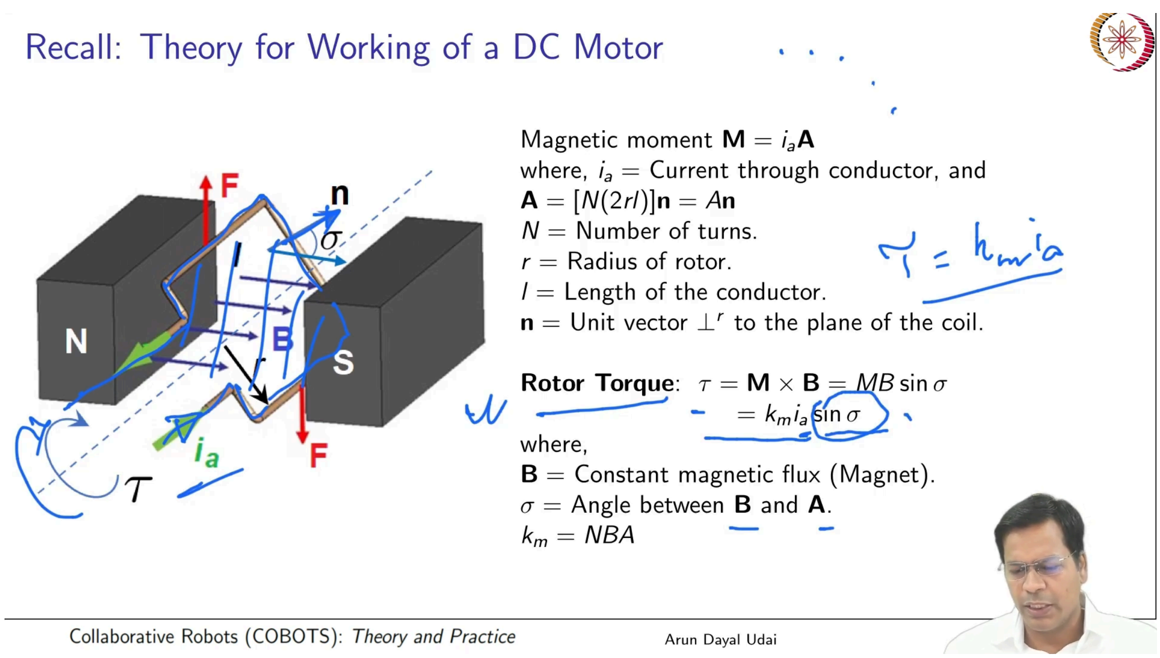
NPTEL Online Certification Courses
COLLABORATIVE ROBOTS (COBOTS): THEORY AND PRACTICE
Dr Arun Dayal Udai
Department of Mechanical Engineering
Indian Institute of Technology (ISM) Dhanbad
Week: 07
Lecture: 31

A Robot Joint: DC Motor Model

Hi, we are now ready with the tools to study control. So far, we have studied linear control systems, specifically linear second-order systems, as we have seen. We have seen how to study their behaviour using the roots of their characteristic equation. We have seen how to analyse such systems using transfer functions and state-space representations as well. So, at least, we are now ready to study Robot Control. So, let us now study controlling any robotic joint to begin with. So, let us start.

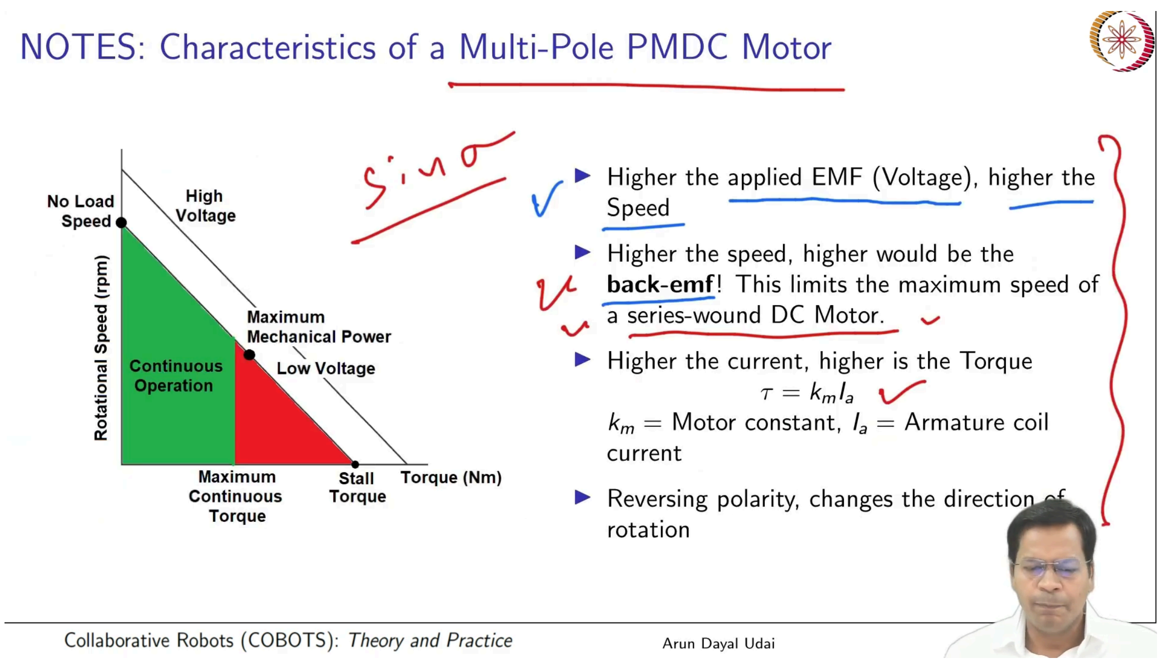


So, let us just recall the theory of the working of a DC motor that we have seen earlier in Module 2. So yes, this is the first thing that we have learned. So, we saw the structure of a DC motor. Basically, it contains a simple armature coil that is shown here, and you see it has a current that flows into there, given by i_a, which is the armature current. It comes out from here. It develops a torque, which is given by tau.

Not many parameters are here to observe. The first thing that I want to point out is this equation is the robot torque, which is the joint torque that the rotor of a motor sees, is proportional to the current, and obviously, there is something called sine sigma. Sigma is the angle between B and A. B is the field direction, and A is the normal to this area. That is the direction of the normal to the area. If this is the area, the normal is this. So, this is the area. So, you see, if you have multiple coils which are there, which are at, let's say, multiple poles and multiple coils, this becomes almost insignificant, and in that case, tau is proportional to the current that is flowing through that. So, this torque basically varies in a sinusoidal manner; it comes up, it goes down, and if you have multiple coils, by the time one of them dies off, the next one gets in. So, you have, let's say, 32 poles or maybe 20 poles or something like that. So, if you have multiple poles, that evens out the torque that is coming due to this change. So, tau is equal to Km into Ia.

$$\tau = k_m I_a$$

So, this is the first thing that we have seen.



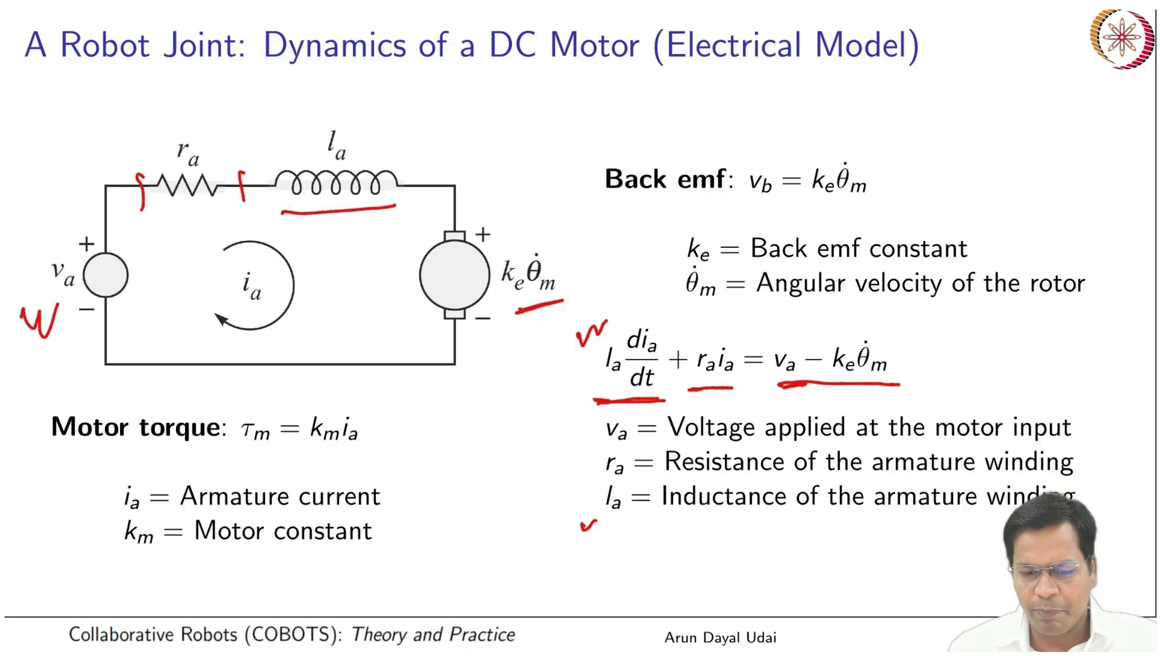
Also, the higher the applied EMF, that is, the voltage, the higher the speed that we have seen. We will now see why it is happening also.

So, this is the first thing that I have pointed out earlier. And the higher the speed, the higher would be the back EMF. Now, what is the back EMF? Just let us go back to the slide once again. So if you are providing current from here, so you see you should feel forces, that is the couple of forces which are there, that is creating a couple basically. So, that is effectively creating the torque which is aligned like this. So yes, if at all you are providing current, it produces torque. If you don't provide any current and the rotor keeps on rotating due to its inertia, let's say while you want to stop your motor. So, in that case, what happens as soon as you stop flowing? This current is the conductor that is moving inside this field. So, the field is always there. So, you see, you have a field that is always there. So, the field is there, and you have a conductor which is like this. So, conductors are there. So, if that is moving, it automatically induces a current in this conductor okay. So, that is there. So, even if you don't provide any current from outside due to its own motion within the magnetic field causes some current to be induced. So, that current is in a reverse direction.

So, cause an effect. So, if this is the cause which is creating this current. So effect will be the current that creates a reverse torque. So as to stop the motion.

