NPTEL Online Certification Courses

COLLABORATIVE ROBOTS (COBOTS): THEORY AND PRACTICE

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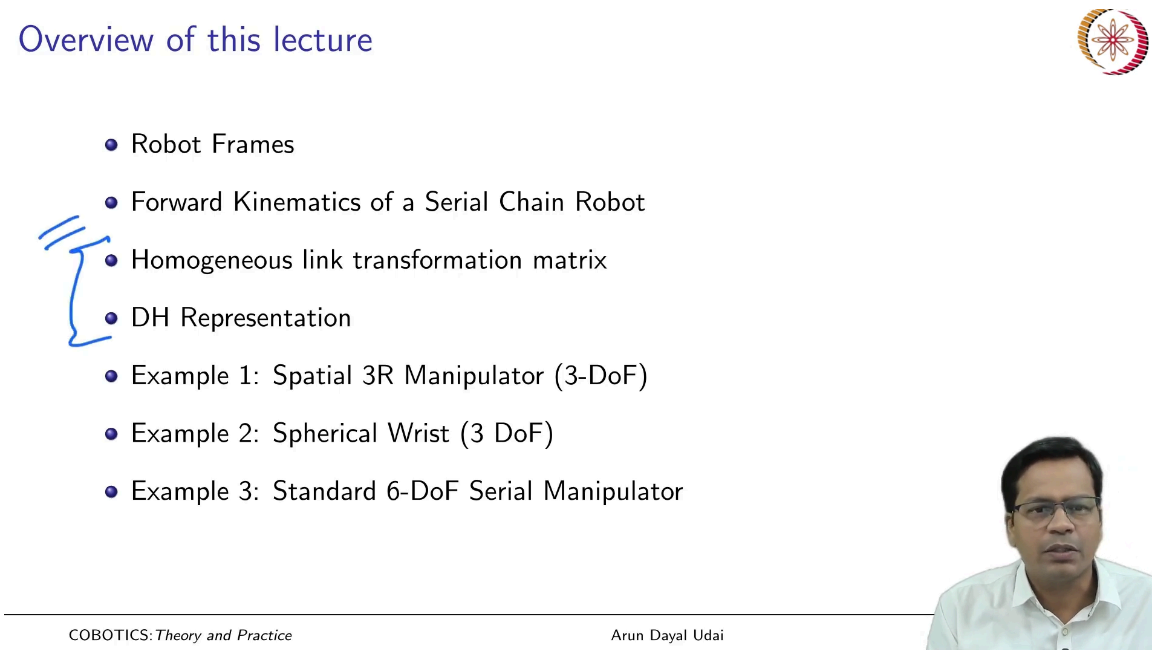
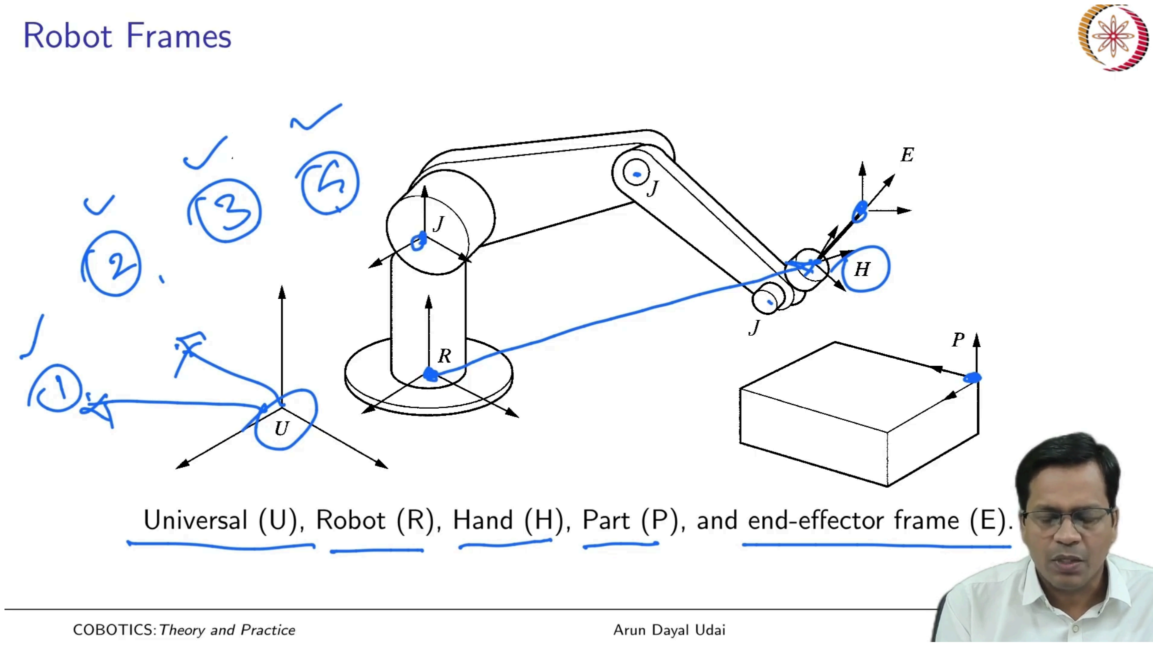
Week: 03
Lecture: 13

Robot Frames, DH Parameters, Link Transformation Matrix, and Forward Kinematics"



Welcome to the third lecture of week three, which is about Transformation Matrices and Robot Kinematics. In this lecture, we will discuss robot Frames, Forward Kinematics of Serial Chain Robots, Homogeneous Link Transformation Matrix and DH Representation, which are essential parts of performing forward kinematics for a serial chain robot. We will discuss these through examples. In Example 1, we will discuss a Spatial 3R Manipulator, which is a 3-degree-of-freedom manipulator. We will discuss the Spherical Wrist, which is also a 3-DoF manipulator. This is part of the standard industrial robot most of the time, as well as in cobots. In Example 3, I will discuss a standard 6-DOF industrial serial robot, which combines both a 3-degree-of-freedom standard spatial manipulator and a spherical manipulator.

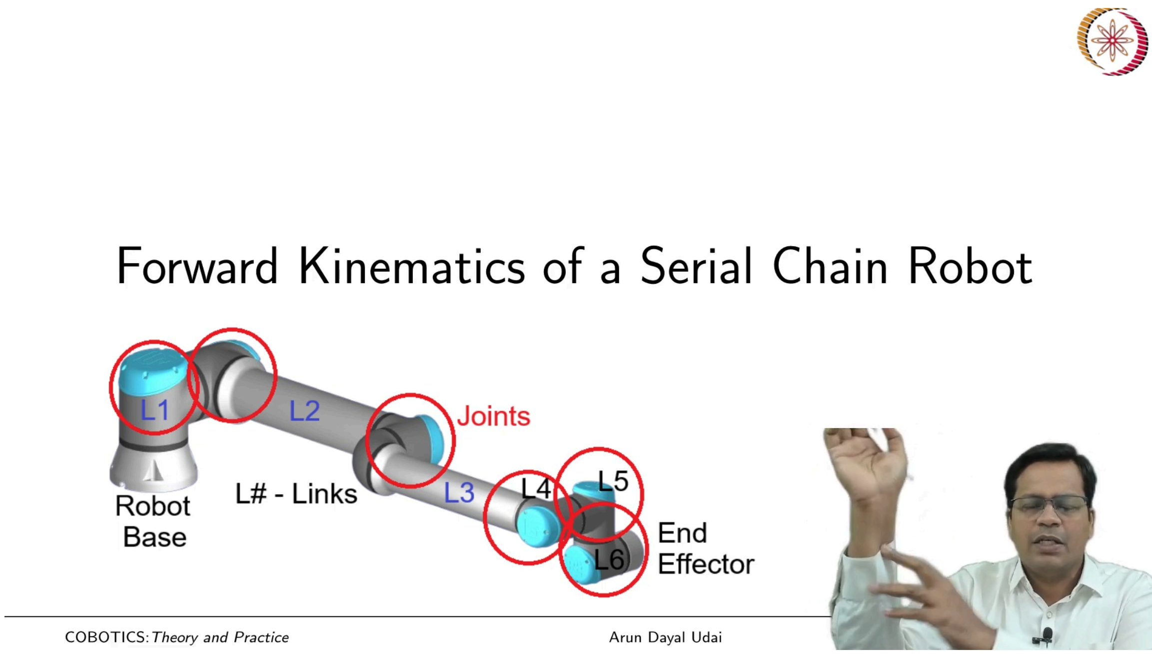


Now, let us move ahead with Robot Frames. A robot consists of multiple frames. The robot itself lies in a workspace, and the workspace has its own frame, known as the universal frame. You may have multiple robots sitting in the same workspace, each of which can be individually addressed with reference to this particular frame. This is the universal frame. You can have the first robot sitting here, the second robot, the third robot, and the fourth robot. So, the base of each robot can be addressed with respect to this frame. Now, the robot itself has a frame. Those were these.

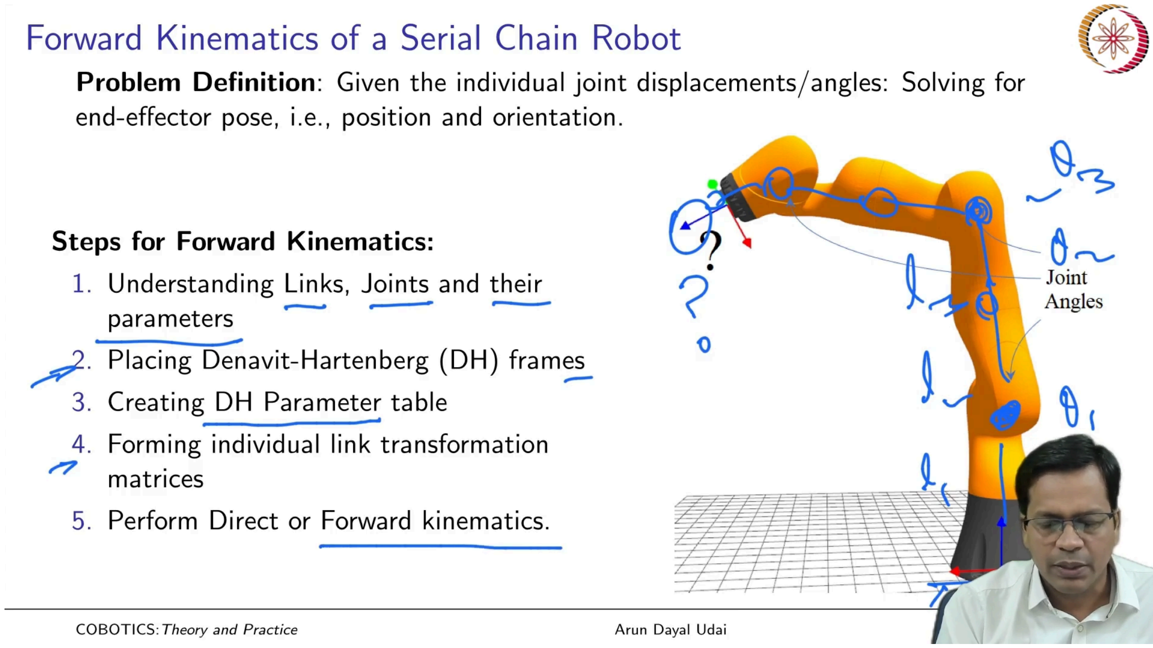
So, this robot frame has all the links, the joints and its end effector, each and every parameter is represented with respect to this frame because this frame is attached to the ground where the whole robot is mounted. Normally, it is sitting at the bottom of the robot at its centre, about which you measure the end effector position, where all the joints are located. So, that is known as a robot frame or robot base frame.

And you also have a hand frame. Where is your hand frame? The hand frame comes here on which you put your tool, okay? So, that is just like the flange of the robot on which you mount your tool. That is also known as a hand frame. You have a part frame, which is any object lying in the workspace. It may have its own frame, which is also known as an object that the robot is handling. So, this has its own frame. This may be a solid object so that this frame can represent the position and orientation of the object with respect to the universal frame, or it can also be with respect to the robot frame.

The final frame is the end effector frame, which is at the tip of the tool. Let us say if it is a welding tool, it becomes the tip of the welding electrode, which actually does the job. So, that is the end effector frame. Apart from these, there may be many other frames that are helping us to do the kinematics, dynamics, controls, and many other things. One such frame is a joint frame. This is the axis about which the links of the robot move. That is known as a joint frame. We will talk about each one of them precisely when it comes.

