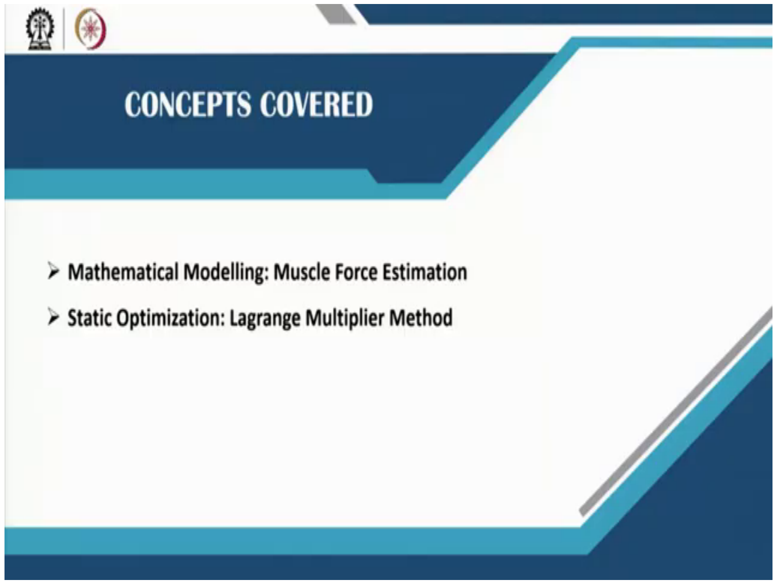
**Biomechanics of Joints and Orthopaedic Implants**
**Professor Sanjay Gupta**
**Department of Mechanical Engineering**
**Indian Institute of Technology, Kharagpur**
**Lecture 23**
**Muscle Force Estimation Using Static Optimization**

(Refer Slide Time: 0:36)

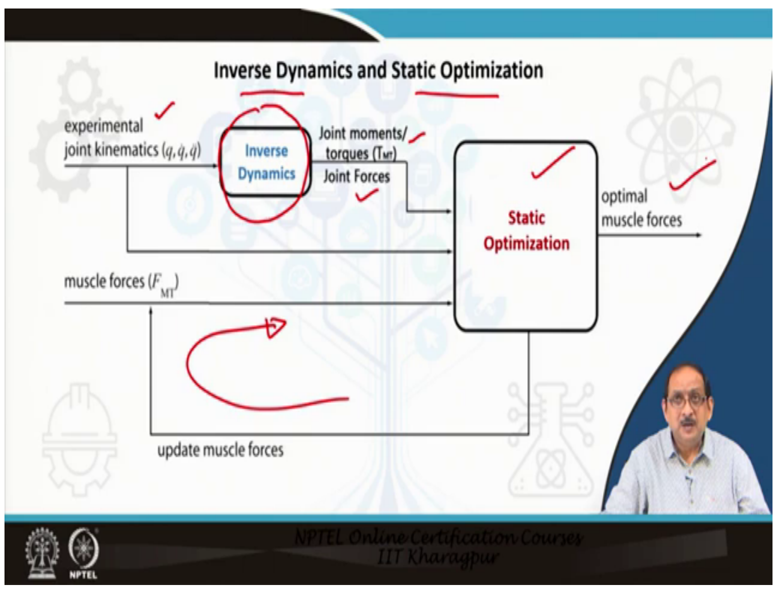

Good morning everybody, welcome to the lecture on lecture 5 on muscle force estimation using static optimization.

(Refer Slide Time: 0:43)




In this lecture, we will be discussing mathematical modeling for muscle force estimation and will primarily focus on static optimization technique based on Lagrange multiplier method.

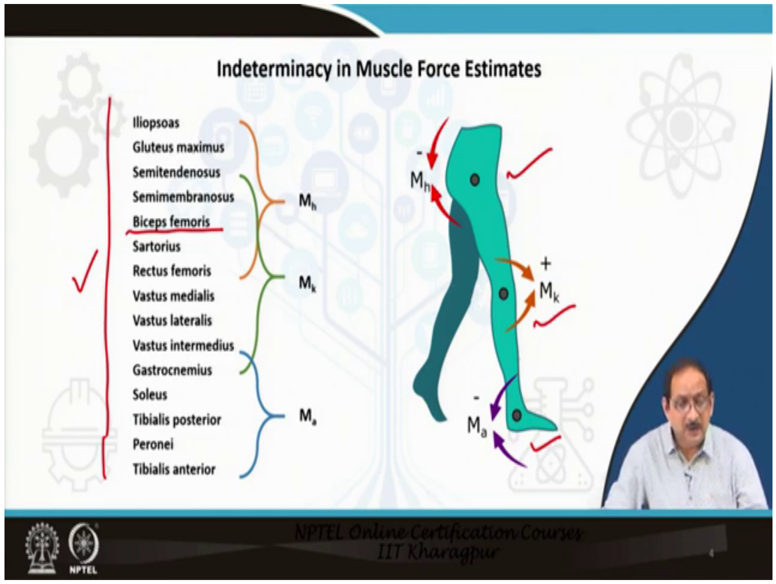
(Refer Slide Time: 1:02)




The figure presented here is the overview of the muscle force estimation using inverse dynamics and static optimization. The information on experimental joint kinematics is used to calculate joint moments and torques, joint moments, or torques and joint forces using the inverse dynamics method.

Now, joint moments, or torques are the net combined effect of individual muscle forces spanning the joint. The contribution of each muscle can be calculated using an optimization scheme, Which we will be focusing on in this lecture. In some procedures, an initial guess of muscle forces is made on which the joint equilibrium is verified, and changes in joint forces are made if necessary. The procedure is repeated until an optimal muscle force distribution is achieved.

