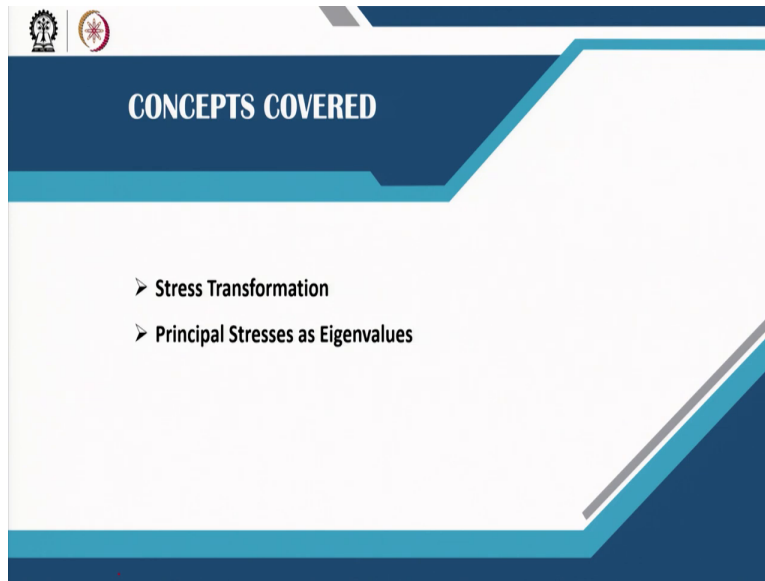


**Biomechanics of Joints and Orthopaedic Implants**  
**Professor. Sanjay Gupta**  
**Department of Mechanical Engineering**  
**Indian Institute of Technology, Kharagpur**  
**Lecture No. 25**  
**Stress Transformation**

Good morning everybody, welcome to the lecture on stress transformation.

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In this lecture, we will be discussing about the following topics the stress transformation and principal stresses as Eigenvalues.

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**Stress Transformations**

- Consider a stress element, which is rotated by an angle  $\theta$ .
- Can we use the transformation rule to obtain the new normal and tangential stresses?
- Stress components depend on the reference coordinate system.
- Stresses aren't vectors, so they can't be resolved using the vector transformations.

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Based on the concept discussed in the earlier lecture, we restate that stress components acting at a point will depend on the orientation of the area on which it acts. So, let us consider a square stress element as shown in the figure on the left, which represents the state of stress acting at a point.

Considering this stress element, we apply a rotation through an angle  $\theta$  as shown in the figure on the right. Can we use the transformation rule to obtain the new stress components or the new normal and tangential stresses? The answer is yes. The stress components depend on the reference coordinate system.

Please note that stresses are not vectors. So, they cannot be resolved using vector transformations, and we need to multiply the stress components with the corresponding area on which it is acting to convert it into force vectors, which can be eventually resolved and transformed using the vector transformation.

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### Stress Transformations

We have to account for:

- Magnitude ✓
- Direction ✓
- the orientation of the area ✓

Note: The same state of stress is represented by a different set of stress components, if the reference axes are rotated.

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Now, to find out the stress components corresponding to a reference system, we have to account for the magnitude of the stress, the direction of the stress components and the orientation of the area on which it is acting. Now, we define a cutting plane as shown here inclined at an angle  $\theta$  equal to the stress element's angle of rotation or the coordinate system's angle of rotation. So, please note that a different set of stress components represents the same state of stress if the reference axis are rotated.

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### Stress Transformation - equations

- The stress transformation is a way to describe the effect of combined loading on a stress element at any orientation.
- From geometry and equilibrium conditions ( $\Sigma F = 0$  and  $\Sigma M = 0$ ),

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos(2\theta) + \tau_{xy} \sin(2\theta)$$

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2} \sin(2\theta) + \tau_{xy} \cos(2\theta)$$

$$\sigma_{y_1} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos(2\theta) - \tau_{xy} \sin(2\theta) = \sigma_{x_1} (\theta + 90^\circ)$$

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Now, let us find out the equations of the set of stress components corresponding to a new orientation or transformation. Now, the stress transformation is a way to describe the effect of combined loading on a stress element at any orientation. So, from geometry and equilibrium conditions, we can find the equations of the normal stresses  $\sigma_{x1}$ ,  $\sigma_{y1}$  corresponding to the new coordinate system  $x1$ ,  $y1$  and the shear stress acting on the plane.

