Engineering Fracture Mechanics
Prof. K. Ramesh
Department of Applied Mechanics
Indian Institute of Technology Madras

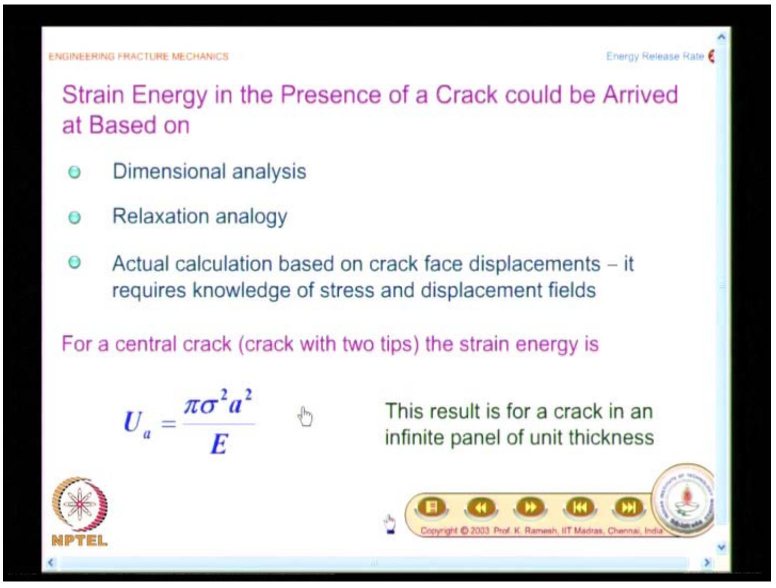
Module No. # 05
Lecture No. # 24
Discussion Session-I

We are half way in the course, we have looked at in detail overview of fracture mechanics, followed by crack growth and fracture mechanisms; then, we also spent some time on energy release rate; then, we had a detailed discussion on crack-tip stress and displacement fields, in that we have completely covered the singular solution; we have started looking at the need for multi-parameter solution.

Before we proceed further, it is desirable I try to clear as many doubts that you have in your minds. So, this we will have it as a discussion session and the class is open for raising questions.

In relaxation analogy why do we use triangular region while evaluating strain energy?

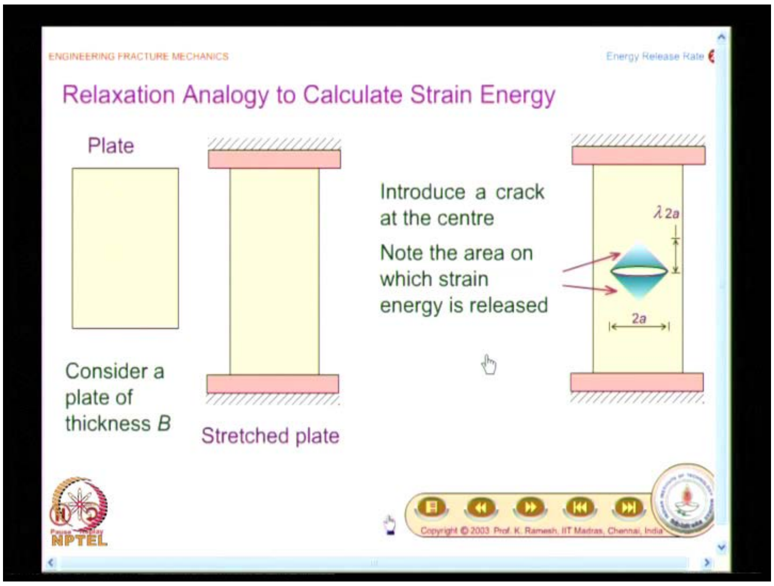
(Refer Slide Time: 01:14)



You know, if you really look at, we want to evaluate strain energy in the presence of a crack. See, it is not a simple task and we saw that it could be arrived based on dimensional analysis, relaxation analogy, and finally, actual calculation based on crack face displacements. In fact, only this would give the intended result, and this was taken up, while we were developing fracture mechanics in the initial stages.

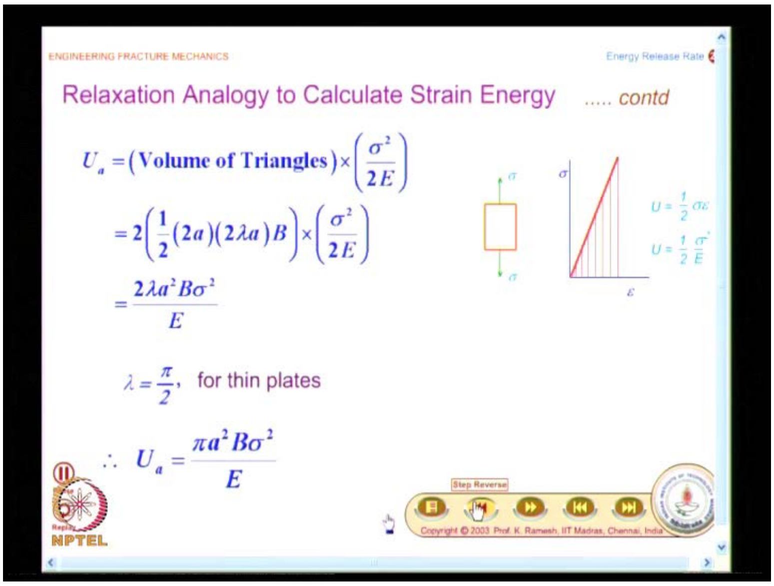
So, the idea is to find out what is the real role of this relaxation analogy; you have to understand that as part of your learning process. And what you have here is, we know from our other considerations, what is the strain energy in the presence of a crack. This result is known; with this result only, we discuss the relaxation analogy. So, the focus of relaxation analogy is not just to get this result, but appreciate the concept that is developed as part of energy release rate.

(Refer Slide Time: 02:34)



The focus is different and let us look at what we have done. What we have done was, we have taken a plate of thickness b and we stretched it, in the set introduced a crack at the centre and what was marked is, note the area on which strain energy is released; marking of this as triangle is only incident.

(Refer Slide Time: 03:23)

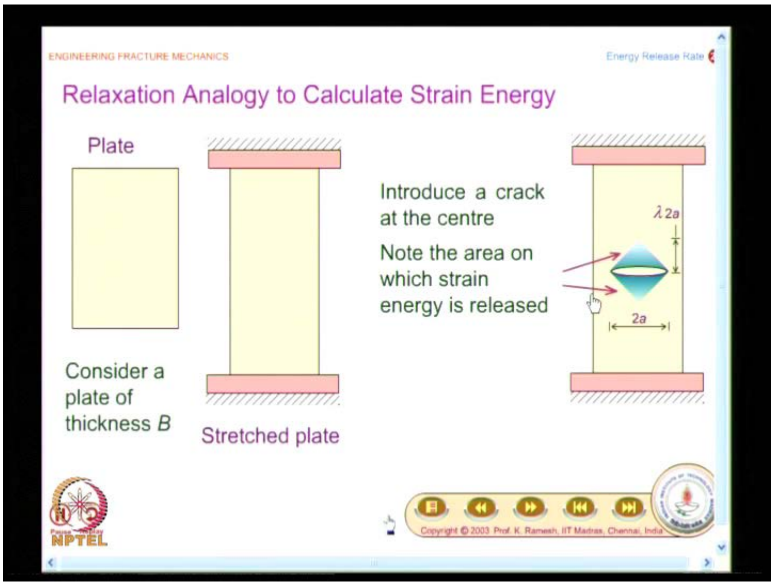


You could take a parabola, you could take a complicated curve, you could do anything of that solve; what it amounts to focus upon is, for the formation of cracks, strain energy in the neighbouring area has contributed to the formation of two new surfaces, that is what is attempted to be shown and what we have also done in the calculations. If you look at the calculation, I say volume of the triangle, I say all this, but I also bring in a factor called lambda, we have not evaluated lambda.

