

**NPTEL Online Certification Courses**  
**COLLABORATIVE ROBOTS (COBOTS): THEORY AND PRACTICE**  
**Dr Arun Dayal Udai**  
**Department of Mechanical Engineering**  
**Indian Institute of Technology (ISM) Dhanbad**  
**Week: 02**  
**Lecture: 09**

**Force/Torque Sensor, Planetary Gearbox and Harmonic Drives**

Welcome back. So, we discussed sensors, especially position, velocity, and acceleration sensors in the last lecture. So, in this lecture, we will discuss Force-Torque Sensors, Planetary Gearboxes, and Harmonic Drives. These are the transmission systems which are there in the cobot.

Overview of this lecture

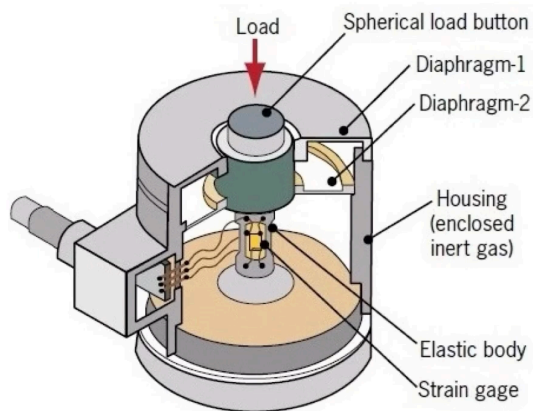


- Force/Torque Sensors
- Planetary/Epicyclic Gearbox
- Harmonic Drives
- Joint Design



So, let us begin. In this lecture, we will be discussing Force-Torque Sensors, Planetary Gearboxes and Epicyclic Gearboxes, that is commonly known as Sun and Planet Gearbox. We'll discuss harmonic drives, which are very widely used in cobots, and the joint design.

## Strain Gauge as Force/Torque and Acceleration Sensor



### Unidirectional Load Cell: Force Sensor

Force sensed by the load cell  $F = \epsilon AE = \frac{\Delta RAE}{RG}$

where,

$E$  = Modulus of elasticity of strain-gauge material

$A$  = Cross-sectional area,  $\epsilon$  = Strain

$G$  = Strain gauge factor =  $\frac{\Delta R/R}{\epsilon}$

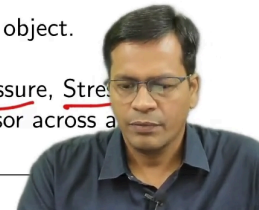
### As an acceleration sensor:

Acceleration  $a = \frac{F}{m} = \frac{\Delta RAE}{RGm}$

where,

$m$  = Mass of the accelerating object.

**Can also sense:** Torque, Pressure, Stress, Strain rate (Differential pressure sensor across a venturi/pitot tube), etc.



So, let us begin with force and torque sensors. So as such, the most commonly used type of Force-Torque Sensor is the Strain Gauge type of force-torque sensor. When you see such sensors, there is a strain gauge rosette which is there inside. So, it looks like this. You see the yellow one, and it has a resistive wire which is inside. You see, it has something like a zigzag, and it comes out. So, on one side of it, you supply the current, and on the other side, it comes out. You apply a potential difference across the electrical lead wires that are there. So, actually, this offers a resistance. So, the whole of this resistance wire is sandwiched between the layers of a resistive foil. So, what happens when this is glued on top of any surface, and that surface gets deformed? So, this strain gauge rosette also gets deformed, and there is some change in the resistance, which is because of the change in the dimensions.