So, it is known as back EMF. So, the higher the speed, the higher the back EMF. So, this basically limits the maximum speed of any series-wound DC motor. In the case of a series-wound DC motor, if back EMF is present, it effectively reduces the incoming current. So, the total current flowing through the armature greatly reduces. At higher RPM, it is very significant, and that causes a reduction in torque. The torque gets reduced. So, that is basically limiting the maximum speed of this. So, that is the reason why speed is limited. In series-wound motors, it is more significant because the same current flows through the armature. So that is there.

The higher the current, the higher the torque that you have already seen in the case of a multipole DC motor. It is directly related without any connection to the sine sigma that you have seen in that case. Reversing the polarity would change the direction of rotation because that would create the forces induced in the coils. So, these forces, this force direction will be reversed if you reverse the direction of the current, and that will create a reverse torque. So, these points should be noted before we begin.



Let us begin with the robot joint. So, we will discuss the dynamics of a DC motor. So, this one, in particular, is the electrical model that I am going to discuss. So yes, this is the applied voltage across the terminals of your motor. Let us say it has its own internal resistance. The coil resistance is given by ra, that is, the armature resistance. Because it is a coil, it is copper wound over a core. So yes, it behaves very much like an inductor also. So, this has an inductance of la, and I know there is a back EMF. So, this is a back EMF constant ke, and that is giving a reverse voltage which is proportional to. You see the back EMF voltage. This gets positive, opposite to this. So, back EMF is there. So, this is the back EMF constant and theta m dot. What is theta m? Theta m is the speed of your armature. So, the higher is the speed, the higher the back EMF. That is what I told you.

$$\tau_m = k_m i_a$$

So, these are the basic relations that you already know from your previous slide. So, i_a is the current flowing through the armature. The motor constant is km.

Now, the back EMF Voltage is proportional to speed, the angular velocity of the rotor, and the back EMF constant is given by k_e.

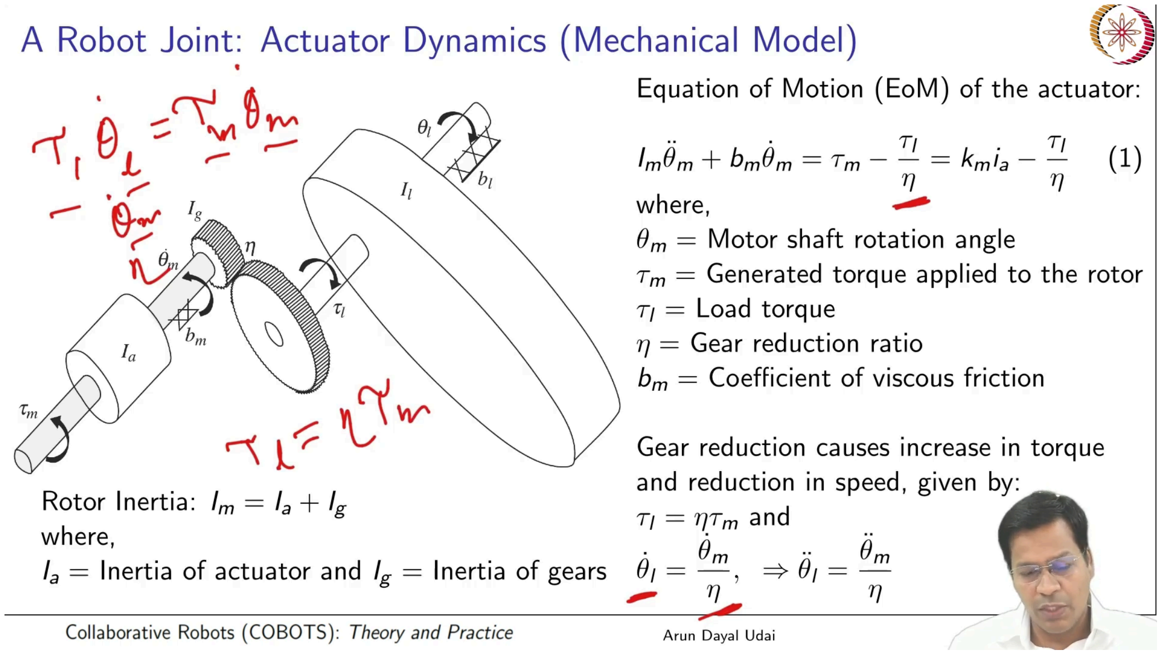
$$v_b = k_e \dot{\Theta}_m$$

So now, putting them all together, what I can write is this closed circuit has got potential across each of them. So, summing them all together should amount to zero. So yes, this is the applied voltage v_a, applied EMF, that is the driving voltage, and this is your back EMF. So subtracting them should give you the effective voltage that is present in this loop; that is what is causing the potentials to be generated across the resistance and across the inductance.

So across the inductance, it is la, that is, the inductance of the armature winding into dia by dt. Rate of change of current. So, the voltage induced across the inductor is inductance, which is the constant into the rate of change of current. So, the higher the rate of change, the higher the voltage that is induced across this inductance. So, this is first. These are basic physics equations that hopefully you have gone through during your higher school also. The voltage across this resistance is ra into ia. This is simply by Ohm's law. So, ia into ra plus Ia dia by dt should be equal to the total potential that is there across this.

$$l_a \frac{di_a}{dt} + r_a i_a = v_a - k_e \dot{\theta}_m$$

So, this is the first fundamental relation, that is, the electrical relation of this DC motor.



Now, let us look into the Mechanical Model also. We call it the actuator Dynamics of the Robot Joint. So, yes, it is constructed like this. So, you have tau m that comes here. That comes here, that is, through the motor, and Ia is basically the moment of inertia of this armature, the whole system including the coil, the solid armature, the core is here, including the shaft everything is here. And you have theta m, which is the joint angle at an instant of time. You have bearings that go here, here, here, here. Altogether, let us say you have a bearing constant here. That is basically the damping coefficient given by bm, and you have a gear ratio over here. There is a transmission. This is the smaller one. This is the bigger one. Overall, it reduces the speed when it comes to this and enhances the torque. So, yes, that torque which is transferred to the link joint is tau l. So, tau m is the motor torque. Tau l is the shaft that connects your link. The link has its own moment of inertia given by Il, and this shaft rotates by an angle theta l. You also have a damping coefficient over here that is bl. So, these are the parameters. Overall, the whole mechanical system can be visualised like this.

So now, let us start forming the dynamic equation for this. So, the rotor inertia will be Armature plus the inertia of the gears. So, gears are also attached with the armature. So, the total inertia Im, which is the inertia of the actuator, should be equal to Ig plus Ia. Armature plus gear total inertia is like this.

$$I_m = I_a + I_g$$

The axis of rotation remains the same. So, it can be simply scalar added.

So, the equation of motion of the actuator can now be given as: the effective torque that is coming from the link side of it. So, if this has a torque which is tau l, which is here, the torque which is coming, the resistance torque which is coming onto the motor shaft is Tau l by eta. So, yes, this is tau l by eta that is reflected out here, and tau m is the driving torque. So, the net driving torque due to both of these is tau m minus, and this is the reaction torque that is coming onto this, that is tau l by eta. So, that comes here.

So, that is actually driving your shaft. So, it is Im theta m double dot, moment of inertia into angular acceleration, plus bm theta m dot, bm is the damping coefficient into angular velocity.

$$I_m\ddot{\theta}_m + b_m\dot{\theta}_m = \tau_m - \frac{\tau_l}{\eta} = k_m i_a - \frac{\tau_l}{\eta} \qquad (1)$$

So, this is the total torque which is here. So, yes, this is the torque which is developed due to the inertia. This is the torque which is developed due to the damping. The total torque should balance out each other. So, that is here and also, tau m is km into ia. You already know. The torque generated by your motor is km into the current. That is flowing through that. The rest remains the same. So, these are the parameters I have already discussed. So, theta m is the motor shaft rotation angle. Tau m is the generated torque applied to the rotor. Tau l is the load torque. Eta (η) is the gear reduction ratio. bm is the coefficient of viscous friction.

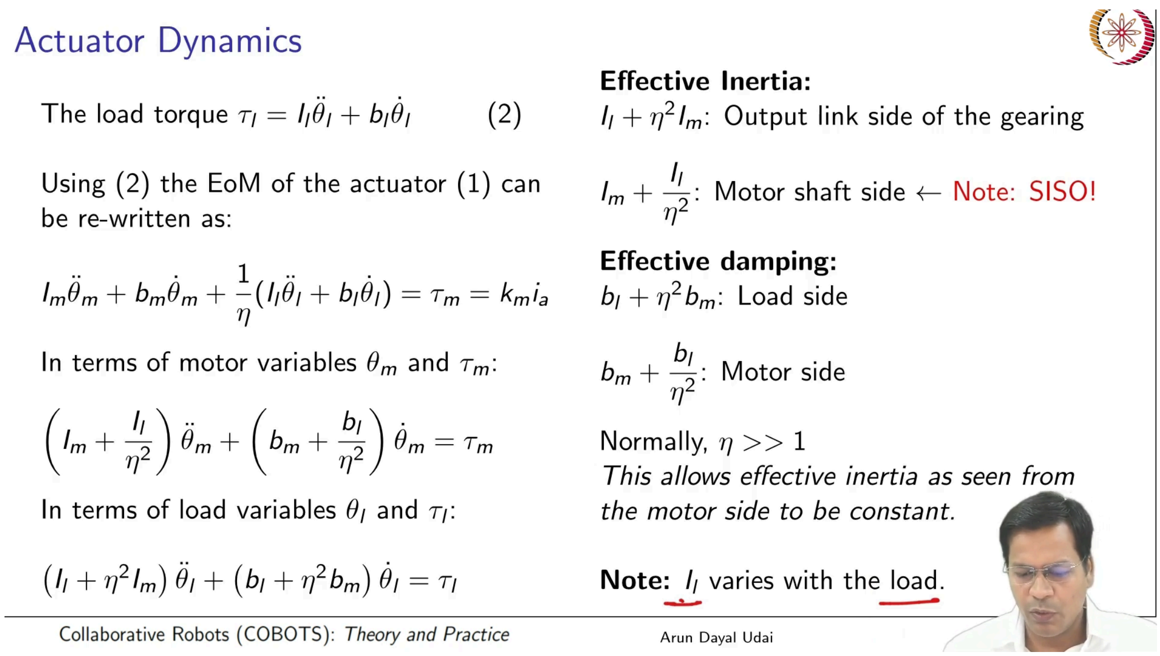
So, yes, the reason why this is here is very clear from here, hopefully. So, yes, angular velocity is eta time reduced. So, if angular velocity is reduced, torque is increased from the l side of it. Because, you know, tau l theta l dot should be equal to tau m theta m dot.

