NPTEL Online Certification Courses
COLLABORATIVE ROBOTS (COBOTS): THEORY AND PRACTICE
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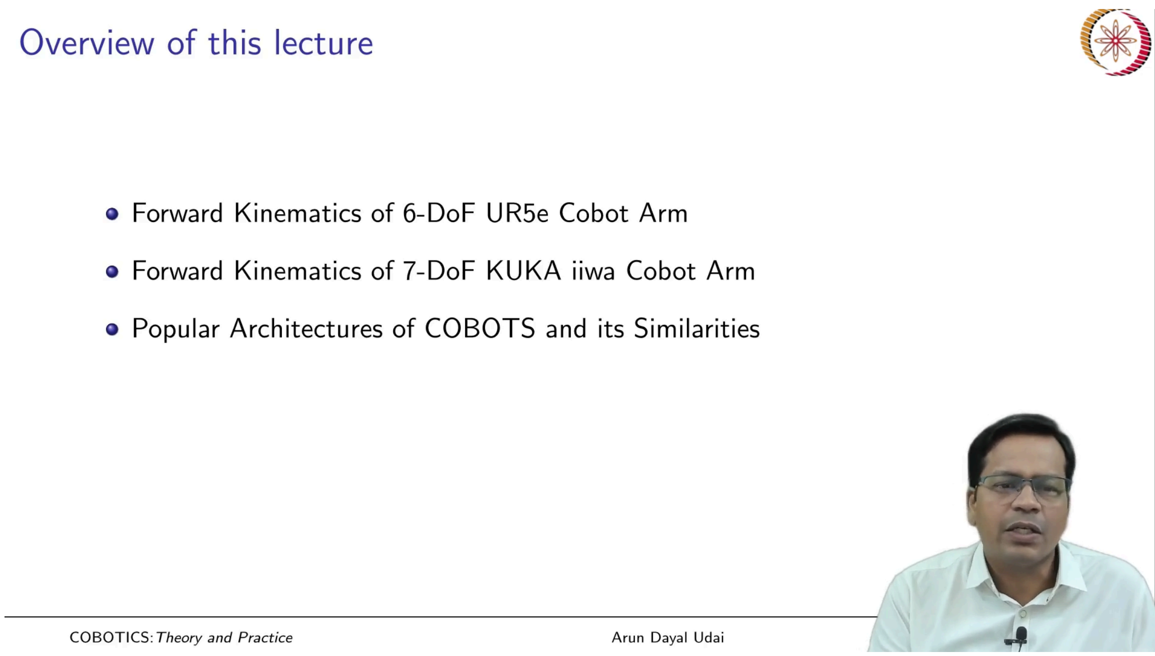
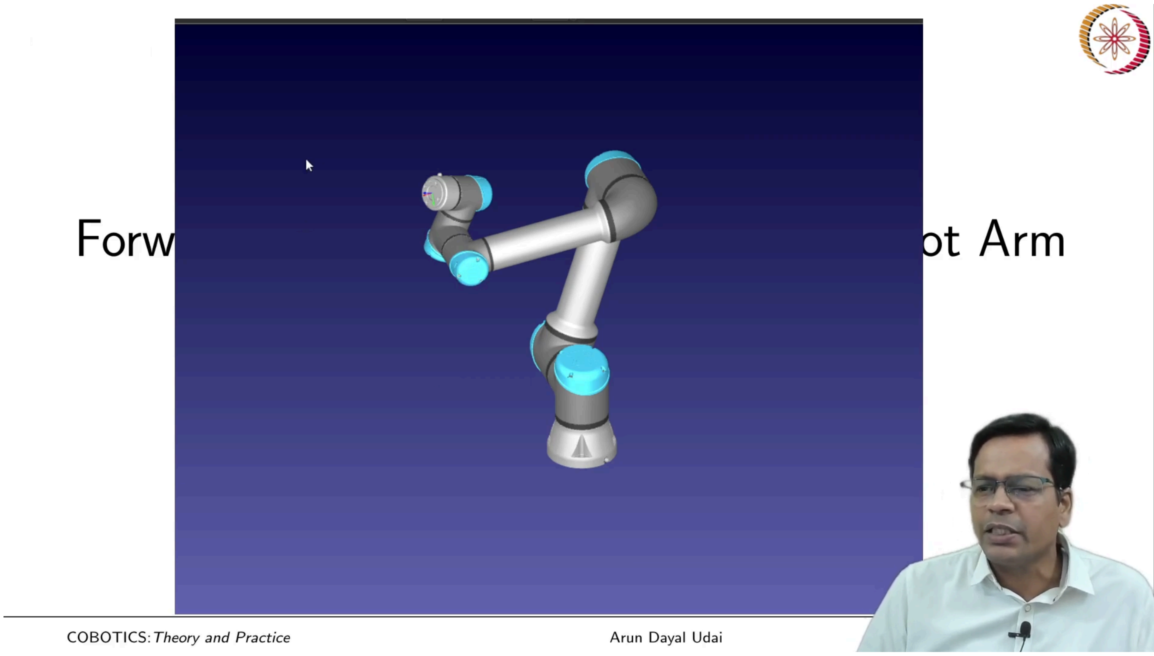
Week: 03
Lecture: 14

Forward Kinematics of Industrial COBOTS

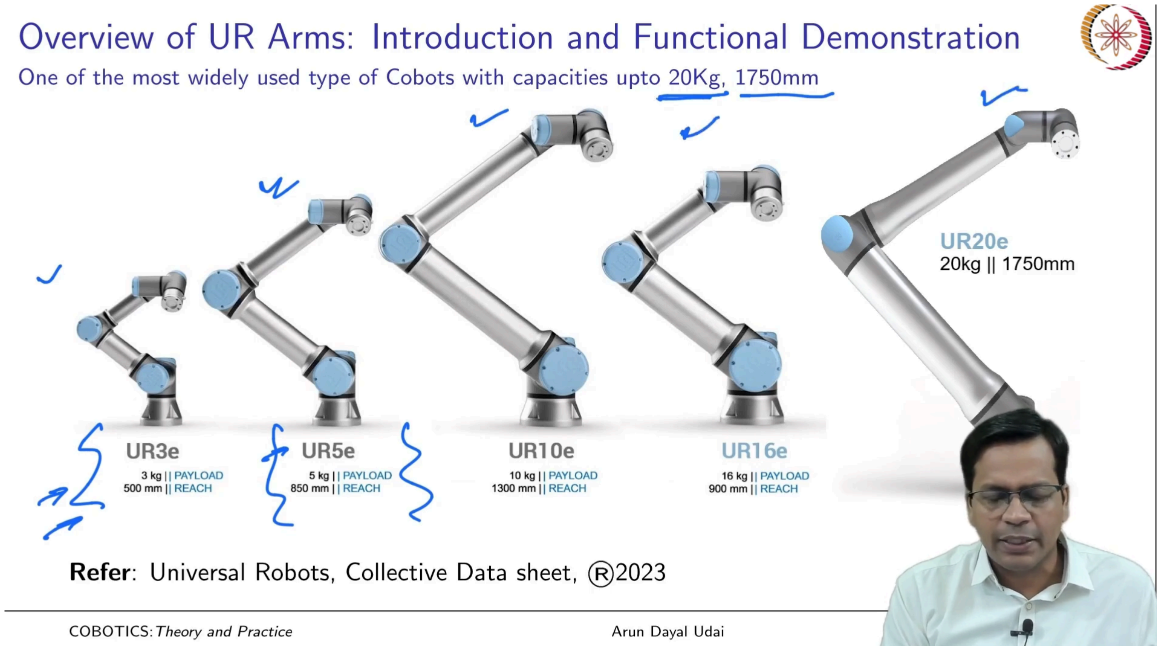
Welcome back to the fourth and final lecture of this module, which is about Transformation Matrices and Robot Kinematics.