Force sensed by such a load cell  $F$  is given by  $\epsilon$ ,  $A$  and  $E$ .

$$F = \epsilon AE = \frac{\Delta RAE}{RG}$$

What is  $\epsilon$ ?  $\epsilon$  is the strain, that is, changing length per unit length if it is a linear change.  $A$  is the area of cross-section.  $E$  is Young's modulus of elasticity of the strain gauge material, so this basically relates to Hooke's law only. The strain gauge factor that is shown here is given by  $\Delta R/R$ , change in resistance per unit resistance

upon epsilon, where epsilon is the strain which was there. So, delta R by R is divided by delta L by L in the case of linear dimensions. So, the force sensed by the load cell F becomes equal to delta R by R AE by G, where most of the terms that you see here are constant.

$$G = \text{Strain gauge factor} = \frac{\Delta R/R}{\epsilon}$$

The total resistance of this strain gauge rosette, once it is selected, remains constant. The gauge factor remains constant, the area is constant, and E is also constant. So, what changes here? The force is reflected directly by the change in resistance. So, these are, if you look at a standard force sensor, that is a linear force sensor, it has a material that is an elastic body which you can see that is here, this one, and on top of it, you see there is a strain gauge rosette which is fixed here. This is the element. And these are all conducting wires. There are loose wires. They are not very tight, so that any deformation will not be affected because of these wires. So that actually carries the potential to these wires. The strain gauge rosette. So, when any load is acted over here, it is transmitted to the elastic body, and that body gets deformed. When that gets deformed, the strain gauge rosette gets deformed, and you see the change in resistance over here in these electrical terminals. If you pack it together, it looks externally like this.

This is a Siemens make. So, the initial resistance is known. The gauge factor is already given by the manufacturer. So, all the dimensions and all the experimental things are already given by the manufacturer. So, this is how it works basically.

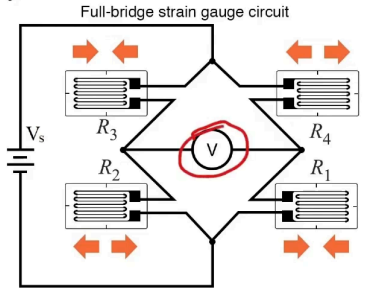
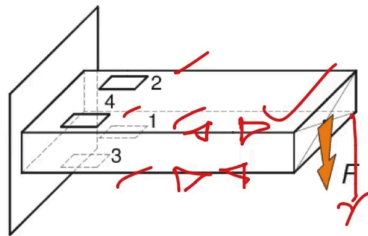
So, if it is used to sense acceleration, what you can do here is the acceleration is equal to force upon m. m is the mass of the accelerating object. So, you can always create a fixture where delta R by R A E by Gm, m is here, that is the mass of the material which is actually getting moved when you test.

$$a = \frac{F}{m} = \frac{\Delta RAE}{RGm}$$

Let us say it is just like a proof mass, which was there earlier in sensors you have learned. So, it may be something like this. So, when there is any acceleration, it would cause a

kind of force if it is moving like this. If it causes a force over here, and here is your material, which may be deformed, and this is where you can put the strain gauge rosette. So, this can be used to sense acceleration also. So, any other secondary sensor could be a torque sensor, pressure sensor, stress sensor, or flow rate sensor; if you can put it on a differential pressure sensor across a venturi or a pitot tube, it can be used to sense all those kinds of physical values.

## Enhancing the properties of Force Sensing



For a balanced Wheatstone bridge:  $\frac{R_1}{R_4} = \frac{R_2}{R_3}$ , results  $V = 0$

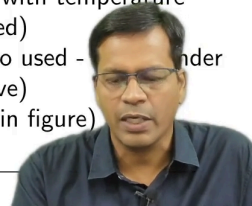
Output voltage due to unbalanced bridge:

$$V = V_s \left( \frac{R_1}{R_1 + R_4} - \frac{R_2}{R_2 + R_3} \right)$$

► Any changes in resistance due to the change in temperature is canceled.

► Configurations:

1. Quarter-bridge strain gauge circuit (only one is used)
2. Quarter-bridge strain gauge circuit with temperature compensation (one bridge unstressed)
3. Half-bridge strain gauge circuit (two used - under tensile load and other in compressive)
4. Full-bridge strain gauge circuit (as in figure)



We can enhance the properties of the Force Sensing. Why is it required? Because a stand-alone sensor may be affected due to the temperature as well. So, this is one of the environmental factors which seriously affects the value of the resistance. So, that needs to be compensated.