See, if I evaluate lambda based on a triangle, then the question is valid - why triangle is so special? We have only indicated, a region close to the crack contributes to formation of two new surfaces. We have said, for thin plates take lambda equal to pi by 2. So, this absolves whether you take a triangle, whether you take a parabola, whether you take a complicated shape, the idea here is the region near the crack has release some part of the strain energy for the formation of two new crack surfaces; and why we look at the relaxation analogy?

See, we are actually discussing what happens in the case of a brittle material, and I said, while discussing relaxation analogy, imagine that you have taken a cycle tube, take a sheet of rubber sheet, and then, pull it; then, you introduce the crack and the crack opens up.

(Refer Slide Time: 04:59)



So, what you will have to look at is, this is only to aid your visualization process, that energy can be released from the system for the formation of crack and there are also many catches here. You know, if you look at, we want to find out strain energy in the presence of a crack how the crack was introduced, we have never discussed it.

See, if I want to find out strain energy in the presence of a crack, it is a very difficult task to do. I said later on when we have developed the displacements, I said you have to do a thought experiment. In the thought experiment, you find there is a crack that crack opens up, what is the force that is required for opening up? That is the way we calculate it.

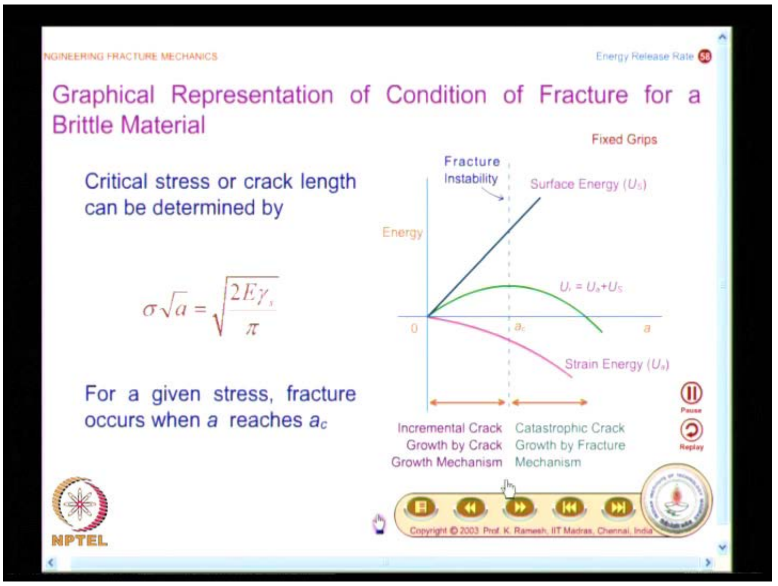
Here again, if you look at, for me to have the crack to propagate, I should have introduced the crack and after some length, the crack would have propagated by itself from the side strain energy of the system and if you visualize it that way, this example is not complete by itself. Here again we are bringing in a thought experiment.

The sheet was stretched and suddenly a crack opens up and you calculate the energy. So, the focus here is for the formation of two new surfaces - strain energy in the neighborhood of the crack would have contributed to this. So, the focus is only to understand that strain energy of the system decreases to form two new surfaces and that is the reason I also mentioned what is logical is, when there is a crack extends by itself, when we found an identity between k and g we did it very systematically. In fact, that is much more appealing to develop energy release rate. And if you really look at the literature also, people found it difficult to calculate the strain energy in the presence of a crack that is the reason why, Irwin suggested you go for complaints approach; when you go for a complaints approach, at least I can perform an experiment and calculate d c by d a.

So, the focus of relaxation analogy was only to give you an impression that strain energy in the neighbourhood of the crack would have contributed to the formation of the two new surfaces. See in fact, even in your design courses when you develop how to understand the concept of shear, suppose you take a shaft and put a key there and put a key there which is subjected to torsion and you want to visualize that shearing action takes place on the key way.

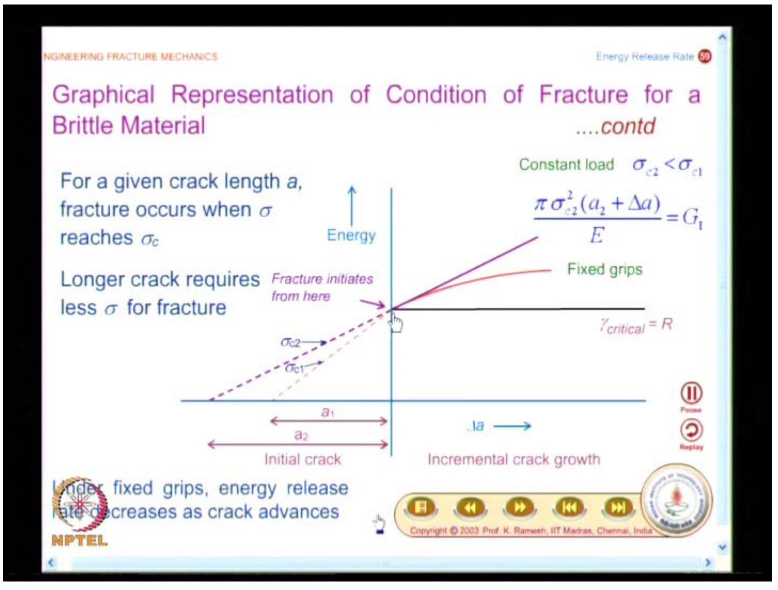
If you are given a strong metallic key, it is difficult for you to visualize. One of the tricks that is employed there is, imagine the key is made of frozen butter, then you can visualize when the shaft is twisted, you will find that shearing action takes place, you are able to feel with the component in question. So, the relaxation analogy has to be looked at only in that perspective; we have not really made the calculation, we have just said lambda equal to pi by 2. So, that absolves, there is no special significance attach to the triangular area. I suppose you have got the answer now. We know that energy release rate is energy required for unit extension of crack, whereas r is a resistance of fracture, but how G and R are equally in stable fracture?