In this lecture, I will discuss the Forward Kinematics of a 6-Degree-Of-Freedom UR5e Cobot Arm, which is a standard industrial cobot. Similarly, there is another robot, which is a 7-degree-of-freedom KUKA iiwa Cobot Arm. So, I will discuss forward kinematics for that, and I will also discuss various other popular robots or cobots that are similar to one of these architectures, such as the UR5e or iiwa, or they may be very similar to standard industrial robots. Those are all cobots. We will discuss the similarities and differences as well so that it helps you to extend the knowledge that I am giving you in this lecture to all forward kinematics.

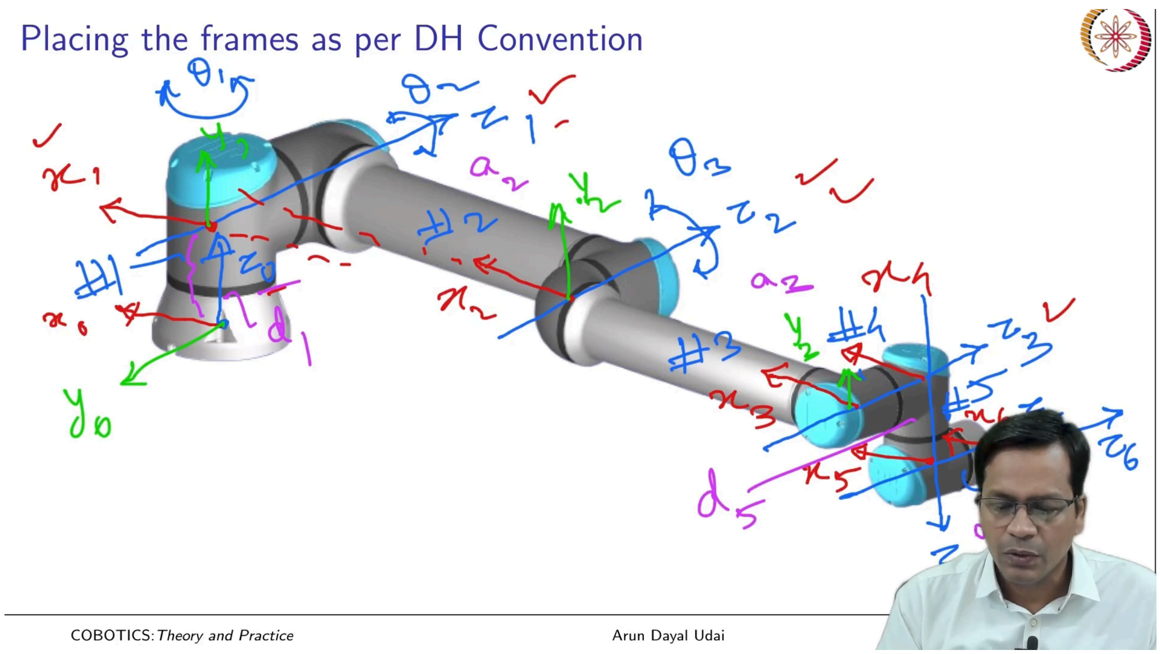


So, let us begin with the forward kinematics of the 6DoF UR5e Cobot Arm. Before we move ahead, let me just show you a short video of what it looks like. This is the UR5e Cobot Arm. So, it is a snake-like robot where you do not see more than two intersecting axes. This is the first axis. Moving this will move the whole arm. This is the second axis, the third axis, and the fourth axis. This is the fifth axis, and finally, the sixth axis is the tool end. What you found here is that any two axes next to each other are intersecting. There are not three axes intersecting, as in the case of a wrist arm. So, if it has a wrist, a spherical wrist at the end, then three degrees of freedom will lead to a position, and the remaining three will actually take it to any orientation. So, that requires three degrees of freedom at the end. They may be intersecting axes, but that is not the case here. So, you see two are intersecting, but ultimately, overall, this can also orient in any way.



So, let us now move ahead with the UR5e arm. UR5e arms are very popular and are the most widely used type of cobots due to their simplicity and wide availability, making them inexpensive. There is a wide range of robots, so that is the reason the maximum payload capacity it now provides is 20 kg, and the maximum reach is as much as 1750 mm. It starts with the UR3e, which has a 3 kg payload with a 500 mm range, and the UR5e, which I am going to discuss now. So, it has a 5 kg payload and an 850 mm capacity. It has a very similar reach to that of a human arm and maybe the same capacity, but yes, this is definitely more precise.

All these robots that you have can be seen as similar, but their shape and size are a little different. So, whatever the technique is to do forward kinematics and place the DH frames in each one of them, they are all the same.



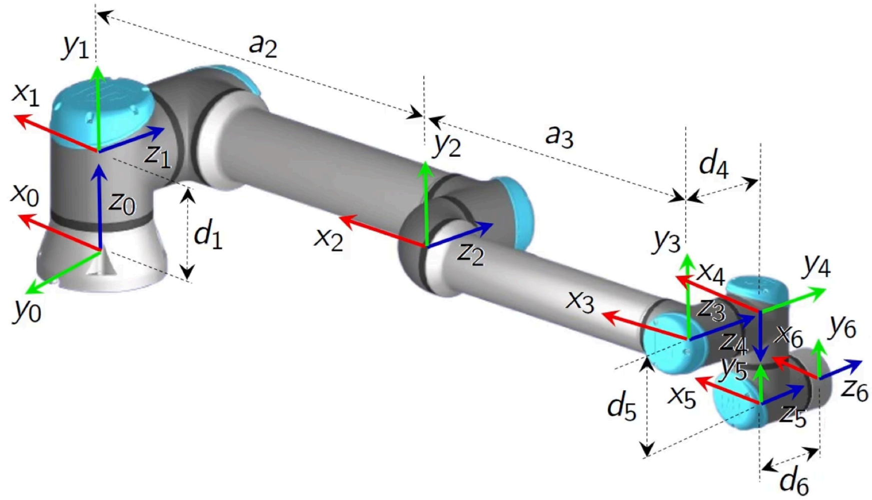
This is the raw robot when all the degrees of freedom are placed in the zero position. Let me just put it once again, so if I put 0 for link 1, 0 for link 2, 0 for link 3, link 4, link 5, and link 6. So, if I put all the arms in the 0 position, it exactly comes to the position that I have shown now. So, now I will go back to my slide.

So, this is the 0 position when the whole robot looks like this, and you see now I'll place one by one each of the axes. I'll also follow the default state of this robot at default coordinates, what it shows in this position. So, initially, I will try to put all the z-axis first. So, whatever BLUE CAPS that you can see at different locations, they are the motors basically that are going to move this robot. This is at its home position. So, for the first frame, I'll place it at the ground, so that is permanently attached to the ground, and that comes like this. So, this is your z0. This will move link 1 about this axis, and link 1 should end up with frame 1, which is z1, and that provides the motion axis for link 2. So, this is your Z1 about which link 2 will rotate. So, if this is the axis for rotation, this is your theta 1. So, this is your theta 2 for link 2. This is joint 2, and then you have link 2 ends up with frame 2 with z2, so that this provides motion base for the third joint angle that is theta 3, and then you have z3, then z4, and then z5, and finally you have z6. Both are collinear. z5 and z6. So, z6 ends up here, and z5 starts much before. So, these are all z-axes. So, this is link 3, this is link 4, you have link 5, and finally, link 6 is here. So, this is how all the z-axes are placed.

Now, how to put all the x-axes? X-axis, you know, all x is the common normal to zi minus 1 and zi. If it is for link 1, I have to place x1, so it should come like this. This is x1; it is the common normal to z0 and z1. I have placed it on this side; it could be on the other side also, but because it is the default state of the robot, I have placed all the frames the way they are in the original controller for this robot. So, in order to make no change here, I will just put x0 in the same direction. So, x0 and, accordingly, y should be like this. This is y0. Let me first put all the x-axes here. So, x2 is like this because it is again the common normal to z1 and z2. Z2 is along the same direction. Now, I will put X3. It comes something like this: X3, the common normal to Z3 and Z2. Now, the common normal to Z3 and Z4, again it should come something like this, that is X4, the common normal to Z4 and Z5, and finally, you have X6 that is here, X6. Z6 is finally the last frame which is here. So, all the common normals are done. So, that creates all the x-axis.