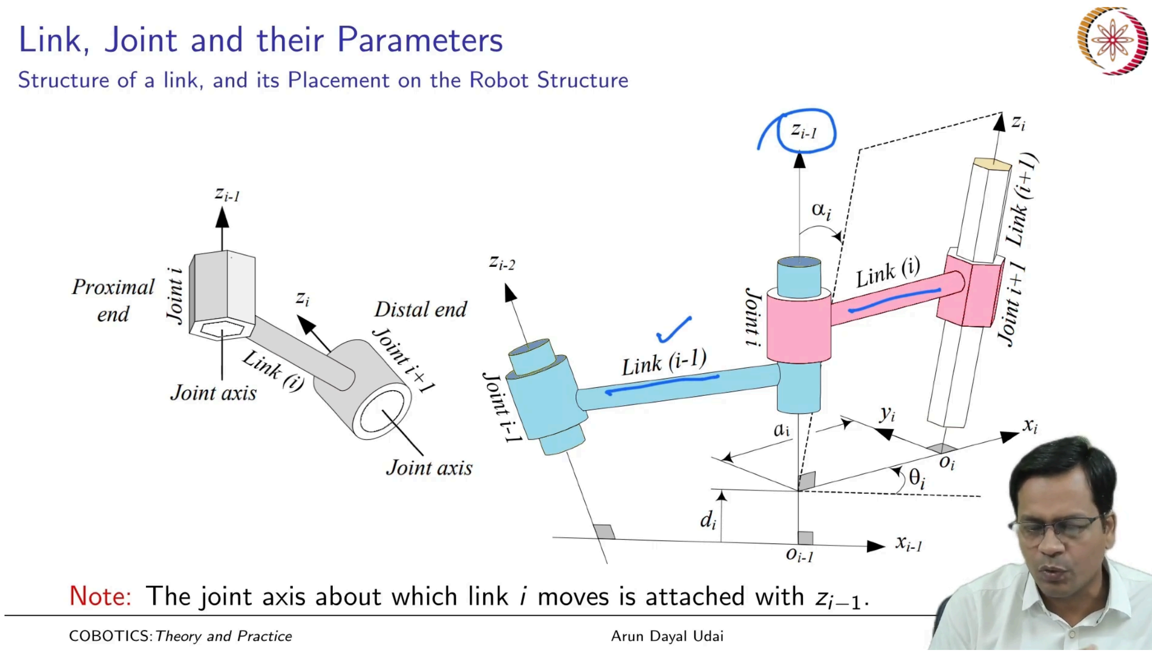


So, let us begin with the Forward Kinematics of a standard serial chain robot. What is a serial chain? Serial chain robots are made in a way that they have a fixed base, and links come one after the other until it terminates. In between, you have joints that connect two consecutive links. So, you have a link, you have a joint, you have a link, you have a joint, and finally, it has the end effector, which terminates or may hold something.



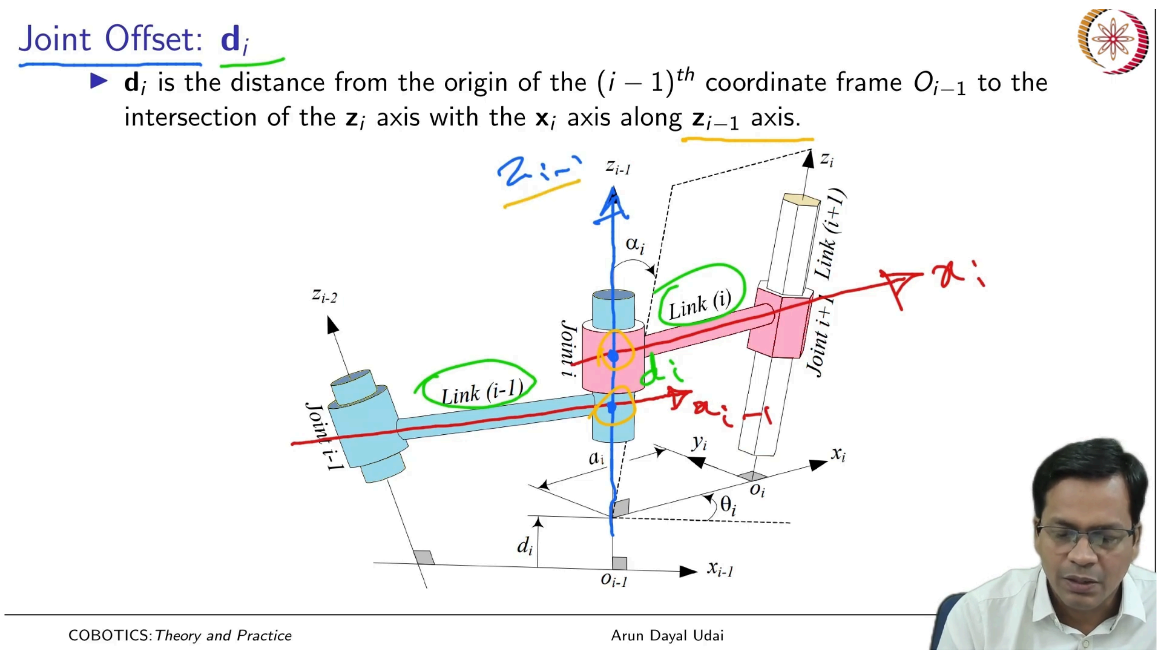
So, the Forward Kinematics Problem of a Serial Chain Robot is defined as given the individual joint displacements or angles, which solve for the end effector pose. The pose here, I would mean the position and orientation. So, let us say this is a robot, this orange-coloured one. So, it is a standard serial chain robot. It has a fixed base here, and it has an end effector that comes here. So, that actually does the job, and you have a link, you have a joint, you have a link, you have a joint, link, joint, link, joint, link, joint, and finally, it terminates. So, it is a serial chain. So, the problem here is you have to find the end effector position, and the joint angles are known. You also know the dimensions of each link. So, you know the link lengths L1, L2, L3, and so on and so forth. You know the link dimensions, and you know the geometry of the link. You know where all the joints are placed. So, knowing the joint angles theta 1, theta 2, theta 3, and each joint angle, you can calculate the end effector position. So, solving this problem is known as a forward kinematics problem. So, the steps for doing forward kinematics are as follows.

Number 1, we will understand the links, the joints, and the different parameters that define them. Number 2 would be placing the Denavit-Hartenberg, commonly known as DH frames. We will place those frames. We will learn what that is first, and we will place them while we do forward kinematics for a few of our examples. We will create a parameter table, which is commonly known as the DH table or Denavit-Hartenberg parameter table. In short, the DH table. The fourth one would be forming the individual link transformation matrix. We will form a link transformation matrix that will transform the points from the start of the link to the end of the link. We will perform the direct kinematics or forward kinematics of the robot. So, these are the steps for doing forward kinematics.

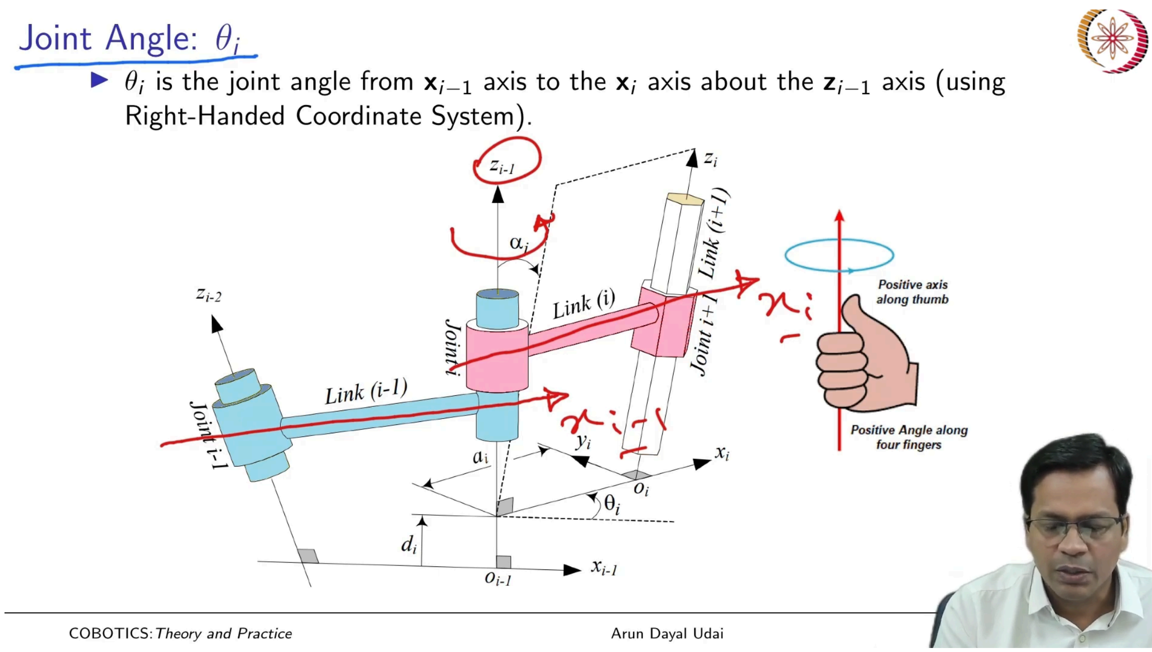


So, let us begin with link joints and their parameters; we will define that. This basically defines the structure of the link and its placement on the robot structure, okay? Not just the link itself; it also defines where and how it is placed on the robot structure. So, a link basically should have a shape that is something like this, as given here. It starts with a joint here, and it terminates with another joint. So, the starting joint, if that joint moves, the whole of the link moves, and this link ends with a joint again, which connects to another link that comes next. So, there is a joint axis here, and there is a joint axis here. The first one is known as the proximal end, and the other one is known as the distal end, and this is your link.

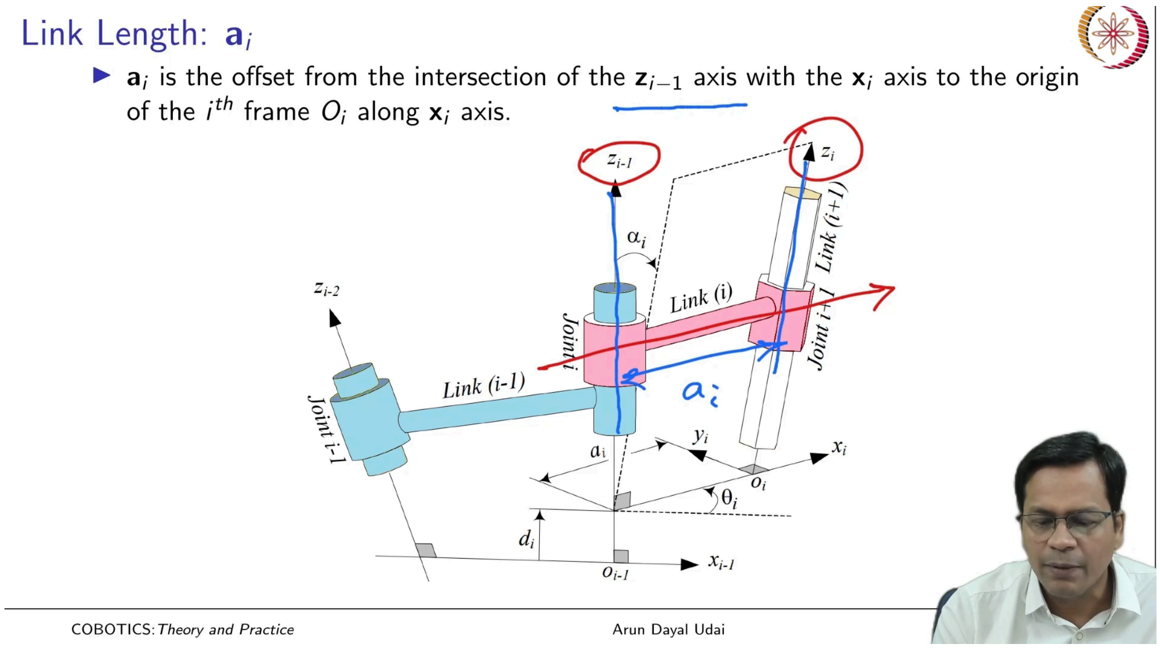
So, now you find the link here in this figure, okay? This is your link. We call it link i, and prior to that, you have a link i minus 1. So, this is your serial chain structure. So, the joint axis about which link i moves is attached to z i minus 1. So, this is the frame or the axis of the frame, which is attached to the link i minus 1. About which link i move over here, I have shown rotation, but it can also be a prismatic joint where it moves in a linear way.