This is angular velocity into torque. Angular velocity into torque. That is, work done at the input should be equal to work done at the output. So, if it is, theta l dot is theta m by eta that is there. So, effectively, tau l becomes equal to eta times of tau m or theta, or tau m side of it will see if the rotor is static. But if it is moving, you see tau l by eta times coming to the motor side of it. That is as a reaction torque.

$$\dot{\theta}_l = \frac{\dot{\theta}_m}{\eta}, \quad \Rightarrow \ddot{\theta}_l = \frac{\ddot{\theta}_m}{\eta}$$

So, yes, I am wiping them all so as to make it very, very clear now, and the same happens to the acceleration also. You see, theta l double dot is equal to theta m double dot by eta, and tau l is eta times of tau m. So, these are some fundamental mechanical relations that should be quite clear. They are simple work input and work output relations that are shown here, and this is the dynamic equation of motion.

$$I_m\ddot{\theta}_m + b_m\dot{\theta}_m = \tau_m - \frac{\tau_l}{\eta} = k_m i_a - \frac{\tau_l}{\eta} \qquad (1)$$



So now, moving ahead, let us see what else we can put in here. So, load torque also drives something. It drives the link.

$$\tau_l = I_l\ddot{\theta}_l + b_l\dot{\theta}_l \qquad (2)$$

So, it is the moment of inertia of the link into the angular acceleration of the link. Got it? And you also have damping over there. So, it should give you the damping coefficient into the angular velocity of the link. So, the total torque is driving all these.

So yes, now equation 2 can be applied to your actuator equation. That was equation 1. So it looks like this. If you remember, I will just put tau l to this or this and bring it over here. So what should I see now? So, it can quickly give me. So, if I substitute tau l over there, it should be placed over here, and that is equal to tau m or km into ia.

$$I_m\ddot{\theta}_m + b_m\dot{\theta}_m + \frac{1}{\eta}(I_l\ddot{\theta}_l + b_l\dot{\theta}_l) = \tau_m = k_m i_a$$

So, this is the overall relation that exists in terms of motor variables, theta m and tau m. I will substitute these two equations once again. So, I will put theta l equal to theta m dot by eta over here. So theta l, wherever I see it. I will put theta m dot by eta. So, that should give me; if I take all of them together and bring them out, similarly, theta l double dot should be equal to theta m double dot by eta. So effectively, eta square is going to come. So, you see this equation.

$$\left(I_m + \frac{I_l}{\eta^2}\right)\ddot{\theta}_m + \left(b_m + \frac{b_l}{\eta^2}\right)\dot{\theta}_m = \tau_m$$

So, it is Im Il by eta square into theta m double dot plus bm plus bl by eta square theta m dot is equal to tau m. So, this is quite a clear straight relation. You can try getting it yourself, also.

In terms of load variables, if I convert. So, I will substitute otherwise. That is theta l I have substituted earlier theta l dot is equal to theta m dot by eta, and theta l double dot is equal to theta m double dot by eta.

$$\dot{\Theta}_l = \frac{\dot{\Theta}_m}{\eta} \text{ and } \ddot{\Theta}_l = \frac{\ddot{\Theta}_m}{\eta}$$

So, I will substitute it; this time,

$$\dot{\Theta}_m = \eta\dot{\Theta}_l \, and \, \ddot{\Theta}_l\eta = \ddot{\Theta}_m$$

I will substitute theta m dot is equal to eta times theta l dot. So, that goes here and here. So, it should be theta l into eta m. It is equal to theta m double dot. So, these two values I will put here, and I can get this relation. So, these are the load-side variables, and this is in terms of motor-side variables.

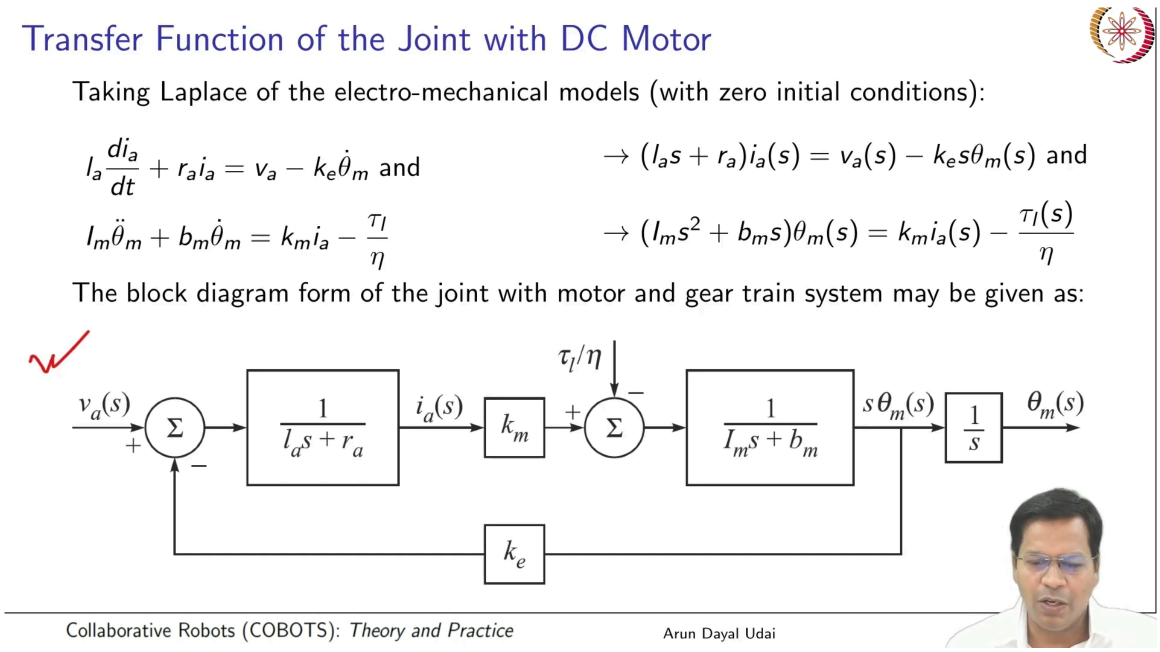
Now, let us see what these two equations tell me. So, the effective inertia is Il, eta square Im. So, this is the effective inertia that goes to the motor. That is what actually comes to the output link side of the gearing. So, the total moment of inertia the output link side will feel is due to its own inertia and due to the rotor shaft inertia, that is multiplied, you see, so more inertia you will feel. So, the total inertia is eta square Im plus Il. So, the output link side due to the gearing, you can see.

Similarly, on the motor side, the motor will see the link inertia reduced by eta square times. If eta is normally ranging, in the case of industrial robots, it ranges from 100 to 300 gear ratio. So, in that case, you can imagine it is reduced up to eta square times. So, all the rotors plus this inertia. Now, you will see this is almost like missing from your inertia. So, motor shaft inertia will only feel this one. Hardly, there is any effect due to the link moment of inertia due to the gearing.

So, that is the reason I told you earlier also. That is the reason why we can drive industrial robots using a single-input, single-output system.

So yes, now, looking at the damping side of it, the same will happen to the damping also. So from the load side, you see you have damping which is here. It has more damping. It is very much damped, naturally damped, and you also see from the motor side again you see, you have less damping due to the load side of it, and total damping has increased also, but to a very less extent is coming from the link side of it. So, the motor side will see this much of damping that is here. So, this is the damping, and normally, you know, eta is very, very greater than one. So, this allows the effective inertia, as seen from the motor side, to be constant. So, from the motor side, the inertia which is there is almost constant. It is hardly affected by the inertia of the links. So this is the reason SISO is quite good to be adapted for controlling industrial robots.

Il varies with the load. The link moment of inertia varies with the load due to the supplementary load. Even with the end effector load, if it is carrying, the moment of inertia is going to change. So, this should be noted.



So, now let us get the transfer function of the joint so that I can get the input and output relation for this DC motor system. That is the whole of the actuator here.

$$l_a \frac{di_a}{dt} + r_a i_a = v_a - k_e \dot{\theta}_m \text{ and}$$
$$I_m \ddot{\theta}_m + b_m \dot{\theta}_m = k_m i_a - \frac{\tau_l}{\eta}$$

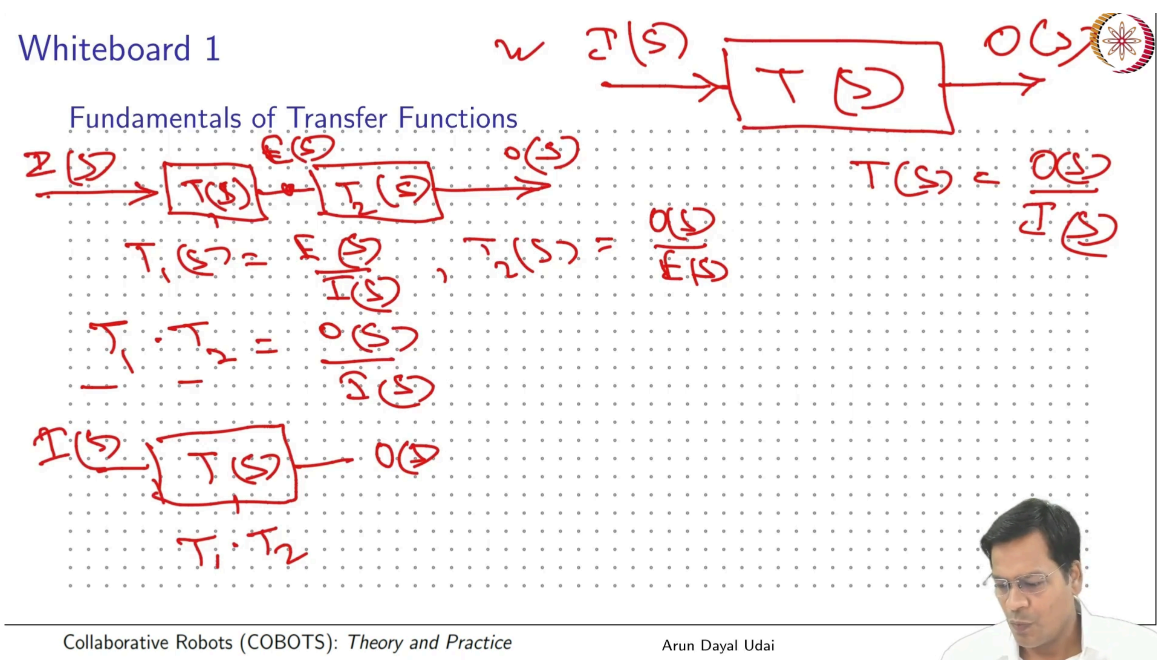
So, these two are the fundamental equations that we have derived just now. This was the electrical relation, and this is your mechanical relation. So, I call it an electromechanical model. So, taking the Laplace of this and this will lead to this.

$$\rightarrow (l_a s + r_a) i_a(s) = v_a(s) - k_e s \theta_m(s) \text{ and}$$

$$\rightarrow (I_m s^2 + b_m s) \theta_m(s) = k_m i_a(s) - \frac{\tau_l(s)}{\eta}$$

You can quickly see it here. So, Ia can be written as IaS times ia is constant. Current is varying. So, IaS into S. So, S goes here. So, S is now taken out with this. Similarly, the Laplace of Ia will be Is. So, taking common, I get to this. On the right-hand side of it, Va becomes Va(S). k is constant. So, again, because it is theta m dot, it is s theta m(s). Taking all the initial conditions as 0. The rotor has started from rest zero velocity from zero position. So, that is the reason this is valid, and then taking the Laplace of this will give me this. So, it is theta double dot. It should give me I'm s squared, and theta ms will come out from both of these. Similarly, from the right-hand side. You have constants which are there. So, theta ls is constant. So, tau ls is constant, and you see this. So, these are the two.