So, in order to take care of that, we can use a kind of Wheatstone base that you can see here, where there are four of such strain gauge rosettes which are here. Each one of them forms the branch of the Wheatstone base. So, when all the resistances are the same, In that case, there is no voltage that appears over here, that is, the Wheatstone bridge voltage can be measured as 0. So, when balanced, in the case of a balanced condition, it is  $R_1$  by  $R_4$  is equal to  $R_2$  by  $R_3$ .

$$\frac{R_1}{R_4} = \frac{R_2}{R_3}, \text{ results } V = 0$$



So, in this case, it is perfectly balanced, and no current flows through it. So, in the case of an unbalanced condition, that means one of the resistances changes due to the strain, So, it can cause some voltage to appear and current to flow from here. In that case, the voltage across these terminals, that is, this and this, that is, the  $V$ , becomes equal to the supply voltage that is over here,  $V_s$   $R_1$  by  $R_1$  plus  $R_4$  minus  $R_2$  by  $R_2$  plus  $R_3$ . So, this is a standard unbalanced bridge formula.

$$V = V_s \left( \frac{R_1}{R_1 + R_4} - \frac{R_2}{R_2 + R_3} \right)$$

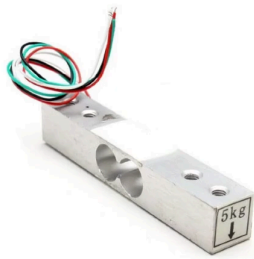
This can be derived using KVL and KCL, that is Kirchhoff's laws. Any change in the resistance due to the change in temperature can now be cancelled. So, one of them is actually fitted on top of the deforming body. And the other one is a dummy one, which is only there to sense the change in temperature. So, temperature affects both arms equally. So, instead of using just one strain gauge rosette, now you can use two or all four of them.

There are various configurations to show this. So, it can be a quarter bridge strain gauge circuit in which only one is used. This is not preferred because it cannot do temperature compensation. The other one could be a Quarter bridge strain gauge circuit with temperature compensation. One bridge is unstressed, so one of them is fixed on the beam, and the other one is not stressed but is there in the same environment. So, there would be no change in that resistance. The one that is not fixed to the object that is getting deformed will see that change in temperature. This is another arrangement. The third one is a Half-bridge strain gauge circuit. Two are used: one is under tensile load, and the other one is under compressive load. Only two are used.

The final one is the full bridge strain gauge circuit, as shown in the figure, in which all four are fitted. So, 1 and 3, if it is forced like this, come under compression, whereas the top two, 2 and 4, will be under tensile load. So, this is elongated, whereas the bottom one will get compressed. So, these are the four ways you can put this to measure the change in voltage. So, voltage change will appear here due to the change in all of them.

## Strain Voltage relations

Strain gauge voltage output ( $V$ ) - Bridge excitation voltage ( $V_s$ )



Standard Load Cell Bar

- In case of Quarter-bridge configuration, Initially  $R_1 = R_2 = R_3 = R_4$

$$V = \frac{V_s}{4} \cdot \frac{\Delta R}{R} = \frac{1}{4} G \epsilon V_s = \frac{1}{4} G \frac{F}{AE} V_s$$

**NOTE:**  $V \propto F$

$G=2$  for general purpose foil strain gauge

Modulus of Elasticity  $E = \frac{F/A}{\epsilon}$  by Hooke's Law.

- In case of Full-bridge strain gauge circuit (rarely used):

$$V = \frac{1}{4} \left( \frac{\Delta R_2}{R_2} - \frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} - \frac{\Delta R_3}{R_3} \right) V_s \text{ or}$$

$$V = \frac{1}{4} G (\epsilon_2 - \epsilon_1 + \epsilon_4 - \epsilon_3) V_s$$



So, now, in the case of a Quarter-bridge configuration, initially,  $R_1$ ,  $R_2$ ,  $R_3$ , and  $R_4$  are all the same, and the voltage in that case would be supply voltage ( $V_s$ ) by four delta  $r$  by  $r$ . If you substitute delta  $r$  by  $r$  with the gauge factor and epsilon, it comes here like this. So, the gauge factor into epsilon is actually equal to delta  $R$  BY  $R$ . Epsilon is the strain, and strain can be replaced by  $F$  by  $AE$ .  $AE$  is the cross-section area, and  $E$  is Young's modulus.  $F$  is the applied force.