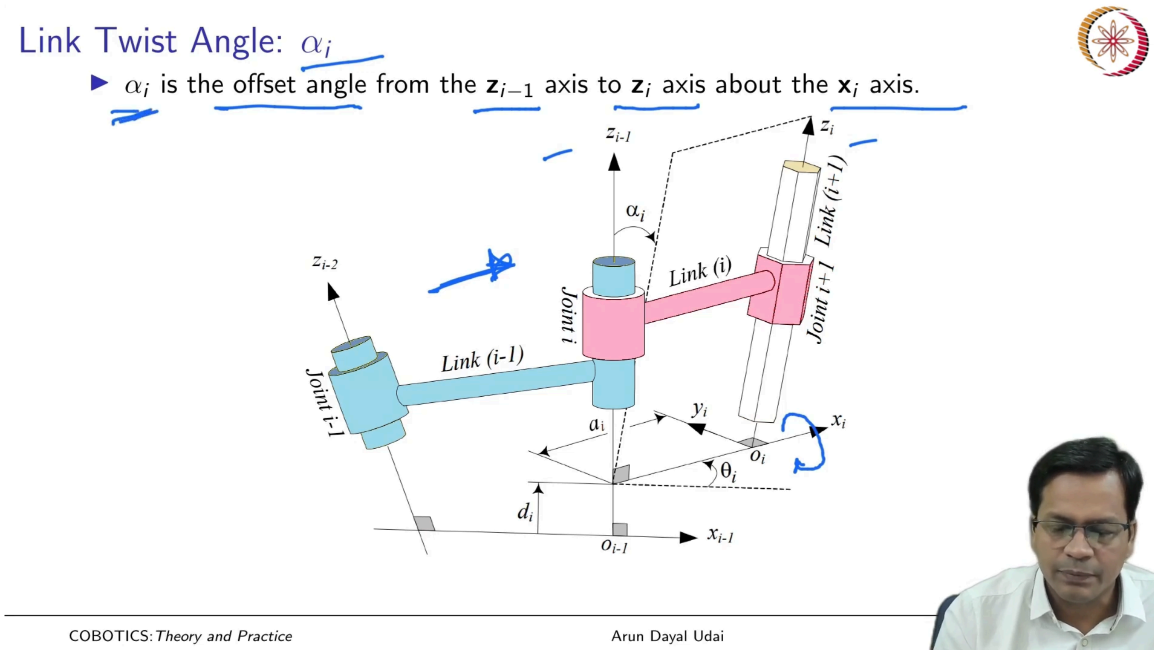
Now, the parameters go as the first parameter is known as Joint Offset. Where is it? Let us say you have a link axis that goes like this. The second link also has an axis that goes like this. So, these are the link axes, and the axis which is here is the axis about which link i rotates with respect to link i minus 1 about the axis zi minus 1. So, this is the axis zi minus 1. So, the common normal, so these two are the points where it intersects the x-axis. So, this is your xi minus 1, and this is your xi. So, the points where it intersects xi minus 1 and xi, so the distance between them is known as the offset. So, this is your distance, which I am talking about. So, this is di. So, it is commonly represented as di. It is the distance from the origin of i minus 1 $(i\text{-}1)^{th}$ coordinate frame Oi minus 1 to the intersection of zi with xi axis that is here along, and the distance is measured along zi minus 1 axis that is like this. So, this is the direction where you have to measure it. So, this is the first parameter.



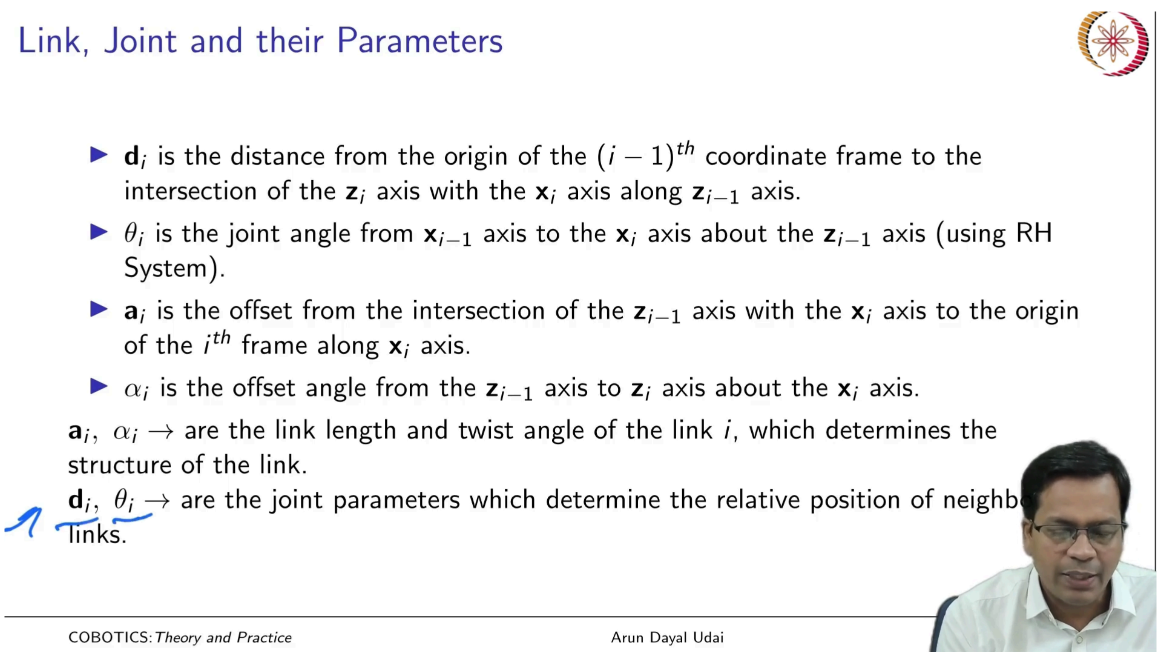
Now, the next parameter, known as the Joint Angle; for a revolute joint, is the Joint Angle. So, theta i is the joint angle from xi minus 1. So, you have xi minus 1 and xi. So, this is xi minus 1 and xi about zi minus 1. So, this is the angle. So, it should be as per the right-hand thumb rule. So, this is the xi angle, and xi minus 1 angle between those two, measured in a plane that is perpendicular to the axis zi minus 1. So, that gives you the joint angle. So, the projection of that, when you look from the top, this is the angle that is theta i.



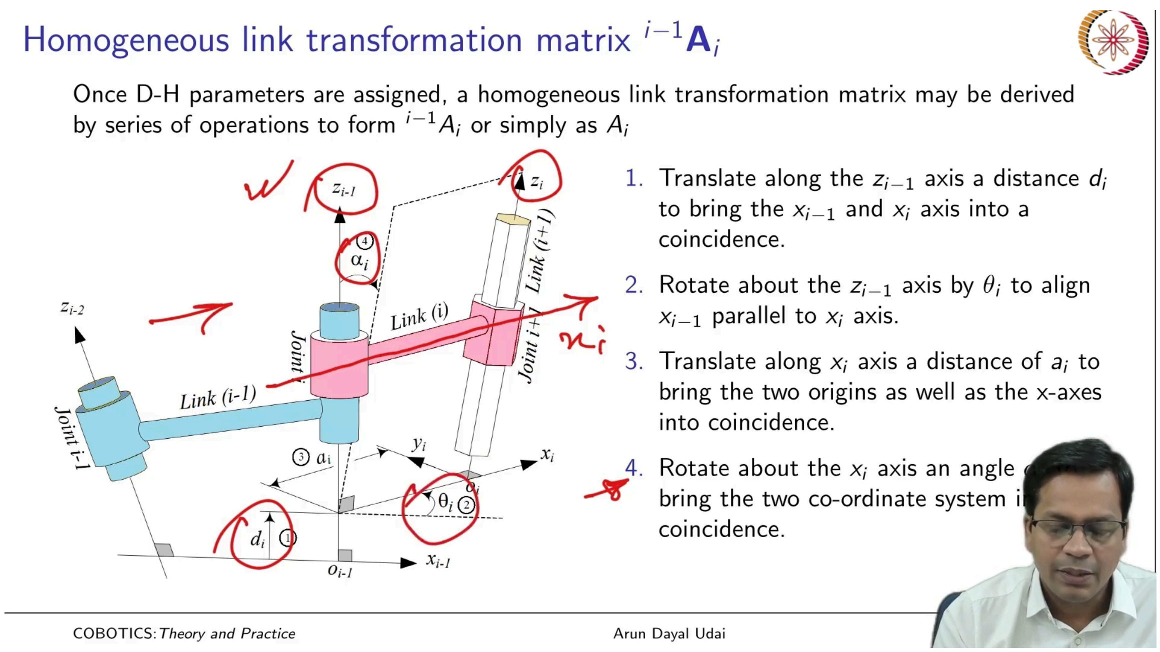
The third parameter is the Link Length. So, this link itself rotates about zi minus 1(Z_{i-1}). It terminates with the frame, which is sitting here with the z-axis again along the motion axis for the next link that will come here. So, this is zi and this is zi minus 1. So, the distance between them, if you draw a common normal to these two, would create an xi-axis. That is known as the xi-axis that comes here. So, that is a common normal to Zi minus 1 and Zi, and the distance between them is the link length, and commonly known as a_i. a_i is the offset from the intersection of the Zi minus 1 axis with the xi-axis to the origin of the ith frame along the xi-axis, okay.



So, what is a Twist angle (α_i)? The twist is given by alpha i. Alpha i is the offset angle from Zi minus 1 to Zi if you look from here, okay. There is a twist angle, so that is measured about the xi-axis. That is measured about the xi-axis from Zi minus 1 to Zi; looking from here, it is alpha i. So, it is mentioned over here that it is alpha i.



So, these are the four parameters: di, theta i, ai, and alpha i. I have just summarised it here. So, ai and alpha i, that is link length and link twist, because it is a solid link. So, this defines the structure of the link. This is not going to change with time. So, what changes are di and theta i. So, di, in the case of a prismatic joint, di becomes the moving axis, and in the case of a revolute joint, theta i is the moving axis displacement. So, these two are the displacements that define the relative position of the neighbouring link. So, these two may vary, but in the case of a revolute jointed robot, which most of the cobots are, only theta i is the variable. di is fixed. So, in that case, the link length link twist, as well as di, is fixed. Only theta will vary.



Now, let us find out the transformation that takes you from O_{i-1} to O_i, that is, from the frame that is sitting over here to the frame that is here. That is, this link itself sits on this frame, and it terminates with the next frame. So, Oi-1 and Oi. Where Oi finally will have the next link inthe series, or it may be the final joint in the case of the last link. So, that becomes the terminating frame. So, we need to know the transformation that takes you from this point to this point.

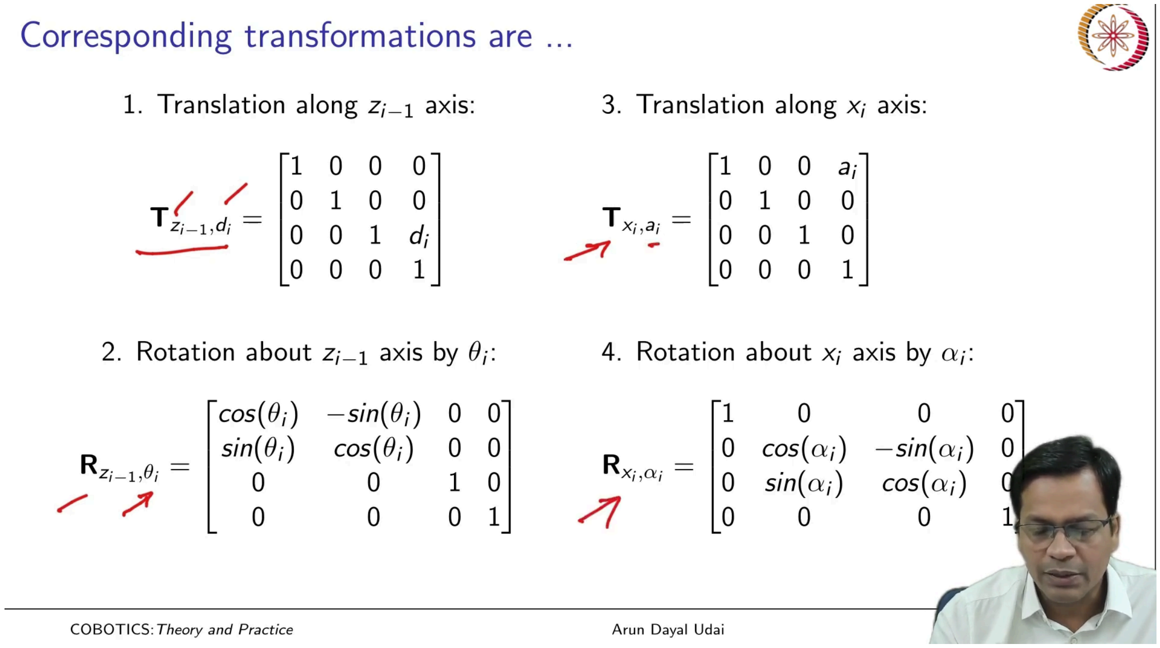
So, the first one we will do is translation along the zi axis. So, let me draw it here. So, if this is your xi-1, then this is your xi. So, this is the distance. So, this is your di. So, it is translation along zi-1 that takes you from xi-1 to xi. So, that distance. So, it is a displacement along the zi axis by a distance di.