(Refer Slide Time: 08:41)



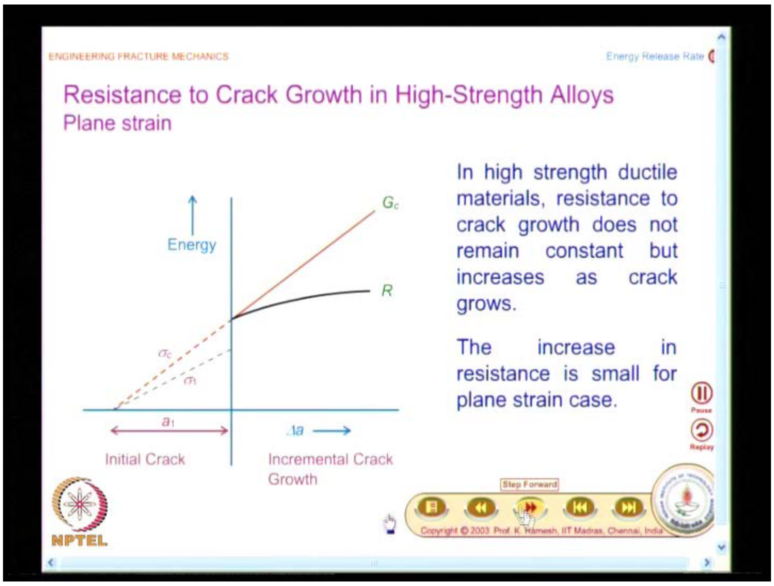
See you will have to look at what way we have developed the fracture theory. This is one form of representation which we had seen, where you find out sigma and root a is related to root of 2 E gamma divided by phi. This is one form of graphical representation.

(Refer Slide Time: 09:05)



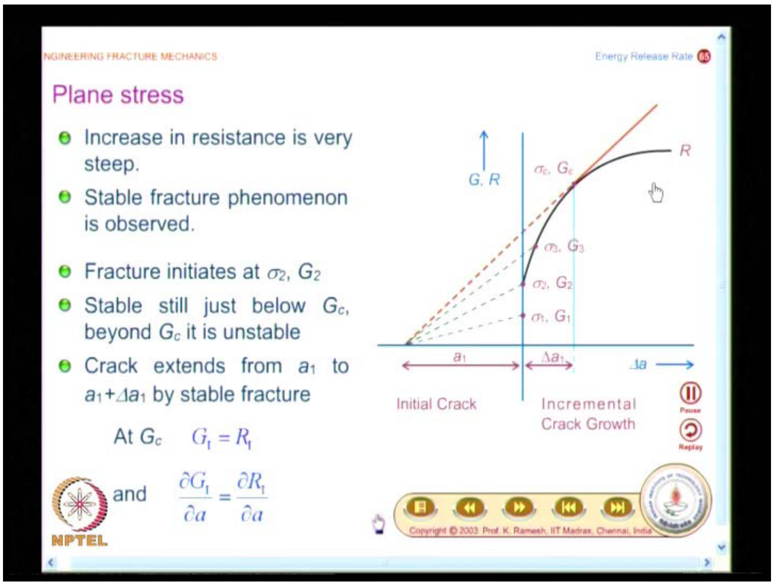
We also saw another form of graphical representation, where we had mentioned, the resistance R is this much. In the case of brittle material as you increase the stress, what you find is, at a particular value of critical stress, you have fracture instability takes place. So, here, G equal to R; for brittle material, there is no problem.

(Refer Slide Time: 09:37)



When we come to ductile materials what we have noticed was, in the case of plane strain R is not a constant, but it varies; but the variation is shallow. Here, again when does fracture instability occurs? When G becomes R that is dictated by what is the stress level that you are imposing on a structure; when it is sigma 1, fracture instability does not occur; only when sigma 1 reaches sigma z, fracture instability occurs. So, you have the condition G equal to R satisfied.

(Refer Slide Time: 10:19)



Let us look at for a plane stress situation. In a plane stress situation what happens? The nature of R curve is totally different; it is very steep. When you have stress as sigma 1, the corresponding energy release rate is 0, which is far below the value of R.

So, nothing will happen to the crack; the crack will remain as such. Suppose I keep increasing the load, when I increase the load from sigma 1 to sigma 2 that is the time, where G becomes equal to R. So, now fracture instability is possible.

Suppose the stress is raised to sigma 3, when stress is raised from sigma 1 to sigma 2 crack would not a propagated, but after reaching sigma 2, crack is ready to propagate. When sigma 2 is slightly increased, not sigma 3 even slightly increased, then you have a situation G equal to R that is satisfied, but the current G is greater than the previous G.

So, to that extent, the crack will have a forward moment, then as these stress level is increased, it will keep on moving like this on the graph and when it reaches G c, any small increase would result in catastrophic failure of the component. So, you have to have the necessary condition is G equal to R, sufficient condition is dou G 1 by dou a equal to dou R 1 by dou a.

So, in this zone, only the necessary condition is satisfied. So, whenever the necessary condition is satisfied, the crack will propagate by a small extent, provided the load is increased; if the load is stopped, the crack also will stop, but once you to come to this position, even a very small pertapation will make the crack to propagate catastrophically.

So, what you will have to really look at is, you have a stable fracture followed by unstable fracture and this where I said, when you are doing a plane stress test, you do not need to introduce a sharp crack like what you do in a plane strain fracture toughness testing; we would see that as part of fracture toughness testing. Because in all plane stress fracture toughness testing, you have a natural crack formation because of stable fracture, then it is followed by unstable fracture.

Now, let us come to the question. In the case of fatigue, the crack takes finite time to grow. In an actual practice, you will have stable fracture followed by unstable fracture there is will be hardly any time. For pictorial representation, it is shown as sigma 2 is so small and sigma z is so high; this kind of a graphical representation is shown to illustrate the point.

So, whenever G equal to R, to satisfy this condition when the energy is released, you will find some small extension of crack has to take place, but that would not be catastrophic that is what is discussed upon. Thank you.

Whether the resistance and fracture toughness are they same?

You know we have looked at the graph for the plane stress as well as plane strain; now you look at the graph of this plane stress. See this is the resistance present in the case of the material, as the crack advances in the case of ductile materials what you find is, you have plastic deformation and this plastic deformation size increases. So, as the crack length increases, the resistance also increases.

So, when we talk about resistance, you have to look at what happens in the case of brittle solids. Brittle solids, there is no plastic deformation; the resistance remains constant, which you could take it as a property, whereas resistance is used as a conceptual appreciation how the material responds.

In the case of plane strain it is shallow; in the case of plane stress, it is very steep like this it is very steep like this and if you look at fracture toughness what do you say? Fracture toughness by definition, it is a material property, and if you look at for plane strain case, it becomes the material property and if you exceed stress intensity factor greater than fracture toughness, you will have catastrophic failure. We will also see in the fracture toughness testing, variation of fracture toughness as a function of specimens.

So, you will find out for different thicknesses what is the value of fracture toughness when you look at the plane stress case. So, when you say fracture toughness, it is a material property. When you say resistance, what actually happens, how the material behaves in actual practice? So, R curve is a concept, which is beautifully developed to explain various things that you come across in fracture mechanics, like stable fracture followed by unstable fracture and the case of intermediate plates, you have this popping behaviour.