To do that, we need to multiply the various stresses by the areas of the faces.

So, we multiply the various stresses by the areas of the phases on which they are acting, and then the forces on the triangular element can be identified after that, considering force equilibrium along  $x1$ ,  $y1$ , we can find out the new stress components corresponding to the new coordinate system.

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**Stress Transformation: Matrix Form**

Problem: The state of stress is given in the following

State of stress in the transformed new coordinate system,  $[\sigma'] = [R][\sigma][R]^T$

Where the rotational matrix,  $[R] = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$

Here  $\theta$  is measured in the anticlockwise direction. The angle between the new and old coordinate systems.

Write the stress in the new coordinate system:  $[\sigma'] = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} [\sigma] \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$

MPa

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The stress transformation can also be expressed in matrix form as indicated here a state of stress in the transform new coordinate system can be found out by using the rotational matrix R and employing the transformation rule. So, R, the rotational matrix, is given here in the slide, where the  $\theta$  is measured in the anti-clockwise direction, and it is the angle between the new and the old coordinate system.

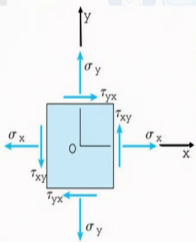


So, the transformation rule for obtaining the new stress components is that we need to pre multiply the old stress components by the rotational matrix and post multiply by the transpose of the rotational matrix to obtain a new stress component. So, the same thing is expressed here:  $\sigma'$  is equal to the rotational matrix multiplied by the old stress tensor, and the rotational matrix transpose.

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**Stress Transformations – Sample Problem**

Problem: The state of stress is given in the following



Stress  
Tensor

$$\begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{bmatrix} = \begin{bmatrix} 22 & 12 \\ 12 & -7 \end{bmatrix} \text{ MPa}$$

Write the stress tensor in the coordinate system which makes 45° with the current coordinate system:

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Now, let us discuss a problem, a state of stress is given.

So, we formed the matrix. So, write the stress tensor in the coordinate system, making 45° with the current coordinate system. So, we have the current stress tensor, and we would like to transform it to a new stress tensor when the coordinate system is rotated


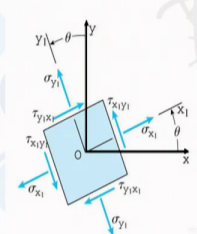
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### Stress Transformations – Sample Problem

State of stress in the transformed new coordinate system,  $[\sigma'] = [R][\sigma][R]^T$

$$[\sigma'] = \begin{bmatrix} \cos(45) & \sin(45) \\ -\sin(45) & \cos(45) \end{bmatrix} [\sigma] \begin{bmatrix} \cos(45) & -\sin(45) \\ \sin(45) & \cos(45) \end{bmatrix}$$
$$[\sigma'] = \begin{bmatrix} 0.707 & 0.707 \\ -0.707 & 0.707 \end{bmatrix} \begin{bmatrix} 22 & 12 \\ 12 & -7 \end{bmatrix} \begin{bmatrix} 0.707 & -0.707 \\ 0.707 & 0.707 \end{bmatrix}$$

Using the rotational matrix and transformation rule, the new stress tensor is,

$$[\sigma'] = \begin{bmatrix} \sigma_{x_1} & \tau_{x_1 y_1} \\ \tau_{y_1 x_1} & \sigma_{y_1} \end{bmatrix} = \begin{bmatrix} 19.494 & -14.49 \\ -14.49 & -4.498 \end{bmatrix} \text{ MPa}$$


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So, we substitute the value of  $\theta$  in this equation and then find out the value of the new stress tensor using the rotational matrix and the transformation rule. So, the new stress tensor is given by  $\sigma'$  and the values are presented in this slide.