The next one would be to rotate about the zi-1 axis by an angle θi to align xi-1 to the xi axis. So, if it starts from here till here. So, from here to here, that is rotation about zi minus 1 axis starting from xi minus 1 to xi by an angle theta i. So, this is a rotation transformation about the z-axis by an angle theta.

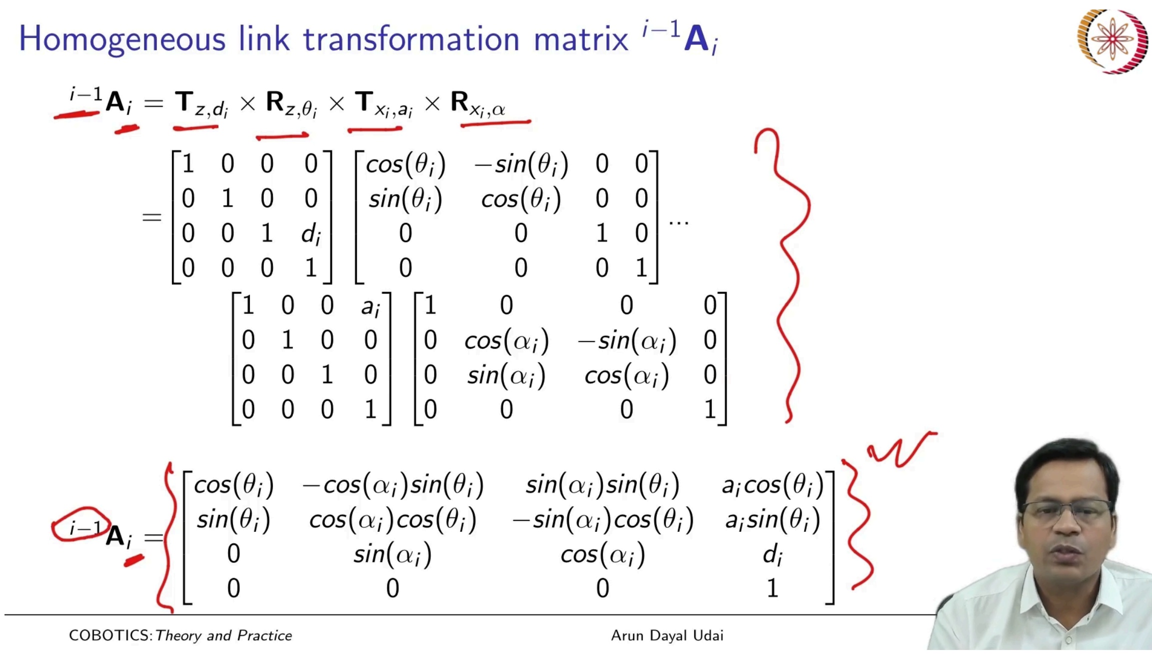
The third one is again translation along the xi axis from zi minus 1 to zi. So, if this is your zi minus 1, this is your zi. So, this is the distance it is telling. So, this is from here to here; that is the link length along the xi axis. Link length is measured along the xi axis from here till here is the translation about the xi axis by a distance ai, that is the third transformation.

The final one is rotation about the xi axis. So, this is rotation about the xi axis that takes zi minus 1 to zi. So, if you look from here, it is rotation about the xi axis by an angle alpha i that would take zi minus 1 to zi. So, the final transformation is a rotation transformation about the xi axis by an angle alpha. So, four transformations are there.

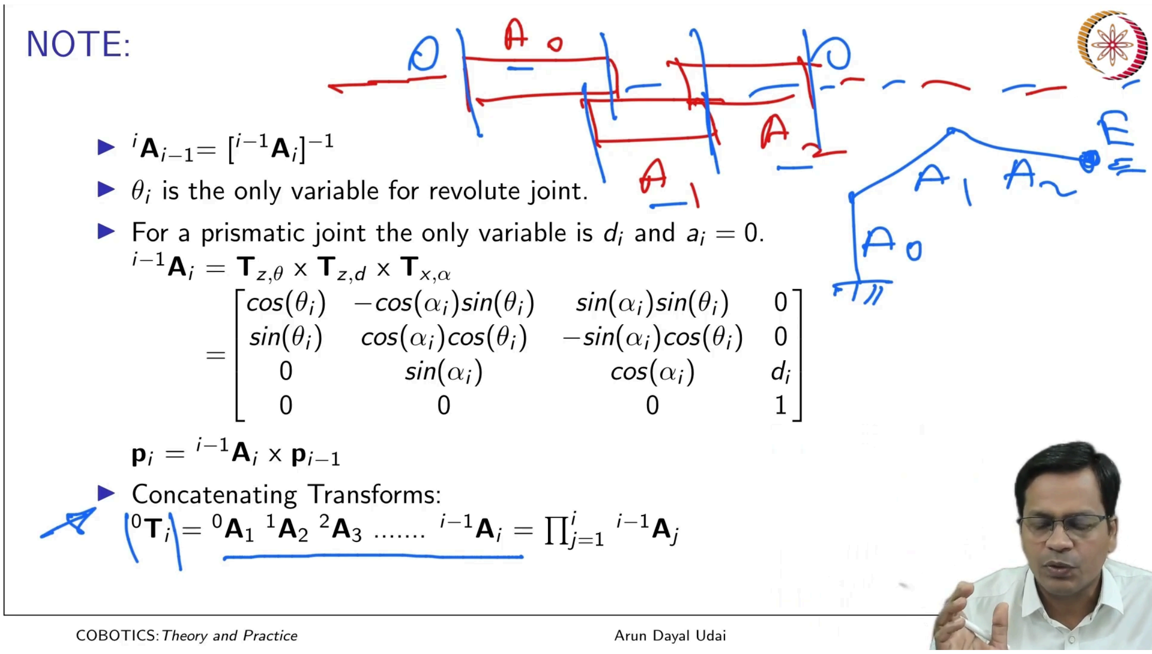
Let me summarise. So, the first one is displacement along zi minus 1 axis by a distance di. The second one is Rotation about zi minus 1 axis by an angle theta. The third one is displacement along x i axis by a distance ai. The final one is rotation about the xi axis by an angle alpha i. Four are there.



So, four consecutive transformations can be written like this: 1, 2, 3, and 4. So, this is a translation about zi minus 1 by a distance di, rotation about zi minus 1 by an angle theta i. Translation about xi by a distance ai, and rotation about xi by an angle alpha i. So, these are the transformation matrices that we studied in the last lecture. So, these are the four, and they are applied one after the other with respect to the previous frame, that is, relative transformation. So, this time it would be post-multiplication.

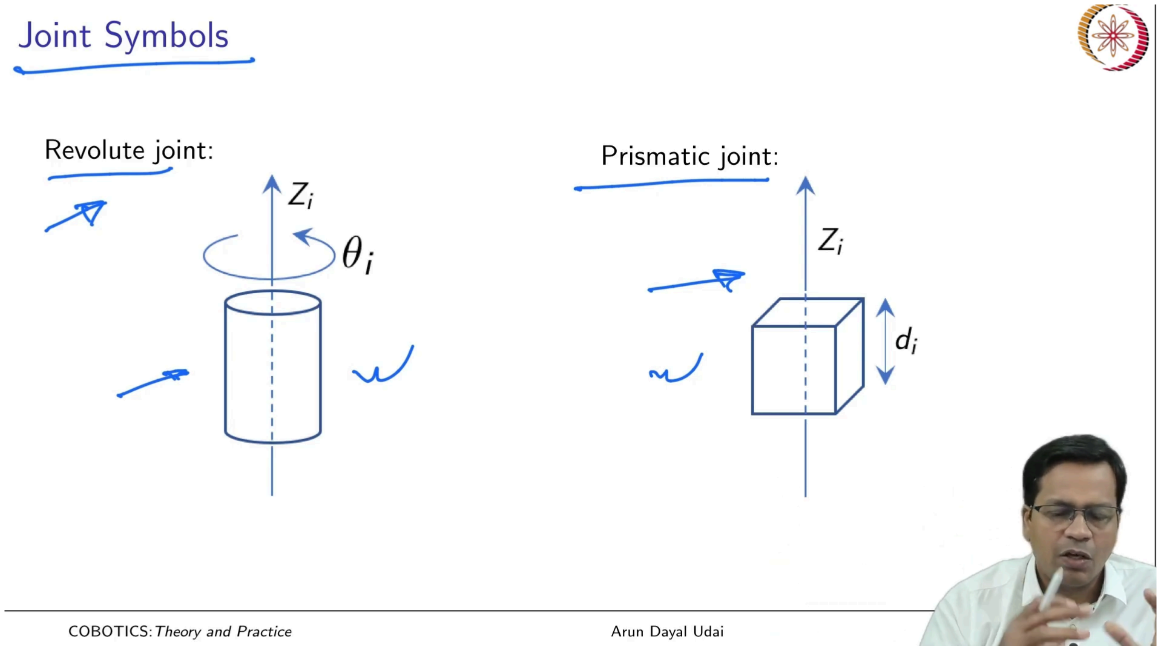


In order to get the final transformation that takes you from the i minus 1th frame to the ith frame, you do post-multiplication, that is, 1, 2, 3, and 4. If you multiply them all together, you get the link transformation matrix. This will take you from the i minus 1th frame to the ith frame. So, this is a combined transformation for all the translations and rotations that define the link shape, that is, apart from theta i, which defines the link position with respect to the previous link. The remaining are di, alpha i, and ai, which give you the link shape and its structure.

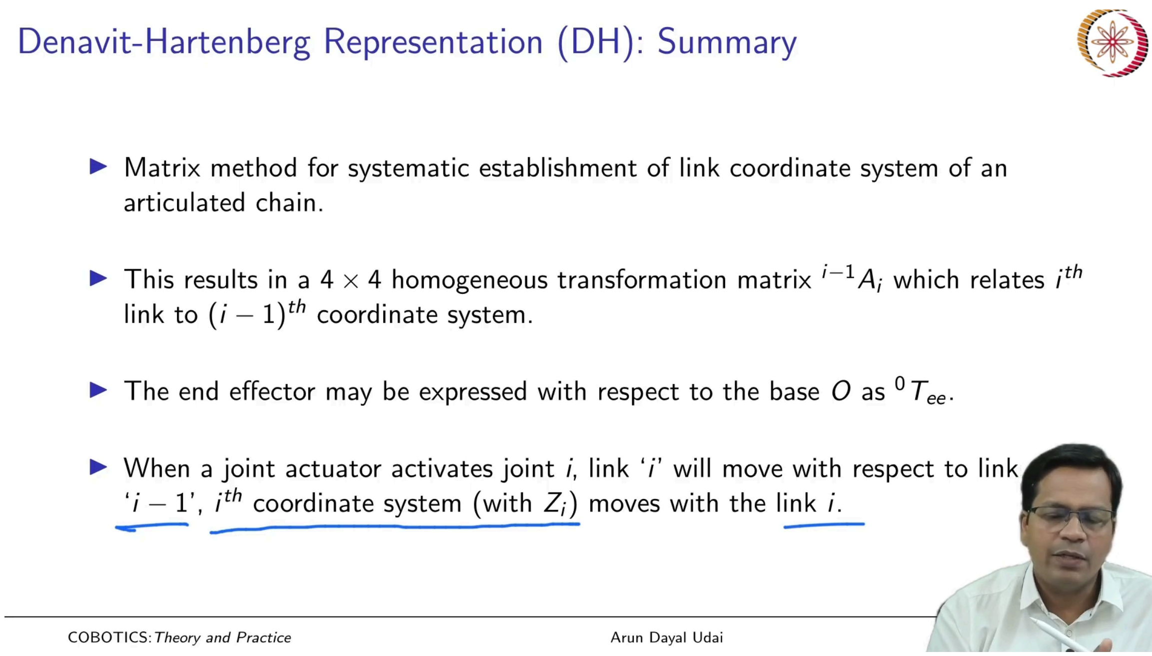


Note here: Ai minus 1 with respect to Ai is equal to the inverse of i with respect to i minus 1. Next, theta i is the only variable for a revolute joint. In the case of a prismatic joint, the variable is di, and ai, which is the length, becomes equal to 0. If you put these values into the previous ai minus 1 ai transformation, it becomes this. So, this can also transform the point which was at pi minus 1. If you multiply it with this transformation matrix, it takes you to the next frame.