(Refer Slide Time: 2:56)

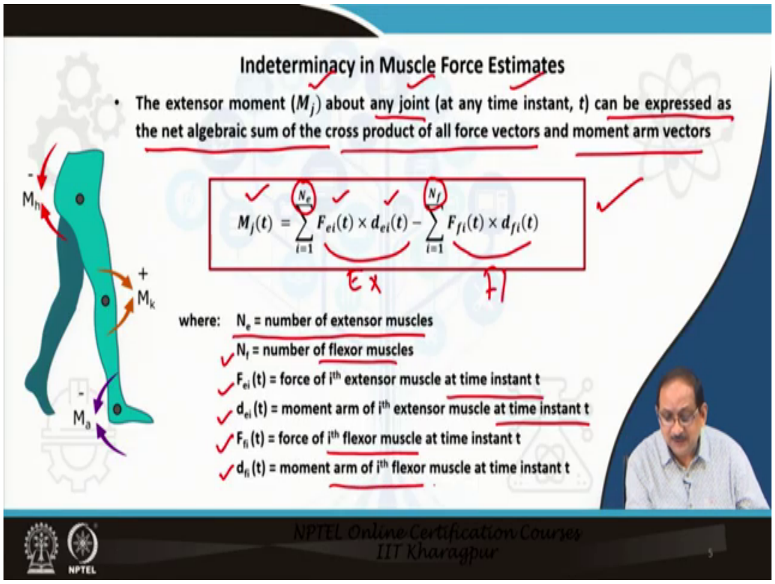



Now, let us consider indeterminacy in muscle force estimates. The figure presents the major muscle forces responsible for joint moments in the lower limb, as observed in the side view presented in the sagittal plane. So, these are the fifteen major muscles that actually contribute to the moments of forces at the ankle joint, the knee joint, and the hip joint.

So, fifteen major muscles contribute to the joint moments across these three joints in the lower limb. At the knee, majorly, there are about nine muscles whose forces create the net moment.

Now, the important thing to note here is the line of action of each of these muscles is different and continuously changes with time. The other point to note is that some muscles like bicep femoris, for example, may contribute towards moments around the hip joint and the knee joint.

(Refer Slide Time: 4:46)



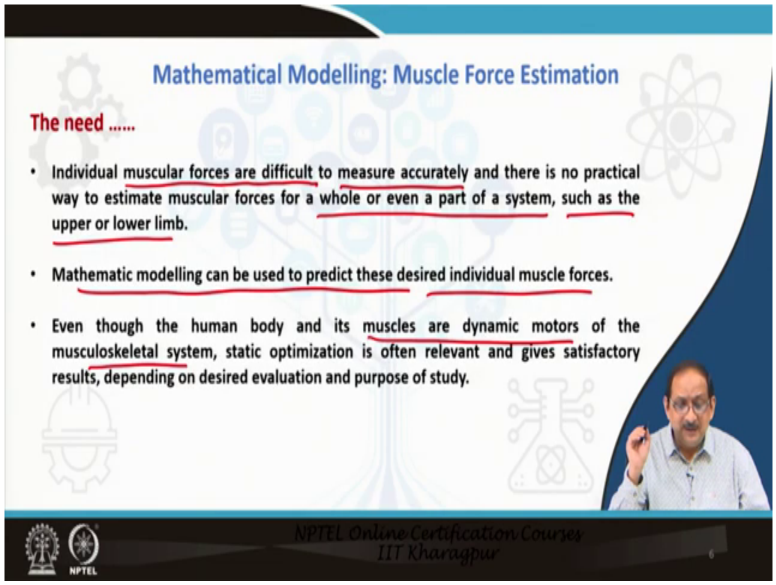

We continue with the indeterminacy of the muscle force estimates. We would like to say that to make valid estimates of the individual muscle forces, it is necessary to consider a detailed anatomical, or kinematic model with lines of action for each muscle relative to the joint center.

Now, the extensor moment $M_j$ about any joint, at any time instant t can be expressed as the net algebraic sum of the cross products of the all the force vectors and the moment arm vectors. So, this is mathematically represented here in this equation. So, we can actually write down, the extensor moment $M_j$ equal to the force vector, cross product with the moment arm of the extensor muscles. And the other part is the same cross product of the flexor muscles with the momentum of the flexor muscles.

So, the sum of the cross products of all force vectors and the moment arm vectors, extensor in this case and flexors in this case is represented together. And we calculate the net extensor moment $M_j$. Now, in this equation on the right-hand side of the equation, we can see $N_e$ is the number of extensor muscles, whereas $N_f$ is the number of flexor muscles.

Now, extension and flexions are just two opposite movements, the force of the $i^{th}$ extensor muscle at any time t, is represented by $F_{ei,}$ and the corresponding moment arm of the extensor muscle is $d_{ei}$. Similarly, the force in the $i^{th}$ muscle of the flexor. So, $i^{th}$ flexor muscle is represented by $F_{fi}$ and the corresponding moment arm of the muscle at any time instant t is given by $d_{fi}$. So, the moment arm of the $i^{th}$ flexor muscle is given by $d_{fi}$.

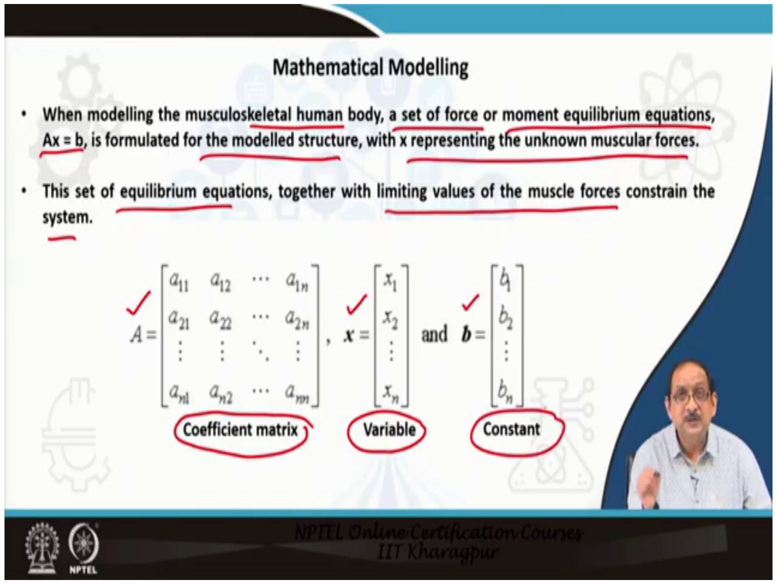
(Refer Slide Time: 8:11)




Now, let us discuss the first important topic on the mathematical modeling of muscle forces, over the years several mathematical modeling techniques in muscle force estimation have evolved. Before going into the details of the procedure, the need for such technique is discussed in this slide. The individual muscle forces are difficult to measure accurately and there is no practical way to estimate muscular forces for a whole, or even part of a musculoskeletal system, such as the upper or lower limb.