So, can I draw all of this? Before I do this. Let me just start with a little fundamental first. Fundamentals of Transfer function. Before I can make you understand this one, I will discuss it a bit.

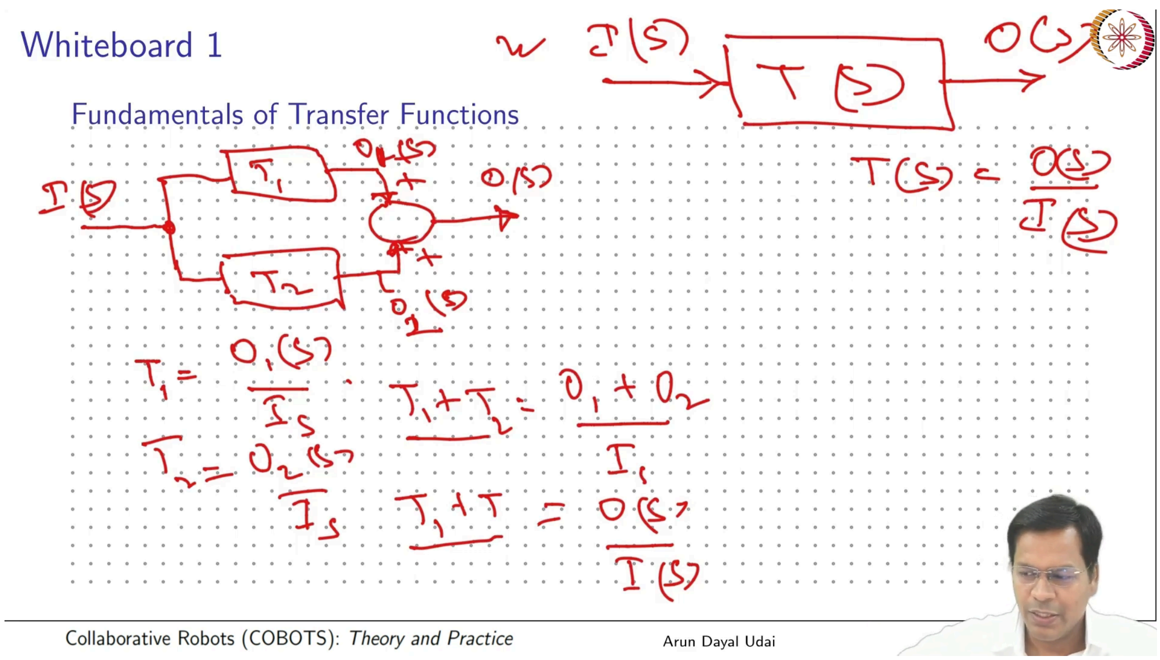


So, let us start. So, transfer function, you know. It is input that is I(S), and you have a transfer function, as you have seen. It is the Laplace of output by input. So, it is output Laplace. So, output by input, transfer function is given as.

$$T(s) = \frac{O(S)}{I(S)}$$

Output by input. This is quiet. So, now again. So, I will extend this idea further. So, if it is in series, you have two blocks, coming one after the other. So, this becomes your input. This becomes your output. You have transfer function 1, and you have transfer function 2. So, you can write it as, let us say, there is something that signal which is tapped over here is this, E(S). So, T1(S) is equal to E(S) by I(S). Similarly, T2(S) becomes equal to O(S) by E(S). So, multiplying them together will give me T1. Let us forget about S now; T1 into T2 should give me E by I and O by E. So, effectively, ES will get cancelled, so it gives me output by input. So, you see, if they are in series, they can be multiplied.

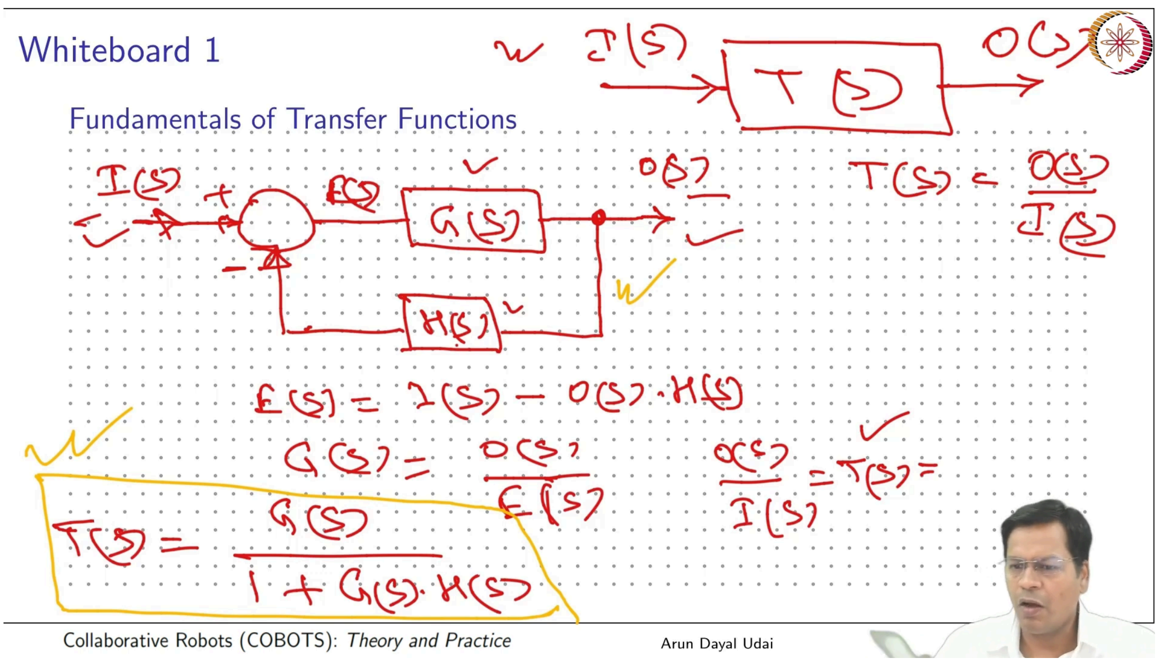
So, you can directly get an equivalent block diagram of the equivalent transfer function as T that is T1 into T2 that will directly relate the output to the input. This is the first thing that you should know. So, this is yours if they are in a series.



What will happen if it is in parallel? So, if it is in parallel, let me just wipe it off. So, I can draw it like this. So, if it is in parallel, you cannot draw the way it is drawn for serial. This is your block. This is another block, and signals from both of them are now added over here, and it gives you a further output. So, this is added from here; this signal coming from here is added. You have T1, and you have T2. This is your output this is your input. So, this time, you see there is a signal which is from here. So, let us say this is

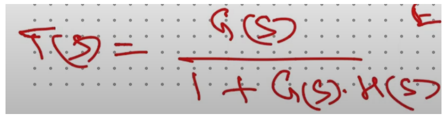
O1(S). this is O2(S) O1(S) and O2(S). So, T1 will be equal to this. Let it be this O2 and this one as O1. So, it is O1(S) by is, and similarly, T2 will be equal to O2(S) by I(S).

So, if I add them together, what will I get? T1 plus T2 will be equal to O1 plus O2 by I(S). So, O1 plus O2 is nothing but output by input. Got it? So, that becomes the equivalent transfer function in this case. So, this is your T1 plus T2 is output by input. So, in the case of parallel. It gets added like this. Now let me again wipe it off, and show you one more example.



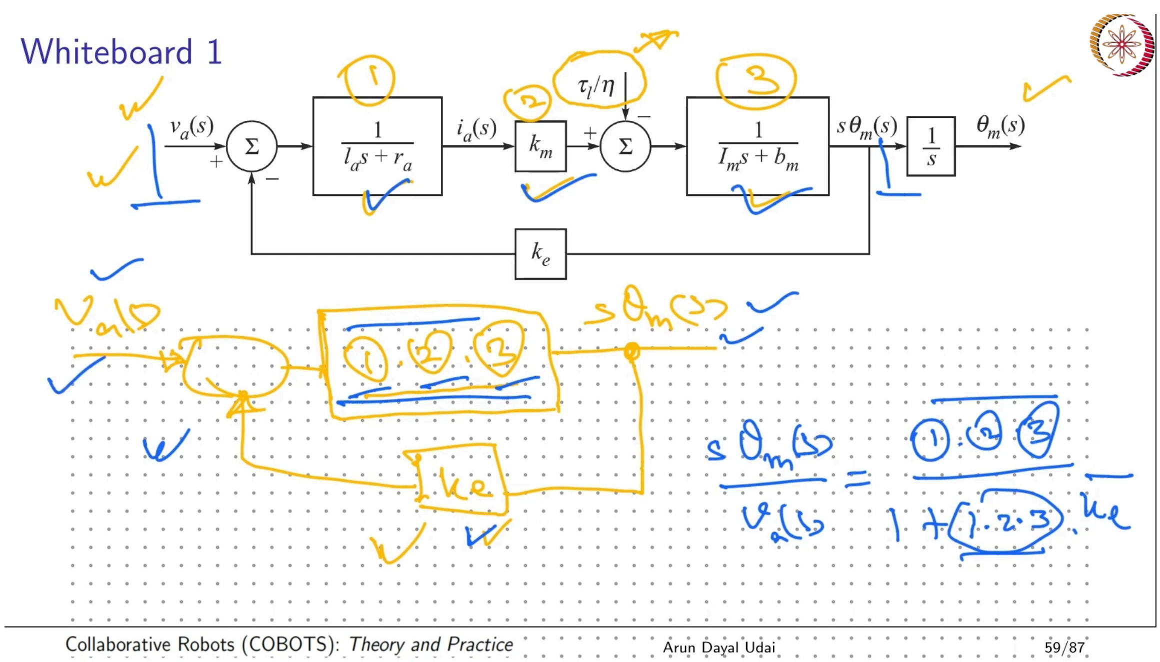
Now, let me draw it in a closed-loop way because the way we want to do it now is a closed-loop application. So, this becomes your input that comes here to the summing block over here that is plus. This goes to the Transfer function, which has G(S), and that is your output. You have a tapping from here that comes through a feedback line, which is here, that is, H(S), and that comes here with a negative sign. So, this is your input, and this is your output.