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### Principal Stresses and Maximum Shear Stress

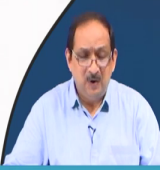
If we are interested in evaluating the failure in a material then we need to determine the following:

- maximum and minimum normal stresses
- maximum shear stress
- orientation ( $\theta$ ) at which these occur

$$\frac{d\sigma_n}{d\theta} = 0$$

These are called the principal stresses ( $\sigma_1, \sigma_2$ ) and maximum shear stress ( $\tau_{max}$ ).

The equations for these can be found from the stress transformation, by differentiation  $\left(\frac{d\sigma_{x_1}}{d\theta} = 0\right)$  and using algebraic operations.

$$\sigma_n$$


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Let us discuss principal stresses and maximum shear stress. If we are interested in evaluating failure in a material, we need to determine the maximum and minimum values of normal stresses

the maximum shear stress and the orientation or the direction or the plane at which these stresses are acting.

Now, the maximum and minimum normal stresses are known as the principal stresses;  $\sigma_1$  and  $\sigma_2$  and the maximum shear stress is known as  $\tau_{\max}$ . The equations for this can be found out using stress transformation as discussed earlier. So, we take the equations of the  $\sigma_x$  and  $\sigma_y$  and  $\tau_{xy}$ , and we differentiate the terms to determine the conditions for which these stresses can be maximum or minimum.

And after that, we use that condition through algebraic operations. We find the expressions for the principal stresses  $\sigma_1$  and  $\sigma_2$  and the maximum shear stresses. Now, to find out the location of the plane of maximum normal stress say for example,  $\sigma_x$  we actually need to take the derivative of the normal stress and equated to zero and then from the algebraic equation, we can actually determine the condition for which the normal stress is maximum or minimum.

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**Principal Stresses and Planes**

The slide illustrates the transformation of a stress element from a standard Cartesian coordinate system (x, y) to a principal coordinate system (x', y'). The original element is subjected to normal stresses  $\sigma_x$  and  $\sigma_y$  and shear stresses  $\tau_{xy}$ . The principal stresses  $\sigma_1$  and  $\sigma_2$  are shown acting on planes oriented at an angle  $\theta_p$  to the original axes. The angle  $\theta_p$  is defined as the angle of the principal stresses.

$\theta_p = \text{planes of principal stresses}$

$\theta_p = \theta_{p1}, \theta_{p2}, 90^\circ \text{ apart}$

No shear stress acts on the principal planes

The conditions defines two values of  $2\theta_p$  differing by  $180^\circ$  and hence, two values of  $\theta_p$  differing by  $90^\circ$

Equations for principal stresses:

$$\tan(2\theta_p) = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

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So, following that procedure, we can actually find the angle in terms of  $\tan 2\theta$  at which the normal stresses can be maximum or minimum so, once we have determined that, we can put the value back in the equation and find out the maximum and minimum normal stresses, which

corresponds to the principal stresses  $\sigma_1$  and  $\sigma_2$  and this is given by the following expression as indicated in the slide.

The planes of the principal stresses are  $90^\circ$  apart. So, you can see  $\sigma_1$  and  $\sigma_2$  is 90 degree apart.

Now, if I substitute this value, the plane's orientation in the equation of the shear stresses, we see that shear stress at this plane orientation of the plane comes out to be zero. So, no shear stress acts on the principal planes, which are the planes on which the principal stress is act. So, principal planes are the orientation on which the principal stresses are acting, but no shear stresses are acting on the principal planes.