Finally, the y-axis can be done that is perpendicular to X1 and Z1 for this axis. So, Y1 should be like this. Similarly, Y2 should be like this. As per the right-handed coordinate system, Z cross X should give you Y. So, as per the right-handed coordinate system, I have placed all the y-axes. The y-axis is not very important here because all the DH parameters are along either the x-axis or the z-axis. So, all the distances that are there along the z-axis are d, that is, the joint offset, whereas all the distances between two z-axes along the x-axis are link lengths. So, you have; I will use a different colour here. So, this becomes your d1, not a1, because this is the distance travelled along the z-axis between two x and if it is between two z-axis and along the x-axis, it is link length. So, if this is d1, you have link length a2 here, this is link length a3 here, and similarly, I will put all the distances. So, this should be your fifth link, so it is d5, and this is your final. Again, it is along the z-axis. So, it should be d6. So, this is all done. So, yes, there are many other distances. It should not become clumsy here.

Placing the frames as per DH Convention



So, I will directly put it like this. So, now it shows all the frames that I have placed exactly in the same way. So, this is all the distances. So, those distances can be taken up from the robot datasheet directly. Let me just vanish for a moment so that you can capture it well. So, this is it.

Creating DH Parameter Table

Using the default states of the robot at its home position (All joints at 0° position)

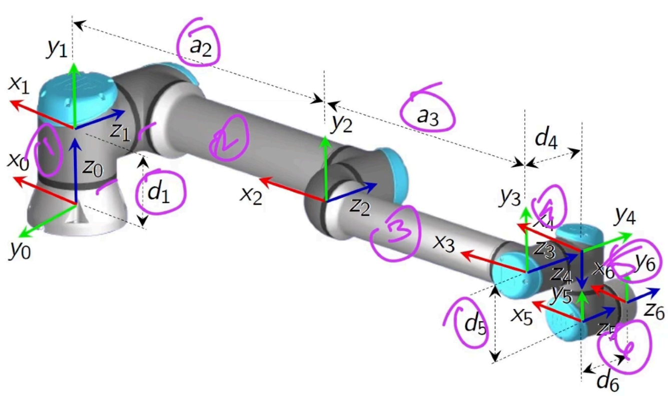
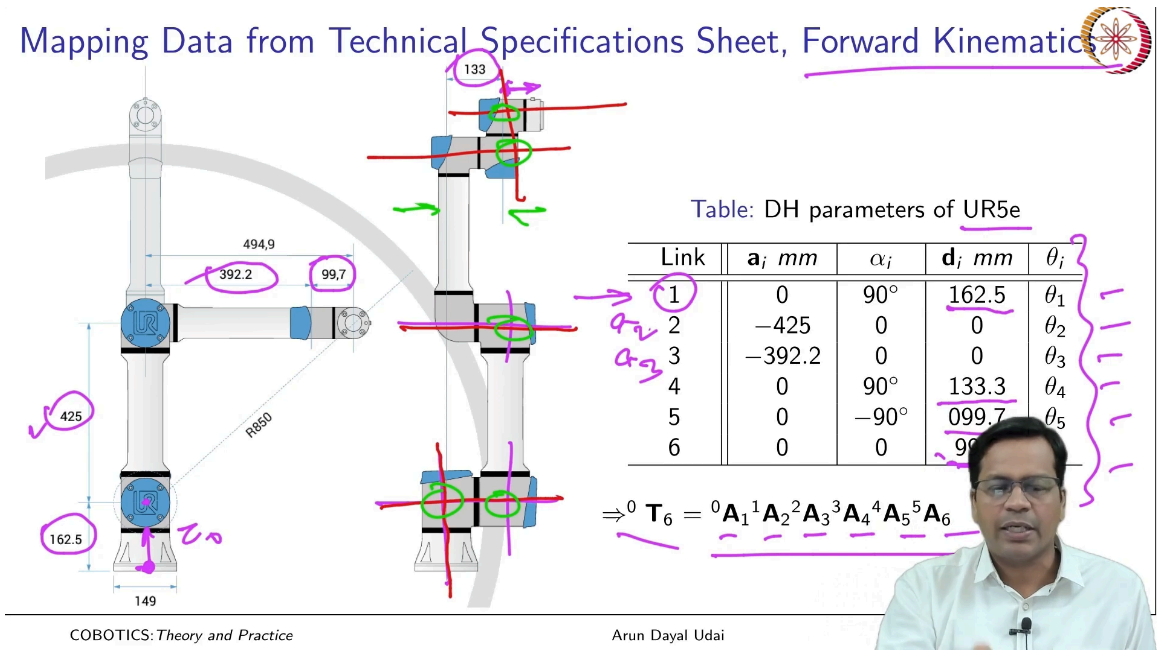


Table: DH parameters

Link	$\mathbf{a}_i$	α_i	$\mathbf{d}_i$	θ_i
1	0	90°	d_1	θ_1
2	a_2	0	0	θ_2
3	a_3	0	0	θ_3
4	0	90°	d_4	θ_4
5	0	−90°	d_5	θ_5
6	0	0	d_6	θ_6

So, now, if I take it to the DH parameter table, it comes up like this. So, The distance between two z-axes is the link length for the first. So, you see, z0 and z1 are intersecting.

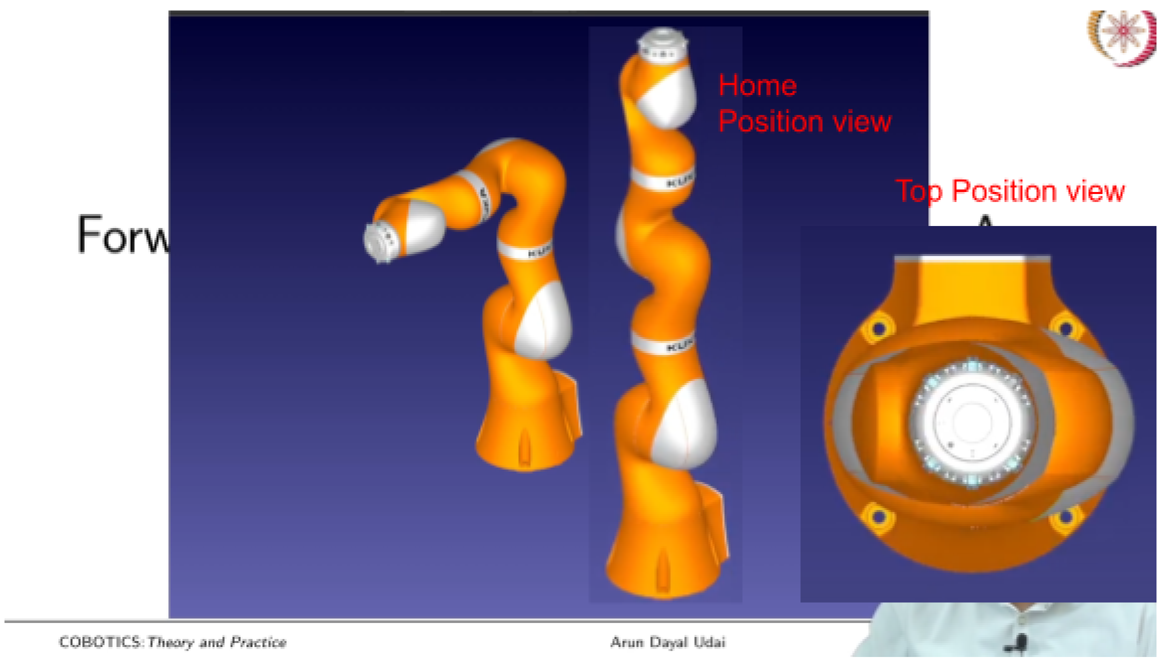
It gives you 0 over here; that is, link length 1 is 0. I will just name it here: 1, 2, this is 3, So you have 4, this is 5, this is 6. So, that is how it goes. So, accordingly, I have just copied the distances between two z-axes as a and the distance along the z-axis as d, which is the offset. So, in this case, this is here, a2 is here, a3 is here. And a few distances I have not mentioned here, but those are there, and that should come like this. These are all the joint variables. Let me vanish again. So, this is the DH parameter table.



