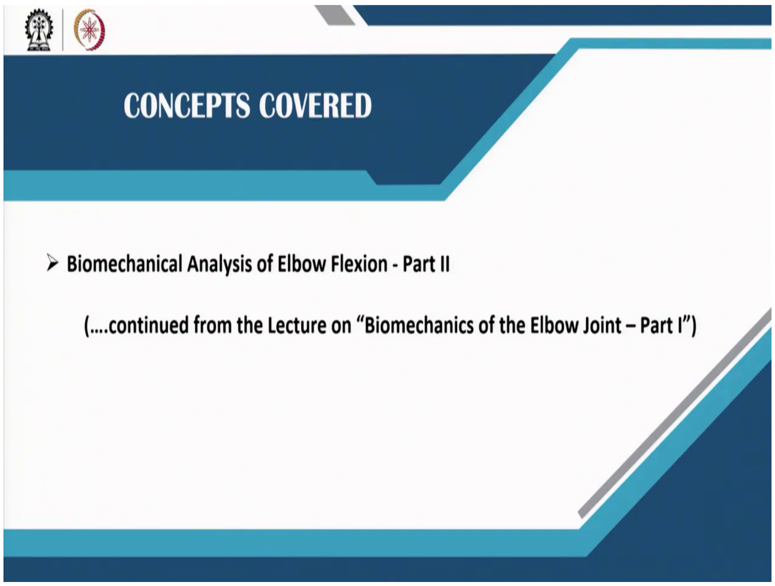
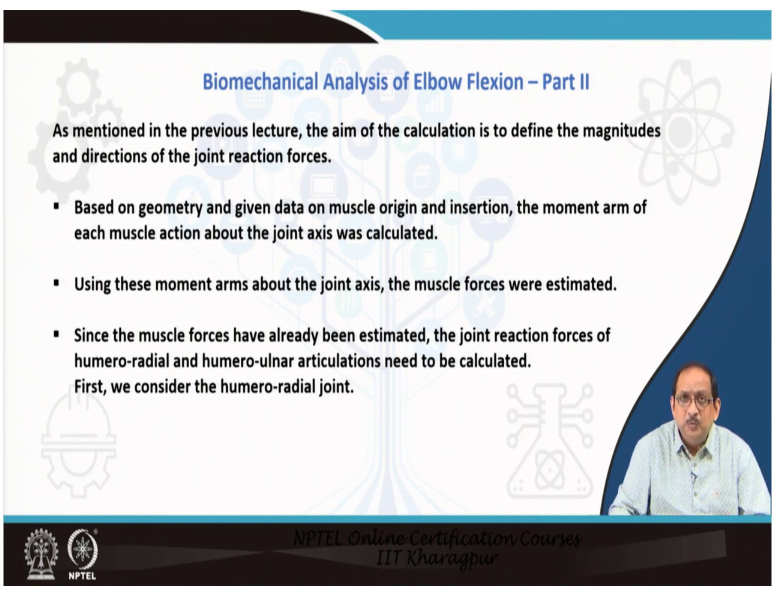
Biomechanics of Joints and Orthopaedic Implants
Professor Sanjay Gupta
Department of Mechanical Engineering
Indian Institute of Technology Kharagpur
Lecture 12
Biomechanics of the Elbow Joint Part-II

(Refer Slide Time: 0:42)



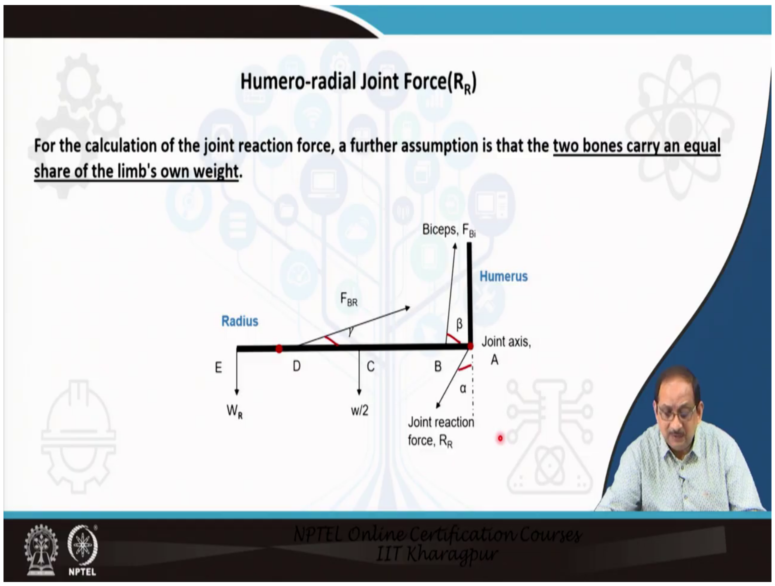


Welcome to the second part of the lecture on the biomechanics of the elbow joint. This lecture is in continuation to the lecture on the biomechanics of the elbow joint part 1 discussed earlier.