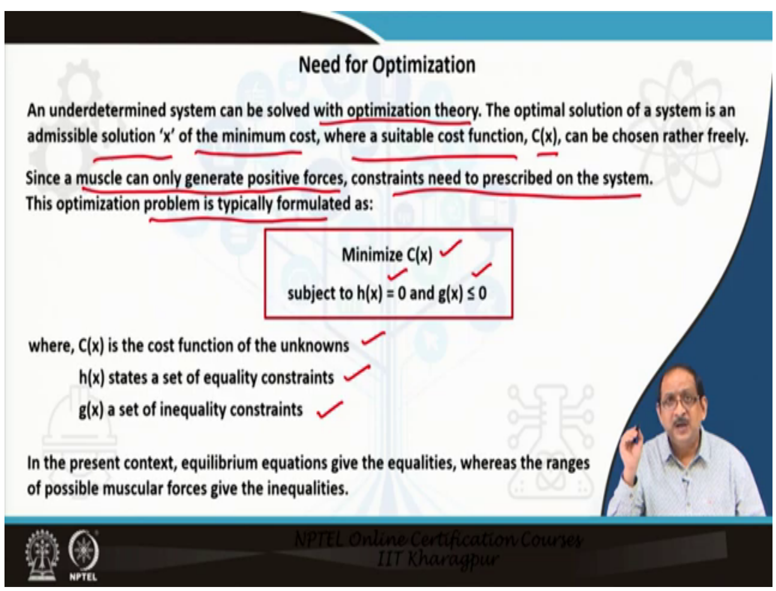
Now, mathematical modeling can be used to predict these desired individual muscle forces. Even though the human body and its muscles are dynamic motors of the musculoskeletal system, static optimization is often relevant and gives satisfactory results depending on desired evaluation and purpose of the study. So, it may be noted that the dynamic forces are then considered as static, as static at every time instant.

(Refer Slide Time: 10:13)




Let us now discuss about mathematical modeling. When modeling the human musculoskeletal body, a set of force, or moment equilibrium equations in the form of A x equal to b, can be formulated for the model structure, where x represents the unknown muscular forces. This set of equilibrium equations, together with limiting values of muscle forces, constrain the system. A is the coefficient matrix of the variable x, and b is the constant.

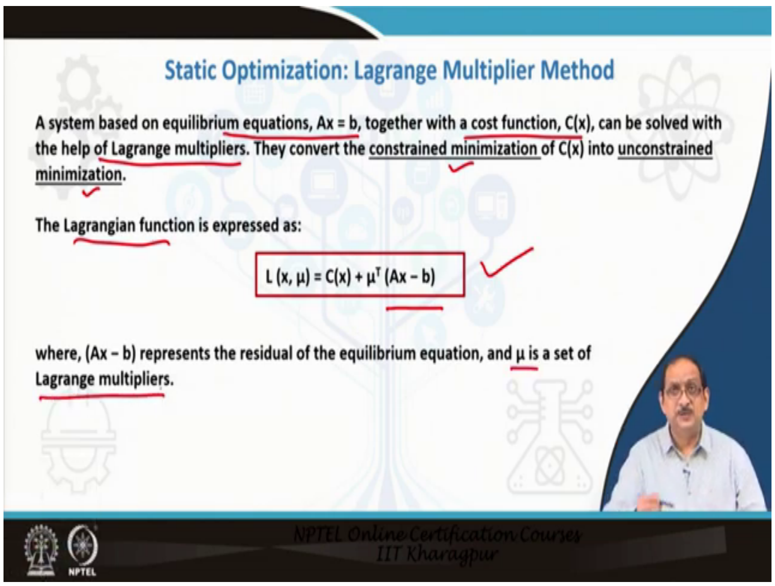
(Refer Slide Time: 11:16)



In mathematics, a set of linear equations or a system of polynomial equations is considered undetermined if there are fewer equations than the unknowns. An undetermined system can be solved with optimization theory. The optimal solution is an admissible solution x, of the minimum cost, where a suitable cost function C(x) can be chosen freely.

Since a muscle can only generate positive forces, constraints need to be prescribed on the system. So this optimization problem is typically formulated as minimizing C(x) subject to h(x) equal to 0 and g(x) less than equal to 0, where C(x) is the cost function of the unknowns, h(x) states a set of equality constraints, and g(x) is a set of inequality constraints.

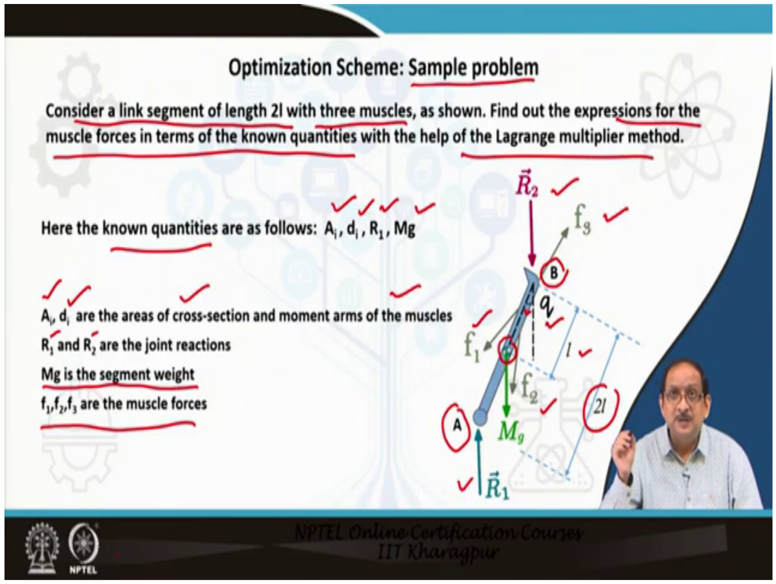
(Refer Slide Time: 13:27)




Now, let us focus on the Lagrange multiplier method in the static optimization technique. The Lagrange multiplier method is a strategy for finding the local maxima and minima of a function, subject to equality constraints. This numerical method does not involve any initial guess.

So, a system based on equilibrium equations as discussed earlier A(x) equal to b, together with a cost function C(x), can be solved with the help of Lagrange multipliers. They convert the constrained minimization of C(x) into unconstrained minimization. The Lagrangian function is expressed as presented in the slide, where A(x) - b represents the residual of the equilibrium equation, μ is a set of Lagrange multipliers.