So, now, if I populate this DH parameter table by looking at the specifications from the technical data sheet of the robot. So, it clearly shows the distances. So, d1 is visible here, which is from the 0th frame to the first frame. That is the distance travelled along Z0. It is 162.5. Next is 425. This you saw it was a2, a3. So, this is a2, and then you have a3 that is here, and then you have 133.3 that is as d. So, this is here, d4 and d5 that is here is 99.7, okay, and d6 finally, so that is the final distance, which is not visible here and not mentioned in the datasheet, but yes, that depends on the type of media flange and many other things, so by default, this distance is a variable distance, and for this robot, UR5e, it is 99.6 from the data sheet; it is there. So, these are all joint angles, which are variable here. So, from the manufacturer's technical specification sheet, I can capture all the distances. Otherwise, you can do kinematic identification experiments, using which you can do forward kinematics. By just multiplying the link transformation matrices.

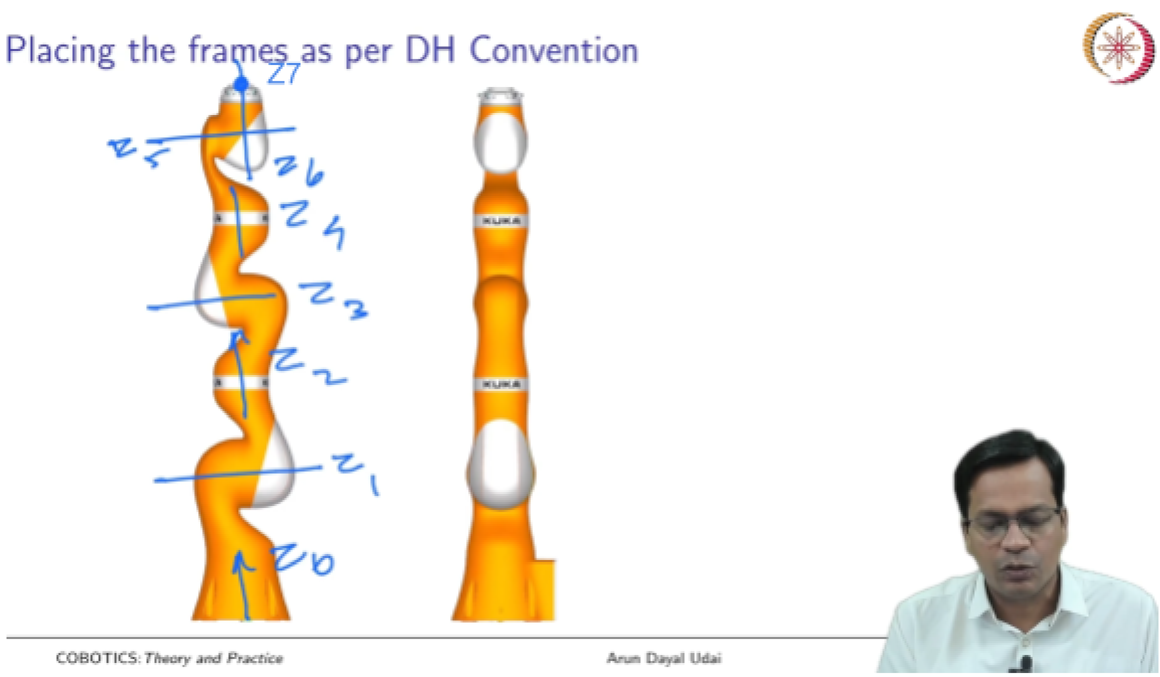
Individual link transformation matrices are calculated based on the table. For link 1, you take d, theta a, alpha from 1, then d, theta, a, and alpha from 2, 3, 4, 5, and 6.

Finally, when you take the product of all those matrices, what you get is the final homogeneous transformation matrix, 4 by 4. The dimension that says the state of the robot it includes the orientation as well as the position. So, this is how you do forward kinematics of your 5e kind of robot. This is a snake type, so if I can mark it here, you see. This is your first axis; this is your second axis, your third axis, your fourth axis, your fifth axis, and finally, the sixth. So, at any point, you see only two intersecting, so it moves like a snake. It goes zigzag like this, and finally. It cannot be placed in a sagittal plane 100%. So, it is always there is some offset from the centerline axis, okay? The centerline axis, or centerline sagittal plane, it goes a little off. So, this is how it is, and forward kinematics is done for this.

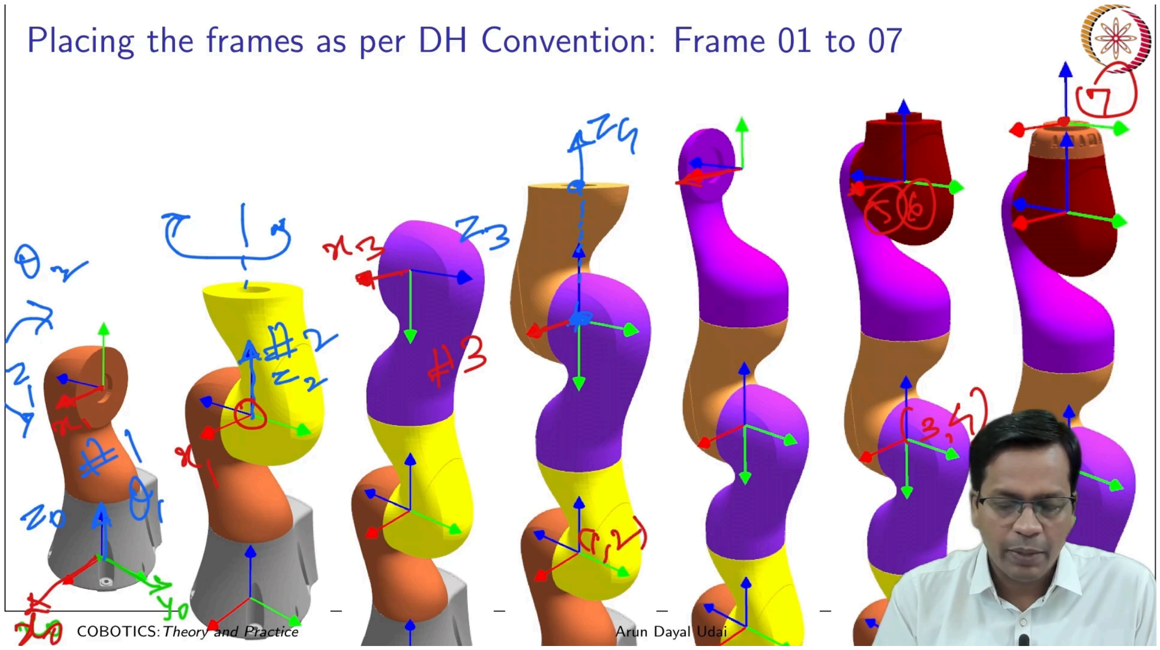


Let us move ahead and do the forward kinematics of the 7-degree-of-freedom Kuka iiwa Arm. This is a 7-degree-of-freedom arm. Let me again show you a video here. S,o this is the robot, which is KUKA iiwa. This is the seventh arm. Let us first move the axes here. So, this is your first axis, this is the second, this is the third, and this is the fourth. This is the fifth. You see, even with the link between two parallel axes, there is a cross-axis. So, that is basically dividing the arm, which used to be quite straight in the case of a solid, standard industrial robot. So, this is the break here. Wherever you see KUKA written here, those are the breaks. So, this is axis 5. So, this is axis 6, and finally, this is axis 7. So, this is how it can go. So, yes, when I put all of them to 0, that is the home position of this robot. So, this is the home position of this robot. You see, this can be made to go in a straight line. If you look at the top view, the top view of this. So, it is exactly in a straight line.