Now, as mentioned in the previous lecture, the calculation aimed to define the magnitudes and directions of the joint reaction forces. Based on the geometry and the given data on muscle origin and insertion the moment arm of each muscle action about the joint axis was calculated. The muscle forces were estimated using the moment arm about the joint axis. Since the muscle forces have already been evaluated, the joint reaction forces of humeroradial and humeroulnar articulations need to be separately calculated.

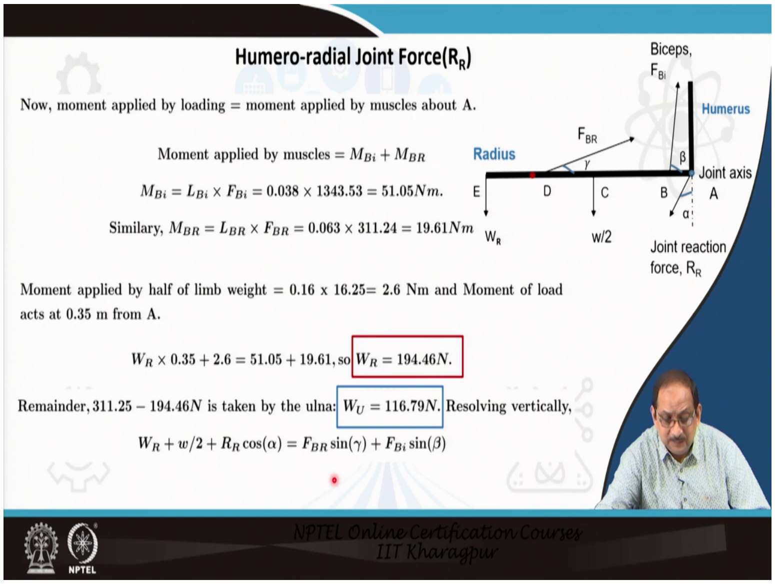
In part one of the lectures, we had assumed that the radius and ulnar to be one bone articulating with the humerus and based on that assumption, we had calculated the muscle forces. Now, we will be separately calculating the joint reaction forces corresponding to humeroradial articulation and humeroulnar articulations. So, we consider the humeroradial joint.

(Refer Slide Time: 3:12)



Now for the calculation of joint reaction forces, the humeral radial joint reaction force, we consider the assumption that the two bones carry an equal share of limbs on weight. The slide depicts the free body diagram consisting of the muscle forces and the limb weight, and humeroradial joint reaction force.

(Refer Slide Time: 4:18)



Now, we had already determined the angles at which these muscle forces were acting. So, we have already the values of gamma and beta. The moment applied by the external loads is equal to the moment applied by the muscles about the joint axis at A. So, the moment applied by the muscles are the moment generated by the biceps and the moment generated by the muscle brachioradialis.

Now, let us come to the moment created by the limb weight and the weight carried by the radius. Now, we do not know the amount of weight carried by the radius. So, this needs to be first determined, but we can use the moment balance equation, the equilibrium equation and determine the value of W_R, which is the weight carried by the bone radius, which comes out to be 194.46.

$$Moment\ applied\ by\ muscles\ = M_{Bi} + M_{BR}$$

$$M_{Bi} = L_{Bi} \times F_{Bi} = 0.038 \times 1343.53 = 51.05Nm.$$

$$Similary,\ M_{BR} = L_{BR} \times F_{BR} = 0.063 \times 311.24 = 19.61Nm$$

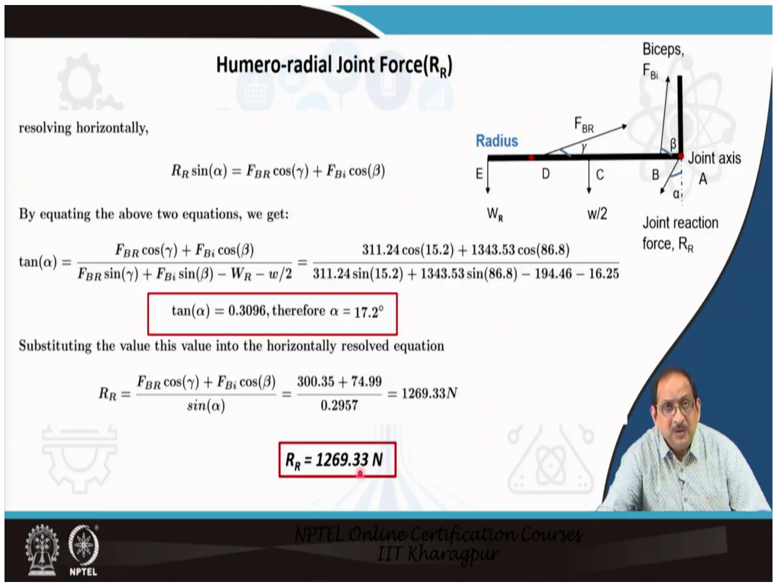
Moment applied by half of limb weight = 0.16 x 16.25= 2.6 Nm and Moment of load acts at 0.35 m from A.

$$W_R \times 0.35 + 2.6 = 51.05 + 19.61, so\ W_R = 194.46N.$$

$Remainder, 311.25 - 194.46N\ is\ taken\ by\ the\ ulna$: $W_U = 116.79N.\ Resolving\ vertically.$

Now, the total weight carried by the elbow joint at 90^{o} flexion was 311.25 N out of which 194.46 N is taken up by the radius bone, the remaining part that is 116.79 N is carried by the ulnar. So, now, we proceed with this data calculated data of the weight carried by the radius bone and we resolve the system of forces along the vertical direction and we can write down the equation based on the laws of equilibrium summation of the forces along the vertical direction.