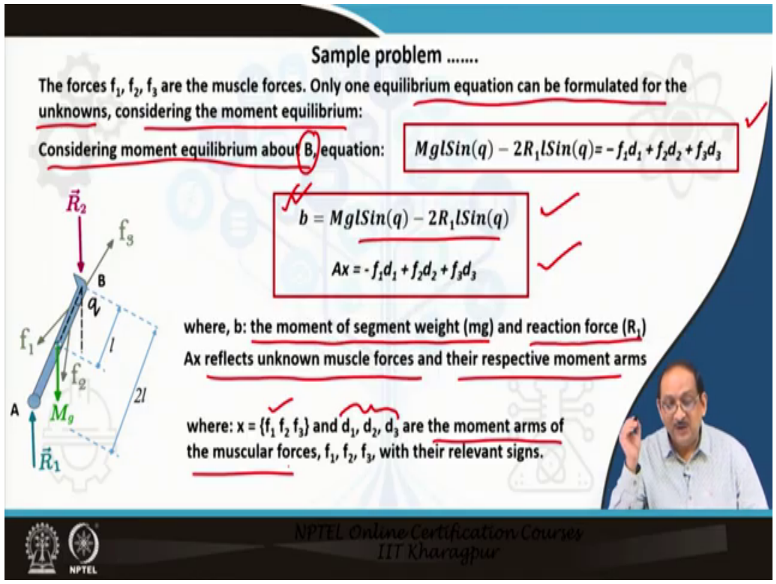
(Refer Slide Time: 14:59)




Let us consider a sample problem to understand, the optimization scheme. So, we consider a link segment of length '2l' as indicated in the figure with three muscles, $f_1$, $f_2$, and $f_3$. We need to find out the expressions for the muscle forces in terms of known quantities with the help of Lagrange multiplier method.

So, here in this problem, the known quantities are $A_i$, $d_i$, $R_1$ and Mg, $A_i$ is the area of cross section of a muscle, $d_i$ is the moment arm of a muscle, $R_1$, $R_2$ are the joint reaction forces as indicated in the figure, $R_1$ is known in this problem, and Mg is the link segment weight, $f_1$, $f_2$, and $f_3$ are the muscle forces as indicated in the figure. The two endpoints of the links are A and B and the angle q is the angle between the axis of the link and the vertical direction. The length L is the distance of the center of gravity of the link, from the end B.

(Refer Slide Time: 16:53)

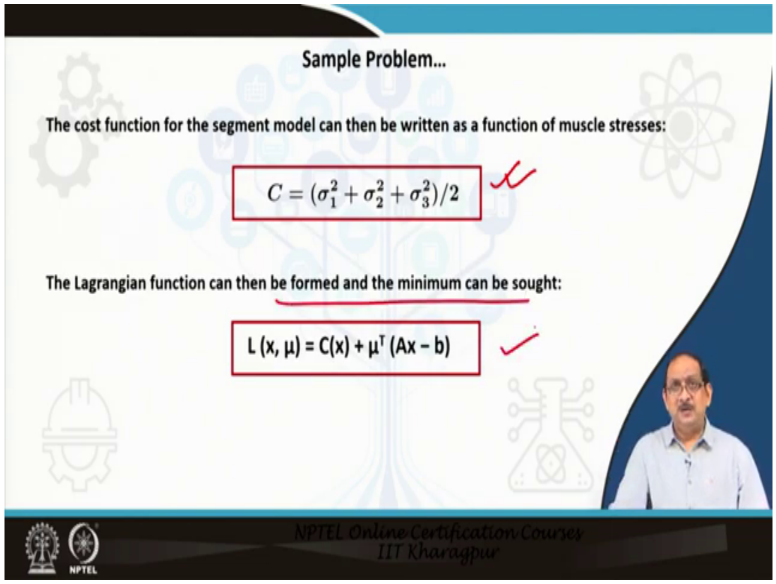



The forces $f_1$, $f_2$, and $f_3$ are the muscle forces as already indicated in the earlier slide. Only one equilibrium equation can be formulated for the unknowns, considering the moment equilibrium. So, considering the moment equilibrium at B, the equation can be written as shown in the slide.

Now, considering the equilibrium equation, the terms b, and Ax can be separately written as shown in the slide. So, b is written here, and Ax contains the forces and the moment, corresponding moment arms are written here in the slide.

So, the b and Ax terms of the Lagrangian function is represented here in the slide, where b actually contains the moment of the segment weight mg, and the reaction force R1, as you can see already in the slide, whereas Ax reflects unknown muscle forces and their respective moment arms. So, x is the set of unknown muscle forces, $f_1$, $f_2$, and $f_3$ and $d_1$, $d_2$, and $d_3$ are the corresponding moment arms of the muscular forces $f_1$, $f_2$, and $f_3$ with relevant signs.

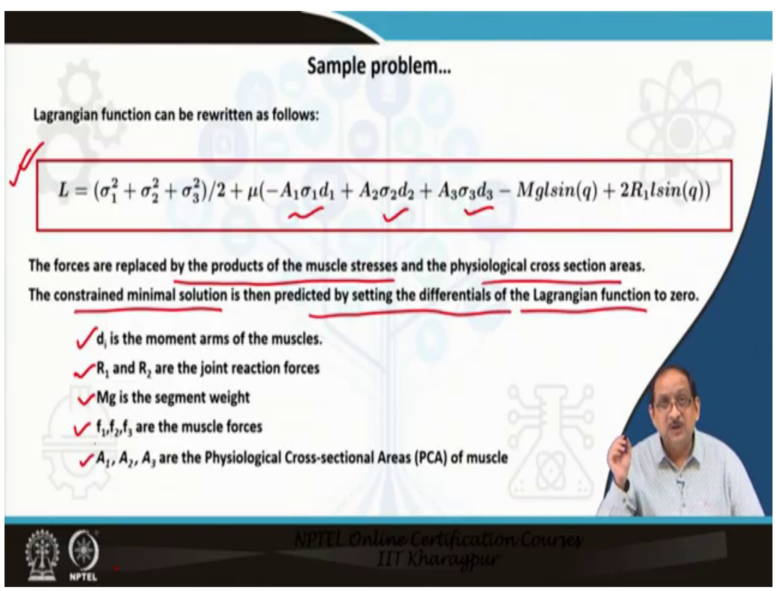
(Refer Slide Time: 19:12)