Now let me see, how what should be the equivalent transfer function that connects the output to the input. So, the signal which is seen here now, let us say if it is E(S). so E(S) is nothing but, E(S) is equal to the difference of these two. So, the I(S) minus O(S) signal is getting multiplied here and is coming here. So, O(S) into H(S). So, that comes here. So that is getting multiplied, and G(S) may be written as O(S) by E(S). So, substituting this E(S) here. So, what I should see now and rearranging that will give me A direct transfer function that relates the output to the input. Output by the input that is the equivalent transfer function of this. So, that is equal to:



So, I will put it here. So that it is clearly visible to you. So, the transfer function now can be evaluated as. G(S)by 1 plus G(S) into H(S). So, this is known as a feedback transfer function. This is the system transfer function. Input and the output. So, the equivalent transfer function this time is this. You need to remember this. We will be using this now. So, this is the reason I have discussed this in so much detail. So, this is your G(S) by 1 plus G(S) H(S). The system structure is very much like this.

There are many other transfer function rules. I won't discuss it here. But yes, this is how you can handle them in blocks. So, that is the beauty of arranging them in transfer functions. So yes, now let us come to our problem. Now, move here.

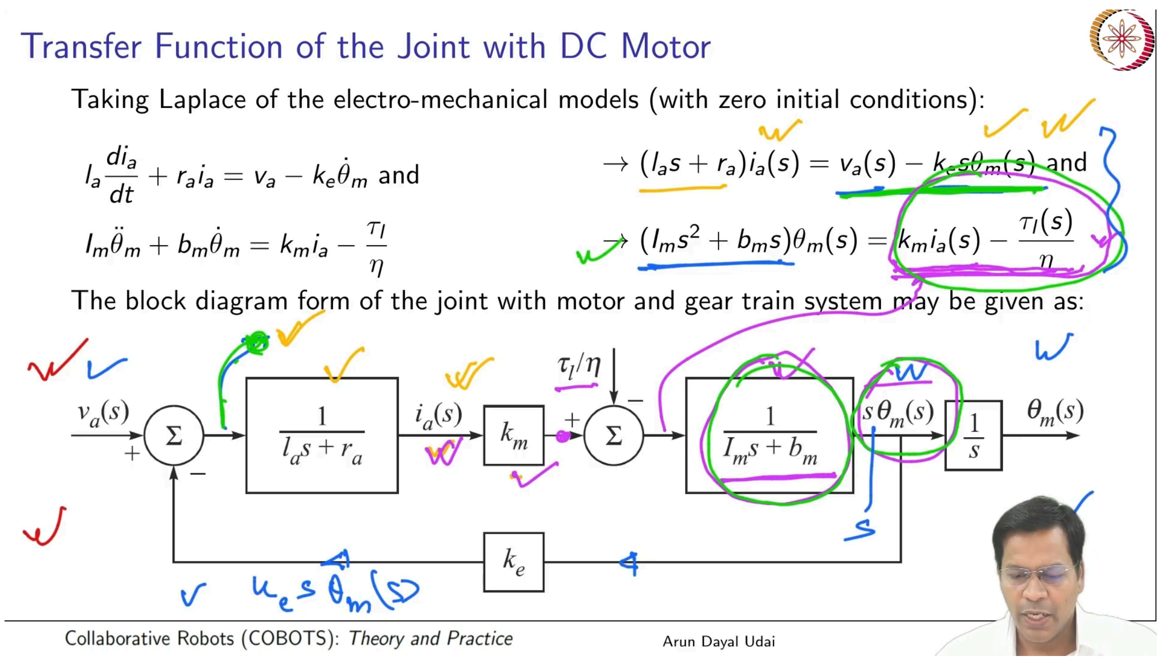


So yes, this is the first thing that I want to discuss. Let me come back to my slide also.

So, this was there. You saw till here. I want to come to this now. How these two equations hold true and they can be clubbed in this manner. Okay, they can be clubbed in this manner. So, that is what I am going to discuss now. So now, when this part is there directly, okay, you have input here. You have output here, and let us forget there is any disturbance coming from the link side of it. So, if this is not there. So I will just write one by one what all systems come in.

So, you see, if I leave this out for now. So this, this, and this are in a series. So, you can draw an equivalent transfer function for all of these together as this. So, it is a product of one, two, and three terms that will come here. One, two, three will come here, and that is. Going out. You see, you have s theta m (s), which is written here, and you have ke, which is here, and you have a block that is shown here. This is your input, Va(s). That is the voltage from the armature. So that comes here. So, this looks very much like a closed-loop system with feedback as ke, and this is coming here, and this is whole. So yes, the total transfer function for all of this system that starts from here till here, from here to here, the equivalent transfer function can be given by this, and that can be calculated as output, that is, s theta m (s) by input Va(s) should be equal to G(s), so it is the product of all these three. So, it is 1 into 2 into the third term by 1 plus 1 into 2 into 3 into ke. Got it? So, I will just make myself vanish a bit. So, this is what I am talking about.

So, output by input should be equal to the product of the terms 1, 2, and 3, which are shown here, here, and here divided by 1 plus G(s) H(s). So, that is ke that comes here, and this one that goes here and here. So, output is this; input is this. So, this is the first thing that is there. Now again, you see what all the equations I want to put here. The first equation, so let me just come back to the previous slide.



So, this is what was there. Just see if it is correct or not. First, let us try and get to this block diagram first. So, the first equation where is it located in the transfer function system? So, this is va minus ke s theta m (s). So, as the theta ms signal is coming to this block, Feedback, so this is getting multiplied with this. So over here, you see you have ke into s theta m (s) that comes here. So, this minus this is getting into this line. So here, you see, you have exactly this one. So I'll just mark them here. So, this value is visible here. So, this into this gain, now the value which is here into this gain should be equal to this. So, that is what is written here. So, if you multiply the value which is here, that is here, into 1 by I_as r_a.

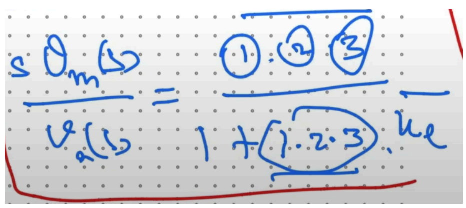
1 by I_as r_a gives me IaS. So, that is I_ar. So, you have understood how this equation is put to the variables which are lying in this line.

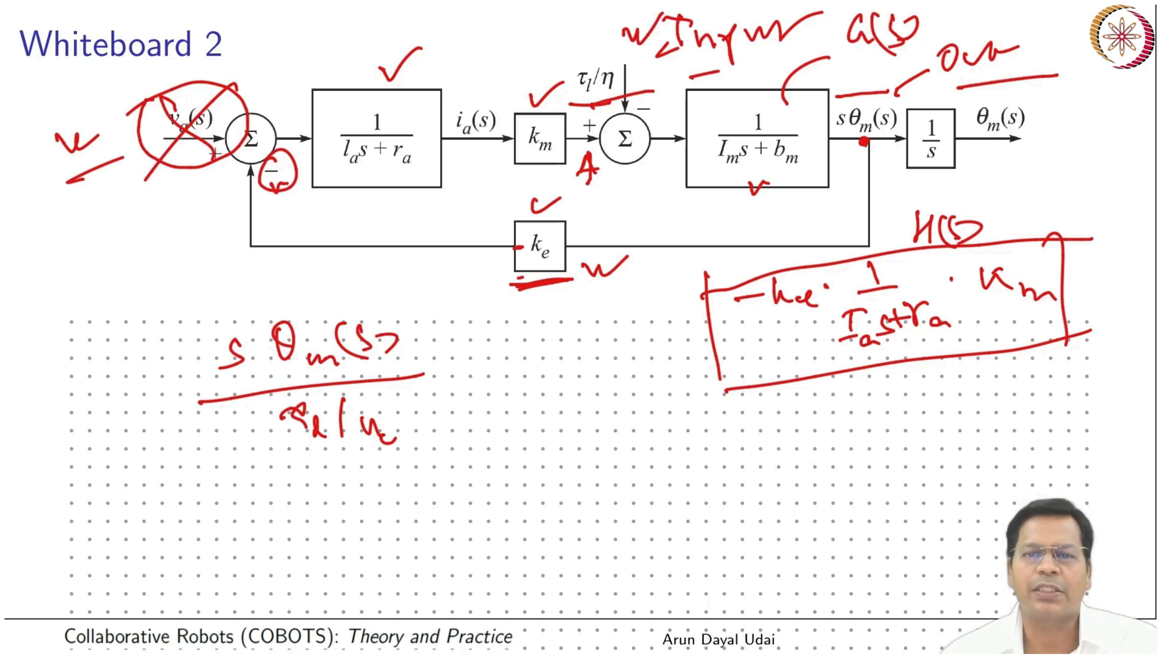
Now, let us see the model which is coming next. Now, km. Let me use another ink this time. So, km into Ia(S) that is getting here minus tau l by eta. So, km Ia(S) minus tau l by eta is now coming here.

$$(I_m s^2 + b_m s)\theta_m(s) = k_m i_a(s) - \frac{\tau_l(s)}{\eta}$$

So, this signal is this one. From here to here. So, this into 1 by I_ms plus bm. So, this is here, and the term which is here is s theta m (S). So this signal. Into this should give me this (sΘm(s)). Got it stepwise. So, I'll tell you once again. So, this into this should give me this. So that is what is put here, and I have brought in this s, which is inside to this. The s which is here, so I have put that inside this bracket, and I should see Im s square plus bms. So, that is what is here. So, these two equations are clearly put to this transfer function, and all of this can now be compacted using the approach that I have discussed here. So, I will make a transfer function of all of this. So, in the first case, I am excluding the disturbance which is coming from outside. That is this term. Take the product of all these three and create a closed loop over here with this one as the system electromechanical system. So, this is electrical. This is the mechanical part of it. So, that is sitting here. This is your output of the closed loop that comes here, and this is your input that is Va(S). This is your feedback.

So, the equivalent transfer function can now be given as this one. That is this part.