(Refer Slide Time: 7:44)



Resolving horizontally, we can write down the equation the R_R, the component of R_R Sin(α) is equal to the brachioradialis muscle force's component along the horizontal direction and the horizontal component of the muscle biceps. The γ and β are the inclination angles of these forces. By equating the above two equations, we can calculate the tan(α), where alpha is the angle of inclination of the humeroradial joint reaction force R_R. So, α can be determined as 17.2°, substituting this value in the equation of horizontally resolved forces, we can determine the R_R to be 1269.33 N.

Resolving vertically,

$$W_R + w/2 + R_R cos(\alpha) = F_{BR} sin(\gamma) + F_{Bi} sin(\beta)$$

resolving horizontally,

$$R_R sin(\alpha) = F_{BR} cos(\gamma) + F_{Bi} cos(\beta)$$

By equating the above two equations, we get:

$$tan(\alpha) = \frac{F_{BR}cos(\gamma)+F_{Bi}cos(\beta)}{F_{BR}sin(\gamma)+F_{Bi}sin(\beta)-W_R-w/2}$$

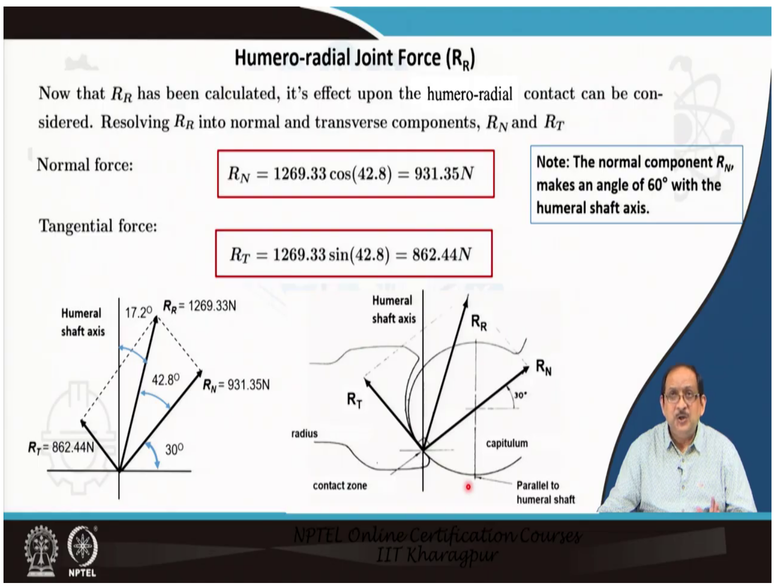
$$tan(\alpha) = \frac{311.24cos(15.2)+1343.53cos(86.8)}{311.24sin(15.2)+1343.53sin(86.8)-194.46-16.25}$$

$$tan(\alpha) = 0.3096, therefore\ \alpha = 17.2^{\circ}$$

Substituting the value this value into the horizontally resolved equation

$$R_R = \frac{F_{BR}cos(\gamma)+F_{Bi}cos(\beta)}{sin(\alpha)} = \frac{300.35+74.99}{0.2957} = 1269.33N$$

(Refer Slide Time: 9:45)



In the earlier slide, we calculated the angle α is equal to 17.20°; that is the angle at which the humeroradial joint reaction force is inclined. Now that we have calculated the magnitude of the resultant humeroradial joint reaction force R_R and its inclination 17.20°; its effect upon humeroradial contact can be considered, resolving R_R into the normal and transverse component.

So, R_R can be resolved into normal component R_N and R_T, the transverse or the tangential components. We can find out the normal and the transverse or tangential force as indicated in the slide. In this calculation, we need to note that the normal component R_N makes an angle of 60° with the humeral shaft. So, the normal component R_N as indicated here, R_N actually makes 60° with the humeral shaft axis. We have calculated R R making an angle 17.20° with the humeral shaft axis, then the remaining angle is 42.80°. So, with this angle 42.80°, we can calculate R_N and

R_T by resolving R_R in the normal direction and the tangential or transverse direction. So, the values of R_N = 931.35 N and R_T = 862.44 N. Normal force:

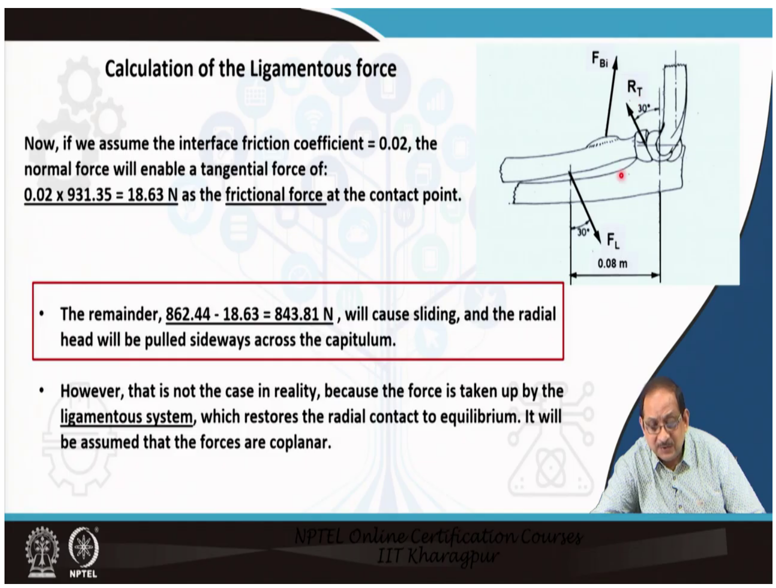
$$R_N = 1269.33cos(42.8) = 931.35N$$

Tangential force:

$$R_T = 1269.33sin(42.8) = 862.44N$$

The figure on the right presents a sagittal section of the humeroradial joint.

(Refer Slide Time: 12:46)



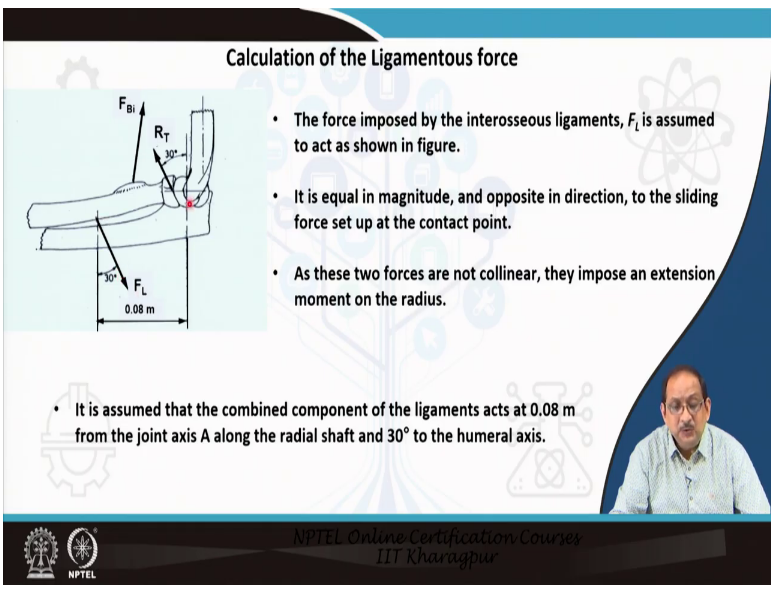
Now comes a critical consideration in the problem that is on the calculation of the ligamentous force. On the right, we can see a figure which presents an arrangement of the articulation, the elbow joint articulation and if we assume an interface friction coefficient of 0.02 we can find out the frictional force at the point of contact as a function of the normal force that has been calculated as 931.35 N.

So, the frictional coefficient multiplied by the normal force will give you the frictional force that is actually being generated at the point of contact. Now, frictional force is along the tangential direction. Previously, the tangential component of the force was 862 N. The frictional force along the tangential direction gives rise to a force 843.81 N that will cause sliding and physically, what will happen?

This unbalanced force of 843.81 Newton along the transverse direction will move the radial head sideways across the capitulum. So, the radial head will be pulled sideways across the capitalism due to the action of this unbalanced force of 843.81 N. However, this is not the case in reality, because the force is taken up by the ligamentous system, which restores the radial contact to equilibrium.

So, it will be further assumed that the forces: F_L and R_T are coplanar.

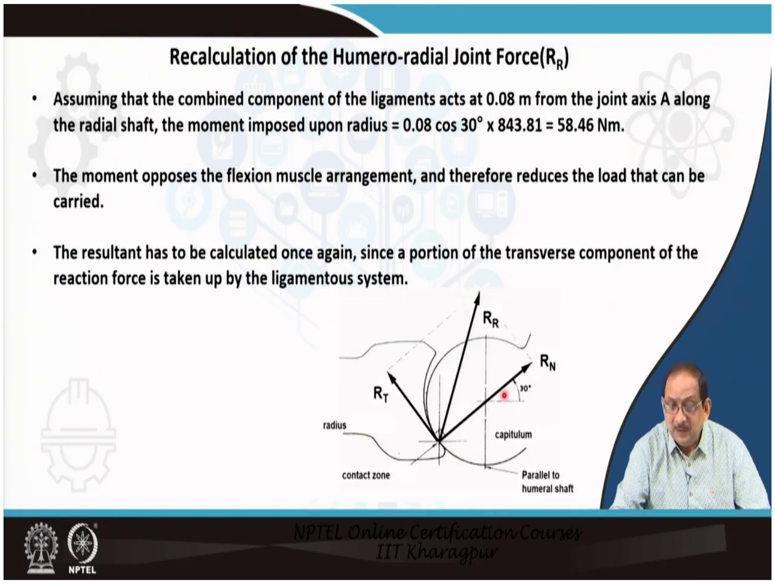
(Refer Slide Time: 16:28)



Now, let's further consider the action of the ligamentous force. The force imposed by the ligament F_L is assumed to act, as shown in the figure. It's equal in magnitude but opposite in direction to the sliding force set up at the point of contact. Sliding force is the remnant force that we had calculated earlier.