So, this is how this robot is, which is not quite possible in any of the UR arms or those types of robots, which are snake robots. So, this is in a straight line; they lie in a sagittal plane also. So, if you move axis 2, axis 4, and axis 6. 2, 4, and 6, the robot moves in the sagittal plane also. So, there is no offset anywhere.



So, now let us move ahead. So, this is how this arm looks like. You see, we have to place all the axes. Quickly, I can place Z like this. So, first, Z0 should lie here. This was our axis. This was another axis. Z0, Z1, Z2, Z3, Z4, Z5 Z6, and finally, you have Z7, which should be placed here. So, this is how all the Z-axis needs to be placed.

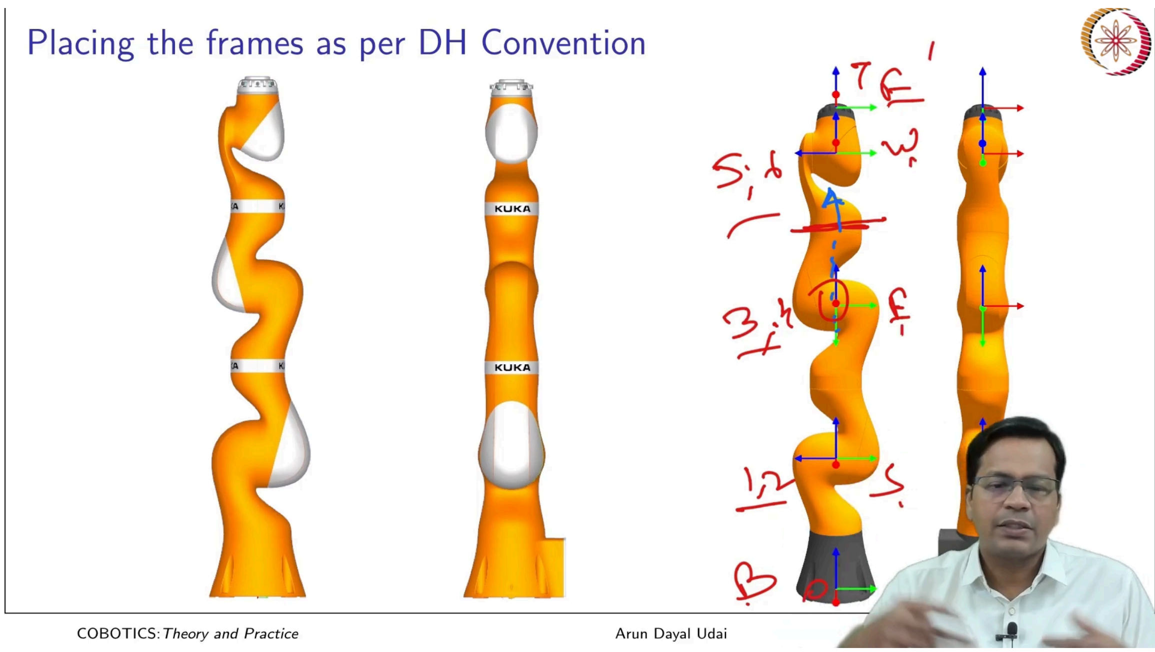


Now, let us start placing the frames as per DS convention from link 1 to link 7, frames 01 to 07. I will place them all. Let us begin with the ground frame and the first link. So, this is my ground frame where it is placed again; it is the default position. So, this becomes my x0, y0, and z0 it should provide the axis of motion for link 1, which is in brown colour. So, this is link 1. Link 1 ends with a frame that provides the axis of motion for link 2. So, this is your link 1, which ends up with z1 here. so this was z0, which was here, and z1 is here. So, this is link 1.

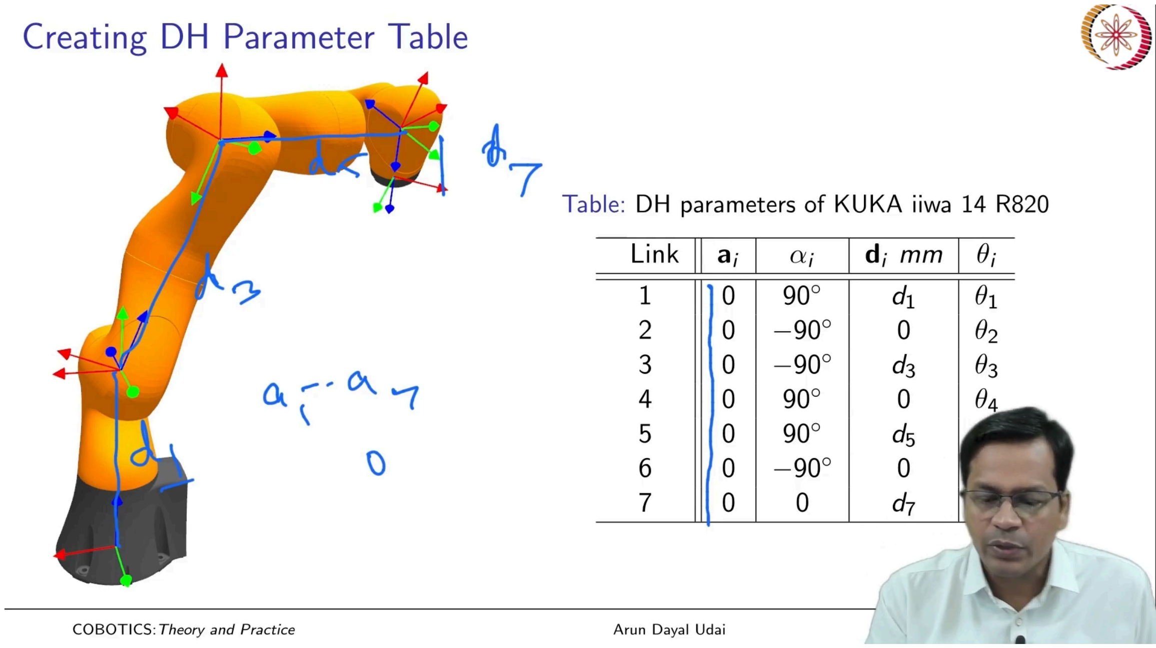
Now, this is your link 2. So, this is moving about this axis. So, this is your theta 2; this was initially your theta 1. So, now this link should provide the axis of motion for the third link that comes next. I know that it moves like this, okay; it moves like this. So, this should have an axis, the z-axis, like this. So, I have placed it like this. So, this is your Z2 that provides motion to link 3. This is how it is. So, link 3. So, this is your link 3. Link 3 should end with its own frame, Z3, which provides motion to link 4. So, this is purple in colour. So, I know it was like this. So, this is your Z3. So, this is how you have to keep placing all the frames next to each other, okay? So, if I keep moving like this, I have placed all the frames like this, okay? Hope you understood it now.

So, each next link should have a Z-axis which is placed on the prior link, and that is how it moves. So, this is your Z3. Okay, then, instead of placing it at the end here as Z4, I have protruded it like this, a little backward, and I have made it over here. Why, you know? Because there is a common point between two Z-axes that should create an X-axis. So, that is the reason. So, this is your X0, this is your X1, X1, X3. So, you see, there is an intersecting axis here of 1 and 2 over this point. So, that is why that colour is not visible here, and accordingly, you see all the x-axes are placed.