$$V = \frac{V_s}{4} \cdot \frac{\Delta R}{R} = \frac{1}{4} G \epsilon V_s = \frac{1}{4} G \frac{F}{AE} V_s$$

So, now you can notice that the voltage output is directly proportional to the force. The higher the force, the higher the voltage that you see here at the bridge. For a general-purpose foil strain gauge,  $G$  is equal to 2.

The modulus of elasticity is equal to  $F$  by  $A$  by epsilon, as per Hooke's law, which you already know.

$$E = \frac{F/A}{\epsilon}$$

So, in the case of a full bridge strain gauge circuit, which is rarely used, it is not required; two of them are quite good enough. One of them can be in tension, the other one could be in compression, or one of them is glued, and the other one is just in that environment. So,

those two are the most commonly used configurations to do temperature compensation. In this case, the full bridge strain gauge circuit, all four resistances are used. So, in that case, it is this. 1 by 4 times of all the strain that will be visible here. The change in resistance is shown here.

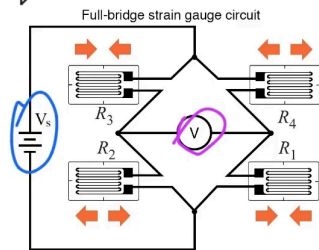
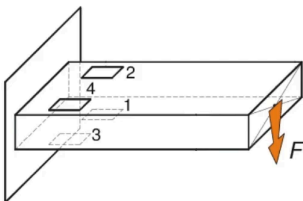
$$V = \frac{1}{4} \left( \frac{\Delta R_2}{R_2} - \frac{\Delta R_1}{R_1} + \frac{\Delta R_4}{R_4} - \frac{\Delta R_3}{R_3} \right) V_s$$

This may be written as one by four gauge factor into epsilon two minus epsilon 1, epsilon four minus epsilon 3.

$$V = \frac{1}{4} G (\epsilon_2 - \epsilon_1 + \epsilon_4 - \epsilon_3) V_s$$

## Strain Voltage relations

Strain gauge voltage output ( $V$ ) - Bridge excitation voltage ( $V_s$ )



- ▶ For Half-bridge: in arms  $4 \leftrightarrow 1$  OR in  $3 \leftrightarrow 1$   
In this case:  $\epsilon_3$  and  $\epsilon_2$  OR  $\epsilon_4$  and  $\epsilon_2$  as zero.

- ▶ **Torque Sensing:** For a  $45^\circ$  four helix gauges on shaft:

$$\tau = \frac{V}{V_s G} \frac{JE}{r(1-\nu)}$$

where,  
 $J$  = Polar Moment of Inertia,  
 $r$  = Radius of the shaft,  
 $\nu$  = Poisson's ratio,  
 $E$  = Young's Modulus of the elasticity of the strain-gauge material.



For a half-bridge that is in arms 4 and 1 or 3 and 1. In this case, epsilon 3 is equal to epsilon 2, or epsilon 4 is equal to epsilon 2 as zero. So, in case you are using only 4 and 1, 2 and 3 become equal to 0. In case 3 and 1 are used, 4 and 2 would become equal to 0. If at all these four strain gauge rosettes are fitted on a shaft which is having a torsional load like this, these gauges can be fitted like this: 1, 2, 3, and 4. In this case, what would happen? It is basically a 45-degree 4-helix gauge on the shaft that is fitted. So, in this case, if it has a torsional load the way I have shown here, What would happen? Number 4 and number 1 would come under compressive load. These two would be in compression.