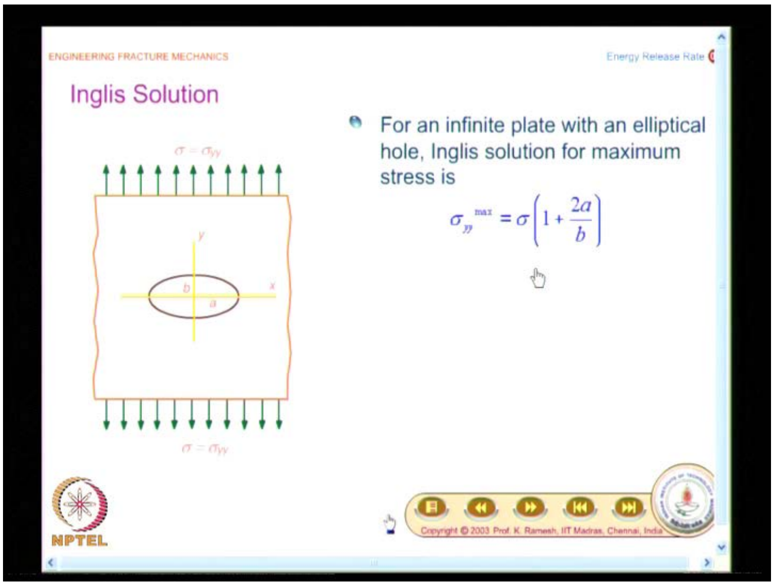
So, R is different to denote the resistance provided by the material. When you say fracture stiffness, you will recognize that as a material property like yield strength; when you say yield strength, you have certain connotation, how to understand that.

So, similarly when you say fracture toughness, you look that as a material property, resistance also behaves as a material, but it is little more elaborate. Because you have R curve as a function of crack length, you do not have that kind of information; you have fracture toughness as the function thickness of the specimen, not related to the crack length. So, this is way we have to look at. yeah

Sir as you define the stress concentration factor as a stress region near the geometrical discontinuities similarly, how can we define the stress intensity factor theoretically?

You know I would like to correct your question also. See you have mentioned stress concentration factor as a stress raiser, it is not so; when you have a geometric discontinuity, that acts like a stress raiser.

(Refer Slide Time: 17:55)



When you have a geometric discontinuity, for example, if you take the case of an infinite plate with an elliptical hole; so, I have a cut-out here. So, this is a geometric discontinuity, because of this what you find is, you will have a maximum stress developed at the tips of this ellipse and that is given as sigma y max equal to sigma into one plus 2 a by b, where a is the semi-major axis of the ellipse and b is the semi-minor axis of the ellipse and what you will have to look at is, how the stress concentration factor was understood in initial stages.

You know, there was lot of debate between mathematicians and engineers. Mathematician said an infinite plate with a small hole is equal to an infinite plate; they did not recognize that because of a cut-out, it would introduce certain changes in the stress field that is point number one. Point number two is the stress levels will increase was recognition by the solution of Kirsch for a plate with a circular hole and Inglis for the case of an elliptical hole. And what you will have to realize is, suppose I do not have a cut-out; when I have a uniaxial stress, I would still have only uniaxial stress field in all the regions; the moment I bring in a cut-out, the stress field changes; it changes to biaxial this is you have to recognize first and the second issue is you have a very high value of stresses developed and that is also noted.

So, people said the moment you have any geometric discontinuity, it is potentially dangerous from the point of your design approach; you have to take that into account. How do you take that into account? You have to have some kind of a methodology by which it says the stress levels are very high, and then, you need to accommodate that in your desire calculation. From that point of view, stress concentration factor was introduced for many problems. You know, if you look at the definition of stress concentration factor from theory of elasticity, it says the maximum stress divided by far field stress that would be greater than three for many finite bodies.

It would approach something like a factor of nine also, but it would not increase beyond that; for many of the common design problems that is how people have look at the stress concentration factor. On the other hand, when you look at the case of an elliptical hole, Inglis alerted, when b tends to 0, the stresses go to infinity.

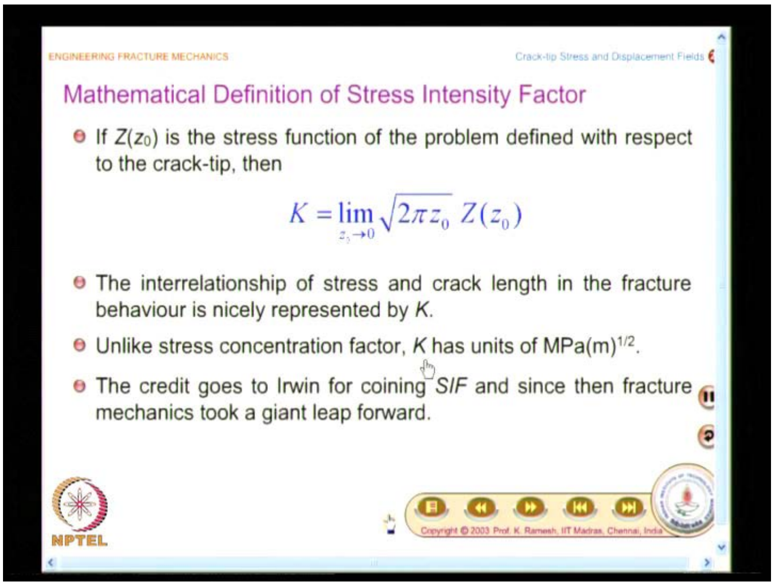
Now, you may ask a question; I think the question has really come, even in fracture mechanics we say sigma y goes to infinity. So, what is so special, there you call it as stress intensity factor, here you call it as stress concentration factor is not valid?

You know, if you really look at, stress concentration factor definition fails when b becomes 0. So, you need to have a different type of parameter to explain what happens in the presence of a crack. What was the outcome of Inglis solution? Inglis said even a very, very small crack, even for very, very small load, the stresses would be infinity and fracture would take place. After having listened to Griffith analysis, now you are very clear, you need to have a finite length of crack which has to be a critical value for a given stress applied, only then crack propagation take place.

So, what to have is, in the case of crack problems, your conventional representation of stress concentration factor is not sufficient; you have to bring in a size factor, size factor here is length of the crack.

So, you have sigma root a is a parameter bundle, which you call it as stress intensity factor, which has dispel the anomaly created by Inglis solution. It has brought in length of the crack is also very important.

(Refer Slide Time: 22:58)



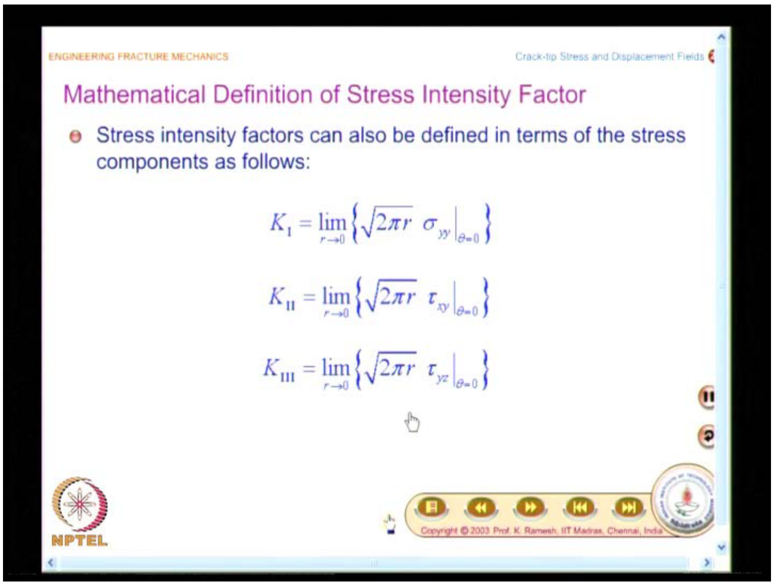
So, you have to look for a critical crack length and a critical associated stress. So, this is the fundamental difference between the stress concentration factor and stress intensity factor and we have also seen a mathematical definition.