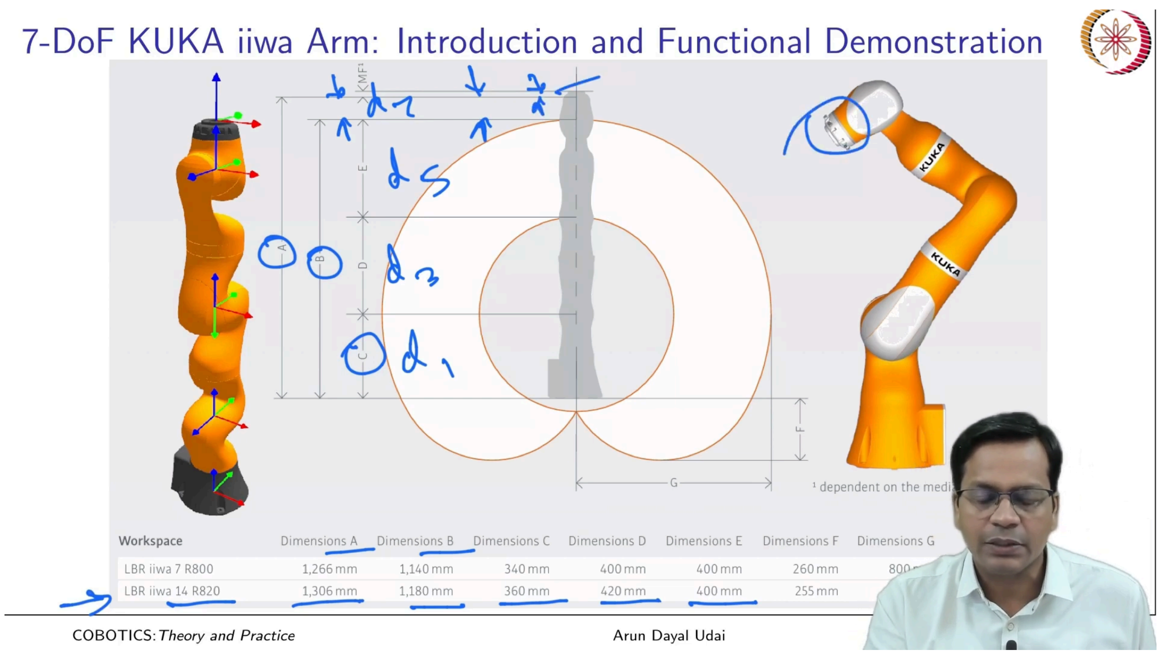
So, this is how you create all the axes till the end. At the end, this is the 7th frame. This is your 6th frame, right? The sixth frame and fifth frame are combined together again. Three and four are combined together. Three and four. 1 and 2 are combined together. 0 is at the ground.



So, ultimately, you can see all the frames are here. This is 0, 1, 2, 3, 4, 5, and 6. Ultimately, 7 goes here. So, instead of putting the frames here, it comes a little backwards so as to find a common normal for the x-axis. The z-axis comes like this, but it is protruded like this so that you see a common normal, and you can create an x-axis over here. Same at every intersection point wherever your axes are intersecting. You see, there are many wrist-like structures. So, this is your shoulder joint. Finally, this is the elbow, and finally, this is the wrist, and finally, this is the end effector. This is your base. So, base, shoulder, elbow, wrist, and end effector. This is a straight robot. It can be aligned in a straight line.



So, now, if I move a bit, all the axes are segregated. You can see the axes very clearly. So, all the distances here that are along the Z axis are d, so this becomes your d1. So, you see, there is no distance between two Z axes, okay? So, that makes all the a0. There is no a; there is no distance between two Z axes. So, this is d1, this is d3, and then you have d5 and, finally, d7 that comes here. So, this is how it is, and all the link lengths a1 to a7 are all 0 because there is no distance travelled between two Z axes along the X-axis. There is no distance that exists. They are all overlapping one over the other. So, this is a unique robot, and there are many robots that are in this architecture also, so we'll discuss this next.



So, if I populate that, so you see you have distances. This is from the datasheet of KUKA iiwa. You see, so this is d here; this was d1, d3, d5, and, finally, this d7. is calculated by taking A minus B that comes here. So, if this is A, this is B. So, for this KUKA iiwa 14, 14 kg payload, 820 reach, it is A and B are taken from here. This is d1, d3, d5, and this d7 distance, this basically depends on the type of media flange which is there also. So, that small gap after this depends on the thickness of the last part.

Populate DH Parameters, Forward Kinematics

Using Manufacturer's Technical Specification sheet, System Identifications, or CAD

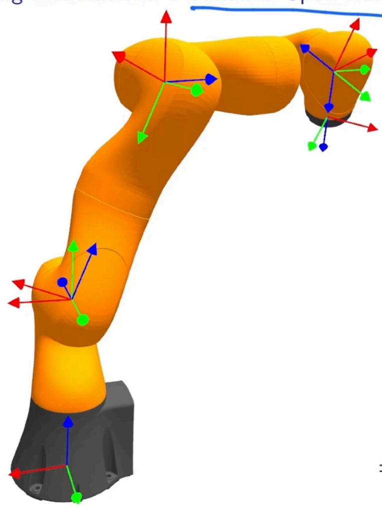
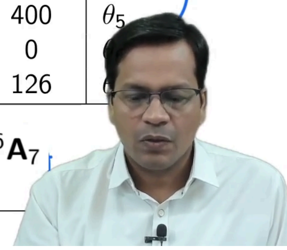


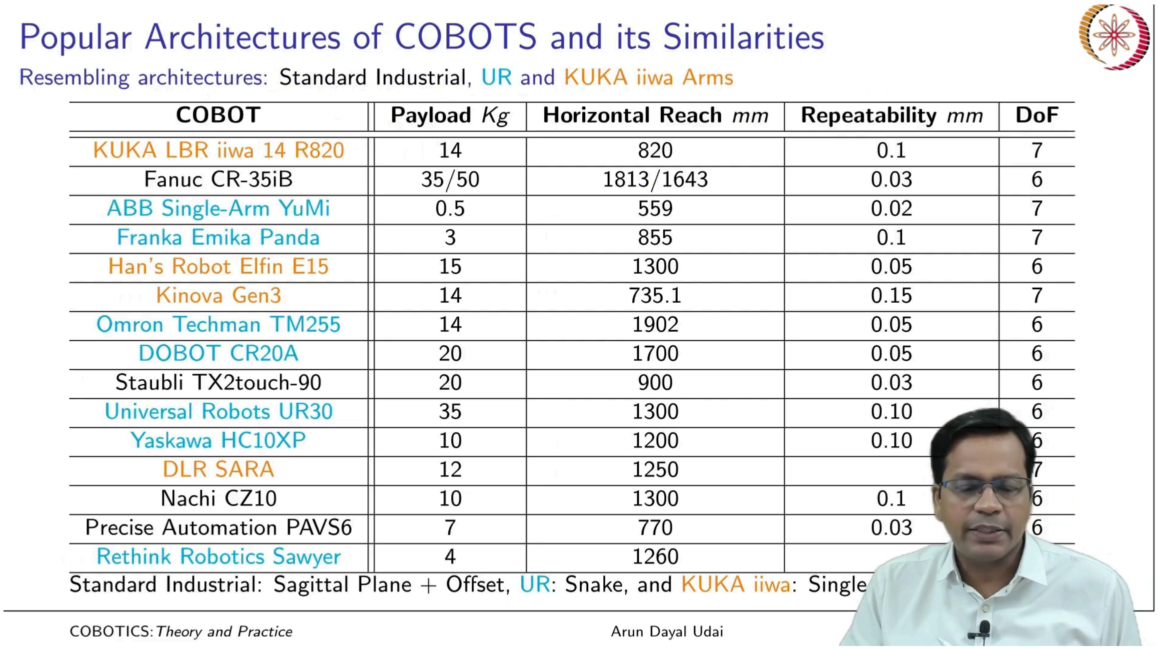
Table: DH parameters of KUKA iiwa 14 R820

Link	$\mathbf{a}_i$	α_i	$\mathbf{d}_i$ *mm*	θ_i
1	0	90°	360	θ_1
2	0	−90°	0	θ_2
3	0	−90°	420	θ_3
4	0	90°	0	θ_4
5	0	90°	400	θ_5
6	0	−90°	0	[illegible]
7	0	0	126	[illegible]

$$\Rightarrow {}^0\mathbf{T}_7 = {}^0\mathbf{A}_1\,{}^1\mathbf{A}_2\,{}^2\mathbf{A}_3\,{}^3\mathbf{A}_4\,{}^4\mathbf{A}_5\,{}^5\mathbf{A}_6\,{}^6\mathbf{A}_7$$



So, this is how once you take up all these values, you populate them here. And finally, you can perform forward kinematics of the KUKA iiwa robot. It can be extracted from the technical specifications of the robot, as I have shown from the manufacturer's data sheet, or using system identification experiments, kinematic identification, or using the CAD model. Nowadays, manufacturers provide all the CAD models on their websites. So, if you calculate all the link transformation matrices. So, they come as 0, A1, 1, A2, 2 to 3, 3 to 4, 4 to 5, 5 to 6, and finally, it is 6, A7, which is also there. So, that ultimately would lead to T0 to 7. So, this is the forward kinematics of the KUKA iiwa.