This one will be in compression. And what would happen to number 3 and number 2? Those two will be in tension. These two will come under tension. So, the deformation occurs accordingly. So, if you reverse the direction of torque, the opposite would happen. In that case, 1 and 4 would come under tension, and 3 and 2 would come under compression. So, this is how the deformation can take place. So, you can have four such gauges again put on a shaft, and you can measure the torque.

$$\tau = \frac{V}{V_s G} \frac{J E}{r(1-\nu)}$$

In that case, the torque is given as tau is equal to V, that is, the voltage that is sensed here, and Vs, that is the source voltage which is here, and then G is the gauge factor for the strain gauge. R is the radius of this shaft. E is Young's modulus of elasticity of the strain gauge material. This is the Poisson's ratio. So, overall, This can be expressed as J, the polar moment of inertia so that you can measure torque. You can have multiple arrangements of these gauges on such systems, and you can have your own derivation for this formula. You can calculate the torque accordingly. So, this is for one such arrangement, and it can measure the torque.

### 6-DoF Force/Torque Sensors

$$\begin{bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} & k_{46} \\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55} & k_{56} \\ k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & k_{66} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix}$$

$$\begin{bmatrix} \text{Force and Moments} \end{bmatrix} = \begin{bmatrix} \text{Gauge Calibration Matrix} \end{bmatrix} \begin{bmatrix} \text{Sensor Output Voltage} \end{bmatrix}$$

**Calibration Matrix:** Links the Gauge voltages to Force/Torque.



← Joint Torque Sensor →

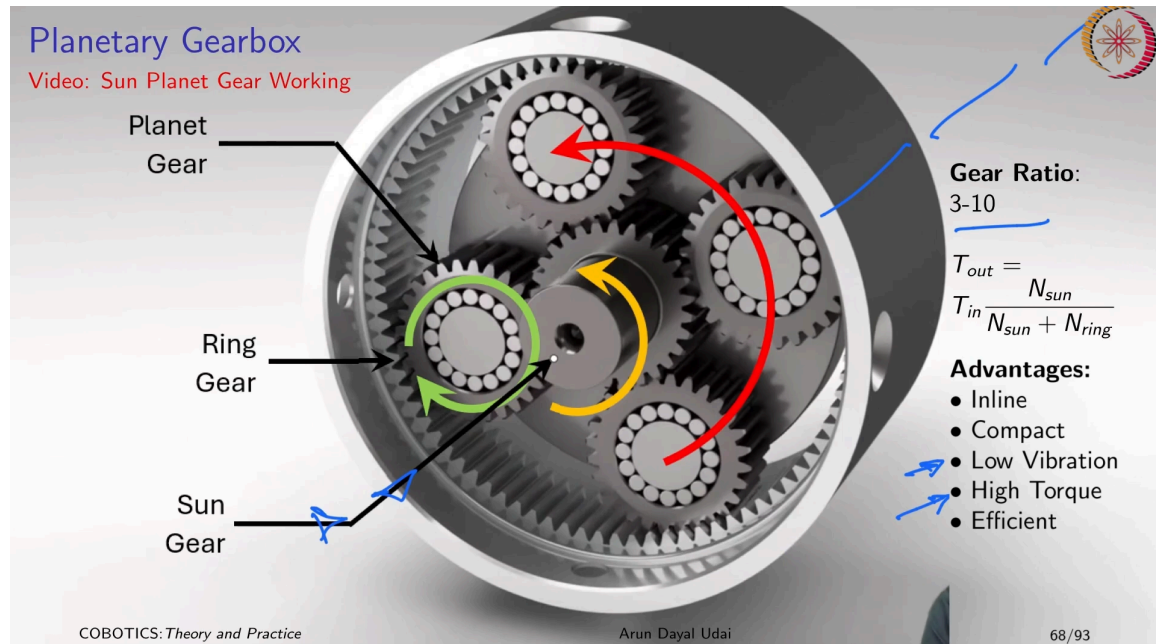
So, now, if at all you have Arrays in a manner when you have six such sensors, such strain gauge sensors, which can give six outputs across some arrangement. It may be a

four-bridge arrangement, which may be something like this. You have so if you can put your strain gauge somewhere over here, somewhere over here, somewhere here, and somewhere here, or maybe some other places also. There are various arrangements that have been experimented out. So, based on the voltages that are seen by different sensors over here. It can have a relationship between the forces  $F_x$ ,  $F_y$ , and  $F_z$ . This becomes a 6D sensor, that is, 3 for the forces along X, force along Y, and force along Z. Z will also deform it. So, there will be multiple deformations even for a single load, unidirectional load along any axis will change the voltages to all the gauges.