So, we have looked at the mathematical definition in two different ways. One way is we have related this to the stress function. In fact, we are going to discuss in the next chapter, if a stress function is given, how to find out the stress intensity factor.

So, you have a mathematical definition. So, limit z naught tends to 0 root of 2 pi z naught multiplied by a stress function capital Z written in terms of z naught.

So, when you solve a problem and find out what is the definition of stress intensity factor of in terms of the parameters of the problem then you will appreciate. So, stress intensity factor really brings in the role of a crack and how it affects the structure? And there are lot of differences between stress concentration factor and stress intensity factor, one of the first thing that you can notice is, it has very funny units of m p a root meter, whereas stress concentration factor has no such units, it just the number. And another one what we have looked at? We have also looked at the strength of the stresses near the crack-tip vicinity slowly dictated by the stress intensity factor.

(Refer Slide Time: 24:25)

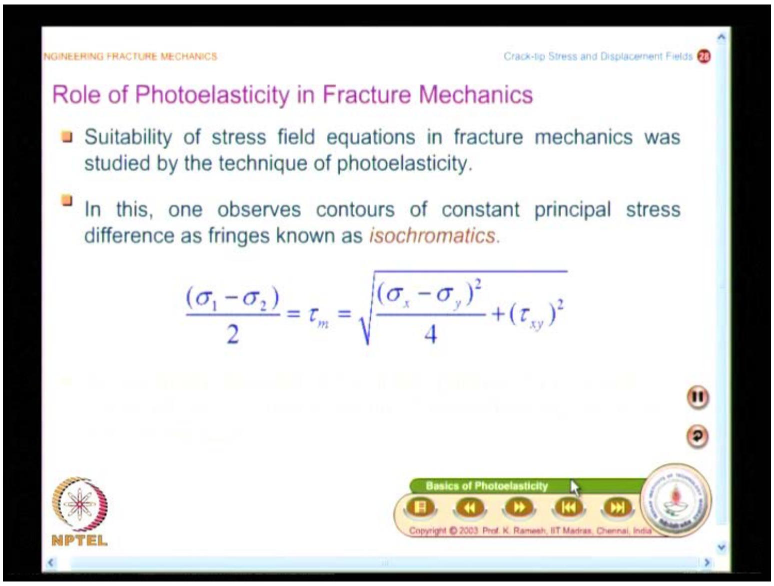


So, from that point of view, you could also define stress intensity factor for each of the modes like this. So, you look at the stress component sigma y y at theta equal to zero multiplied by root of 2 pi r, take the limit r tend to zero. So, you have a mathematical definition how to get stress intensity factor K 1, how to get stress intensity factor K 2, here you take the role of a shear stress tau x y; for the stress intensity factor K 3, you look at the anti-plane shear stress tau y z. So, this brings out what is the essential difference between stress concentration factor and stress intensity factor.

Sir when you have shown the photo elastic fringes, fringes nearer to the crack-tip are not a forward tilted, but fringes away from the crack-tip or forward tilted, whether this forward tilting indicates the crack advancing direction and why the fringes nearer to the crack-tip are not forward tilted? You know this is good question. You are primarily getting this kind of a doubt, because I find half the class is exposed to experimental stress analysis and half the class is not exposed to experimental stress analysis. So, I will use this opportunity to explain certain fundamentals of photo-elasticity that would aid your understanding.

So, we will look at that, as well as the specific question you raised regarding the fringe pattern.

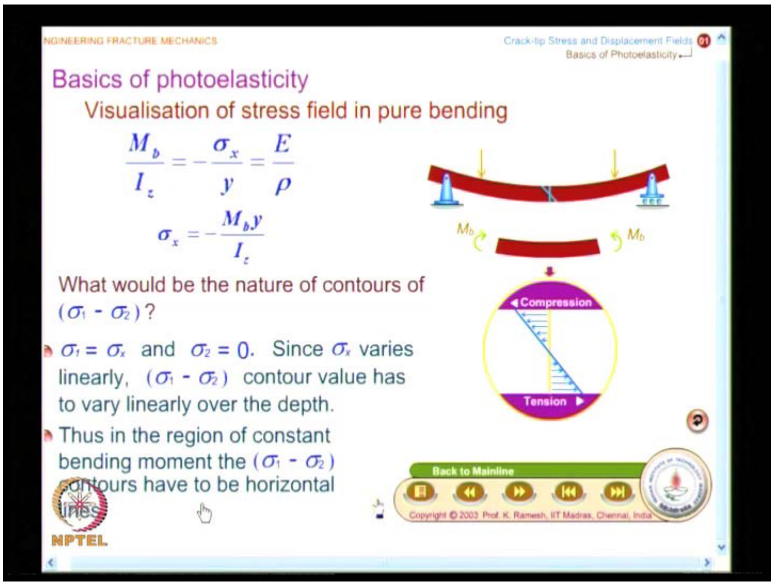
(Refer Slide Time: 26:00)



First of all, let us look at how photo-elasticity is using fracture mechanics. I have always been mentioning, whatever the stress field that we obtain, whether they are suitable for a given situation was established by the technique of photo-elasticity, then the question comes - how?

So, we have to know what the photo-elastic fringes really represent. In photo-elasticity, one observes contours of constant principal stress difference which are labelled as fringes and these are known as isochromatics and how are they mathematically related? I have the maximum shear stress is given as square root of sigma x minus sigma y whole square divided by 4 plus tau x y whole square. Suppose I solve a problem analytically, then I would get expression for sigma x, sigma y as well as tau x y. So, from that solution it is possible for you to calculate the quantity maximum shear stress, then you can also plot this as contours; when you plot them as contours, you can compare it with experiment, and then, see whether your analytical equations are calculated.

(Refer Slide Time: 27:49)

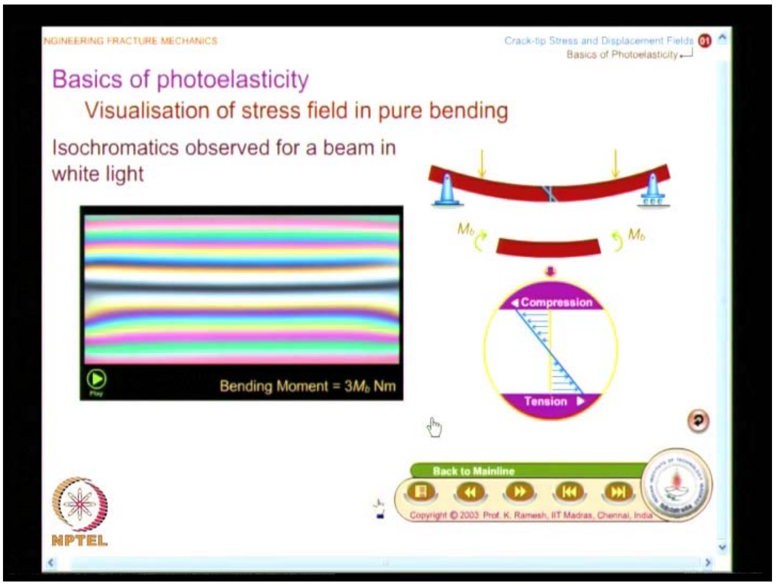


So, before we go into that kind of an approach, we will look at how we understood what photo-elasticity is. I took up a very simple problem of a beam under four point bending. You know for this problem from your strength of materials approach, you know the stress field as a closed form solution in the region away from the points of loading.