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**Maximum Shear Stress and Direction**

$$\tan(2\theta_s) = -\frac{(\sigma_x - \sigma_y) / 2}{\tau_{xy}}$$

$$\tau_{\max ip} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{\max} = \frac{|\sigma_1 - \sigma_2|}{2}$$

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2}$$

$\theta_s = \text{planes of max shear stress}$   
 $\theta_s = \theta_{s1}, \theta_{s2}, 90^\circ \text{ apart.}$   
 $\theta_{s1} = \theta_p + 45^\circ \text{ and } \theta_{s2} = \theta_p - 45^\circ$   
 $\tau_{\max ip} = \text{max in-plane shear stress}$

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Let us, now find out maximum shear stress and its direction following the similar procedure as done in case of the normal stress, we take a derivative of the shear stress and equate it to 0 to find the condition for which the shear stress can be maximum. So, it comes out to be  $\tan 2\theta$  equal to an expression consisting of  $\sigma_x$ ,  $\sigma_y$  and  $\tau_{xy}$  and if we substitute this expression in the original equation of the shear stress.

We can actually find the maximum shear stress given by the expression in the slide. So, the maximum shear stress actually can be related to the principal stresses as well.

So,  $\tau_{\max} = (\sigma_1 - \sigma_2) / 2$ .

Now, in the case of the maximum shear stress, we have obtained the directions given by the expression here. Now, when we compare this orientation of the maximum shear stress with the maximum normal stress used to earlier, it is observed that the maximum shear stresses occur on orthogonal planes, bisecting the angle between the principal planes at 45 degrees to the planes of principal stresses.

So, you can see here that the planes on which the shear stresses are maximum are basically at 45 degrees with the principal planes and the  $\tau_{\max \text{ IP}}$  as given here, is the maximum in-plane shear stress.

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**Principal Stresses as Eigenvalues**

- For a stress tensor, there exists at least one orientation in which  $\sigma_{ij} = 0$  for  $i \neq j$ , and  $\sigma_{ii} \neq 0$  for  $i=j$  (i.e., stress components are non-zero only on the principal diagonal).
- The directions corresponding to this stress state are called principal stress directions. Consider a principal stress direction corresponding to a unit vector with directions  $n_1, n_2$  and  $n_3$ .

The slide features a blue header with the title, a white background with faint icons of gears, a tree, and a molecular structure, and a video inset of a man in a light blue shirt speaking. The bottom of the slide has the NPTEL logo and the text 'NPTEL Online Certification Courses IIT Kharagpur'.

Let us, now come to the second topic that is principal stresses as Eigenvalues. Now, for a stress tensor, there exists at least one orientation in which  $\sigma_{ij} = 0$ , for which  $i$  is not equal to  $j$  and  $\sigma_{ii}$  is not equal to 0 for  $i=j$ , that means, that stress components are nonzero only on the principal diagonal if we consider the stress tensor the directions corresponding to this stress state are called principal directions. So, we can consider a principal stress direction corresponding to a unit vector with directions  $n_1, n_2$  and  $n_3$ .

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**Principal Stresses as Eigenvalues**

- The normal stress,  $\sigma$  is an **Eigenvalue** of the stress tensor, while the unit vector corresponding to the normal stress,  $n$ , is an **Eigenvector**.
- Since the stress tensor is  $3 \times 3$  (for 3D), there are **three Eigenvalues (the principal stresses)** and **three corresponding Eigenvectors (the principal stress directions)**.
- The plane normal to this unit vector is called a **principal stress plane**. Projecting the stress tensor onto the **principal stress plane results in a principal stress**.

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = \sigma \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$$

Eigenvalue Problem Representation

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The normal stress  $\sigma$  is an Eigenvalue of the stress tensor while the unit vector corresponding to the normal stress  $n$  is the Eigenvector very important the normal stress  $\sigma$  is an eigenvalue and the unit vector or directions corresponding to the normal stress  $n$  is an Eigenvector. But, if we consider a 3D problem, then the stress tensor is expressed as 3 by 3 matrix.

So, there are 3 Eigenvalues: the principal stresses and 3 corresponding Eigenvectors: the directions of the principal stresses. The plane normal to this unit vector is called the principal stress plane projecting the stress tensor onto the principal stress plane results in principal stresses. So, eigenvalue problem can be represented as shown in this slide.