Now, the cost function for the segment model can be written as a function of muscle stresses, it is actually written as one half of the sum of the squares of the muscle stresses, as presented in the slide. The Lagrangian function can then be formed, and the minimum can be sought, based on the Lagrangian function as presented in this slide.

$$C = (\sigma_1^2 + \sigma_2^2 + \sigma_3^2)/2$$

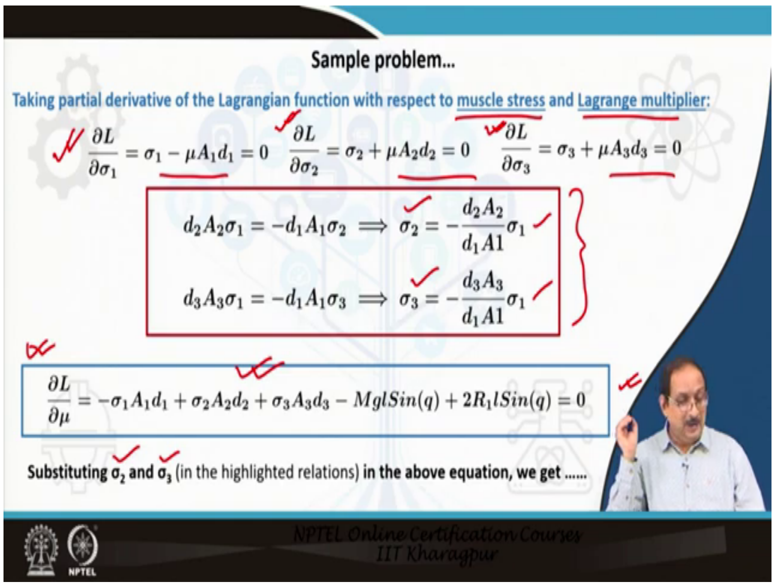
(Refer Slide Time: 19:55)



The Lagrangian function can be rewritten as follows, the Lagrangian function is indicated in the slide. The forces are replaced by the products of muscle stresses and the physiological cross-sectional area. The constrained minimal solution, please note here very important statement is then predicted by setting the differentials of the Lagrangian function to 0.

$d_i$ is the moment arms of the muscles. $R_1$, $R_2$ are the joint reaction forces, Mg is the segment weight, whereas $f_1$, $f_2$, and $f_3$ are the muscle forces and $A_1$, $A_2$, and $A_3$ are the physiological cross-sectional area PCA of the muscle.

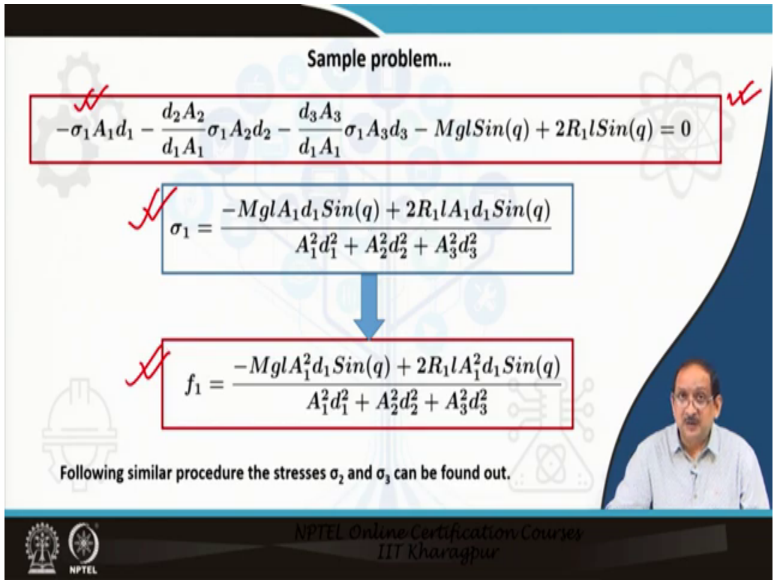
(Refer Slide Time: 21:01)




Now, taking the partial derivative of the Lagrangian function with respect to muscle stress, with respect to muscle stress and Lagrangian multiplier. We can write down the following equations. So, this is with respect to the derivative, partial derivative with respect to muscle stresses, $\sigma_1$, $\sigma_2$ and $\sigma_3$, whereas this equation is formed by taking the partial derivative of the Lagrangian function with respect to Lagrange multiplier and setting it to 0.

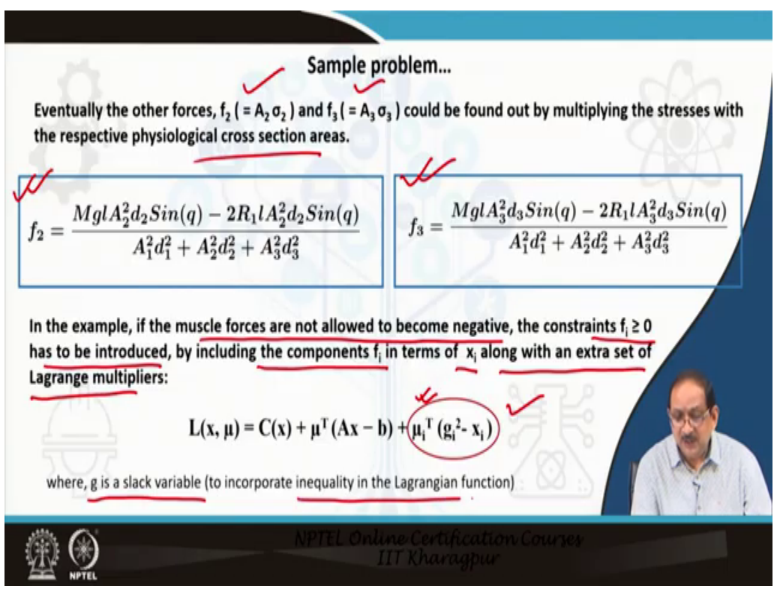
So, from the three partial derivatives as indicated here, equating to 0, we can express $\sigma_2$, and $\sigma_3$, in terms of $\sigma_1$. And thereafter, substituting $\sigma_2$ and $\sigma_3$, in the, from these highlighted relations, in the above equation as I have indicated, we can find out $\sigma_1$.

(Refer Slide Time: 22:56)




So, we have substituted the $\sigma_2$, and $\sigma_3$ in terms of $\sigma_1$ in the equation as indicated in the earlier slide. So, we have only one unknown $\sigma_1$ in the equation, and we can solve out $\sigma_1$. So, when we multiply $\sigma_1$ with the corresponding physiological cross-sectional area of muscle $A_1$, we can easily find the muscle force $f_1$, as indicated in the slide.