Concatenating them, let us say you have the first link, you have the second link, you have the third link, and so on and so forth. So, A0, A1, A2, if there are multiple links like that, then one by one, each of these transformations will take you from this to this. This takes you from this to this. This takes you from this to this. Finally, it takes up like this. So, if you have to go from here to here, you multiply them together, okay? So, that is what it is telling. If you have to do forward kinematics for a robot, you have to reach the end effector from the base. Each link has its own transformation matrix. So, what will you do? You will multiply them together and finally get the state of the end effector. So, the final matrix that will come out of multiplying all of the link transformation matrices will give you the final 4x4 homogeneous transformation matrix, which finally gives you the state of the last frame, which is also known as the end effector frame.

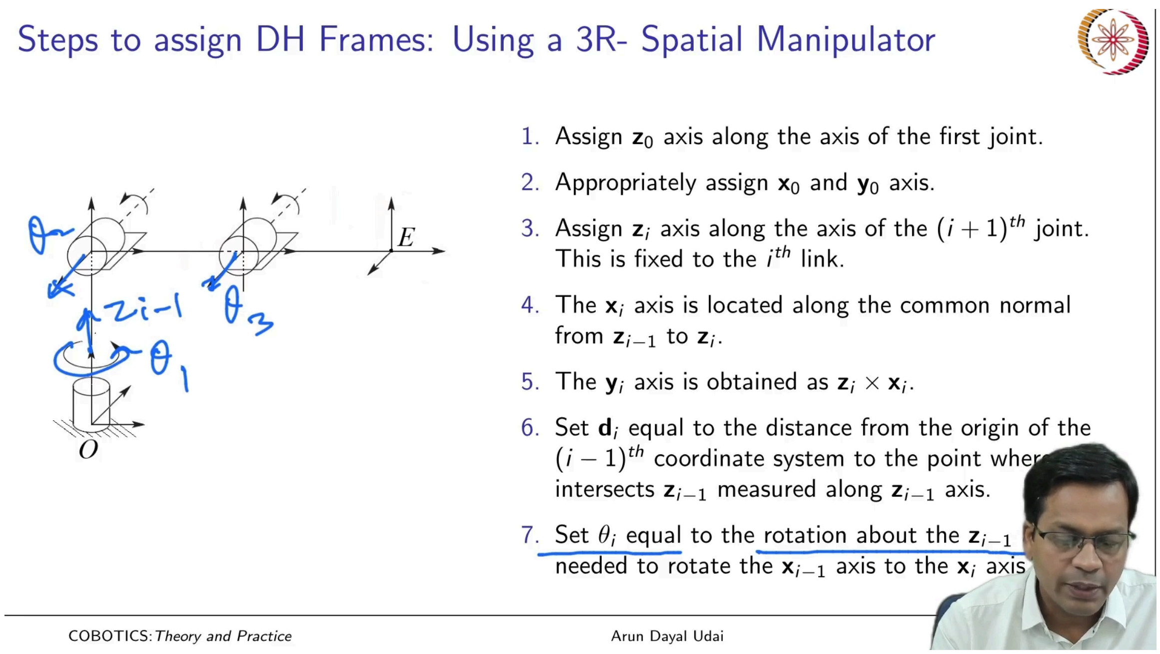


Now, I will be using a Joint Symbol like this to represent any revolute joint and a kind of symbol that is like a cube to represent all my prismatic joints. In the case of a cobot, there is no prismatic joint because that needs to be back-drivable. There are many other things to be discussed. We will be doing it. So, it does not allow a prismatic joint in a standard cobot. Because they are not necessarily, and they are not easily back-drivable. But it is not that it is not possible at all. So, most of the time, we will be discussing revolute joints in this course. So, we will use this kinematic representation to represent a joint, which is a revolute joint and the whole structure with sticks. Instead of drawing it in a complex geometric shape, we can directly draw using this.



So, let us summarise Denavit-Hartenberg's Representation (DH) here. So, it is a matrix method for the systematic establishment of the link coordinate system of an articulated chain. This results in a 4x4 homogeneous transformation matrix $^{i-1}A_i$ that relates the i^{th} link to the (i-1)th coordinate system. So, this frame is fitted at the end of the i-1th link. The end effector may be expressed with respect to the base 0. Finally, when you take the product of all those link transformation matrices, it takes you from the base frame, that is, 0, to the end effector.

When a joint activates, link "i" will move with respect to link "i-1". The ith coordinate system with zi moves along with link i. So, every link i will end with frame i, and that moves along with link i.

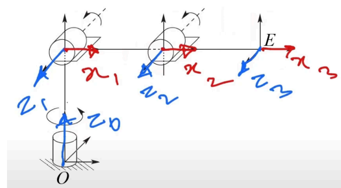


So, let us assign the DH frame and do forward kinematics now. So, we will do it for a 3R spatial manipulator. How does it work? Let us just see the motion here.

Now, let us see the video. How is this structure? So, this is moving joint 1, which moves link 1, link 2, and link 3. Now, joint two is moved, which moves links 2 and 3 only. Whenever joint 1 moves, it moves Link 1 about its own axis, so it may not be that noticeable here. Links 2 and 3 are moving. Only link 3 is moving. Joint 3 is in action. This is to make it very clear how this 3R spatial manipulator looks. Moving joints 1, 2, and 3 simultaneously will move the entire robot. Moving joints 2 and 3 simultaneously. Now, I have moved them all and, put the arm in a static position and moved joint 1 only. This is to show you that this is a plane on which links 2 and 3 lie, and that plane moves about axis 1. So, this is how it is.

So, let us do the DH frames first. We will put all the DH frames here. So, starting from here. So, assign Z0 here, the Z0 axis along the axis of the first joint. So, Z0 comes here; this is Z0, about which link 1 will move. Now, appropriately assign x0 and y0. So, x0 is put here, y0 is put here, and z0 is here. This is your link 1.

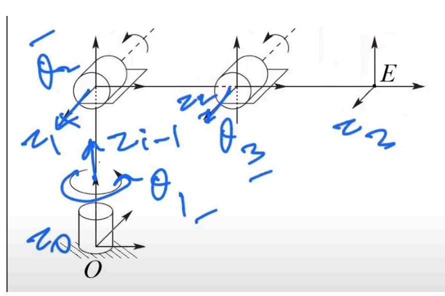
Now, assign the z1 axis along the axis of the $(i+1)^{th}$ joint. Assign the zi axis along the axis of the $(i+1)^{th}$ joint. So, likewise, you keep on putting all the z-axes. So, this was your z0. At the end of link 1, you have the next z1, okay? This is the frame that moves along with link 1, okay? When it rotates about its axis, and similarly, z1 is the axis of rotation for link 2. Link 2 terminates with the frame with z2 over here. Similarly, this is link 3, which finally terminates with the frame with z3 over here. This is how all the z-axes are placed. This is your end effector, and this is the robot base. Once all the z-axes are placed, place the xi-axis, which is located along the common normal to zi minus 1 and zi.



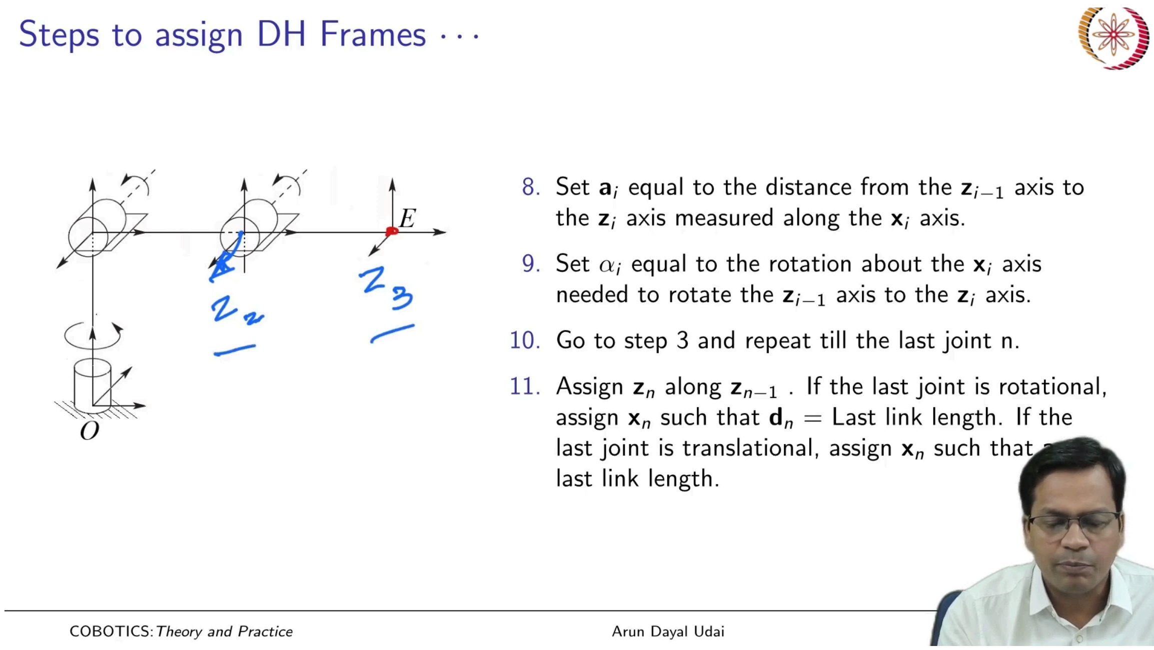
So, if this is your z0 and this is your z1, the common normal to that is, so this is your x1. Similarly, because this is your Z2, the common normal will be making this x2, and the last one, because this was your z3, so this becomes my x3. Now that I know all x and z, I can quickly get my y-axis.

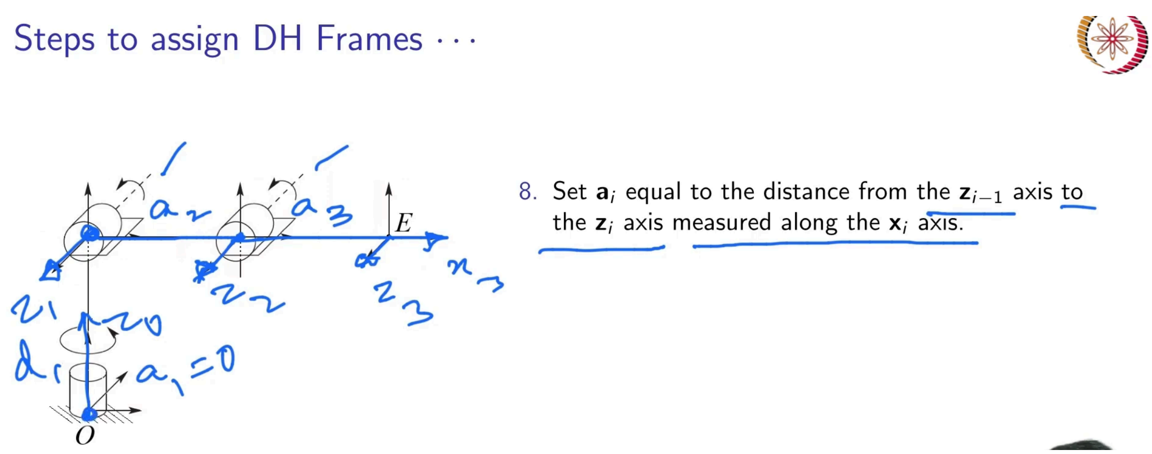
So, the y-axis is simply obtained by z cross x. So, y can be quickly drawn here. These are my Y-axes. So, these were my z-axes. So, these were your x-axes, got it? This is how you placed all the x, y, and z-axes, and this was your initial x0; this is your y0. So, this is how all the frames are placed. So, di is equal to the distance from the origin of the i minus 1th coordinate system to the point where xi intersects zi minus 1.

So, over here, you have distance measured along zi minus 1. This is your zi minus 1 till zi, and how much is that? So, this distance is your di. So, over here, it is the first link. So, it is d1. So, this is present here. But you see, there is no other di. There is no distance measured along the z-axis across any of the other links for link 2 and link 3. That is not there.