So, that is the reason it is a complex relationship. There is a gauge calibration matrix, which is a 6x6 matrix that relates the voltages that are seen by the six strain gauges to the forces and moments that are appearing here, so three-axis force and all three-axis moments can be related to the voltage using a 6x6 gauge calibration matrix. Sensor output voltage force and moment are here.

So, such a calibration matrix Relates the sensor output voltage to the forces and moments. A six-axis four-stop sensor looks like this when you can see  $F_x$ ,  $F_y$ ,  $F_z$ ,  $M_x$ ,  $M_y$ , and  $M_z$ . So, you have a built-in electronic circuit inside that actually does the conversion. It has a pre-calibrated gauge calibration matrix inside. It checks the voltages, multiplies them by the gauge calibration matrix, and finally, it gives you directly the force and the movements. So, this is how this can be designed.

So, a torque sensor may look like this. So, it is a simple arrangement in which you see there are strain gauge rosettes which are fitted at this location over here, over here, and here also, so all the four places it is there. So, this is a single-axis torque sensor about the axis, which is like this. So, this directly can be seen, and it looks like this, and you have an electronic circuit which is here that takes up the signal does some filtration, conditioning of the voltages, and finally, it multiplies with the gauge calibration matrix. If at all, it is required in the case of a six-DoF sensor, and it gives you a pre-calculated form of the forces and the movements. So, a joint torque sensor which is there in the cobot looks like this.



So, now let us come to the transmission that is used in Cobot. So, it can be a planetary gearbox or it can be a harmonic drive. So, we will look at them one by one.

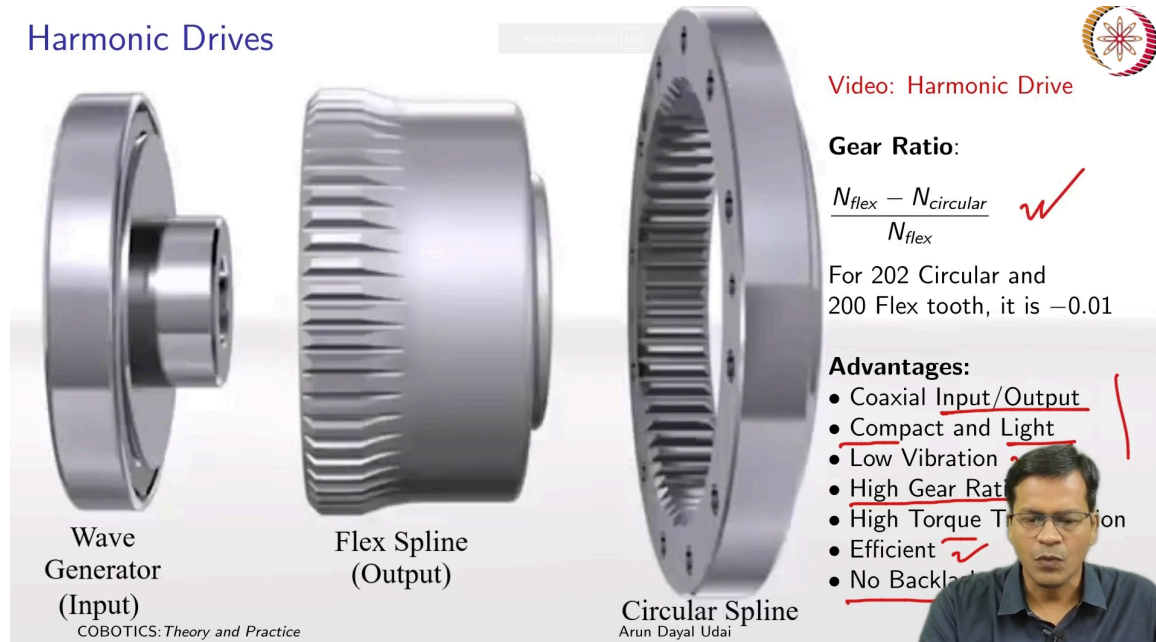
So, let us see. So, a Planetary Gearbox is commonly known as a Sun and Planet gear system. In this case, you see you have a rotating shaft. This is the sun gear from where you can provide the input. So, this rotates like this, let us say, as it is shown in the yellow over here if this rotates like this. So, all the planets that are there, planet gears which are there, there are four over here, 1, 2, 3, and 4, they will rotate opposite to this. So, if the yellow one is rotating like this. This green one will rotate in the opposite direction, and this will now roll over the gear, which is known as Ring Gear. So it rolls over that. And finally, each one of them will move like this. So they move like this. All four of them will move like this. So, the frame holds these four gears, those are the planet gears. So, that frame rotates. That becomes the output shaft. So, the input shaft is the sun gear. The output shaft becomes the frame that holds the planet gears.