Popular Architectures of COBOTS and its Similarities

Resembling architectures: Standard Industrial, UR and KUKA iiwa Arms

COBOT	Payload *Kg*	Horizontal Reach *mm*	Repeatability *mm*	DoF
KUKA LBR iiwa 14 R820	14	820	0.1	7
Fanuc CR-35iB	35/50	1813/1643	0.03	6
ABB Single-Arm YuMi	0.5	559	0.02	7
Franka Emika Panda	3	855	0.1	7
Han's Robot Elfin E15	15	1300	0.05	6
Kinova Gen3	14	735.1	0.15	7
Omron Techman TM255	14	1902	0.05	6
DOBOT CR20A	20	1700	0.05	6
Staubli TX2touch-90	20	900	0.03	6
Universal Robots UR30	35	1300	0.10	6
Yaskawa HC10XP	10	1200	0.10	6
DLR SARA	12	1250		7
Nachi CZ10	10	1300	0.1	6
Precise Automation PAVS6	7	770	0.03	6
Rethink Robotics Sawyer	4	1260		

Standard Industrial: Sagittal Plane + Offset, UR: Snake, and KUKA iiwa: Single

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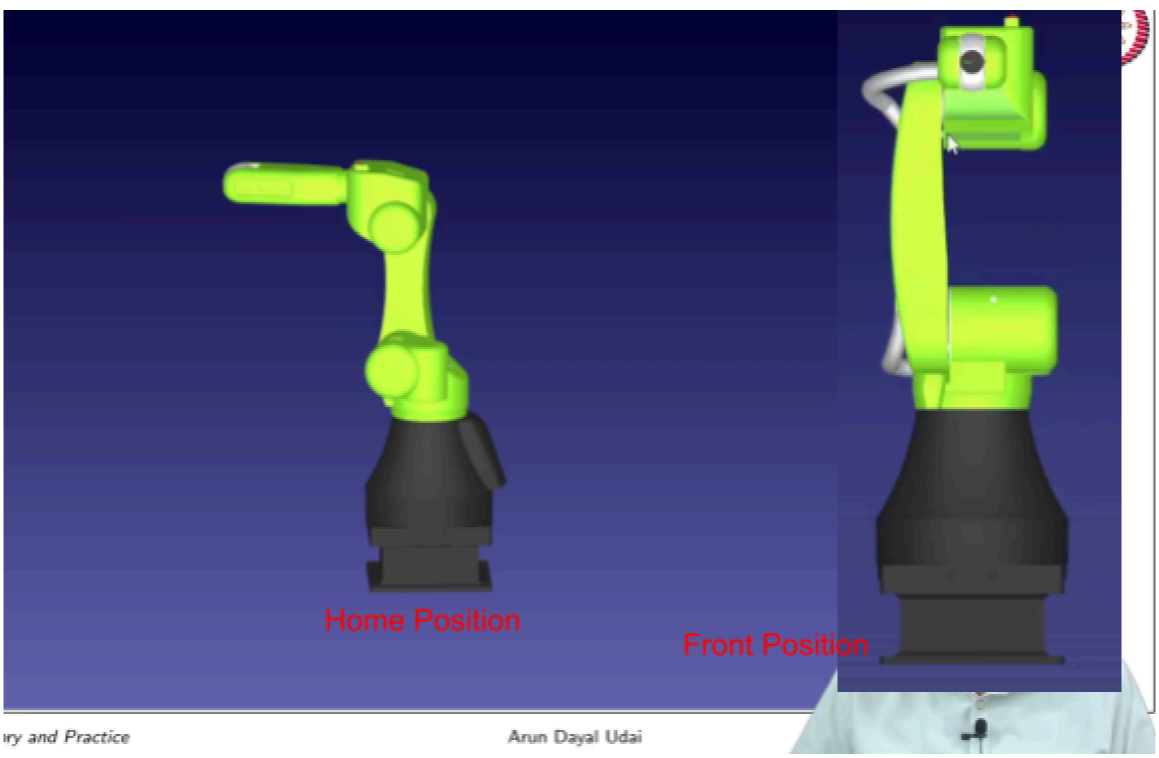
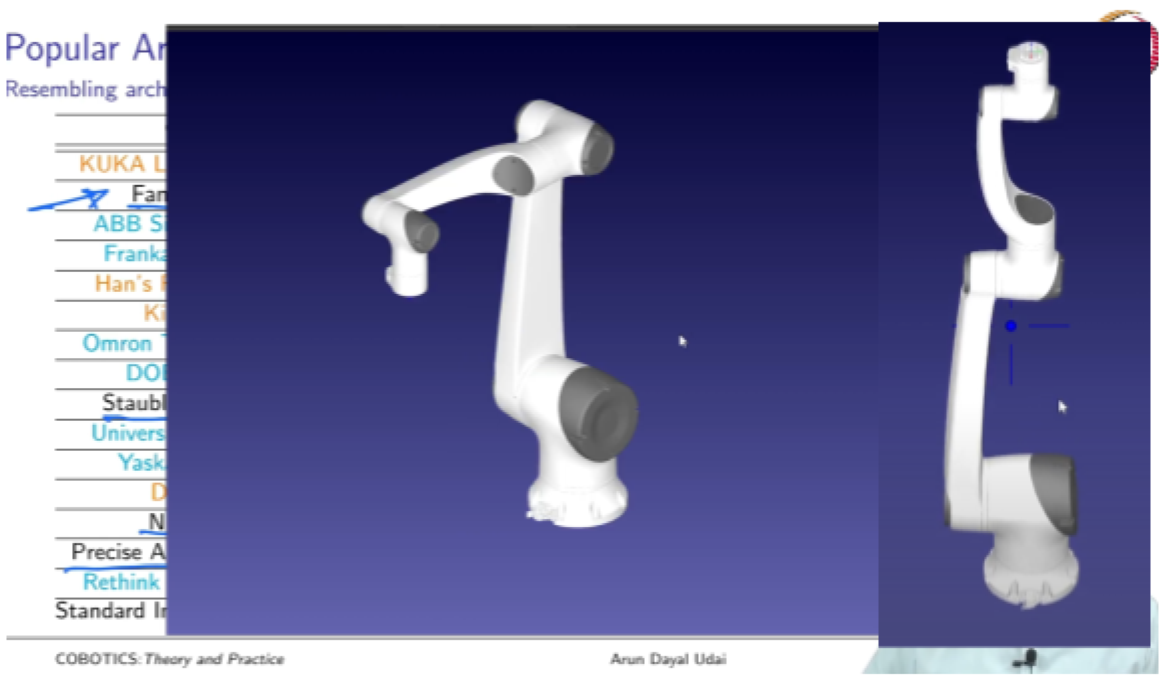
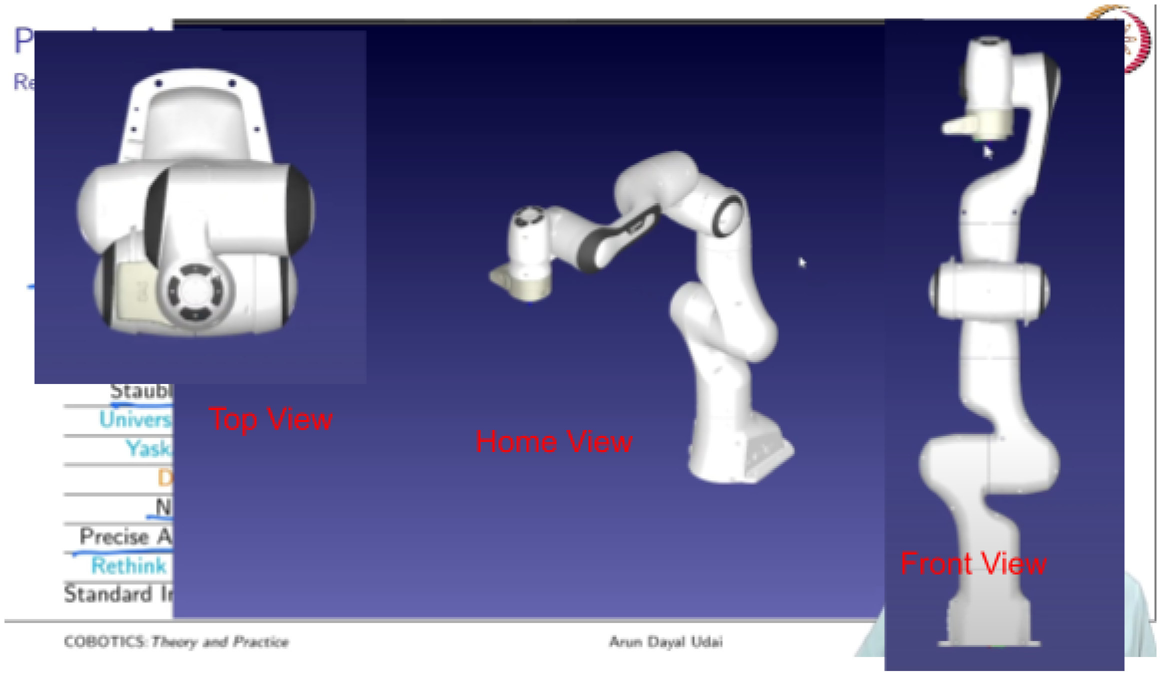
So, now, there are many similarities between the KUKA iiwa and various other arms that look similar. Let me just open one by one.