Now, as these two forces (Ligamentous force and the sliding force) are not collinear, they will impose an extension moment on the radius. Now, it is assumed that the combined component of the ligament act at a distance of 0.08 meter from the joint axis at A and along the radial shaft.

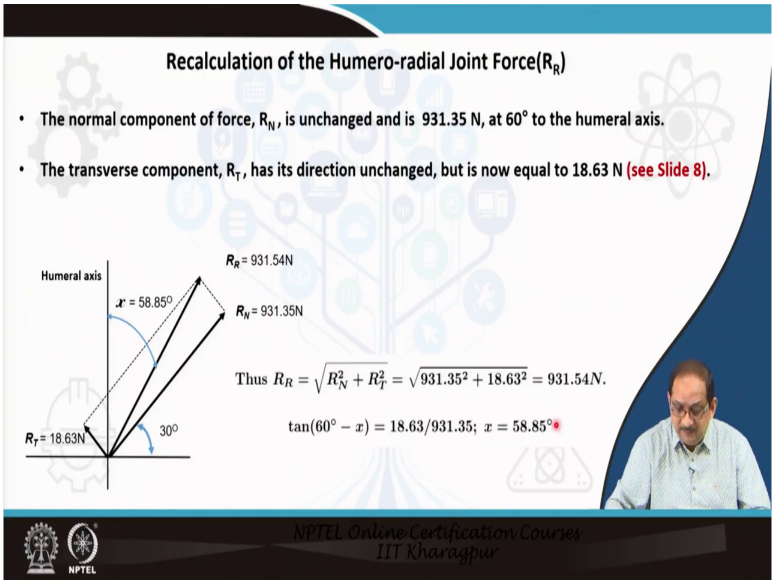
(Refer Slide Time: 18:31)



Now, we should recalculate the humeral radial joint reaction force considering the effect of the ligamentous force. So, assuming the combined component of the ligaments act at a distance 0.08 m from the joint axis along the radial shaft, the moment imposed upon the radius is calculated as 58.46 Nm. This moment opposes the flexion muscle arrangement and, therefore, reduces the load carried.

The resultant has to be calculated once again since the ligamentous system takes up a portion of the transverse component of the reaction force.

(Refer Slide Time: 20:08)



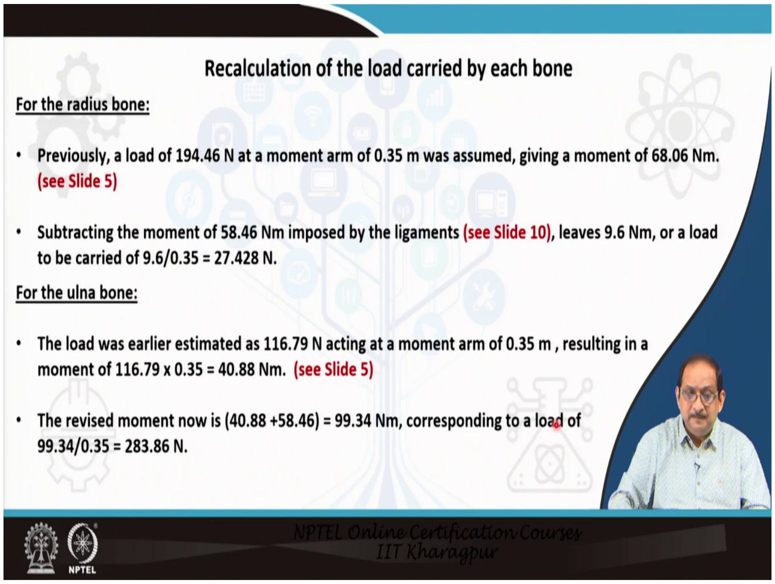
Now, the normal component of the force R_N as shown earlier in the figure. it remains unchanged and is calculated as 931.35 N acting at an angle 60°to the humeral axis. So, the normal component of the joint reaction force remains unchanged in direction and magnitude. However, the transverse component or the tangential component R_T has its direction intact, but now, the magnitude is equal to the frictional force that has been calculated earlier in slide 8. Please visit slide 8 for calculation on the frictional force that is 18.63 N and this frictional force is the tangential component of the joint reaction force.