The stresses vary linearly and we have identified what would be the nature of contour sigma 1 minus sigma 2; this is the way I introduced the photo-elasticity for you, and we have been able to establish the contours of sigma 1 minus sigma 2 have to be horizontal lines.

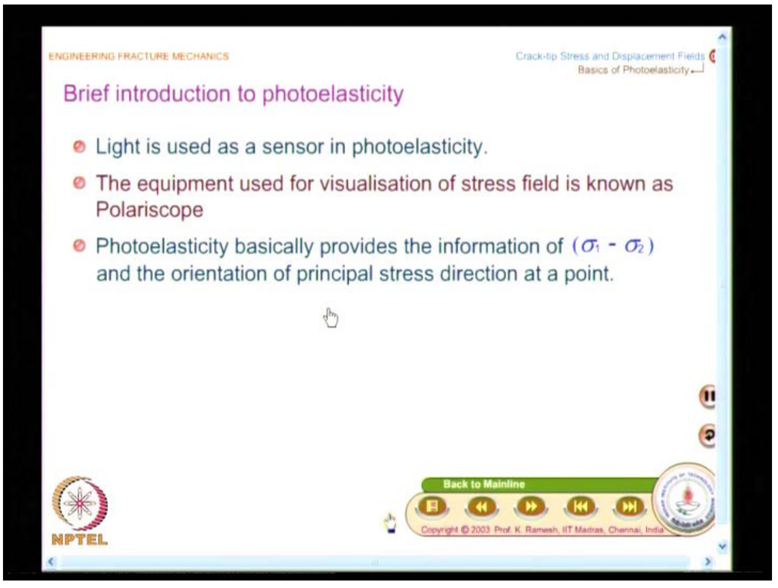
This we have done it based on analytical calculation, because the field is known, you know the solution; so, it is simple for you to establish how the contours will look like, then what we did? We directly looked at the fringe patterns obtained for the similar problem and how do they look like - a horizontal.

(Refer Slide Time: 28:52)



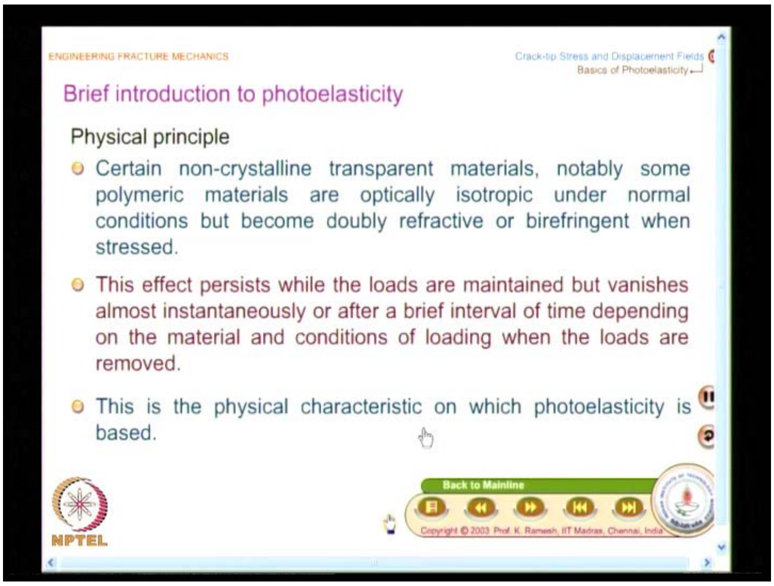
So, from that, we have concluded that, whatever the contours I get in photo-elasticity are contours of sigma 1 minus sigma 2 and what I will do is, I will go a step further and then give this information as well.

(Refer Slide Time: 29:21)



In the case of photo-elasticity, light is used as a sensor and the equipment that you used for visualization is a polariscope and in-fact, photo-elasticity provides, two basic information, it provides contours of sigma 1 minus sigma 2 as well as the orientation of principal stress direction.

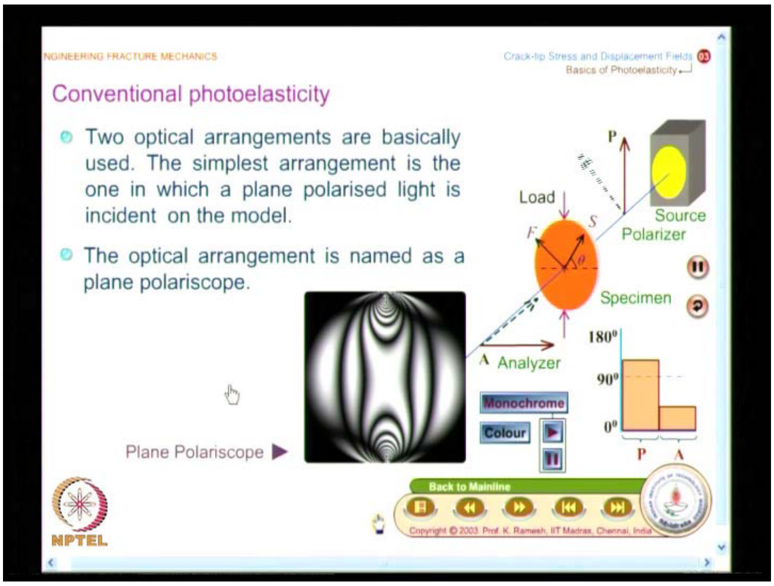
(Refer Slide Time: 29:58)



As we are not required to use this in this course, I do not emphasize it earlier, but now we will also have a brief look at that. And what is the physical principle? Each of these techniques is governed by a basic physical principle and what you find is certain non-crystalline transparent materials, notably some plastics are optically isotropic under normal conditions, but become doubly refractive or birefringent when stressed.