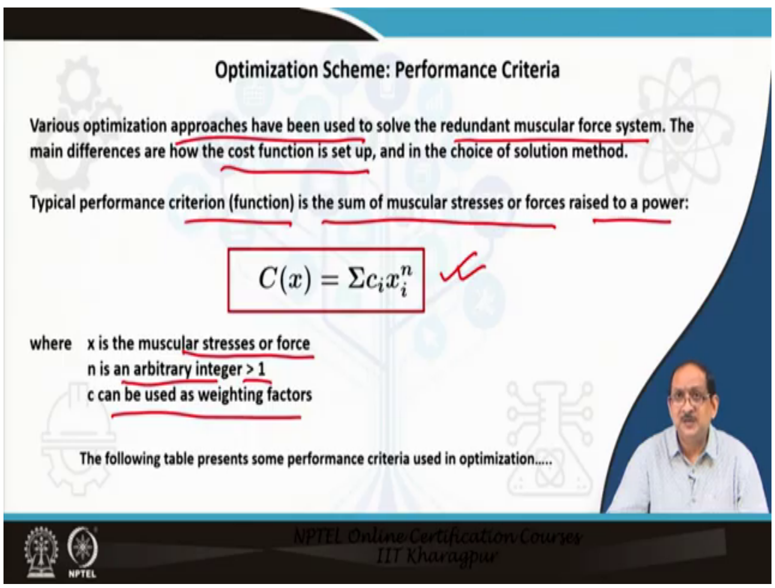
(Refer Slide Time: 23:41)



Eventually, the other forces $f_2$ and $f_3$ could be easily found out by multiplying the stresses with the respective physiological cross-sectional area. So, $f_2$ and $f_3$ has been found out as indicated in the slide.

Now, in the example, if the muscle forces are not allowed to become negative, which is an important condition, the constraints $f_i$, greater than equal to 0 has to be introduced, by including the components $f_i$ in terms of $x_i$ along with an extra set of Lagrange multipliers. This is an additional set of Lagrange multiplier presented in the slide to include the constraint condition that forces in the muscle cannot be negative. Now, in this equation, g is a slack variable, which is used to incorporate inequality in the Lagrangian function.

(Refer Slide Time: 25:28)

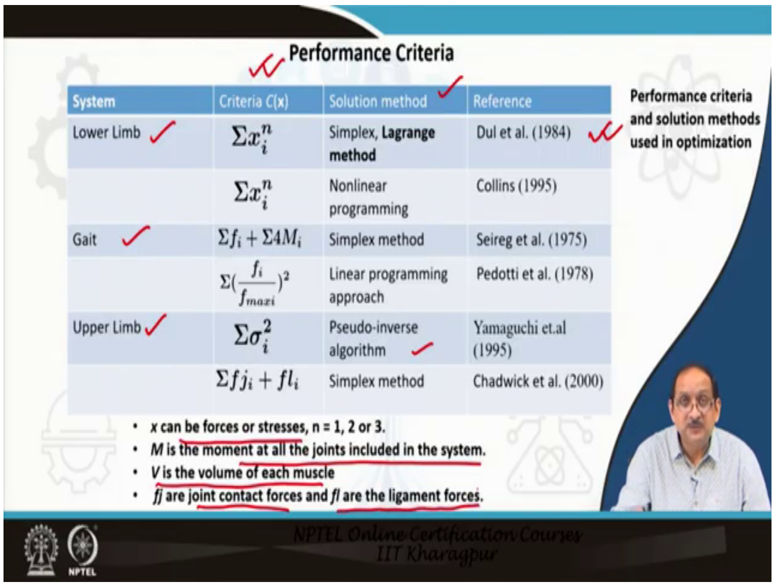



Now, in a biomechanical context and with regard to an optimization scheme, the cost functions are sometimes called performance criteria, to represent the performance, which minimizes the activation of the muscular system. The assumption is that the body selects muscles for a given activity according to the performance criteria chosen.

Various optimization approaches have been used over the years to solve the redundant muscle force, or muscular force system. The main differences are how the cost function is set up and in the choice of the solution method.

Typical performance criteria or function is the sum of muscular stresses, muscle stresses or forces raised to a power, as indicated in the slide, where x is the muscle stress or force, n is arbitrary integer greater than 1 and c can be used as a weighting factor. The table presented in the following slide will summarize different performance criteria used in different optimization studies carried out over the years.

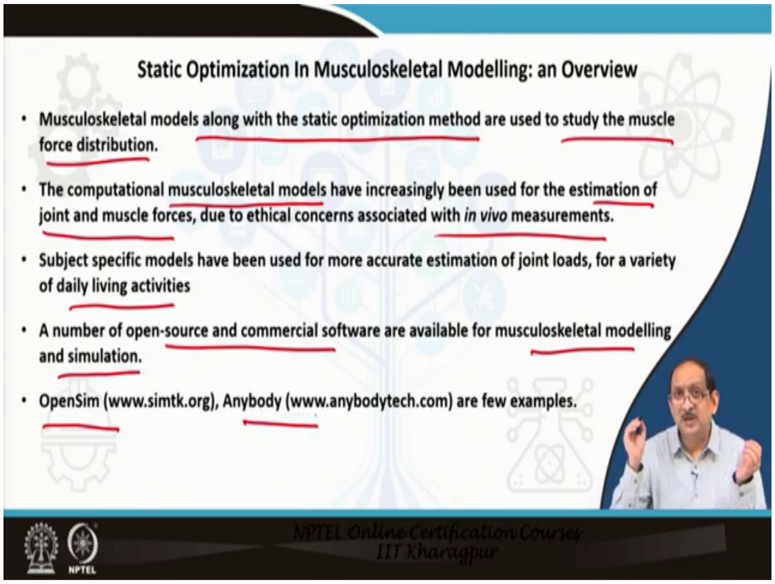
(Refer Slide Time: 27:22)




So, here we present a summary of the performance criteria and the solution methods used in the optimization. So, the system here is lower limb, gait, or upper limb, and we can see that there is a host of methods simplex, Lagrange method, non-linear programming, pseudoinverse algorithm. So, the reference of these performance criteria are indicated on the right-hand side of the table.

