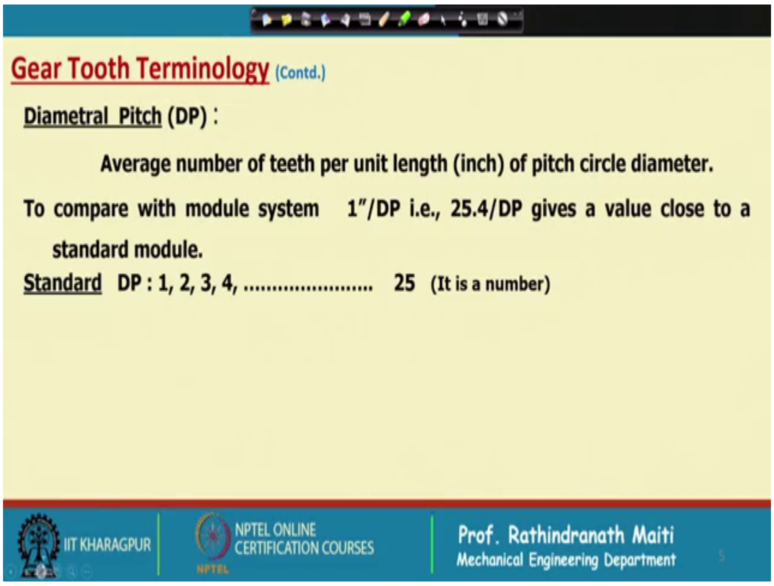

Now, diametral pitch in A P S system or (Refer Time: 13:17) system instead of module the another parameter was there which is called diametral pitch. Now diametral pitch is average number of teeth per unit length in inch up with circle diameter; that means, if we consider the gear and if we considers it is diameter, on that if we consider the how many teeth it is exactly how many teeth will be there, that the total teeth on that. So, average of that; that means, teeth number divided by the or diametral that we give the teeth number per inch that is called D P.

So, D P has no dimensions it is a number. To compare with module system 1 inch by D P that is 24.5, because 1 inch is equal to module 24.5 by D P gives a value close to a standard module. Now this standard D P diametral pitch are 1 2 3 4 up to 25 and it is a number. If we compare with module then 1 D P means 24.5 divided by 1 is 25.4 equivalent module is 25.4 this means that it is very close to 25 module.

Suppose if we consider 10 D P in that case it is equivalent to 25.4 divided by 10 2.5 4, which means it is very close to 2.5 module 2.5 module. So, in that case in in in that way we can compare that what will be the equivalent size of the case this means that 2.5 module 100 teeth straight tooth spur gear will be equal of in size hundred tooth 10 D P gear.

Now, this tooth shape is already shown, but again I will repeat that in a tooth first of all this is the rim on the rim. So, this is the rim thickness on the rim.

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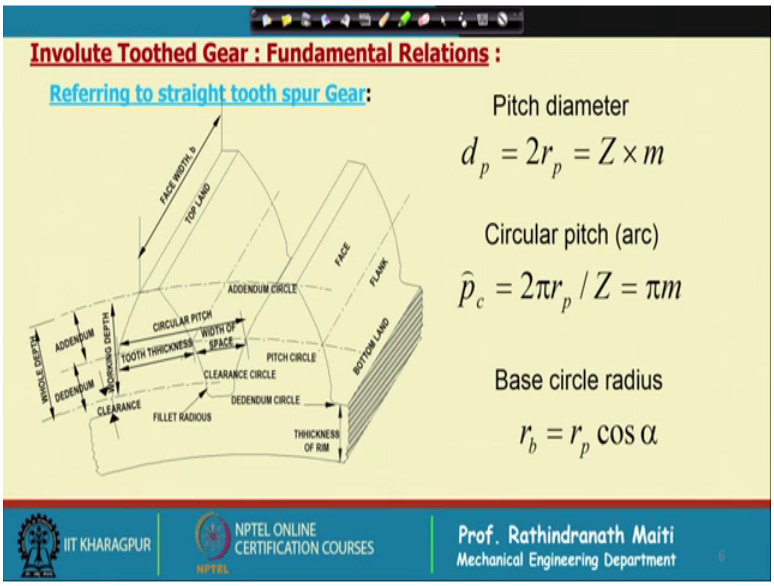

This is the tooth profile 2 teeth here it is shown side by side and the standard tooth thickness as pitch circle is given by this distance it is our 2 thickness usually expressed in calculations. And this is the speech of the weight width of the space or gap this is actually equal although it is not showing proportional, it is equal and then this distance is called circular pitch this is all in arc tooth thickness not the straight, which is it is written here, but it is actually arc tooth tooth thickness..