So, this is the FANUC robot CR-35iB. You see this robot. It is very much similar to an industrial robot. It looks exactly the same. So, there is an offset here. So, between this and this, there is a small gap. So, if you see axis 1, this is the axis, and axis 2, this is the axis. So, what happens? You see both. This one has a fixed gap. So, that is the offset between Z0 and Z1. So, that is the link length across that. So, this style is very popular among all industrial robots. So, this arm looks like a standard industrial robot, not like a snake or not like a KUKA iiwa that I have shown. If you set all the axes to 0, it should look something like this. So, this is the home position of this robot. So, this is what I have marked in black colour over here that is Fanuc CR-35iB, Staubli TX2touch-90, Nachi CZ10, Precise Automation PAVS6 etc. All these are black ones. So, they all belong to that category. So, they can be put in a sagittal plane. So, if it is in a home position if you look from the front. So, there is a plane that is the sagittal plane of the robot on which the whole of the robot can move. So, this is what I mean.

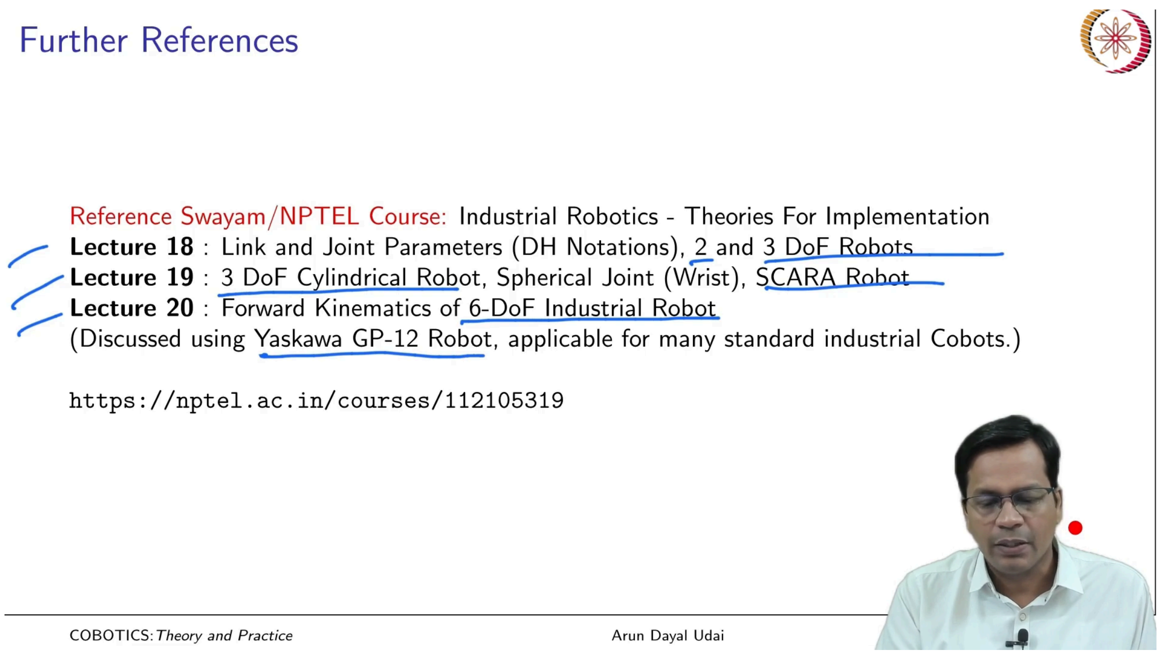


Then the next case is this again appears like an industrial robot, but it is not. If you put all the axes to 0 degrees and look at this robot from the top, you see again the topmost point and the bottom frame; they are in a straight line. So, that is what it is. So, I can make this robot go like this. So, if you look from here, it is like this. So, it is in a straight line again. So, this is exactly similar to the KUKA iiwa robot, although it looks quite different, but Kinematically, they are the same, and you can put the DH exactly the same; the DH parameters will be there. Only the dimensions will vary for d. All the ds which will be along this axis. But this is a 6-DoF robot. 6 degrees of freedom robot. Just 1 degree less than iiwa. But it is very much similar to KUKA iiwa.



So, the next one I will open is Franka Emika Panda. So, this should be like UR arm. Yes, it is. If you just see, put all the axes to 0 degrees and look at the robot from the top. You see, they are not in the same line. They are a little bit different. And from the front, it looks like this. You see, it has the first axis. It tends to be very similar to Kuka iiwa right up till here. But after that, it has changed a little bit. So, it is the centerline axis of this robot. So, the end effector does not lie on that. So, that is the difference. So, that makes it a little different from KUKA iiwa.

Why is that important? It will be more relevant when you start solving the inverse kinematics of that. So, that is why I am coming to this. So, I have opened up a few robots for you, but you can always download these CAD models from the manufacturer's website and check these architectures. A few of them are 7 DoF, and quite a lot of them are 6 degrees of freedom. The payload varies, with a maximum of up to 35 kg, which is for FANUC. The reach is maximum for this. So, the standard industrial robot is the sagittal plane plus offset. UR would be like a snake kind of, and iiwa would be like a single inline one. So, these are a few architectures.



I would suggest you further go to these references that I have mentioned for you, which are from my other course, Industrial Robots: Theories for Implementation- lectures 18, 19, and 20. They also discuss the DH parameter placement and forward kinematics of a few more robots apart from the one that I have discussed here: 2DOF, 3DOF SCARA, 3DOF cylindrical, and 6DOF industrial robots. I discussed the Yaskawa GP-12 there, but that is also applicable, you know, for quite a lot of standard industrial cobots that are of the same kind. This is the link for the course. That is all for this lecture.

In the next lecture, we will discuss Inverse Kinematics, Velocity Relationships, and Acceleration Analysis.

Thanks a lot.