Again, look at it otherwise. This time, let us say it is driving by itself. That is, it is being driven by the load. So, in that case, you are not applying any external voltage. So, this is totally absent. So, in that case, the load torque will be visible. With the gear reduction ratio, it is coming onto the system. Mind it. So, this is always there. So, this is the driving torque, and the whole system is now getting driven by that.

So, all the things that go in series are s times theta m (s) into k_e. So, this is your output.

$$output = s\Theta_m(s)$$

So, in the feedback, all the things that are getting in. So, this becomes your input. This becomes your output. So, the output is s theta m (s). So, s theta m (s) by input you have tau l by eta that comes here.

$$input = \frac{\tau_l}{\eta}$$

So, that is visible from here output by input. Feedback is, this time, positive. So, if you apply G(s) by one plus G(s) H(s). So H(s) is this, k_e. So, that is basically over here. It goes with a negative sign. So, you have to take the product of this (k_e) coming to this (1 by Ia s plus ra) and this (km). So, the whole of this is in series now. The whole of this is in series. So, the minus of k_e will be visible because you see you have a negative sign over here. So, ke into so you have total terms which are there in the Hs side of it, that is the feedback side, will be ke into 1 by minus ke into 1 by Ia s plus ra into km. So, all these will be there as H(s) blocks.

$$H(s) =- k_e \frac{1}{I_a s+r_a} k_m$$

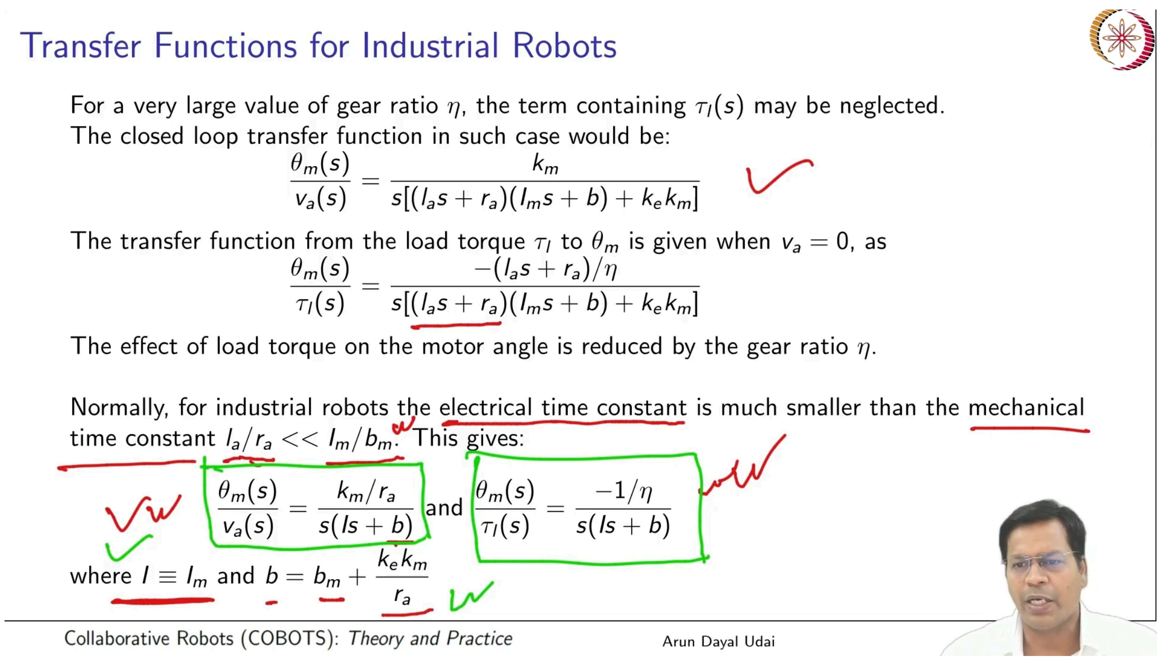
This is your input. This is your system. This becomes your G(s).

$$G(s) = \frac{1}{I_m s+b_m}$$

So, G(s) is here, if you remember. So, this is your G(s), and that is your output and input.

So, now you can write the closed-loop transfer function. Got it? So, there are two approaches to it. First, the robot is being driven by the input voltage, which is appearing over here.

In the second one, you have stopped giving any voltage, and the robot is now being driven by the link weights. That is the torque that is coming from the link side of it. It may be because of the external force, due to the payload, or due to the weight of the links itself. So, that is coming here.



So now, there are two things. As I have said, this is the first one. When you see, there is motor displacement due to the voltage that is there. This is the forward transfer function. That is motor is driving your system. In this case, for a very large value of gear ratio eta, the term containing tau ls may be neglected. That is what is neglected. As I have said, so anything that is coming from here is now neglected. So, that is neglected, and the closed-loop transfer function is given by this.

$$\frac{\theta_m(s)}{v_a(s)} = \frac{k_m}{s[(l_a s + r_a)(I_m s + b) + k_e k_m]}$$

This is the first case. In the second case, You don't apply any voltage, v_a becomes equal to 0 and it is getting driven by the load torque itself. In that case, this is your transfer function.

$$\frac{\theta_m(s)}{\tau_l(s)} = \frac{-(l_a s + r_a)/\eta}{s[(l_a s + r_a)(I_m s + b) + k_e k_m]}$$

ke and km you see, both were appearing in the product, so that is visible here, and that is this one. So, the effect of load torque on the motor angle, is now reduced by the gear ratio eta. So that is always there. This we have seen earlier.

So, normally for industrial robots, the electrical time constant is very much smaller compared to the mechanical time constant. Time constants are the terms that are written here. l_a by r_a is the electrical time constant. It is very much smaller than I_m by b_m.

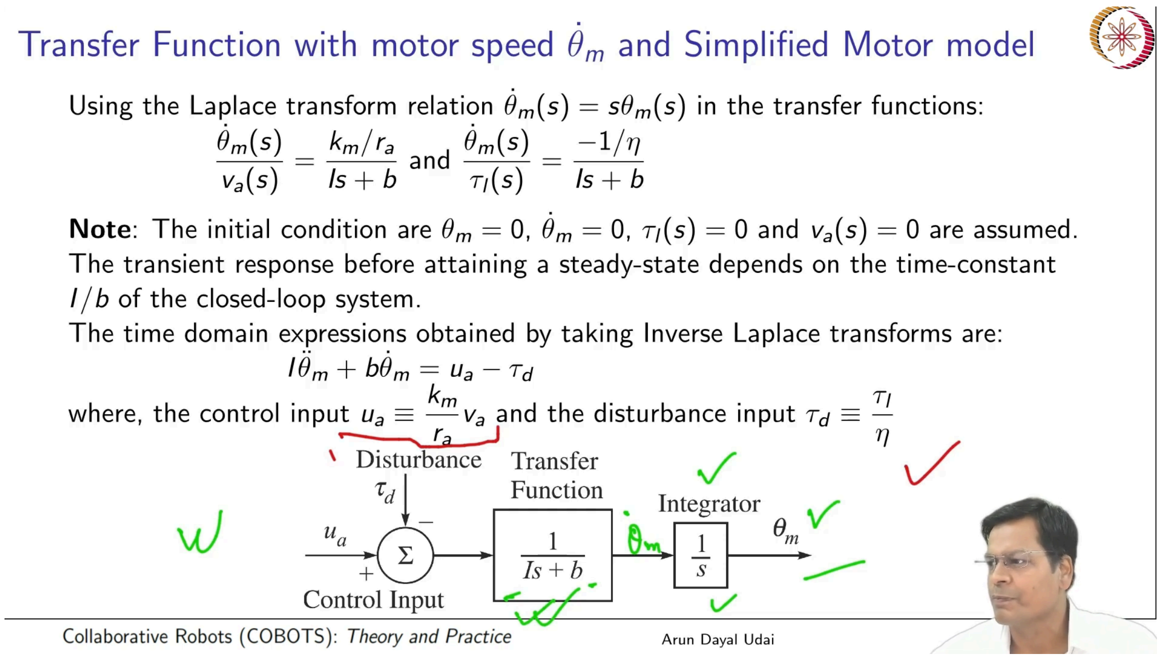
I_m by b_m so that is your moment of inertia by the damping coefficient. So, that is very significant compared to this. So, I can ignore that. Taking dividing all of this by r_a and ignoring the terms that contain la by r_a, you get to this.

$$\frac{\theta_m(s)}{v_a(s)} = \frac{k_m/r_a}{s(Is+b)} \text{ and } \frac{\theta_m(s)}{\tau_l(s)} = \frac{-1/\eta}{s(Is+b)}$$

Okay, and this is from the second one. So this is from the first one, this is from the second one. Got it?

$$I \equiv I_m \text{ and } b = b_m + \frac{k_e k_m}{r_a}$$

The moment of inertia is only Im. b, which is here, is the sum of the moment of inertia. It is the sum of bm and k_e k_m by r_a. So, those terms, if you substitute, you can get to a very simple term which is like this. So, I will just highlight them. So first is this. The next equation is this, where I is this, and b is this. I and b are these. This becomes your very fundamental equation for any industrial robot. This is your transfer function in both cases when you are driving using the voltage and when you are being driven by the load, not applying any voltage to the motor. So, these are the two transfer functions.



So now, using the Laplace transform of these two transfer functions that we have obtained. This one and this one.

$$\frac{\dot{\theta}_m(s)}{v_a(s)} = \frac{k_m/r_a}{Is+b} \text{ and } \frac{\dot{\theta}_m(s)}{\tau_l(s)} = \frac{-1/\eta}{Is+b}$$

So, I can get to this transfer function, taking initial conditions as 0. All these initial conditions are assumed to be 0, and this is your relation.

So, the transient response before attaining a steady state depends on the time constant, I by b, of the closed loop. That is basically the part of the equation which is there over here, that becomes your characteristic equation for this transfer function, which is in the denominator. So, I by b basically, if you take B common. So that gives you the root, and that only governs your system. That only governs your system. That is the input and output relation.