And then this is the addendum, addendum is nothing, but the tip circle which I have expressed earlier and height from the standard pitch circle to the tip is called addendum. And from pitch circle to the root up to this point, it is called the dedendum and the distance from the 1 module, say this we will consider equal to the addendum and there will be a gap. This means that if the mating gear mates then tip will reach here. So, from that point to the bottom point the clearance is called the clearance.

Now, this is again face flank topple and face width all over which already discussed. Now the pitch diameter as mentioned is D P expressed by D P into twice into pitch circle radius r P. This is for straight tooth spur gear is directly given by number of teeth into the module, again in this module is the normal module in case of state tooth per spur here it is simply called module. And then circular pitch this will be the twice pi into r P; that means, a circle radius. This means that periphery of the total periphery of the pitch circle diameter divided by number of teeth, because the piece is from this point to this point, this point to this point means this is nothing, but this periphery which is 2 pi r P divided by the number of teeth and as r P is equal to Z into m therefore, this is this relation will become nothing, but pi into m.

So, pi into m is a distant, which is called circular pitch. This it is in independent of number of teeth; that means one a module is selected then for that the gear cut with that module or the gears with this this module of any number of teeth will have the equal circular pitch; that means, at all. So, pitch circle the thickness of the teeth will be same for irrespective of number of teeth.

Now again base circle is given by r b is equal to r P into cos alpha, if we remember the geometry the base circle can be expressed in terms of the pitch circle radius into cos of pressure angle.

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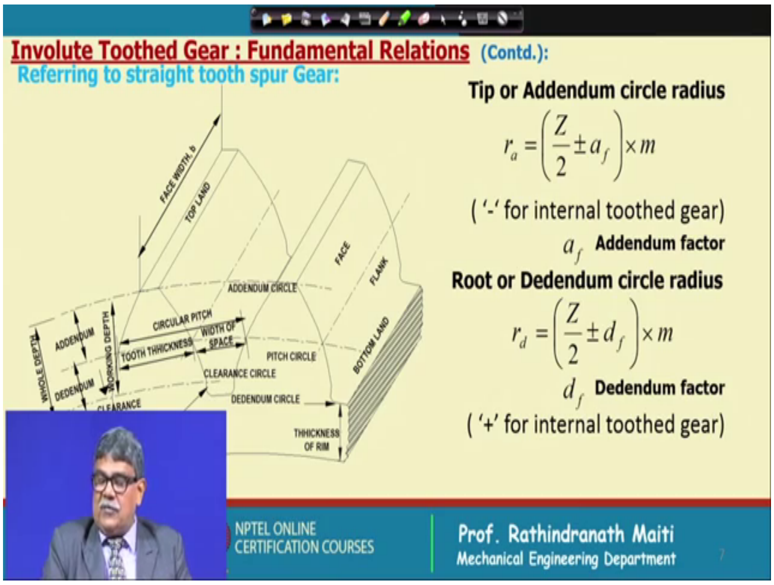

Now, diff or addendum circle radius is given by the number of teeth divided by 2 plus minus addendum factor addendum factor as I told this in case of standard here it is 1. So, Z by 2 plus minus a 4 1 what we can put in case of standard gear into the module? Now root or the dedendum circle radius will be given by rd Z minus z divided by 2 plus minus d f that is an dedendum factor into the module. Now here in both cases minus sign for internal tooth gear; that means, in case of internal tooth addendum circle will be less than pitch circle and in case of and the dedendum circle will be higher than above the pitch circle.

So, it will be plus sign there. So, this we should remember we should not make a mistake to calculate these diameters.