So, here you can see the criteria expressed in the second column, the x can be forces or stresses and n can vary from 1, 2, 3; M is the moment of all the joints included in the system, and V is the volume of each muscle, $f_j$ are joint reaction forces or joint contact forces and $f_l$ are the ligament forces.

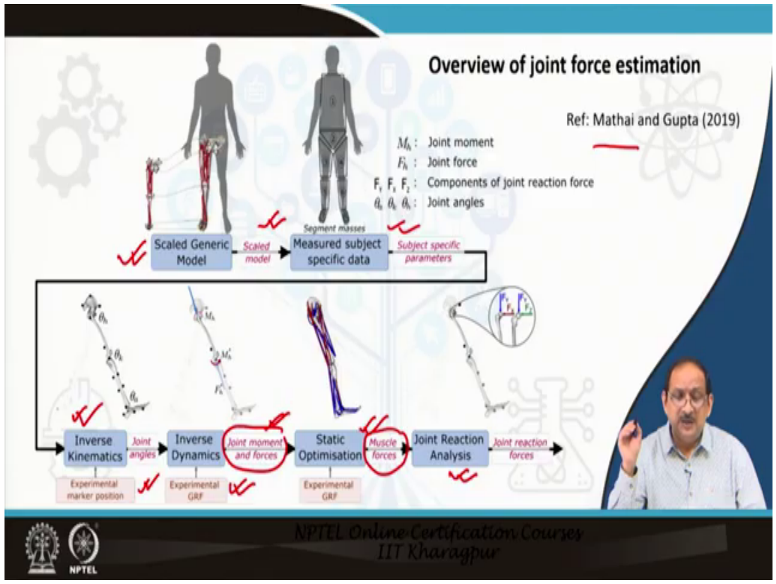
(Refer Slide Time: 28:40)




Now, let us summarize the static optimization used in musculoskeletal modeling, a few important points. So, we would like to give you an overview of this technique used in musculoskeletal modeling. The musculoskeletal models, along with the static optimization method, are used to study the muscle force distribution. The computational musculoskeletal models have increasingly been used to estimate joint and muscle forces, due to ethical problems associated with in vivo measurements, of joint contact forces and muscle forces.

Subject-specific models have been used for more accurate estimation of joint loads for various daily activities. A number of open source and commercial software are available for musculoskeletal modeling and simulation. Eminent open-source software is OpenSim and a commercial software is anybody for examples.

(Refer Slide Time: 30:21)

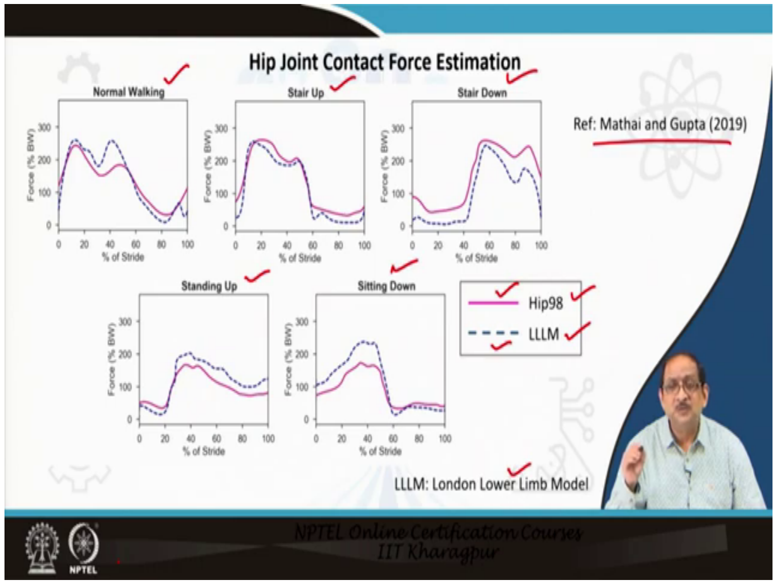



In this slide an overview of the joint force estimation has been presented, the schematic gives an overview of the joint force estimation using the musculoskeletal model. First, a generic validated musculoskeletal model was scaled to the subjects, dimensions and masses. Once the subject specific parameters are included in the model as you can see here, inverse kinematics or inverse dynamics method can be utilized to obtain kinematic information of segments from marker data as explained to you earlier.

As you can see here, ground reaction force data and kinematic information joint forces and moments can be calculated using the inverse dynamics approach. Thereafter, a static optimization procedure can be employed to estimate the individual muscle forces. Considering the joint equilibrium, the joint reaction forces can be recalculated as indicated in this schematic overview.

Now, this is a study by one of my PhD students (32:26) named Mathai and the study has been published in journal of engineering in medicine. Now, in this study, we evaluated the efficacy of few eminent musculoskeletal models regarding estimation of the hip joint reaction force.

(Refer Slide Time: 32:57)

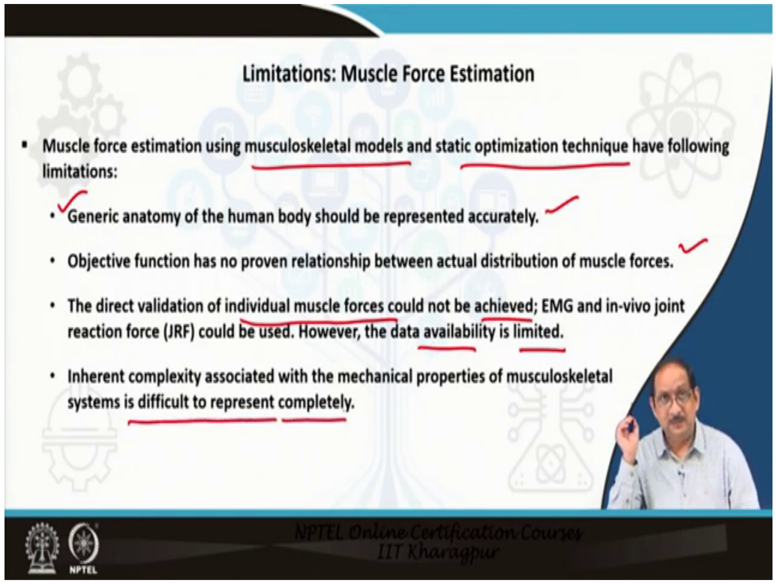



The results of one of the musculoskeletal models, the London Lower Limb Model, is presented in this slide. In this slide, the joint forces were estimated for various activities like a normal walking, stair up, stair down, standing up from a chair, and sitting down during the stride cycle, as indicated in the figures presented here.