So, once we have these two data, we can recalculate the revised humeroradial joint reaction force as 931.54 N and the angle x, inclination of this resultant force with the humeral axis can also be calculated, which is 58.85°.

$$R_R = \sqrt{R_N^2 + R_T^2} = \sqrt{931.35^2 + 18.63^2} = 931.54N.$$

$$tan(60^\circ - x) = 18.63/931.35;\ x = 58.85^\circ$$

(Refer Slide Time: 22:01)

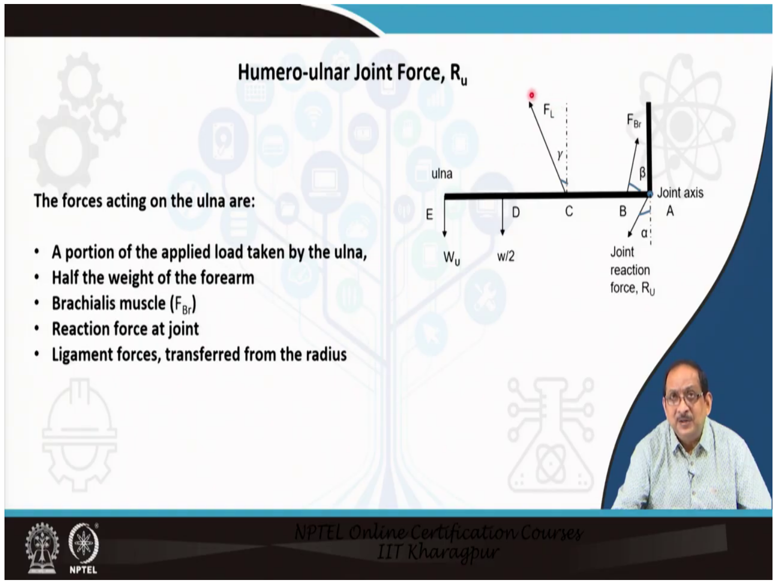


The next important step is recalculation of the load carried by each bone because now, we have the ligaments acting in the system. So, separately we will consider the radius bone and the ulnar bone. Previously, external load of 194.46 N at a moment arm of 0.35 was assumed at giving rise to a moment of 68.06 Nm. So, for the detailed calculation you can refer back to slide 5.

Now, if we subtract the moment of 58.46 Newton meter imposed by the ligament, see calculation in slide number 10, then we are left with an unbalanced moment of 9.6 Nm. If we divide this moment by the distance 0.35m, then we recalculate the load carried by the radial segment of the bone which is 27.428 N.

For the ulna bone, the load was earlier estimated to be 116.8 N acting at a moment arm of 0.35 m resulting in a moment of 40.88 Nm. For detailed calculation, please see earlier slide number 5. Now, the revised moment can be calculated as 99.34 Nm corresponding to a load of 283.86 N.

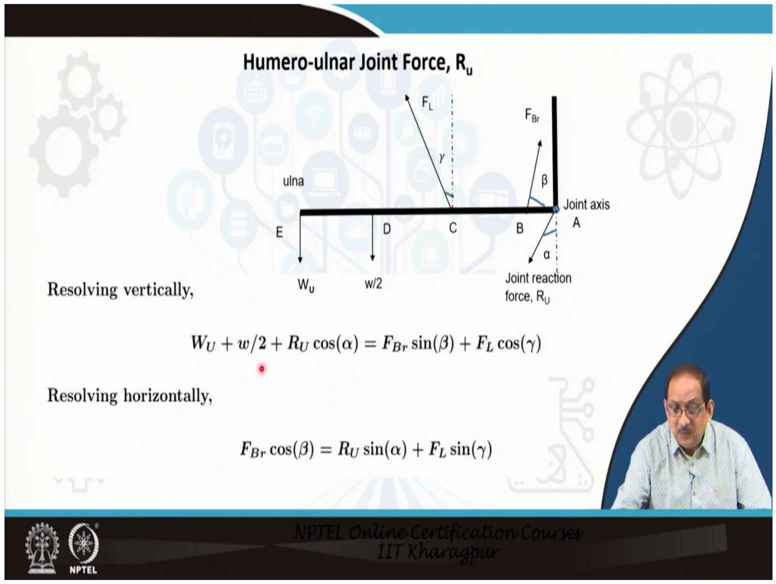
(Refer Slide Time: 25:23)

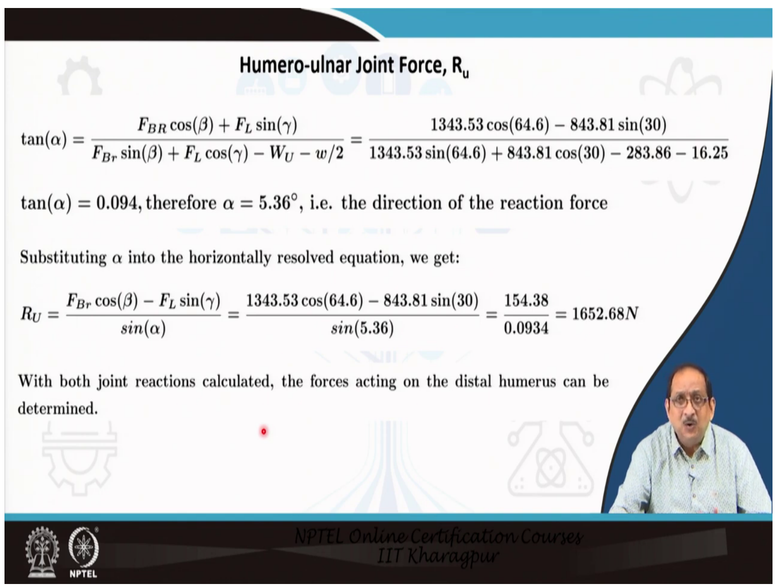


Now, let us consider the humeroulnar joint reaction force. We can see the free body diagram of the arrangement humeroulnar force. Here, we have a portion of the applied load taken by the ulnar that we have calculated just in the preceding slide, the half of the weight of the forearm as discussed earlier. We need to determine the reaction force, R_u.