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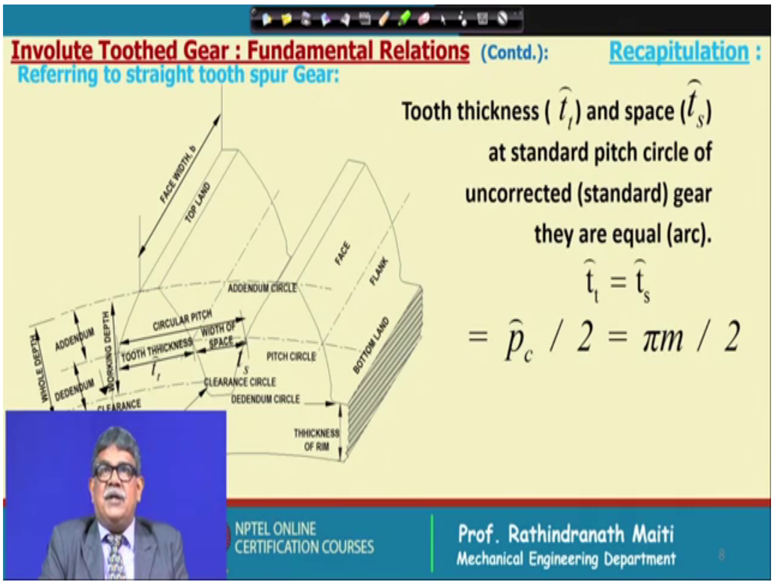

Now, tooth thickness arc tooth thickness and space arc space thickness, you can call at standard with circle of uncorrected or standard gear they will be equal; that means, d t cap is equal to t s cap and both are equal to the circular pitch divided by 2 that is pi m by 2.

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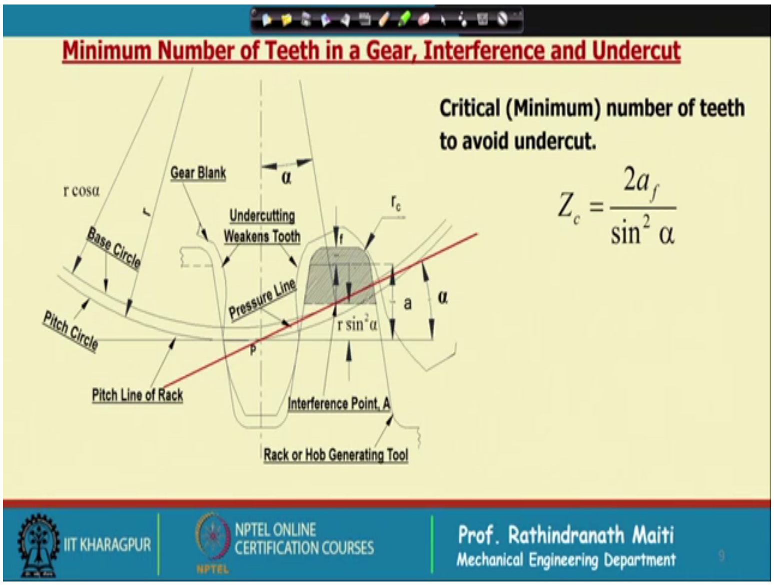

Now, this is already in earlier lecture I have shown that what should be the critical number of teeth to avoid undercut this is given by Z c is equal to twice into addendum factor divided by sin square of this standard pressure angle. Even if we use there the gear corrections, but always this we have to consider the pressure angle because the tooth will be cut on the standard pressure angle.

Now, if we consider a rack cutter a rack or robbing generated tool. Now why we should have a little bit idea about the rack cutter and the hob cutter, the rack cutter is like a rack with the teeth which is used for cutting the gears, but in that case we should remember the addendum we of the cutter will have the height of the standard addendum plus the clearance. That means, the in in that case above the pitch line of the rack cutter the height of the teeth will be the equal to the dedendum of the standard gears whereas, in the opposite side it will be slightly more than dedendum to give some clearance during the gear cutting.

So, that is there are standardization, but as well the depending on the manufacturer they give this corner radius of their own or it can specify it what will be the corner radius. This corner radius will generate the this root profile, which is called torque this root profile, which is called torqued. If we look into this there it is the pressure line or line of action is shown, this is the base circle and this is the pitch circle we have drawn it here and what we find if the number of teeth is very less and cutter is too much is going inside there will be undercut of the gears.

In fact, if we take even the tooth number is a critical number or a slightly more it will have a little bit amount of undercut, but which is which will not affect the strength of the teeth, but if we take the less number of teeth there will be too much undercut.