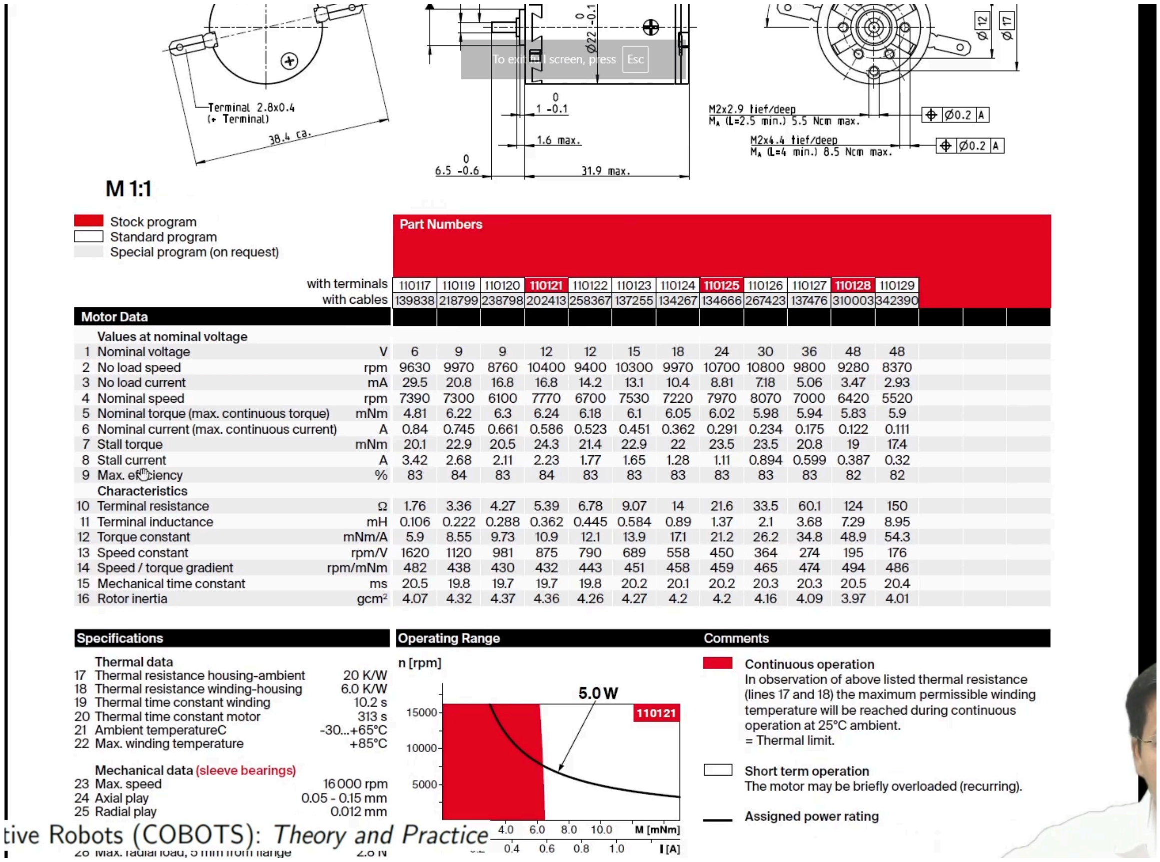
So, the time domain expression for this. It may be obtained by taking the inverse Laplace. You know, you can obtain the time domain solution. So, this is your differential equation for that.

$$I\ddot{\Theta}_m + b\dot{\Theta}_m = u_a - \tau_d$$

And you have now u_a minus t_d. What are those? t_d is the disturbance input that is from the load side of it. I am seeing this load as a disturbance now. This can have a payload, this can have a supplementary load, or it can have some sort of external force as noise. So, that is now my disturbance force. This is your control input, which is there. That is what should be desired by the voltage which is here, and this is your System which is getting driven. So now, effective inertia I. You already know what that was, and the b terms. So, that was written here. So, b and I are here. So, that is what is here. And now you can write your system in a very simple form like this. So, here you were getting theta m dot. So, if I integrate it, that is 1 by s, which as a Laplace integrator, can be written as 1 by S. So, it should give me an angle. So, this gives me my angle. This is my transfer function of the system. Inertia and damping are present here. This is your disturbance, and you have control input u_a, which is shown over here. So, this is your u_a, which is here. So, this is your system.

the control input $u_a \equiv \frac{k_m}{r_a} v_a$ and the disturbance input $\tau_d \equiv \frac{\tau_l}{\eta}$

That is very, very simple. It becomes a single-input, single-output system. You have ignored the load that is coming due to the torque from the load side of it.



M 1:1

Stock program
Standard program
Special program (on request)

Part Numbers

	with terminals	110117	110119	110120	110121	110122	110123	110124	110125	110126	110127	110128	110129
	with cables	139838	218799	238798	202413	258367	137255	134267	134666	267423	137476	310003	342390
Motor Data													
Values at nominal voltage													
1 Nominal voltage	V	6	9	9	12	12	15	18	24	30	36	48	48
2 No load speed	rpm	9630	9970	8760	10400	9400	10300	9970	10700	10800	9800	9280	8370
3 No load current	mA	29.5	20.8	16.8	16.8	14.2	13.1	10.4	8.81	7.18	5.06	3.47	2.93
4 Nominal speed	rpm	7390	7300	6100	7770	6700	7530	7220	7970	8070	7000	6420	5520
5 Nominal torque (max. continuous torque)	mNm	4.81	6.22	6.3	6.24	6.18	6.1	6.05	6.02	5.98	5.94	5.83	5.9
6 Nominal current (max. continuous current)	A	0.84	0.745	0.661	0.586	0.523	0.451	0.362	0.291	0.234	0.175	0.122	0.111
7 Stall torque	mNm	20.1	22.9	20.5	24.3	21.4	22.9	22	23.5	23.5	20.8	19	17.4
8 Stall current	A	3.42	2.68	2.11	2.23	1.77	1.65	1.28	1.11	0.894	0.599	0.387	0.32
9 Max. efficiency	%	83	84	83	84	83	83	83	83	83	83	82	82
Characteristics													
10 Terminal resistance	Ω	1.76	3.36	4.27	5.39	6.78	9.07	14	21.6	33.5	60.1	124	150
11 Terminal inductance	mH	0.106	0.222	0.288	0.362	0.445	0.584	0.89	1.37	2.1	3.68	7.29	8.95
12 Torque constant	mNm/A	5.9	8.55	9.73	10.9	12.1	13.9	17.1	21.2	26.2	34.8	48.9	54.3
13 Speed constant	rpm/V	1620	1120	981	875	790	689	558	450	364	274	195	176
14 Speed / torque gradient	rpm/mNm	482	438	430	432	443	451	458	459	465	474	494	486
15 Mechanical time constant	ms	20.5	19.8	19.7	19.7	19.8	20.2	20.1	20.2	20.3	20.3	20.5	20.4
16 Rotor inertia	gcm²	4.07	4.32	4.37	4.36	4.26	4.27	4.2	4.2	4.16	4.09	3.97	4.01

Specifications

Thermal data	
17 Thermal resistance housing-ambient	20 K/W
18 Thermal resistance winding-housing	6.0 K/W
19 Thermal time constant winding	10.2 s
20 Thermal time constant motor	313 s
21 Ambient temperatureC	-30...+65°C
22 Max. winding temperature	+85°C
Mechanical data (sleeve bearings)	
23 Max. speed	16 000 rpm
24 Axial play	0.05 - 0.15 mm
25 Radial play	0.012 mm

Comments

Continuous operation
In observation of above listed thermal resistance (lines 17 and 18) the maximum permissible winding temperature will be reached during continuous operation at 25°C ambient.
= Thermal limit.

Short term operation
The motor may be briefly overloaded (recurring).

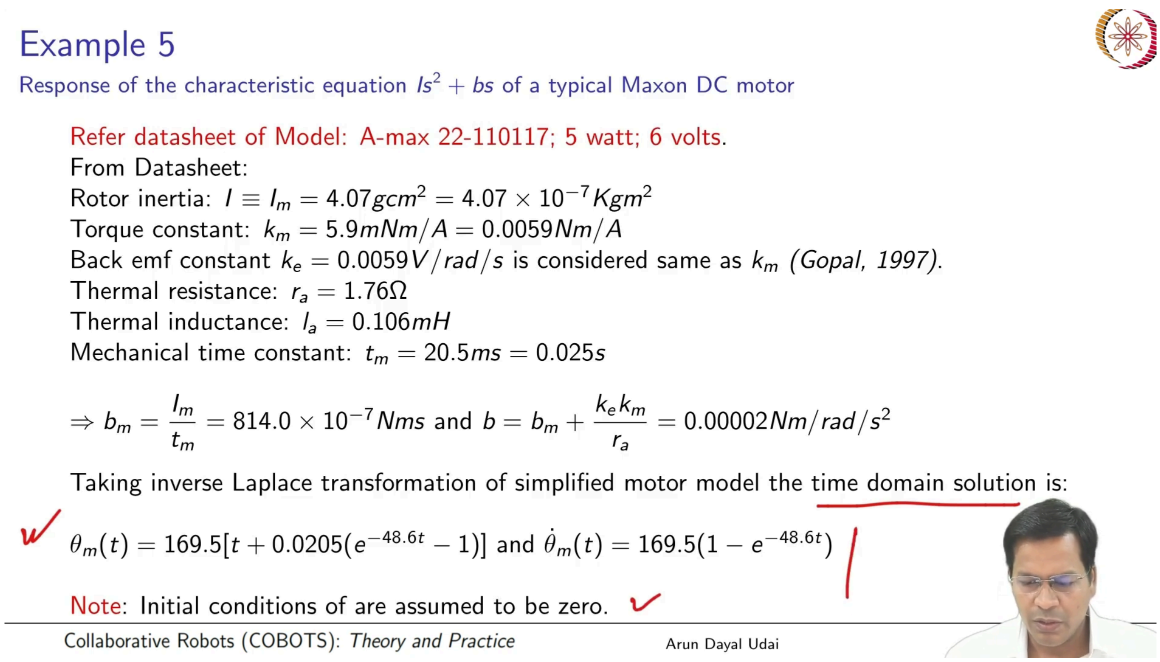
Assigned power rating

tive Robots (COBOTS): *Theory and Practice*

So now, let us take one small example by using a datasheet of a Maxon motor. So, let me just show you that datasheet first.

So, this is the datasheet that I am using now. So, you see, there are many motors here. The parameters that we have just discussed are mentioned here against the different motors listed.

So, I will be talking about the motor, which is 110117. Let me see where it is. So, it is the first one: 110117. So, it has various constants, which are mentioned here. The specific one I will be using is the moment of inertia of the shaft. Where is it? So, you see, the time constant is here, the torque constant Km is here, terminal resistance, rotor inertia, that is, the moment of inertia. But you see, you have to mind the units that are here; convert them to the appropriate SI system if you are working with that. Terminal inductance and resistance. These are inductance l, and r_a is the resistance. Torque constant km, speed constant. So, you see, this is the datasheet that I am using that is, for the motor which is manufactured by Maxon.



So now, let me come back to my slide. So, this is it. So, I have just copied all the values and converted them to appropriate units. Torque constant, back EMF constant all these parameters were there. You can just come back and see that.