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**Nontrivial solution: Characteristic equation**

The only nontrivial solution can be obtained by setting the determinant of the coefficients of  $n_1, n_2$  and  $n_3$  equal to 0.

The condition for a nontrivial solution to exist is given by the so-called characteristic equation:

$$\begin{vmatrix} (\sigma_{11} - \sigma) & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & (\sigma_{22} - \sigma) & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & (\sigma_{33} - \sigma) \end{vmatrix} = 0$$

Solution of the determinant results in a cubic equation that can be solved for the 3 roots ( $\sigma_1, \sigma_2, \sigma_3$ ), corresponding to the 3 principal stresses.



Now, let us discuss about the non-trivial solution and the characteristic equation. The only non-trivial solution can be obtained by setting the determinant of the coefficients of  $n_1, n_2, n_3$  equal to 0. So, we need to set the determinants of the coefficients of  $n_1, n_2$  and  $n_3 = 0$ ; the condition of a non-trivial solution to exist is given by the so, called characteristic equation, which is represented in the slide, solution of the determinant results in a cubic equation that can be solved for the 3 roots:  $\sigma_1, \sigma_2, \sigma_3$  corresponding to the 3 principles stresses.

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**Stress Invariants**

In the characteristic equation:

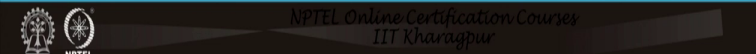
$$\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0$$

the coefficients of the cubic equation, are known as the stress invariants ( $I_1, I_2, I_3$ ).

Since the values of these coefficients determine the principal stresses, they do not vary with changes in the coordinate axes.

$$I_1 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz}$$

$$I_2 = \begin{vmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{xy} & \sigma_{yy} \end{vmatrix} + \begin{vmatrix} \sigma_{yy} & \tau_{yz} \\ \tau_{yz} & \sigma_{zz} \end{vmatrix} + \begin{vmatrix} \sigma_{xx} & \tau_{xz} \\ \tau_{xz} & \sigma_{zz} \end{vmatrix}$$

$$I_3 = \begin{vmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_{yy} & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{vmatrix}$$




Now, we define stress invariants by considering the characteristic equation that is formed by taking the determinant. So, the characteristic equation is represented by a cubic equation of the

$$\sigma^3 - I_1 \sigma^2 + I_2 \sigma - I_3 = 0$$

where the coefficients of the cubic equation  $I_1$ ,  $I_2$ ,  $I_3$  are known as the stress invariants. Since the values of this coefficient determine the principal stresses, they do not vary with the changes in the coordinate axis.

So, the stress and invariants do not vary with the changes in the coordinate axis. Now, first invariant  $I_1$ , second invariant,  $I_2$  and the third invariant,  $I_3$  is represented as shown in the slide. The first invariant  $I_1$  is shown here as some of the normal stresses. Now, it defines a useful relationship that the sum of the normal stresses for any orientation in the coordinate system is equal to the sum of the normal stresses for any other orientation.

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**Principal Directions**

The principal directions can be solved by substituting separately each of the principal stresses  $\sigma_1, \sigma_2, \sigma_3$  each into the set of three linear equations and solving for the direction cosines  $(n_1, n_2, n_3)$  subject to the condition that the sum of the squares of the direction cosines is equal to 1.

$$\begin{bmatrix} \sigma_{11} - \sigma & \tau_{12} & \tau_{13} \\ \tau_{12} & \sigma_{22} - \sigma & \tau_{23} \\ \tau_{13} & \tau_{23} & \sigma_{33} - \sigma \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = 0$$

$$n_1^2 + n_2^2 + n_3^2 = 1$$

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The principal directions can be solved by substituting the principal stresses separately:  $\sigma_1, \sigma_2, \sigma_3$  into the set of 3 linear equations formed from this eigenvalue problem and solving for the direction cosines  $n_1, n_2$  and  $n_3$  subject to the condition that the sum of the squares of the direction cosines is equal to 1 as stated here in the slide. There are several methods to solve the system of linear equations like Gauss elimination, which could solve  $n_1, n_2$  and  $n_3$ .