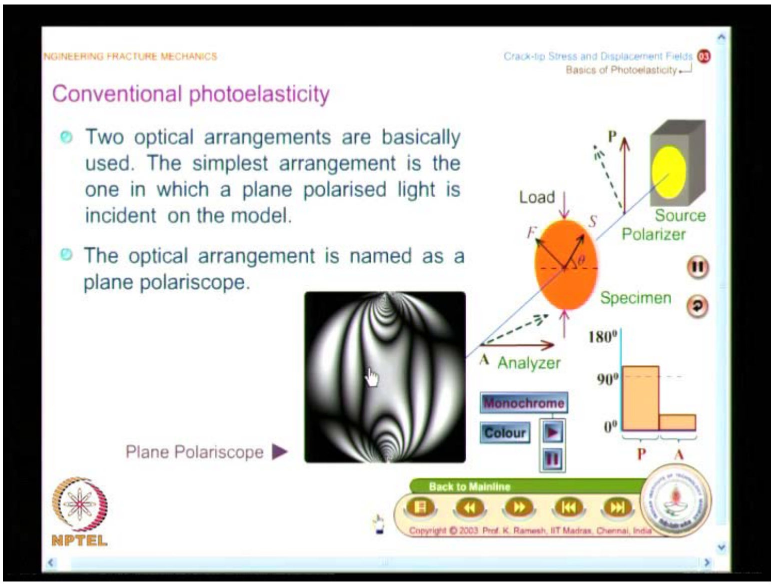
So, this is the basic physical principle and a crystal by nature, it is a birefringent material, whereas here the birefringent is effect is temporary. As long as the loads are maintained, it will behave as doubly refractive; when the loads are removed, it will behave as optically isotropic.

(Refer Slide Time: 31:08)



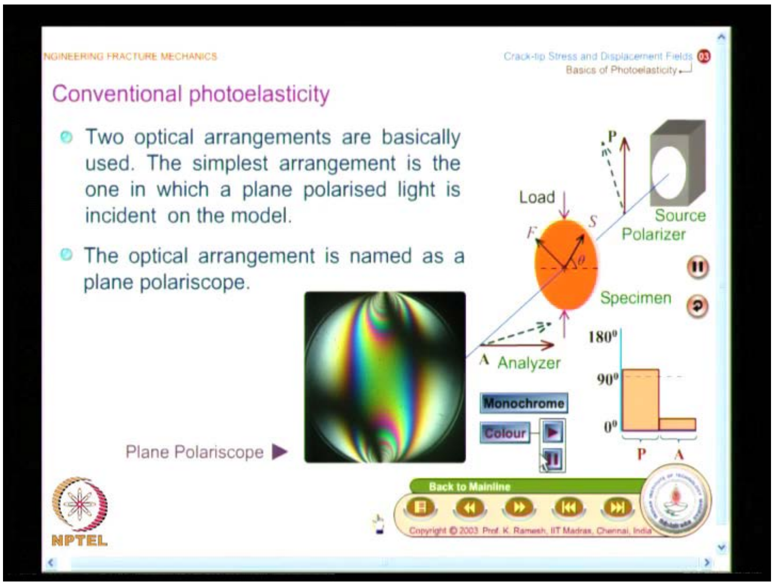
So, whatever the loads that are introduced, that really causes this to behave in a birefringent manner. So, by looking at the birefringent's effect, we can calculate this stress, that is the basis of what photo-elasticity is, and how does the equipment look like? I have equipment like this; I have some piece of optics, we will not get into that; I have a disk under diameter compression and when I rotate these elements, I would see one stationary contour and another moving contour. And I said photo-elasticity provides contours of sigma 1 minus sigma 2 as well as principle stress direction.

(Refer Slide Time: 31:41)



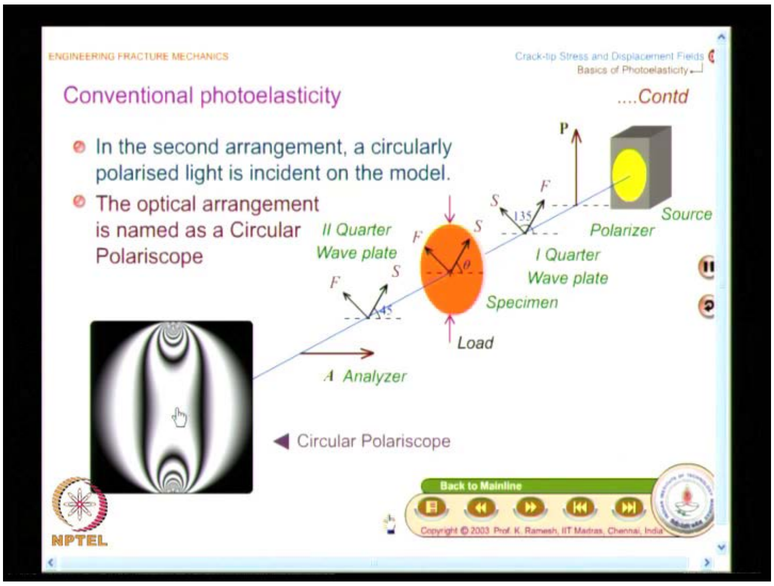
So, in a simple arrangement called a plane polariscope, you will see isochromatics as well as isoclinic. You know, I have taken this to show when I use the monochromatic light source, I would see the fringe patterns as black and white, because for all our analytical calculation, we have only plotted black and white .

(Refer Slide Time: 32:04)



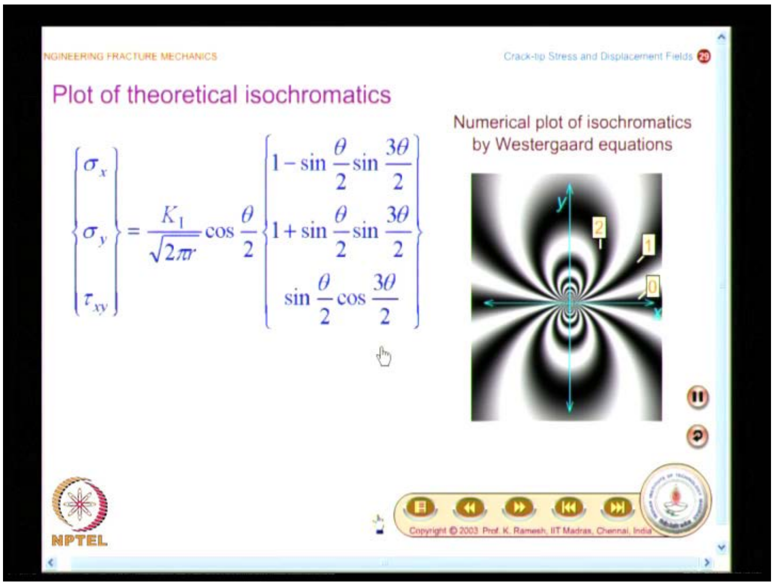
Suppose I change the light source to white light source, you will see these contours as coloured and one set of contours that are remaining black.