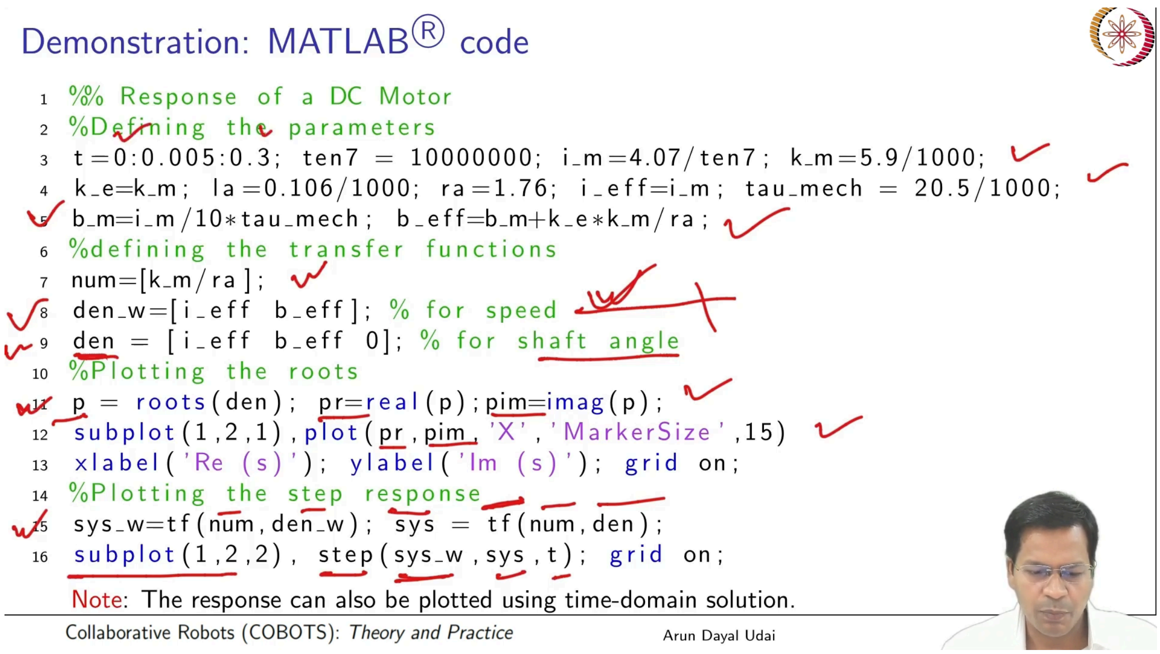
So, this is considered based on the torque constant, which is given in Gopal's book, that is, controls book, and r_a; if it is not given, you can take some standard value. Thermal resistance that is the resistance only. Inductance is here. The mechanical time constant, t_m, is here. This is the mechanical time constant, so you know it is 20.5 milliseconds, which was there. So, now, let me use this. So, it is bm is nothing, but I_m by mechanical time constant t_m. So, the time constant is i by b, you know that. So, b_m you can obtain bm, that is the damping equivalent damping like this, and total b will be bm plus K_e K_m by r_a. That is what you have derived just now. So that is calculated like this.

$$b_m = \frac{I_m}{t_m} = 814.0 \times 10^{-7} Nms \text{ and } b = b_m + \frac{k_e k_m}{r_a} = 0.00002 Nm/rad/s^2$$

So, Ke and Km are here, and resistance is known. So, the total viscous coefficient is here. So, now taking inverse Laplace of the simplified motor model that we have seen just now. In the time domain solution, it can be written as this.

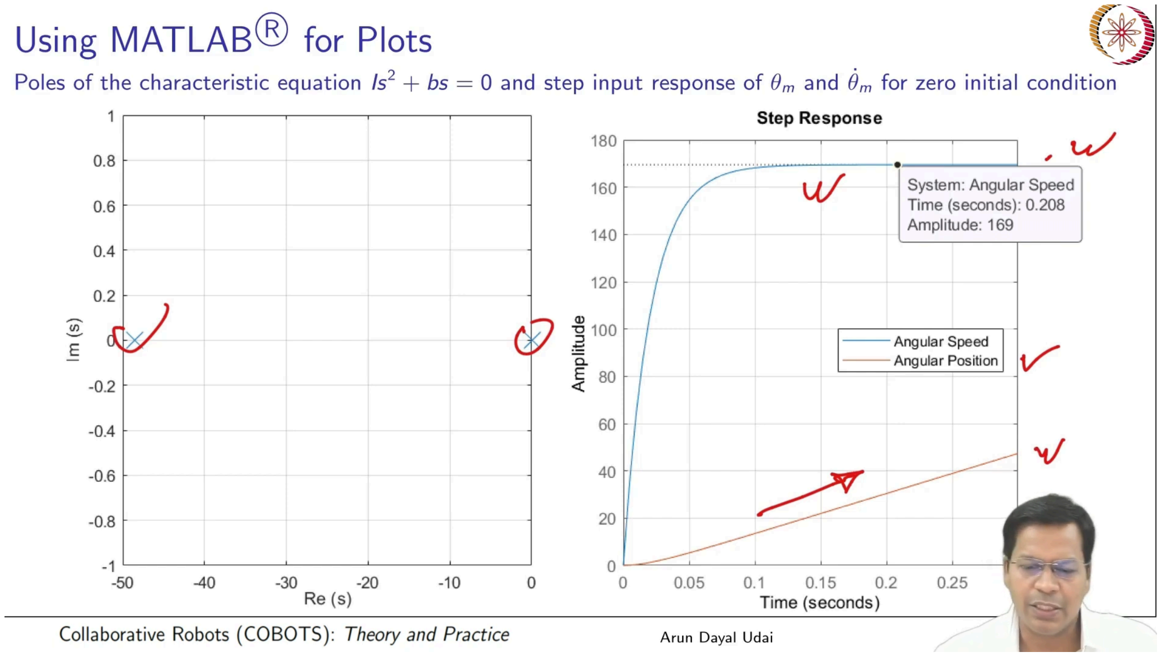
$$\theta_m(t) = 169.5[t + 0.0205(e^{-48.6t} - 1)] \text{ and } \dot{\theta}_m(t) = 169.5(1 - e^{-48.6t})$$

Initial conditions are assumed as 0 when you take the inverse Laplace. So, this is the behaviour of your system. You can just plot this and see.

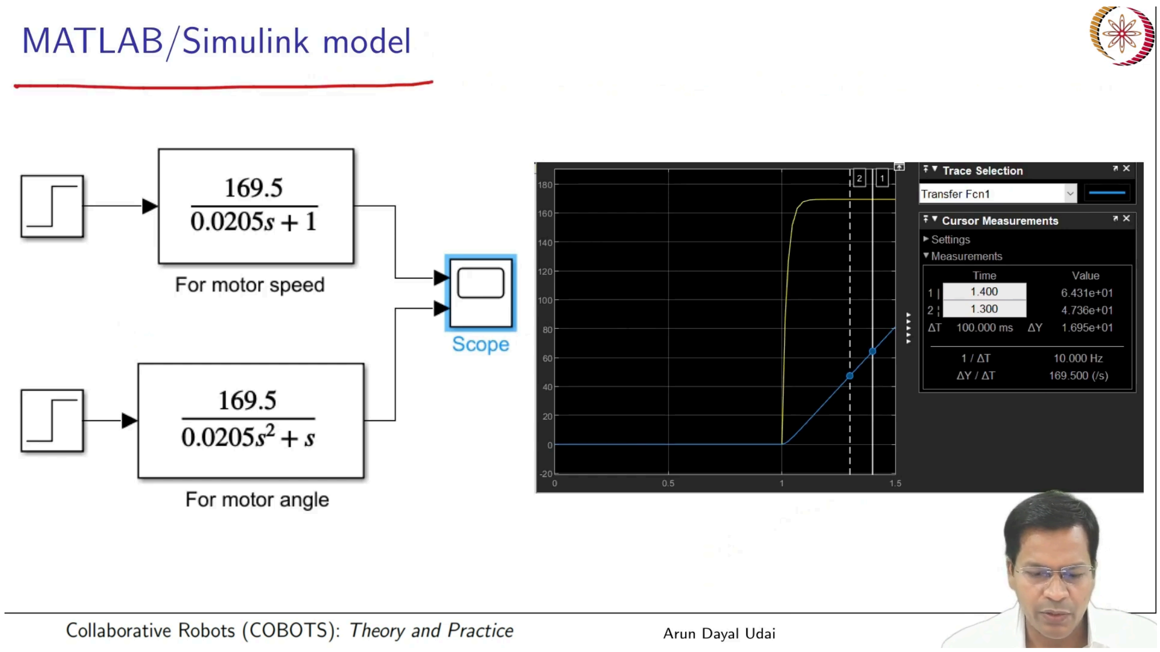


Using MATLAB, you can define the systems. All the constants are defined as b_eff, b_m is here. Everything that we did analytically just now, we can put it into MATLAB and get things done. The numerator has the coefficients which are like this. K_m by r_a. The denominator is for the speed you have. The denominator is given by the polynomial I and b. Similarly, the denominator for the shaft angle is I, h, and 0 was there. So, these are there. So, you have the roots of the denominator, which are here. That is for the shaft angle I am storing here. Real and imaginary parts are segregated here, and I have done the plots for real and imaginary, which is the poles plotting I have done.

Again, for the omega part that is d theta by the dt part that is theta dot part, you have the numerator you have the denominator, which is given by for the speed it is given by this. You have created your system using the transfer function with a numerator and denominator like this and plotted the step input of that. So, this time, it is step input for your system for speed and system for angle against time. Time is varying from 0 to 0.3 seconds only. This is your step input.



Let us see how does it look like. So, you have poles given by one of them will be here otherwise here. Both are on the real number line towards the negative side of the complex plane, and you see, you have for system angular speed it goes something like this. In the case of angular position, as soon as you apply the voltage, it gradually picks up and keeps on changing almost like linear. So, it goes like this. In the case of angular speed, it picks up and goes there. So this is how you can analyse any motor using your standard analytical techniques like the transfer function method I have used. I have drawn the poles and saw everything is okay. It is not unstable at all. So, it is done.



Using the MATLAB/Simulink model, you can also define your system motor speed and the motor angle transfer function. Put the step inputs and plot them here, and you can study it this way also. Without getting into programming, you can do this. But this one (previuous slide) is very well convertible to Python code also. So, that can be done.

That's all for today. So you see, we have now studied the way how transfer functions and Laplace transforms. Those tools can be used to study the robot joint, you know. So your robot joint You see how it behaves. I have basically considered the DC motor model here. But you know, the fundamental principle, tau is equal to Km into I, is the same because the conductors are lying in the same manner even in a synchronous motor. So, the motor equation should not vary much. There are other parameters also, which can be a little different. But yes, the analysis should be somewhere around this.

So, okay, that's all for today. Thanks a lot.