But, one crucial thing included here is the ligament force transferred from the radius. So, the ligament acts between the radius and ulnar, and its line of action is indicated as F_L.

(Refer Slide Time: 26:24)





So, we can resolve the forces based on the free body diagram and apply the laws of equilibrium. So, thereafter, we can actually calculate the inclination of the angle of the humero ulnar joint reaction force, which can be calculated using the two equations and the data already available to us.

Resolving vertically,

$$W_U + w/2 + R_U cos(\alpha) = F_{Br} sin(\beta) + F_L cos(\gamma)$$

Resolving horizontally,

$$F_{Br} cos(\beta) = R_U sin(\alpha) + F_L sin(\gamma)$$

Equating both the equations and rearranging, we get

$$tan(\alpha) = \frac{F_{BR} cos(\beta) + F_L sin(\gamma)}{F_{Br} sin(\beta) + F_L cos(\gamma) - W_U - w/2}$$

$$tan(\alpha) = \frac{1343.53 cos(64.6) - 843.81 sin(30)}{1343.53 sin(64.6) + 843.81 cos(30) - 283.86 - 16.25}$$

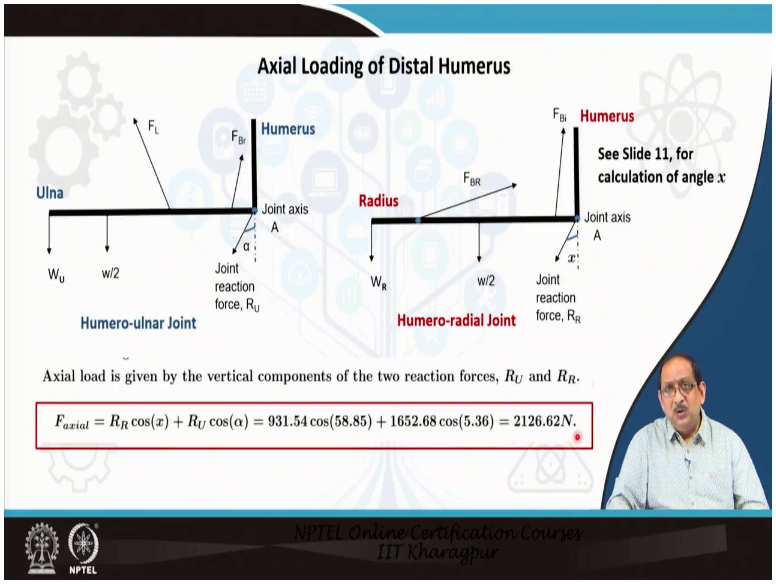
$$tan(\alpha) = 0.094, therefore\ \alpha = 5.36^{\circ},\ i.e.\ the\ direction\ of\ the\ reaction\ force$$

Substituting α into the horizontally resolved equation, we get:

$$R_U = \frac{F_{Br} cos(\beta) - F_L sin(\gamma)}{sin(\alpha)} = \frac{1343.53 cos(64.6) - 843.81 sin(30)}{sin(5.36)} = \frac{154.38}{0.0934} = 1652.68N$$

You can see the updated load W_u has been considered, which is 283.86 N, and of course, the limb weight, half of the limb weight is 16.25 N. So, with this new data, we calculate the angle, α as 5.36° and after that, we can calculate the magnitude of the reaction force R_u as 1652.68 N. Now with both reactions forces estimated, the forces acting on the distal humerus can also be determined.

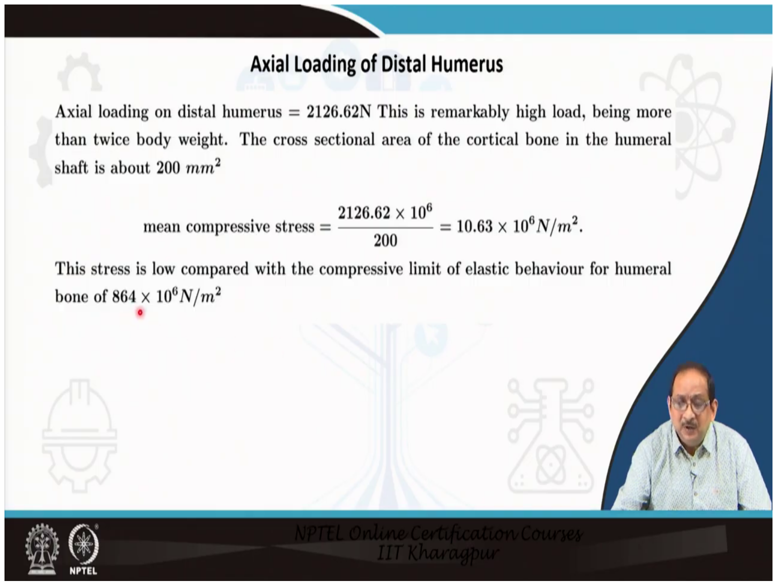
(Refer Slide Time: 28:01)