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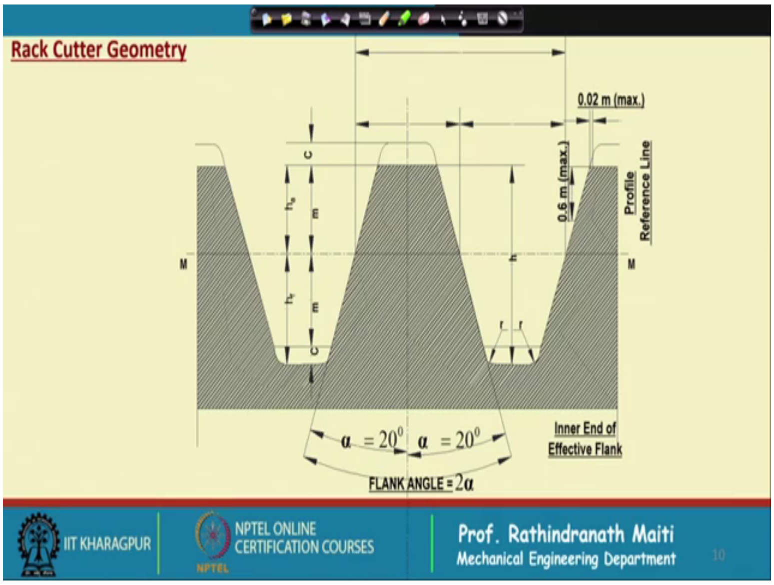

Now, this is again the proportion of a standard rack cutter these all these dimensions will be given when this rack cutter is selected and from we must know this geometry to find out the actual profile of the teeth.

However it is not required by the design engineer usually, because this is for the manufacturing purpose except what is the if radius if one can find out what will be the troquid at the root.

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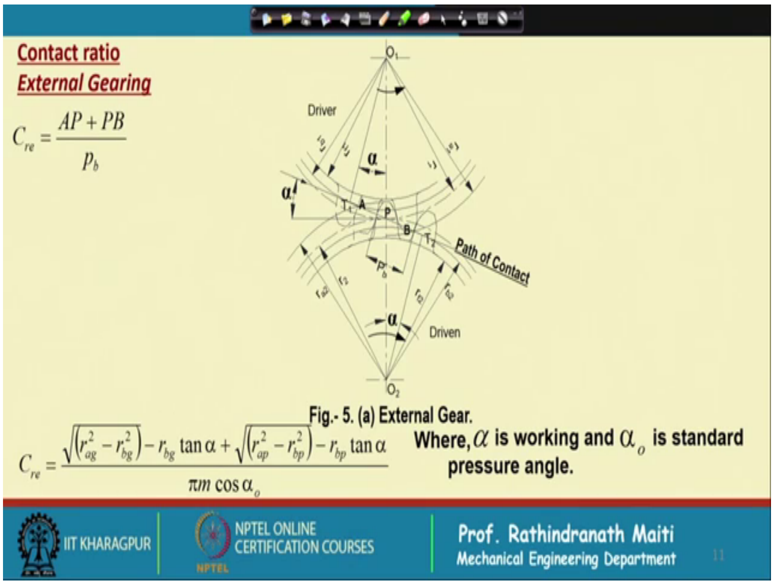

Now lastly, I would like to discuss about the contact ratio, now contact ratio is the we can say the number of pair is coming in contact during the whole action along the pitch line along the line of actions. Now if we consider the first tooth contact say let us it is at this point or maybe, if we consider from this point this is the tooth contact. And then this tooth will move towards the pitch point or if it is rotating in this direction we can say it is starting from a it will reach the point P. And then finally, it will go out, but it can be seen that while this point is going out of contact by the time another pair has such come in contact. Now who can we find found out that we can consider the first this length A P.?

A P is the distance from first point of contact one pair is coming in contact from that point, to the pitch point and v b is the distance when the a pair is coming out of contact. Now that total distance is the distance of contact along the line of action that we can compare with what we will compare from here with that, what is the distance at the root of 2 profiles which is called a base pitch P B is called a base pitch.

So, we shall compare with that distance with the base pitch; that means base pitch is the distance of true profile involute profile fixed on the base circle; we are comparing with that the total line of total length of contact. That gives us a number that number say it is 1.2 1.3 or 2 like that this means that say suppose 1.4; that means, if we consider that a length of 140 then for the 40 length there will be at least 2 pair in contact and from 100 lengths there will be 1 pair in contact.

So, this is required that if we if this contact ratio becomes one in that case when one is leaving the contact, then another it is supposed to come in contact, but due to the manufacturing error or due to the expansion of the gears that may not always match as a point and that will give us use dynamics..