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### Sample Calculations: Eigenvalue Problem

The following sample problem is solved using the concept of eigenvalue problem.

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_{yy} & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{bmatrix} = \begin{bmatrix} 22 & 12 & -15 \\ 12 & -7 & 3 \\ -15 & 3 & -17 \end{bmatrix} \text{ MPa}$$

- Step 1: Finding the coefficients of the characteristic equation:  $\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0$

$$I_1 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz} = 22 + (-7) + (-17) = -2$$

$$I_2 = \begin{vmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{xy} & \sigma_{yy} \end{vmatrix} + \begin{vmatrix} \sigma_{yy} & \tau_{yz} \\ \tau_{yz} & \sigma_{zz} \end{vmatrix} + \begin{vmatrix} \sigma_{xx} & \tau_{xz} \\ \tau_{xz} & \sigma_{zz} \end{vmatrix} = \begin{vmatrix} 22 & 12 \\ 12 & -7 \end{vmatrix} + \begin{vmatrix} -7 & 3 \\ 3 & -17 \end{vmatrix} + \begin{vmatrix} 22 & -15 \\ -15 & -17 \end{vmatrix} = -787$$

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### Sample Calculations: Eigenvalue Problem

$$I_3 = \begin{vmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_{yy} & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{vmatrix} = \begin{vmatrix} 22 & 12 & -15 \\ 12 & -7 & 3 \\ -15 & 3 & -17 \end{vmatrix} = 5363$$

- Step 2: On substituting the coefficients into the characteristics equations, we get the roots (eigen values) as:  $\sigma_1 = 30.085$ ,  $\sigma_2 = -7.149$ ,  $\sigma_3 = -24.936 \text{ MPa}$
- Step 3: To find the principal plane corresponding to the 1<sup>st</sup> principal stress,

$$\begin{bmatrix} 22 - 30.085 & 12 & -15 \\ 12 & -7 - 30.085 & 3 \\ -15 & 3 & -17 - 30.085 \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = \begin{bmatrix} -8.085 & 12 & -15 \\ 12 & -23.085 & 3 \\ -15 & 3 & -13.085 \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = 0$$

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Let us, now undertake some sample calculations on an eigenvalue problem. The following sample problem is solved using the concept of the eigenvalue problem. So, a 3D state of stress is given here. So, the first step is finding the coefficients of the characteristic equation which is stated here.

So, from this characteristic equation, we can identify  $I_1$ ,  $I_2$  and  $I_3$ , and we can actually go back to the earliest slide. So, I can find the values of  $I_1$ ,  $I_2$  by substituting the values from the givens

stress tensor. I can also find out the value of  $I_3$  by substituting the values of the stress components.

Now, let us come to the second step on substituting the coefficients in the characteristic equations, we can obtain the roots or the Eigenvalues, we can obtain  $\sigma_1$  we can obtain  $\sigma_2$  and we can obtain  $\sigma_3$ . So, this is an important step where we determine the principal stress values.

The next step, number 3 is to find out the directions to find out the principal plane corresponding to the first principal stress, say  $\sigma_1$ . So, in the eigenvalue problem, we substitute the  $\sigma_1$  value and find out the orientation of the principal plane. So, we set the left-hand side of the eigenvalue problem equal to 0.

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Sample Calculations: Eigenvalue Problem

• Step 4: On solving the system of equations and imposing the constraint\* discussed previously, we get the principal plane's normal vector as:

$$\begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = \begin{bmatrix} -0.921 \\ -0.276 \\ 0.276 \end{bmatrix}$$

\*  $n_1^2 + n_2^2 + n_3^2 = 1$

Similarly, the principal plane normal vectors can be found out for the other principal stress values.