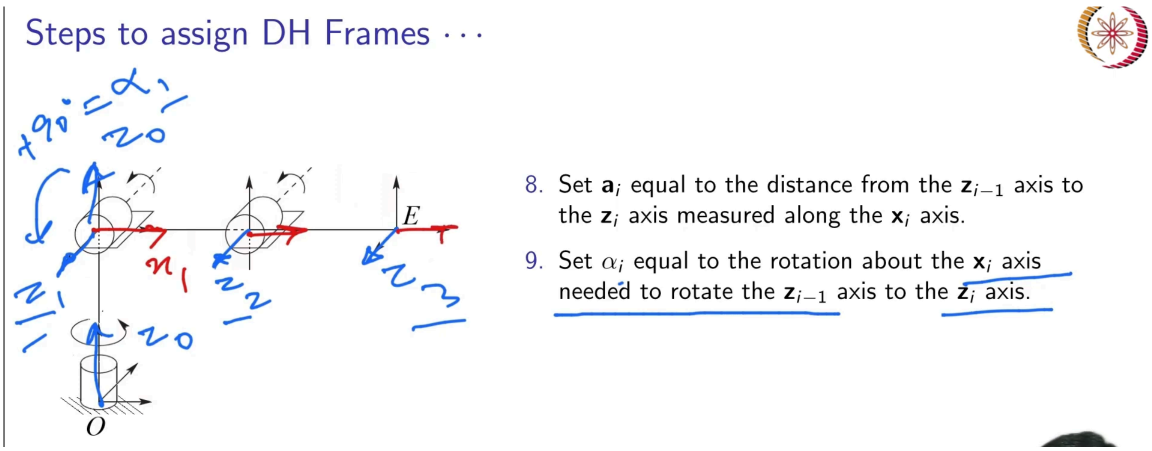


Now, set theta i. Theta i is equal to the rotation about the axis zi minus 1. So, this is your zi minus 1. This is your theta 1, which is the first joint angle. So, if this was your z, this is theta 2. This is your z, this is theta 3. So, these are the joint angles. So, over here, all three were revolute joints. So, theta i is the rotation about the axis zi minus 1. So, this was your z0. So, this is theta 1, this is z1, theta 2, z2, theta 3, this is z3, and there is no further link here.





So, now, ai is equal to the distance from the z_{i-1} axis to the zi axis measured along xi. This is actually the link length. So, this is your z0; this was your z1. You see, there is no distance between them. Both the zi intersect at this point so that link length a1 becomes equal to 0. There is no distance between them for the next one. Z1 and Z2 have a common normal, and you see this is your a2 from here to here again; you have a common normal. That is x3 coming here, and this is your a3. So, these two are link lengths, and this was your joint offset that takes you from O0 to O1.



So, alpha i is equal to the rotation about the xi axis that is needed to rotate from zi-1 to zi. So, there is no angle between z1, z2, z2, and z3. But when you look at this one, this is z0 and z1. If you measure about X1, you see Z0 comes from this to this. So, you have to follow the right-hand thumb rule. So, it is about x1 you are measuring. So, it is plus 90 degrees. It can be taken from z0 to z1. Got it? So, this becomes my alpha 1. So, it is alpha 1, and for others, there is no angle between the z-axis. So, z1 is parallel to z2, and z2 is parallel to z3 at all times. So, there is no twist about the x-axis. So, the x-axis is here, so there is no twist about that, okay? So, the link twist is missing for links two and three.

So, finally, you keep doing this, repeating till the last joint. You have to keep doing this for all the links. Assign Zn along Zn minus 1. So, last, just because this is Zn minus 1, this was Z2. So, the last frame will be Z3, and this is normally kept parallel to this.

Example 1: Spatial 3R Manipulator (3-DoF)

Video demonstration

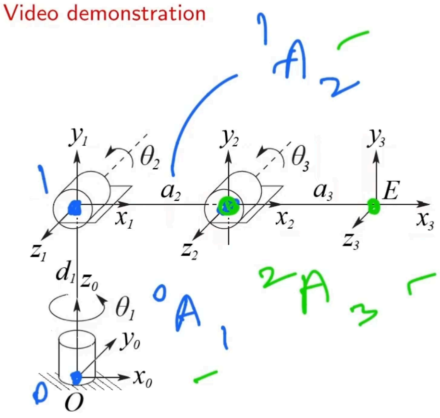
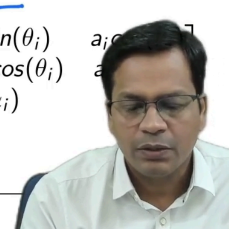


Table: DH parameters

Link	$\mathbf{a}_i$	α_i	$\mathbf{d}_i$	θ_i
1	0	90°	d_1	θ_1
2	a_2	0	0	θ_2
3	a_3	0	0	θ_3

Using link transformation matrix: ${}^{i-1}\mathbf{A}_i =$

$$\begin{bmatrix} cos(\theta_i) & -cos(\alpha_i)sin(\theta_i) & sin(\alpha_i)sin(\theta_i) & a_i \\ sin(\theta_i) & cos(\alpha_i)cos(\theta_i) & -sin(\alpha_i)cos(\theta_i) & a \\ 0 & sin(\alpha_i) & cos(\alpha_i) & \\ 0 & 0 & 0 & \end{bmatrix}$$

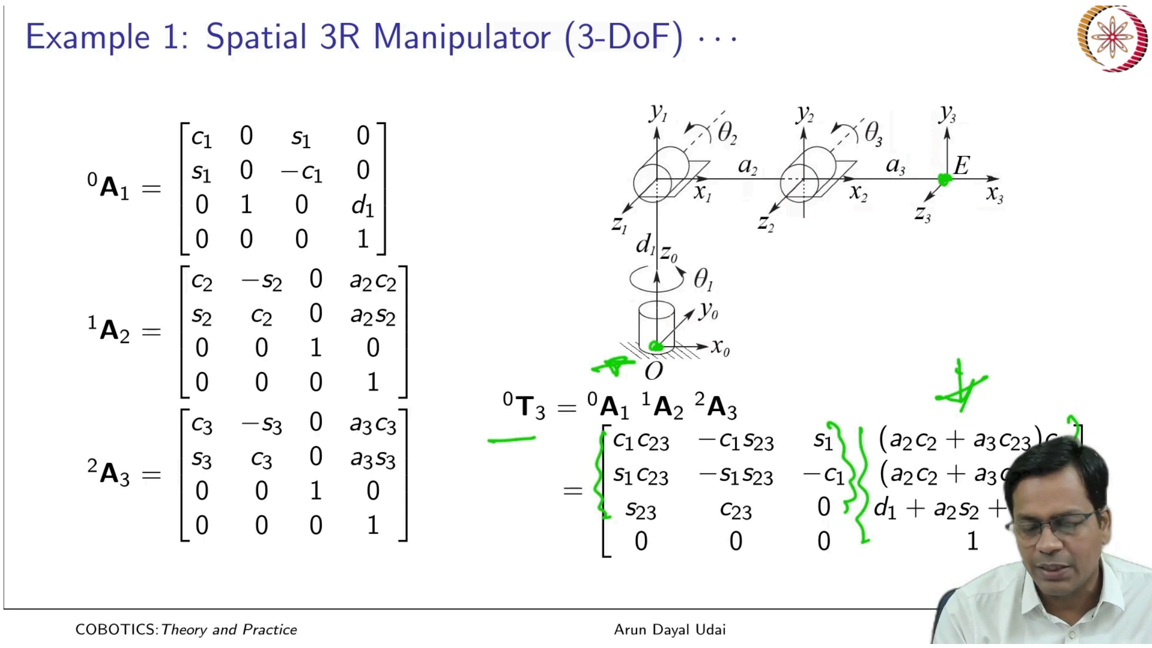


So, I have already shown you the video here, that is, this one, how it is working. So, these are all the parameters I have summarised here. So, d1 is placed here, a2 and a3 are here, and the joint angles are theta 1, theta 2, and theta 3. And finally, all the frames are placed right here. So, I will switch it off here, the video.

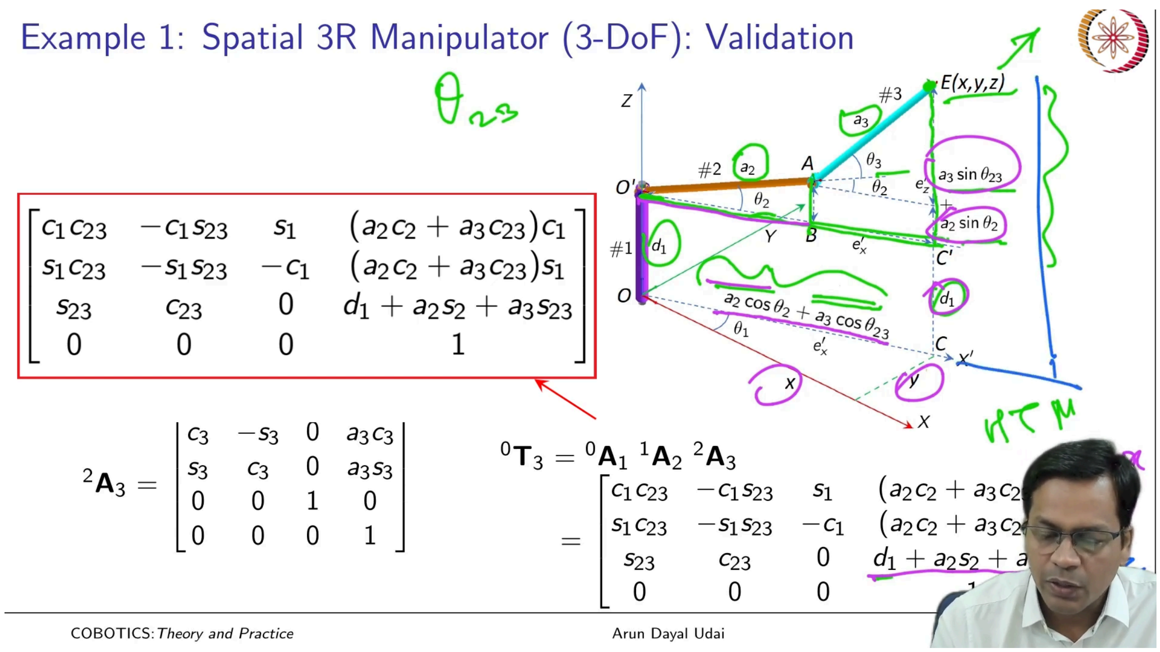
Now, the DH parameter table will be tabulated as link 1. So, that is for the first link. So, you have parameters a1 equal to 0. There is no distance between z0 and z1. So, that makes a1 equal to 0. Alpha 1 is the angle between z0 and z1, measured about x1. So, it came to 90 degrees. d1 is the distance measured along z0 from this frame to this frame. That is d1, and theta1 is the joint variable.

The next one is the distance between z1 and z2 measured along x. So, that is the link length. There is no angle between z1 and z2 measured along x1. So, the twist is 0. There is no distance because both the joints are in a plane. So, there is no distance travelled along the z-axis. So, that makes the joint offset equal to 0, and theta 2 is the joint variable. Similarly, a3 is the distance between z2 and z3 from here to here, measured along x3. So, that is a3. The twist is 0 again, d is 0 again, and theta 3 is the joint variable. So, this is how the whole set of DH parameters is completed, and we tabulated them in a DH parameter table.

So, we'll use the link transformation matrix here, Ai-1 Ai, which will take you from this to this. So, A10 ($^0A_1$) would mean it is taking you from the 0th to the 1st frame. Similarly, A21 would mean it takes you from this to this frame. Okay, and the final one would take you from here to here, that is A32 ($^2A_3$). Okay, so there are three link transformation matrices, and you have to substitute each one of them to get $^1A_0$, $^2A_3$, $^1A_2$. All three matrices will be obtained, and these are known as link transformation matrices.