So, it is essential to calculate that what is the contact ratio now what is acceptable contact ratio it is 1.4 for industrial gears this means that the truth proportion addendum height dedendum all. These things are standardized considering such concept contract ratio tooth thickness and it is such that for a standard gear of involute gear, that contact ratio will be within 1.4 tooth thickness at the standard pitch circle is half of the circular pitch etcetera etcetera.

Now, we shall further discuss on that contact ratio when we will learn a little bit about the gear tooth corrections, but at the present moment I would like to say that this contact ratio for external tooth gear can be expressed in terms of the addendum circle radius a g and r b g is the base circle radius of the gear and alpha is the standard pressure angle of course, this relation with respect to the standard pressure pressure angle sorry. These in this case alpha we have considered the working pressure angle. So, we can consider the working pressure pressure angle and once the standard pressure angle is finalized in that case r b g is fixed because the tip number is fixed r a g is the physical dimension of the gear blank.

So, we will know this dimension easily only this alpha need to be calculate for the depending on the where it is meeting. So, we will consider we will calculate this part and this part, which is again the addendum circle radius of the pinion and base circle radius of the pinion and this is the working pressure angle, divided by the pi m into cos off standard pressure angle, which is nothing, but the base pitch.

So, alpha is the working and alpha 0 is that standard pressure angle. So, this is in case of we can without the derivation we can consider this formula for the calculating of the for calculation of the contact ratio of external gears..

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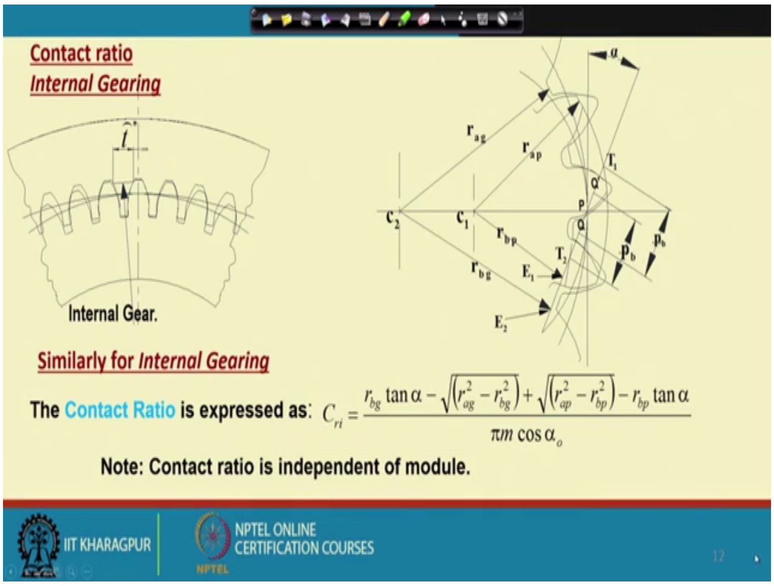

Now similarly for internal gears this expression can be given by can be given by this formula, we shall further discuss on this contact ratio when we will learn about the gear tooth corrections. Now it can be shown that this contact ratio is independent of module.

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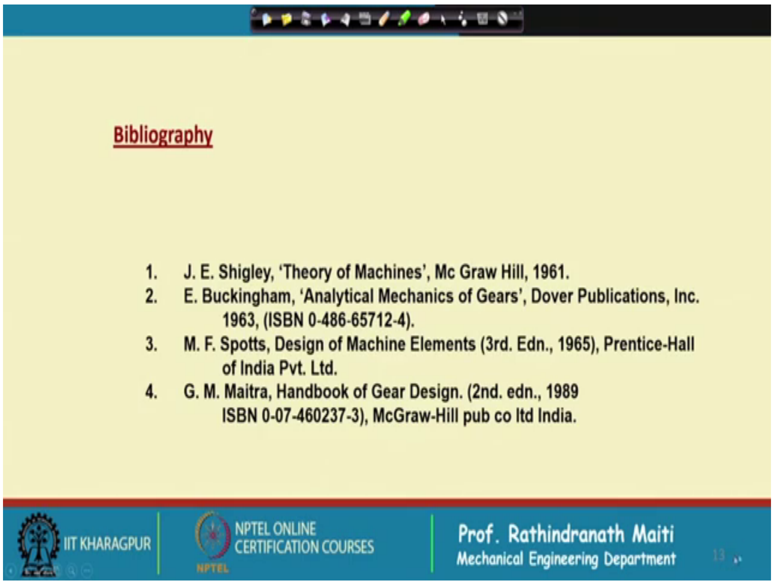

So, thank you for listening.