Let us see a quick video on this so you can see how it is rotating. Let me just wind it up once again. You see how it is working. Got it? So, the frame that is holding the planets is rotating. Finally, that appears as an output shaft. So, the output shaft is rotating along with the planets.? And the sun becomes the input.

So, this is how it works and the gear ratio lies between 3 to 10. If it is 3, that means the sun gear is very, very small. In that case, the output becomes very, very slow. But if the sun gear size is increased, it can go up to 10 times the gear ratio. So, input to output, it actually becomes 10 times the reduction. It is really a reduction. Basically, the output becomes less than the input. The output angular velocity is less than the input. So, it depends on the size of the sun. You cannot make it too big or too small so that limits the gear ratio to 3 to 10. And it becomes almost like a direct drive because there is hardly any loss over here.

So, these are the advantages. The input shaft is over here, along with the sun gear centre axis. So, that is the input shaft, and the output shaft, you saw, is the centre of all the four planet gears. That becomes in line with the input shaft, so the output and input become in line. This is very, very compact, and you can have multiple stages of these kinds of gears. So, the output can again get into the next stage of the sun gear and finally, the output of the planet gear of the second stage becomes the output. So, you can have multiple such modules one by one in series, and you can have a higher gear ratio also. Because it is aligned to the centre, there is very little vibration, and a high amount of torque can be transferred. They are very, very efficient as compared to the standard gearbox, which has gears, gears, and gears, they are one after the other in a serial chain. So, this is the working of a planet gear, and this is the gear ratio. This is how it works. You have seen the video.

## Harmonic Drives



Now, let us come to harmonic drive. This is again very commonly used in cobots. It consists of three major parts. The first one is the Wave Generator which is the input shaft, and it is not circular as it appears. It is actually elliptical. This is elliptical, and the second part is the Flex Spline, which is the output that finally would become the output from here. From here, it takes the output shaft is connected, and it is known as flex because this is made up of a flexible drum kind of. It is flex. You will see the working also, and finally, it is having a Circular Spline. This is rigid, and it has internal gears.

So, now let us just run the video once again. So, this is the complete assembly that is shown. All the three parts are dismantled now. This is an ellipse. You see, this is an ellipse. It is elliptical, and this is the input shaft as well. You have noticed it now. This will get into the flex spline. This is the flex spline. It is like a drum, and it is flexible also and this is the circular spline that has the internal gear, and this is rigid. Now, when everything is assembled, see, this goes into this one, and when this will rotate, it will deform the flex spline. It will make it elliptical, and the axis of the elliptical rotates inside, and that rolls over the internal gear of the circular spline. As it rolls, it also moves because the number of teeth in the flex spline is less as compared to the circular spline. Teeth are less. See, it is rolling, and the input shaft is shown, and you can now see the output shaft that is rolling, and it is rotating very, very slowly.