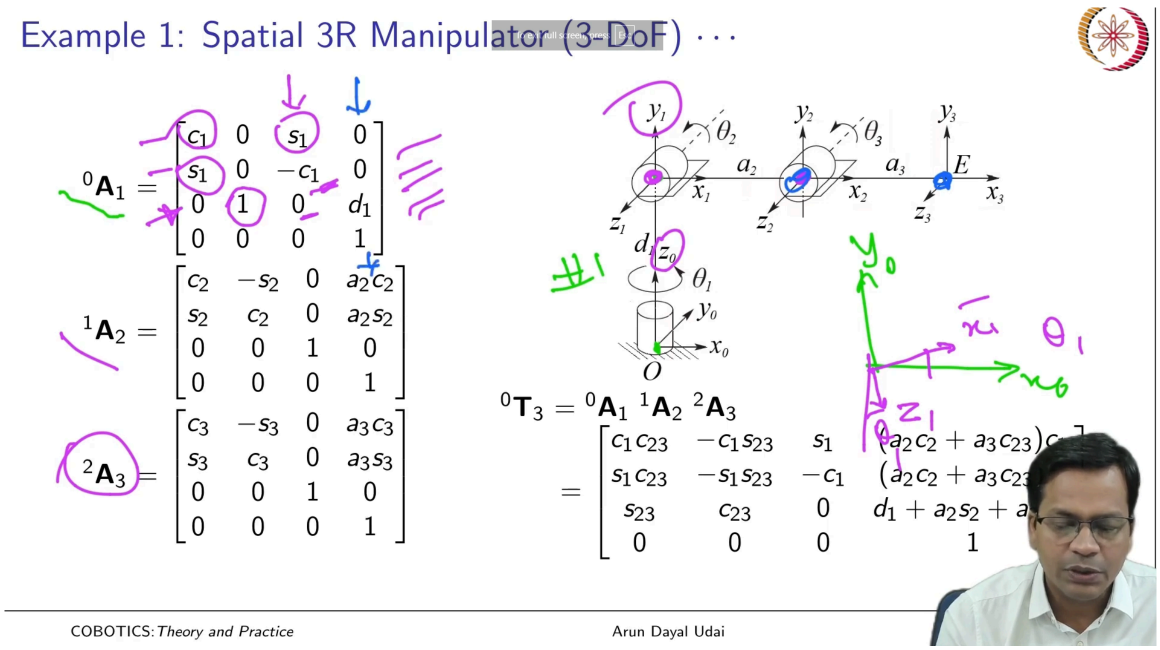
So, these are those. So, if I take the product of all those, I can quickly get the state of the end effector frame here. What it says is the position over here, that is, the position of the end effector with respect to O, which is the robot's base frame. And these three would tell me the orientation of the end effector with respect to the base frame. Is it that trivial to validate it as well? We will try doing it now. So, let us do it.



So, now, quickly comparing the values here, whether this point gives me that is the end effector point, that is, x, y, z, gives the location over here in the homogeneous transformation matrix. So, you see, just by taking trigonometric projections here and adding the components, you can directly get the values. Let me just check. See, this is your link length a2, a3, theta 2, theta 3. There will be two components here: the horizontal component. Like this, so you see this arm has link 2 and link 3 that is lying in a plane, okay? So, this is that plane. So, there are two projections. The horizontal one would be a2 cosine theta 2 plus a3 cosine theta 2 plus theta 3. Theta 2 3 ($\boldsymbol{\theta}_{23}$) would mean theta 2 plus theta 3.

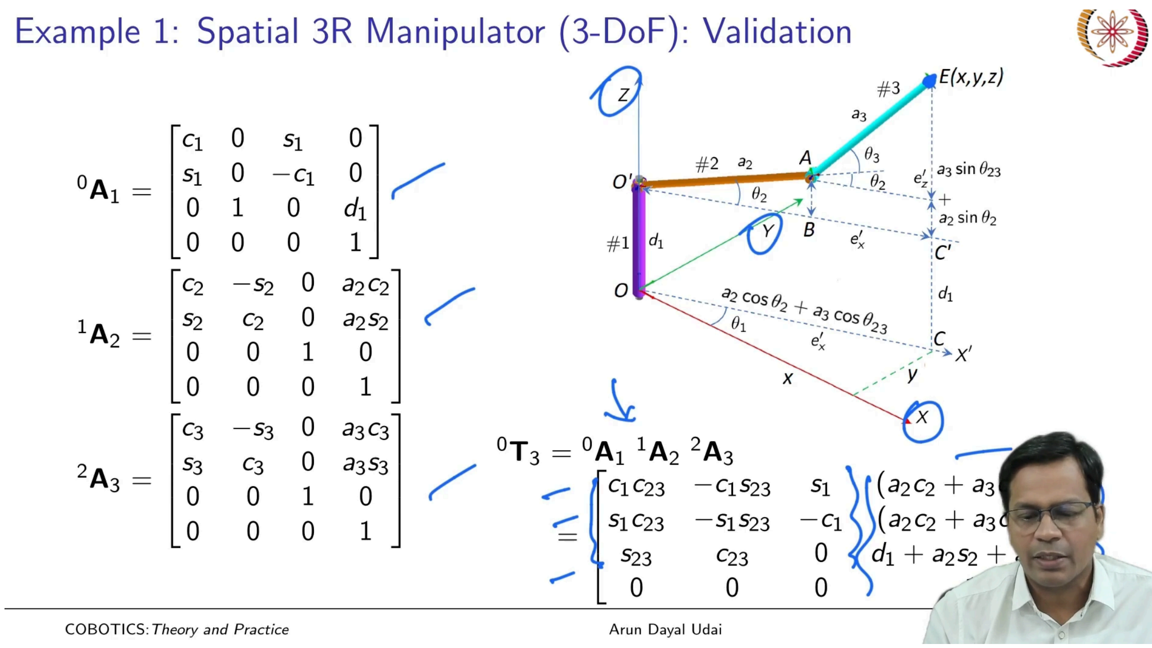
If we sum this distance, that is, from here to here and from here to here, that is quickly this one and this one. So, this becomes my horizontal component. Now, the vertical component is this, that is, a2 sin theta 2 plus a2 sin theta 2 plus theta 3. So, those two are the horizontal and vertical components. So, that is quite trivial here.

Now, this d_1 is equal to d_1. So, now, you can quickly get at least the vertical distance, that is, the z coordinate here. So, you see, it is available here, that is d_1 plus this plus this. So, overall, you got till this. Got it? So, that is validated. Because this is the horizontal projection. So, x and y would be simply components of this along x and along y. So, that gives you this, that is x, this is y, and this was your z, whole of this. Got it? So, this is your z. So, you see, you can quickly validate. So, this is your end effector position. Can we also calculate the orientation here? Let me just check that. Is it possible as well?

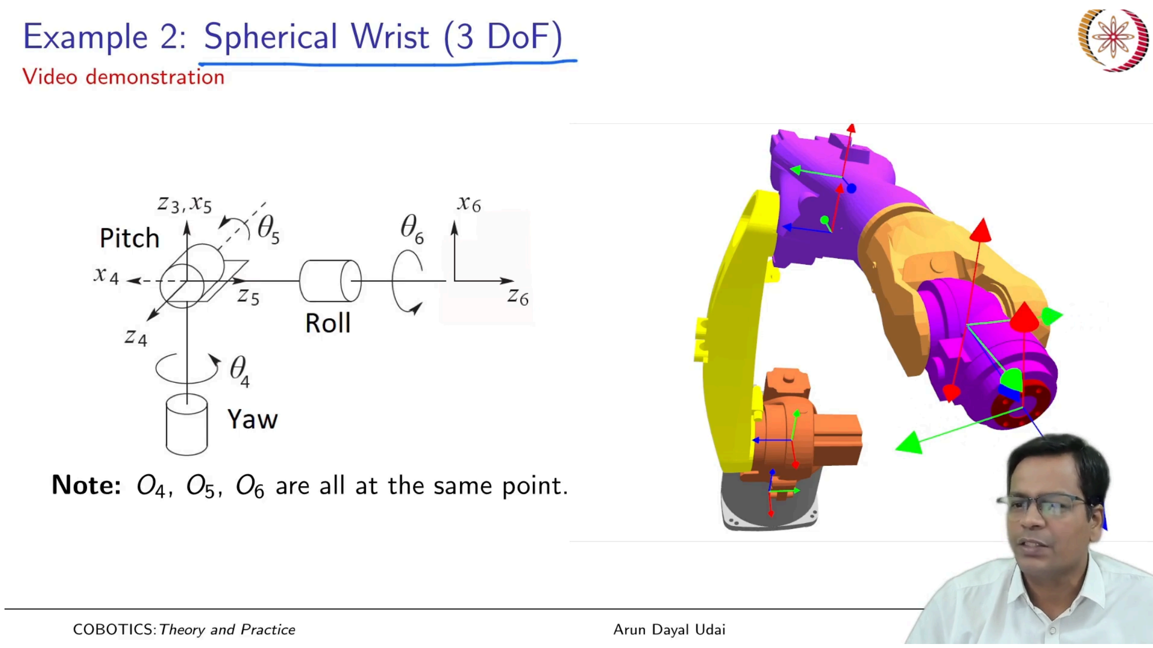


So, let us go back to the previous figure here. So, over here, what you found is ${}^{0}A_{1}$. What does it mean? It is transforming from; this is the link transformation matrix 1. It takes you from this point to this point. Let us look at it from the top. What does it look like? It tells you X0 here, Y0 here. What about the first frame? It tells you, if you look from the top, you are z1 is here, and x1 is here. That is rotated by an angle theta1. So, now you look at the projections. What do you see? x1, projection of x1 along x0, along y0, along z0. So, x1 along x0 would be cosine theta1, x1 along y will be sine theta1, and x1 along z, z is perpendicular to this plane, so it becomes equal to 0. So, quite clear. Now, what about this column that is, for z. So, this again is making an angle theta1, okay. So, this time, you see, projection along x0 for z1 would be sin theta 1. For projection along y for z1 would be minus cosine theta 1. Mind it, all the vectors here are x0, y0, z0, x1, y1, z1. They are all unit vectors. So, this is here, and then finally, it is in a plane, so that makes it 0.

What about the z-axis, okay? Z-axis projections. Along z0, y1 is always lying. So, you see a continuous projection of y along z as 1; the other two projections would become equal to 0. So, this is the way you can validate each one of these matrices. So, if you have to validate this, you have to see this on this. If you have to validate this, you have to see this on this. Okay, so those are these orientation matrices. Okay, definitely, this gives you the relative position as well, okay? That we have anyway verified over here, so we won't do it again.



So, the final one would be this one. So, the product of all these basically gives you this as orientation. What does it tell you? That is the projection of the last frame, x, y, and z, along the global base frame, that is, x, y, and z. Projection of these, so this is the projection of the end effector along x, along y, and along z, and this is your position so that you can validate each one of them, each one of them. You can validate the position also, or you can just put the value for a particular value and trigonometrically calculate it. It would not always be possible, at least for higher degrees of freedom robots.

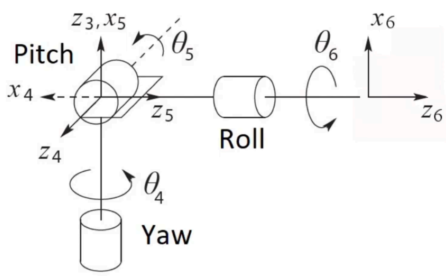


Now, let us move ahead. We will talk about the spherical wrist. Where have you seen it? Let me just show you once again here. So, if this is a robot, this is a robot here; it terminates with a spherical wrist. So, the first three axes basically would take you to any position, that is x, y, z position, as you have seen in the previous example, okay. This is a positioning robot that takes this end effector to a position. The next one would be to orient it, okay?

Now, that I have reached there, I will orient it, align it with the object, and do some valuable work. To align it, you need three further rotations, as you have seen in Euler angle systems earlier, okay? So, this is one of them; it is the roll-pitch-yaw system. So, you see what it does, basically. Axis 4 is like this, axis 5 can take you like this, and finally, axis 6 is the axis that moves the flange. This attaches the tool; this is where the grippers are attached. So, if I remove all the frames here, you can see that. So, this is axis 4, this is axis 5, and axis 6. All three axes intersect at a point, which is known as the spherical wrist. Okay, so all three axes intersect here. So, axis 4, axis 5, and axis 6 intersect here. This is what I am talking about. These three axes would finally orient this to align it with the object that this robot needs to handle.

Example 2: Spherical Wrist (3 DoF)

Video demonstration

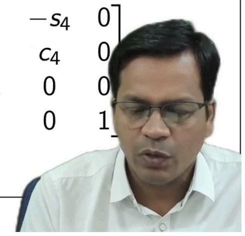


Note: O_4, O_5, O_6 are all at the same point.

Table: DH parameters

Link	$\mathbf{a}_i$	α_i	$\mathbf{d}_i$	θ_i
4	0	$-90°$	0	θ_4
5	0	$90°$	0	θ_5
6	0	0	d_6	θ_6

$$
{}^{3}\mathbf{A}_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$



Axes 1, 2, and 3 take you to a position; axes 4, 5, and 6 actually take you to a particular orientation. So, this is your displacement along axis 4, this is your displacement along axis 5, and finally, you have that, which is your axis 6. So, these motions are along. So, if it is theta 4, it is measured about the previous frame, which is Z3. Similarly, if it is theta 5, it moves along theta 4; theta 6 moves along Z5, okay. So, all three axes intersect here, okay? So, that becomes my wrist centre point, okay.

Now, let us do the DH parameters for this. This can be quickly found, so you have seen it here in the previous case. You have to attach this wrist on top of the previous one, so that is the reason I started with four. One, two, three is already over in the last example, okay. So now, this is your Link 4. The fourth link comes, and it ends here with the final frame, Z4. Z4, actually, the fifth link is moving about Z4, and Z5 is here, which actually moves the sixth link here, and finally, it ends at the end effector over here.