Now, the broken blue line, represents the variation of the joint reaction force during different activities, and this result corresponds to the London Lower Limb Model, whereas the pink line corresponds to the in vivo measured data of the Hip 98 database. So, we are actually comparing the London Lower Limb Model results and the measured in vivo data.

So, the figure compares the hip joint forces estimated from the musculoskeletal model and the in vivo measured data of Hip 98. So, it is observed from the figures presented in the slide, that the hip joint force estimated using the musculoskeletal model London Lower Limb Model is very well compared to the in vivo measured force at the hip joint of Hip 98 database. Please refer to the results presented in the paper as indicated in the slide for more details on joint reaction forces and muscle forces for different activities.

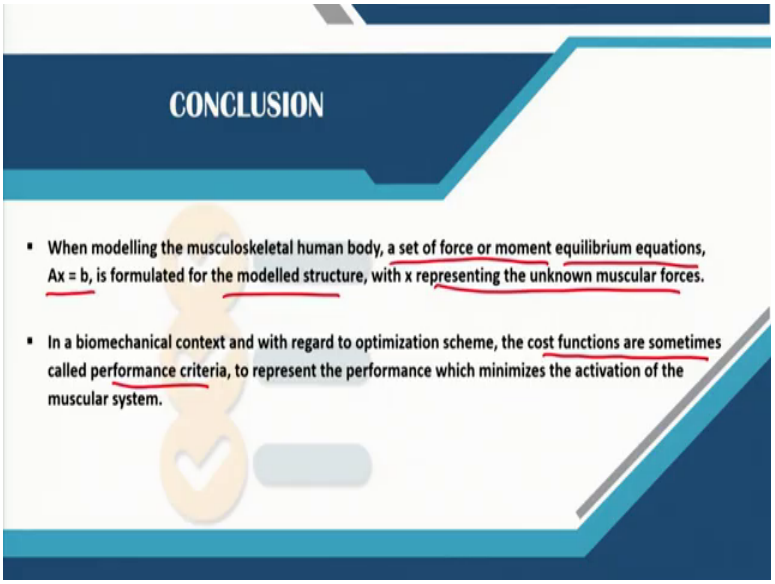
(Refer Slide Time: 35:25)




Let us now, state the limitations of this muscle force estimation method. So, muscle force estimation using musculoskeletal models and static optimization techniques have the following limitations. The generic anatomy of the human body should be represented accurately, as far possible.

Objective function has no proven relationship between the actual distribution of muscle forces. The direct validation of individual muscle forces could not be achieved, EMG and in vivo joint reaction forces could be used for validation, however generally the data availability is very limited. Inherent complexity associated with the mechanical properties of musculoskeletal systems is difficult to represent completely.

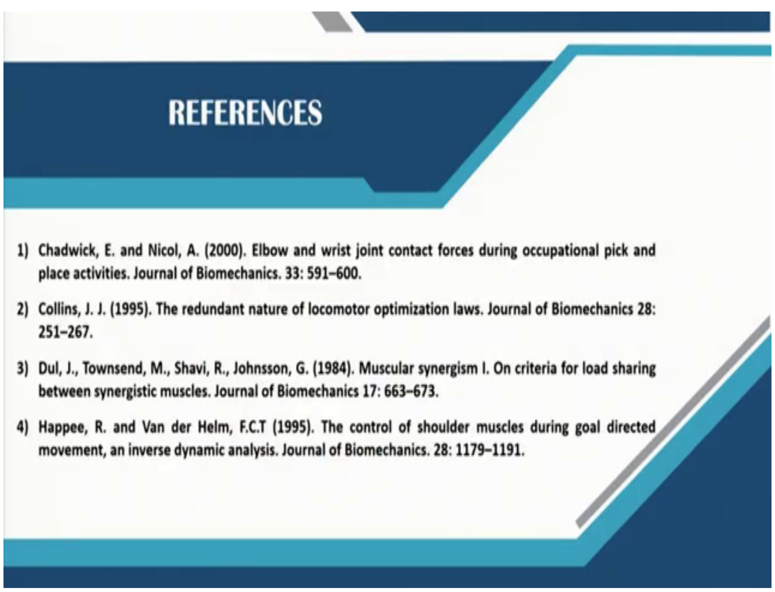
(Refer Slide Time: 36:38)

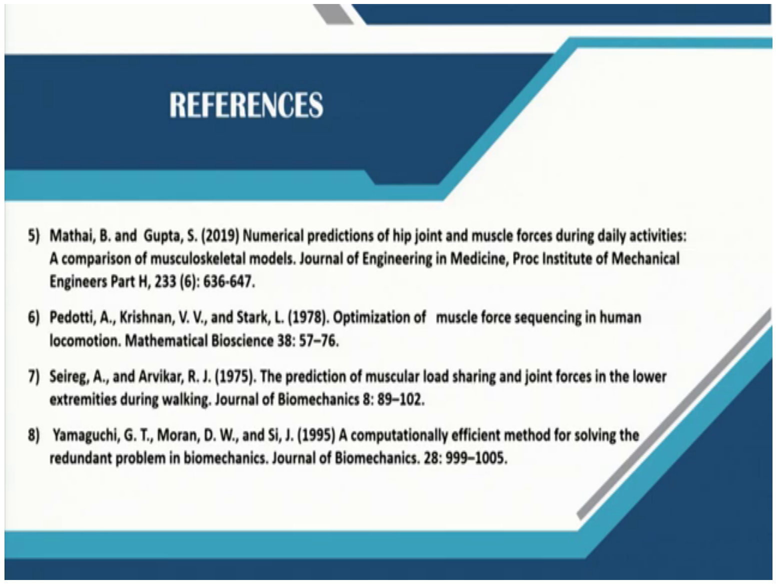



Let us come to the conclusions of the study. When modeling the human musculoskeletal body, a set of force or moment equilibrium equations in the form Ax = b can be formulated for the model structure, with x representing the unknown muscular forces.

In biomechanical context and with regard to optimization technique, the cost functions are sometimes called performance criteria, to represent the performance, which minimizes the activation of the muscular system.

(Refer Slide Time: 37:19)






The references are listed here in two slides, based on which the lecture was prepared. Thank you for listening.