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So, in step number 4 we are solving the system of equations imposing the constraint that is the sum of the squares of the direction cosines is equal to 1.

$$\text{So, } n_1^2 + n_2^2 + n_3^2 = 1$$

that has been discussed previously. So, solving the system of equations with these constraints allows us to get the principle planes normal vector as a set of  $n_1, n_2, n_3$ .

Similarly, if we substitute  $\sigma_2$  in the eigenvalue problem, we will get another set of  $n_1, n_2, n_3$ .and we can also get another set of  $n_1, n_2, n_3$  corresponding to  $\sigma_3$ . So, there are several methods to

solve this system of linear equations like the Gauss elimination method, which could be used to solve for  $n_1$ ,  $n_2$  and  $n_3$ .

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**Maximum Shear Stresses**

- The maximum shear stresses is equal to one half of the difference of the principal stresses and it occur at 45 degree angles with respect to the principal stress planes.
- $\sigma_1, \sigma_2$  and  $\sigma_3$  are the principal stresses.

$$\tau_1 = \frac{\sigma_2 - \sigma_3}{2}$$
$$\tau_2 = \frac{\sigma_1 - \sigma_3}{2}$$
$$\tau_3 = \frac{\sigma_1 - \sigma_2}{2}$$

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Now, the maximum shear stress is equal to 1 half of the difference of the principal stresses and we have found this out earlier and it occur at  $45^\circ$  angle with respect to the principal stress plane. So,  $\tau_1, \tau_2$  and  $\tau_3$  can be determined using the principal stresses as shown here in the slide. So, at  $\sigma_1$  and  $\sigma_2$  and  $\sigma_3$  are the principal stresses.

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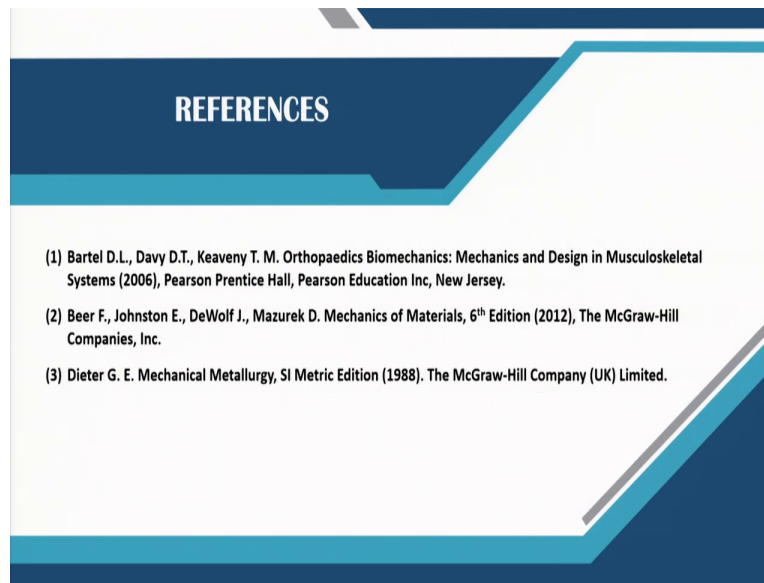
**CONCLUSION**

- Stress components at a point can be transformed to a new set of stresses corresponding to a new reference system using the transformation rule and the rotation matrix.
- Principal stresses are the maximum and minimum normal stresses at the point.
- Principal stresses act on principal planes (orientations), where there are no shear stresses.
- The maximum shear stresses is equal to one half of the difference of the principal stresses.
- Stress invariants do not vary with changes in the coordinate axes.

Let us conclude this lecture that stress components at a point can be transformed to a new set of stresses corresponding to a new reference system using the transformation rule and the rotation

matrix. Principal stresses are maximum and minimum normal stresses acting at a point the principal stresses act on principal planes orientation where there are no shear stresses. The maximum shear stress is equal to 1 half of the difference of the principal stresses, and these stress invariants do not vary with changes in the coordinate system or coordinate axis.

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The references are listed here based on which the lecture has been prepared, and thank you for listening.