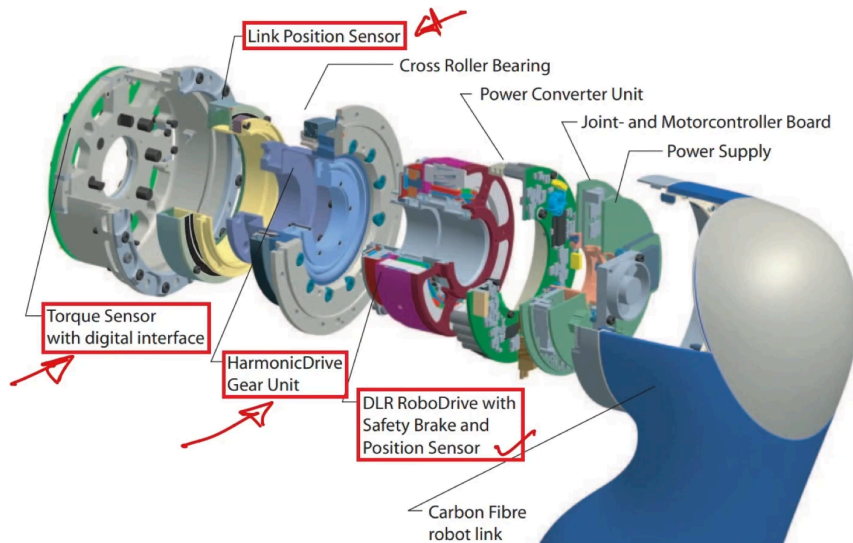


Now, the gear ratio of this would be number of teeth in the flex spline minus the number of teeth in the circular spline, which is here divided by the number of teeth in the flex spline.

$$\text{Gear Ratio: } \frac{N_{flex} - N_{circular}}{N_{flex}}$$

So, for the circular spline, let us say there are 202 teeth. Spline teeth, and in the flex spline, there are 200 teeth. In that case, if you put that, you will get it as minus 0.01. The minus sign indicates that the input and output are directed opposite. If it is a clockwise input, the output shaft will rotate counterclockwise, and the gear ratio is one by 100 times. Got it? So, it has a high gear ratio in such a compact form. So that is the beauty of such harmonic drives.

So, there are many advantages. Again, you see that both the input and output shafts are coaxial. They are very compact and light, and you see, it has a centre hollow. You can pass the wire through its centre. So, if you have a BLDC motor and a harmonic drive, you can create a closed-loop circuit that is the controller with a centre hollow design. So, if it is all assembled together, it creates a centre hollow servo drive. This also has very little vibration, and you saw that a very high gear ratio can be created in a compact way, and High Torque Transmission is also possible. They are very efficient, and practically, there is no backlash. This is one of the most advantageous things that is looked at when you make any robot. In the case of other gear systems, you would need some other arrangements to take care of the backlash using some preloaded torsional spring or some kind of other mechanical arrangements. But this does not require any such arrangement, and it practically has almost no backlash. So, this is how a harmonic drive works, and this is the gear ratio. These are some of the advantages. The only thing is the manufacturing of this flex spline is not that easy. And that is the reason very few manufacturers are there.



*Albu-Schäffer, A. et al. "The DLR lightweight robot: design and control concepts for robots in human environments." Ind. Robot 34 (2007): 376-385.*

COBOTICS: Theory and Practice

Arun Dayal Udai



Altogether, we have already discussed the Torque Sensor, we have discussed the link Position Sensor, Harmonic Drive you have seen just now; it can be a planetary gear also. You have also seen a BLDC motor with a Hall Effect position sensor. It can go somewhere over here, and now we need to discuss the controller and other aspects that will come later in this course. So overall, we have covered till here.

That's all for this lecture. In the next lecture, I'll discuss Industrial Field Bus, Drives, Devices, Safe Workspaces, Safety Triggers, Workspace Monitoring, Marking Forward and Zones.

Thanks a lot.