Let me just recapitulate this. You see, this is your fourth one, okay? And you have the fifth one, the fifth theta five, and finally, this is your theta six, okay? So, you start with zero. Frame 1, 2, 3, 4, 5, and 6, that is the last frame. So, if you just can see this, because there is no distance between Z3 and Z4, that makes A4 equal to 0. The angle between them, measured about X, is minus 90. The offset again, because it is intersecting, there is no offset. There is no distance that is travelled along the Z-axis.

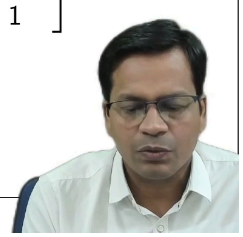
So, for The fifth one, for link 5, what you see here, it is ai becoming equal to 0, okay, because there is no distance between Z4 and Z5, okay. So, that makes it 0, and coming from Z4 to Z5, it is plus 90 degrees, which is measured along this. So, that is plus 90 degrees, and di, because both the axes are intersecting; okay, again, there is no distance travelled along the Z-axis here, so that makes this offset equal to 0, and theta 5 is the joint angle, that is the joint variable here, okay.

Again, the sixth one, that is the sixth link, what you see here, ai, that is the distance travelled between Z5 and Z6 because both of them are collinear. So, there is no distance between them, that is measured along the X-axis. So, that makes A6 equal to 0. There, both the axes are parallel. So, there is no twist angle here, and d6 is the distance from here to here, that is, till the tip. So, that is d6, which is the distance travelled along this axis. Okay, from this centre point to the end effector, that is, d6, and finally, theta 6 is the joint variable. So, this makes the complete DH parameter table for the spherical wrist, okay? If you put them in the transformation matrix, that is the link transformation matrix ai, i minus 1 individually, so you get A3 to 4, A4 to 5, and A5 to 6.

Example 2: Spherical Wrist (3 DoF) ···

$$ {}^4\mathbf{A}_5 = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^5\mathbf{A}_6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ \Rightarrow {}^3\mathbf{T}_6 = {}^3\mathbf{A}_4\,{}^4\mathbf{A}_5\,{}^5\mathbf{A}_6 = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 & c_4s_5d_6 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 & s_4s_5d_6 \\ -s_5c_6 & s_5s_6 & c_5 & c_5d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$



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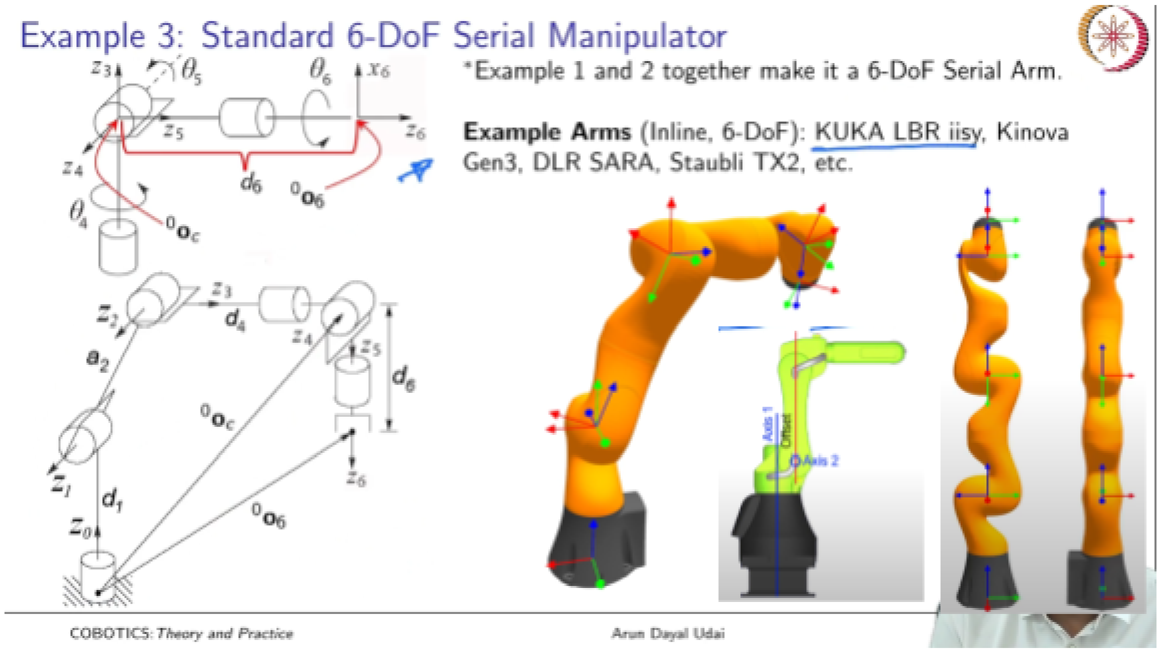
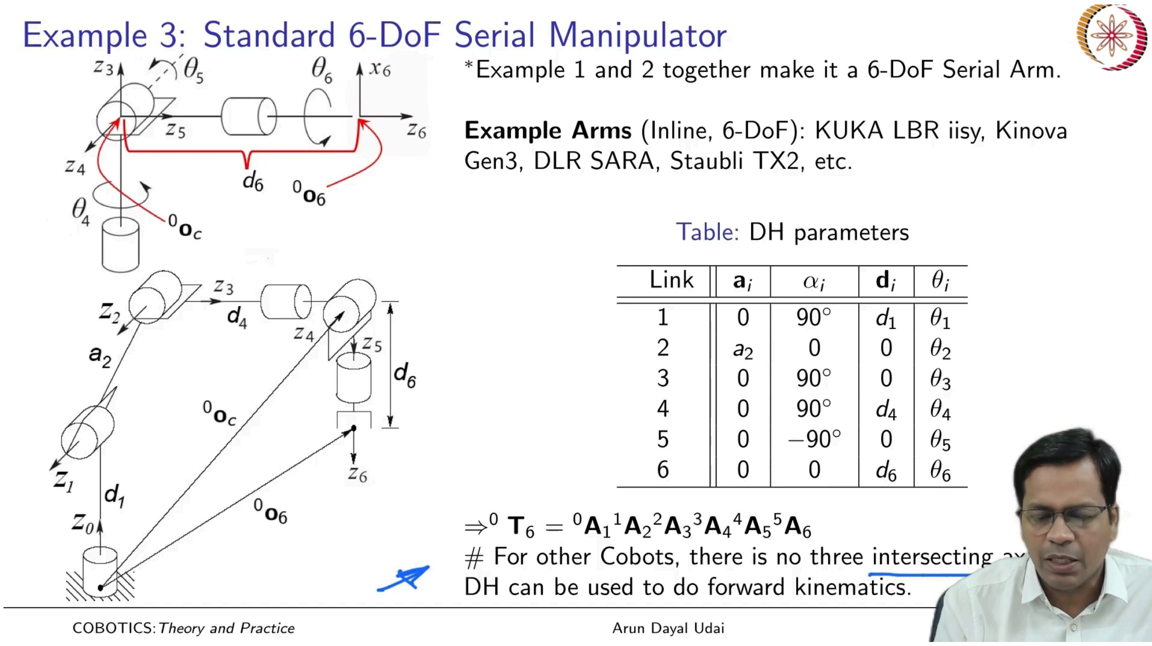
If you take the product of them, this gives you the complete pose of the end effector with respect to the third frame because you see it is three here, from three to four, four to five, five to six. Take the product, it gives you from three to six, so from the third frame till the end effector frame. So, this is the transformation. This, again, can be validated, but it is not that trivial. This gives you the position at least, which may be verified. So, that is the wrist of the robot.



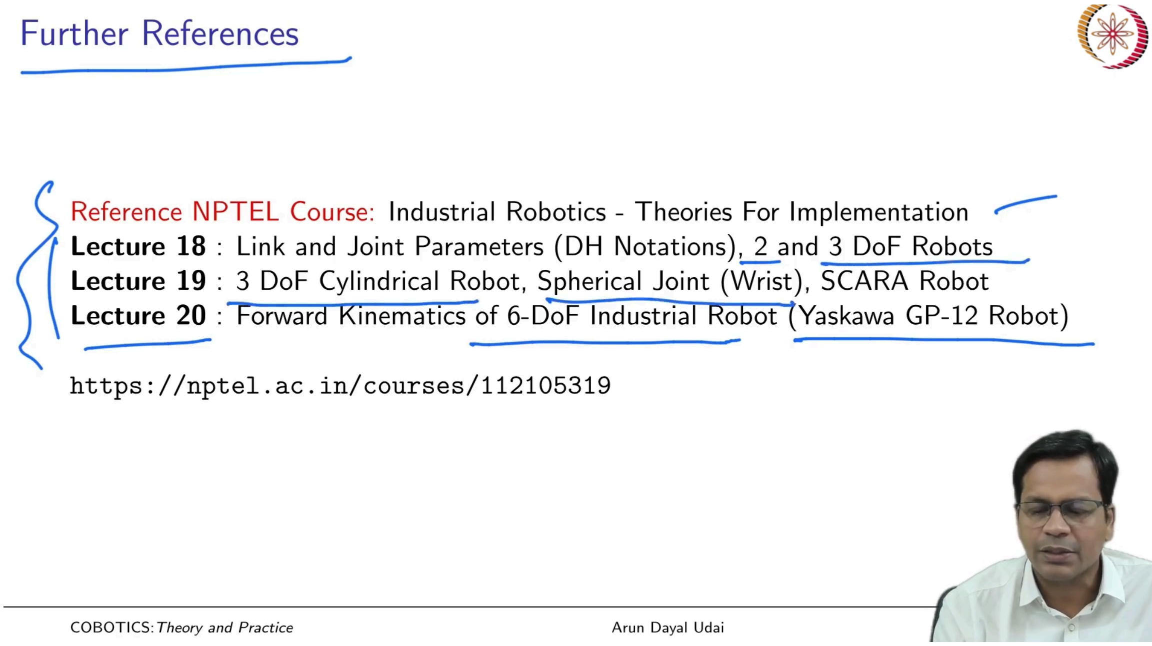
So, now, if you put them together, let me just put it here. So, this is your spherical wrist, and prior to that, you had the thread of the arm, okay? That is a special arm which was here, right till here. So, that was a special arm. If I put this on top of this, it becomes a spherical wrist on top of a 3R spatial manipulator; it becomes a 6 degrees of freedom arm.

So, there are many such arms which are there. One of them is KUKA LBR iisy, Kinova Gen3, DLR SARA, Staubly TX2, and many similar arms are there. If you set them at the home position, it becomes a robot that is in a straight line. So, that is the beauty of it. But yes, most of the industrial robots have the base offset. That means the first axis and the second axis have some distance between them. The first one is a vertical axis. The next one is the horizontal axis. The distance between them exists. That is to make it a little forward so as to do most of the forward tasks. The tasks which are in front of the robot. But in the case of Cobot, we tend to put them in line because it may have a symmetrical workspace all around.



If you tabulate them in the DH parameter, it becomes very much like this. Okay, so you have the first three for the 3R Special robots, that is, the positioning robot, and the rest one is for the 3R spherical wrist. Okay, so that comes here. The only difference that comes here is the axis. So, if you look carefully, the last link at Z is like this. Now it is like this. Others are exactly the same because now it has an axis that comes like this. So now I have to put Z3 like this. So, that makes it that little bit of modification in the DH table; otherwise, it is all the same. So, you see, Z3 is now like this. So, Z3 and Z4 are here, and Z5 is here. Finally, Z6 is the last frame. So, that makes it complete.

So, when you do put all the DH parameters to the link transformation matrix and take the product, you should be getting the end effector pose, that is, the position and orientation, which is the homogeneous transformation matrix that would represent the state of this end-effector frame. So, it gives you this as well as the orientation of the frame, which is sitting here with respect to the robot base. For all other cobots, there are no intersecting axes in the wrist. Most of the cobots, you see, have only two axes which are intersecting next to each other. There is never a place where all three axes are intersecting. That is what I mean. Normally, for UR arms and many other similar arms, we will talk about that later.



So, yes, you can always have some further materials that are here in my course, Industrial Robotics: Theories for Implementation. Lectures 18, 19, and 20 you can refer to. Lecture 20 discusses the Yaskawa GP-12 Robot. Quite a lot of cobots are also of this type, and their forward kinematics can be done in a similar way. Others are 2-DOF and 3-DOF robots. The 3-DOF cylindrical robot wrist is again discussed here. So, a 3-DOF spherical wrist is also there. The SCARA robot is there. So, you can look at these. This is the link for the course. That is all for this lecture.

Thanks a lot. In the next lecture, I will discuss the Forward Kinematics of Industrial COBOTS.