We have come to a stage that we have two separate joint reaction forces; one is the humeroulnar joint reaction force, the other is the humeroradial joint reaction force. Now, if we want to calculate the load at the distal end of the humerus, we need the vertical components of the two joint reaction forces.

$$F_{axial} = R_R cos(x) + R_U cos(\alpha) = 931.54cos(58.85) + 1652.68cos(5.36) = 2126.62N.$$

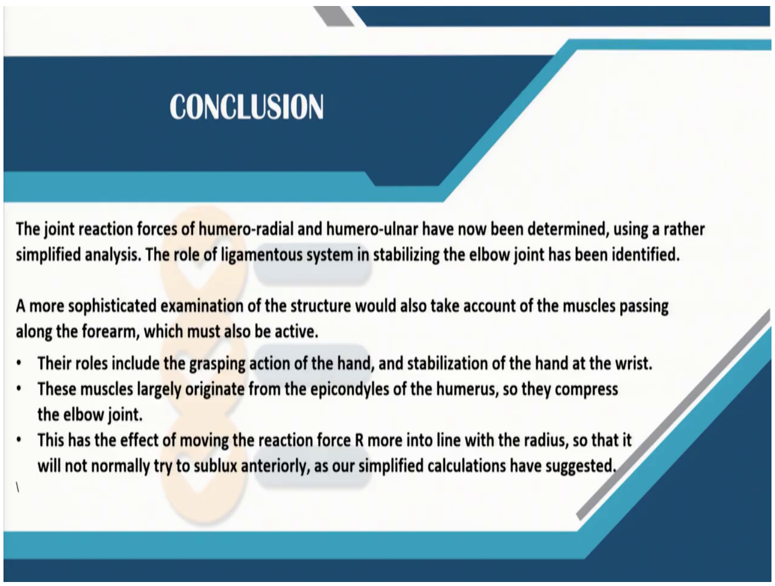
(Refer Slide Time: 29:51)



Now, if we consider the cross-sectional area of the cortical bone in the humeral shaft, the cross-sectional area is about 200 mm^2. So, the mean compressive stress can be calculated as the axial force divided by the cross-sectional area, and it comes out to approximately 10.63 MPa. This stress is lower than the compressive strength of the humeral bone, which is about 864 MPa.

$$mean\ compressive\ stress = \frac{2126.62 \times 10^6}{200} = 10.63 \times 10^6 N/m^2.$$

(Refer Slide Time: 30:58)

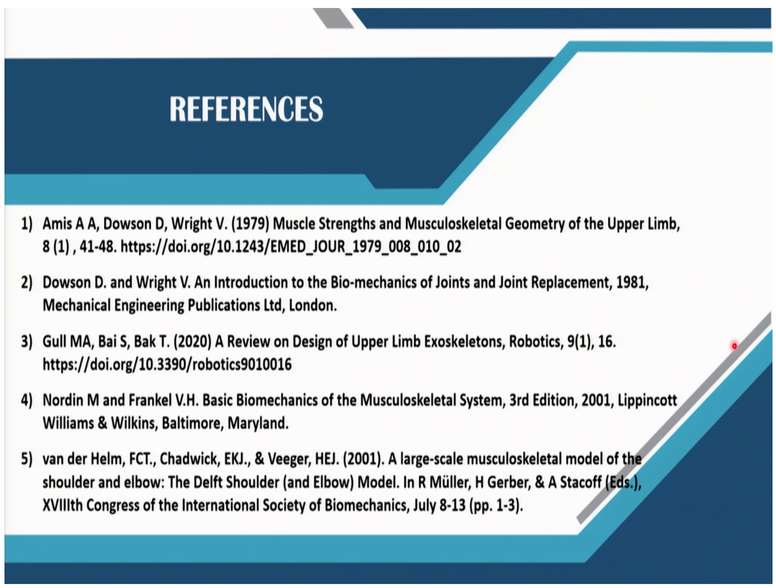


So, let us come to the conclusion of this lecture, both part one and part two together. What did we learn from the biomechanical analysis of the elbow joint, the joint reaction force both humeroradial and humero ulnar separately has now been determined using a rather simplified analysis. The role of the ligamentous system most importantly, in stabilizing the elbow joint has been identified.

However, a more sophisticated examination of the structure would also take into account the muscles passing along the forearm even though we did not account for these muscles. Their role includes the hand's grasping action and stabilization at the wrist. These muscles originate from the epicondyles of the humerus, so they compress the elbow joint.

So, this has the effect of moving the reaction force R more in line with the radius bone so, that it will not normally try to sublux anteriorly as our simplified calculations have suggested.

(Refer Slide Time: 32:49)



I hope you could follow the calculations presented in this lecture. The list of references are indicated in these slides. I would like to thank you for listening.