(Refer Slide Time: 32:22)



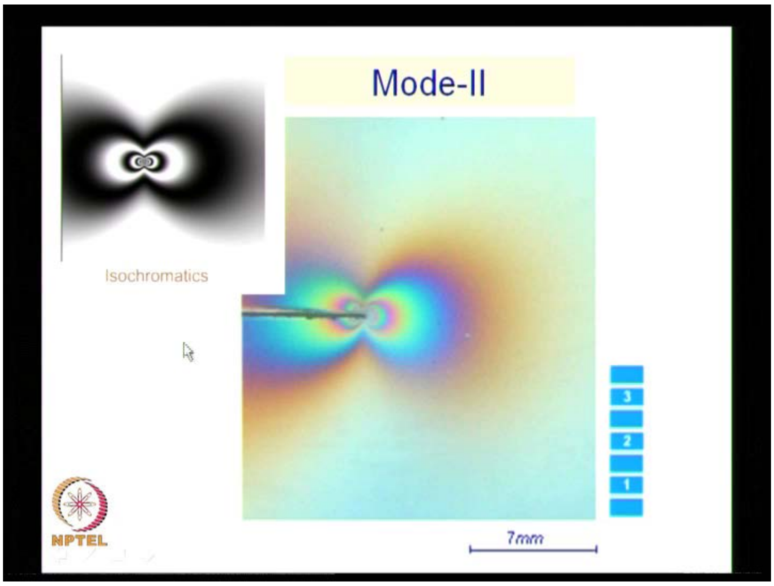
So, that is the technique to identify quickly what is an isochromatic and what is an isoclinic. There is also another optical arrangement that you have, where you have two more additional optical elements, which help you to remove the isoclinic and you will see only isochromatic and this is the kind of optical arrangement that we have used in fracture mechanics to get the relevant fringe pattern and what you will have to notice is, the experimental fringes, the thickness varies; if you see here, the thickness is very broad here. When you say a contour, you will have only that has a line; what you have to do is, when we are plotting the fringe order, we will have to give a small variation and analytically plot that, you will automatically get a thickness variation. You have papers written on that, you could look at the references. So, this gives you, if you know sigma x sigma y and tau x y, it is possible to plot.

(Refer Slide Time: 33:21)



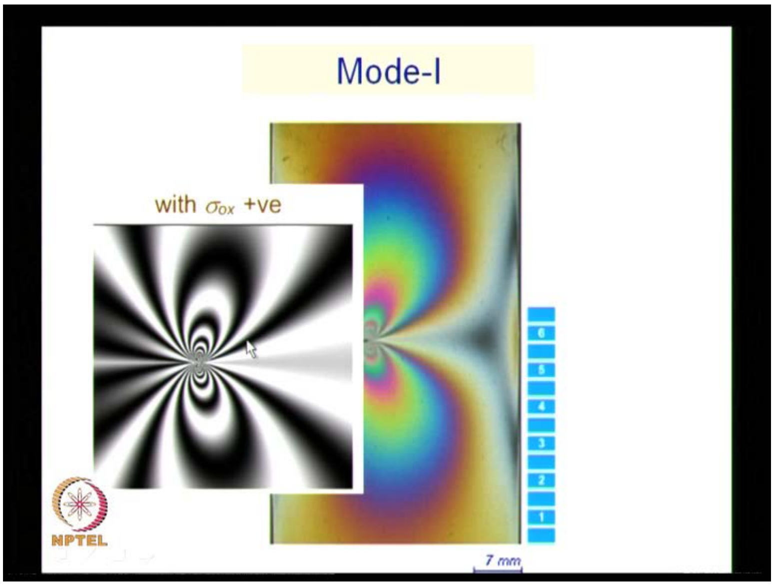
Now, let us look at what was the solution from Westergaard. The Westergaard solution was something like this; when I plotted the fringes, how I have got? I have got the fringe patterns symmetrical about the x axis as well as the y axis. In fact, if my experimental fringes are also like this, we would not have discussed modified Westergaard equations, and then, now we are looking at generalized Westergaard equations, all of them you would not have looked at.

(Refer Slide Time: 33:58)



What we saw in the case of mode 2? In the case of mode 2, I have a fringe pattern like this, which is experimentally recorded. When you look at Westergaard solution, I have this; the singular stress field itself gives the fringe pattern; it is very similar to what is there in the experiment and in fact, the second term was 0 here, that also I mentioned; we would see that when we develop the generalized Westergaard equations as well as William's corner function.

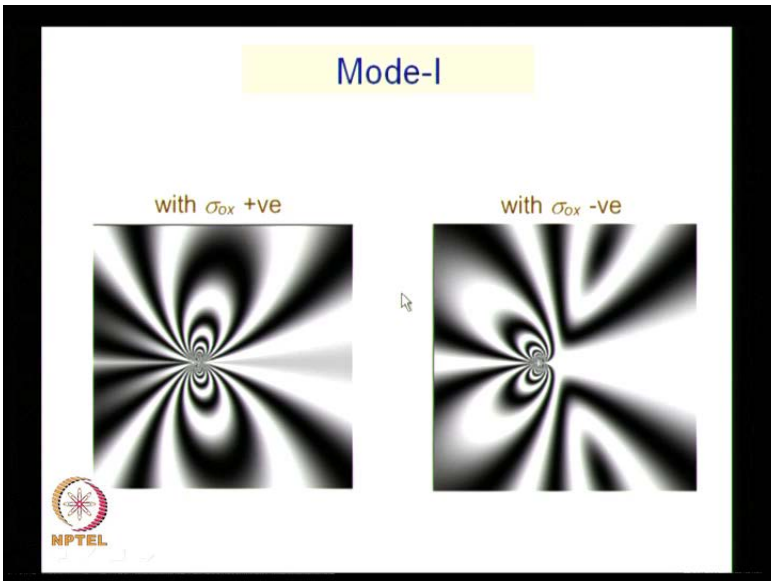
(Refer Slide Time: 34:29)



The moment go to mode 1 when I have a short crack like this, I have my fringes tilted and they are tilted forward in this case and how that tilt was? If you look at very carefully, I have the tilt like this; as you go close to the crack-tip, they are becoming straight.

So, by raising this question, I appreciate that you have observed this phenomenon. What way we can interpret? Suppose I have fringes which are straight, then I could say, I could still use Westergaard solution in that zone and process the experimental result.

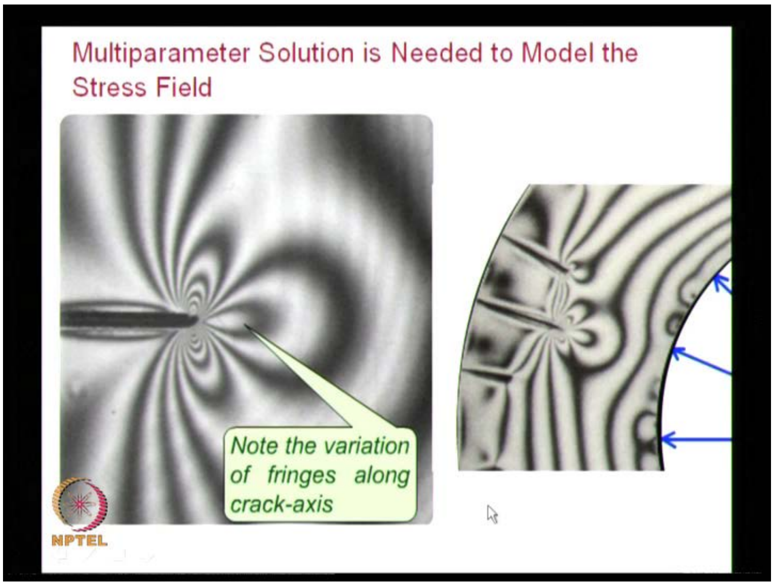
(Refer Slide Time: 35:32)



When I have a tilt, then I have to have Irwin's modification, only with that I can interpret the data and if you really look at the fringe pattern, the zone, where it is straight, so small, it is very difficult to collect data from that zone, and we have also seen one more aspect- in the case of RDZB specimen, that is, rectangular double cantilever beam specimen, I have the fringes tilted backward.

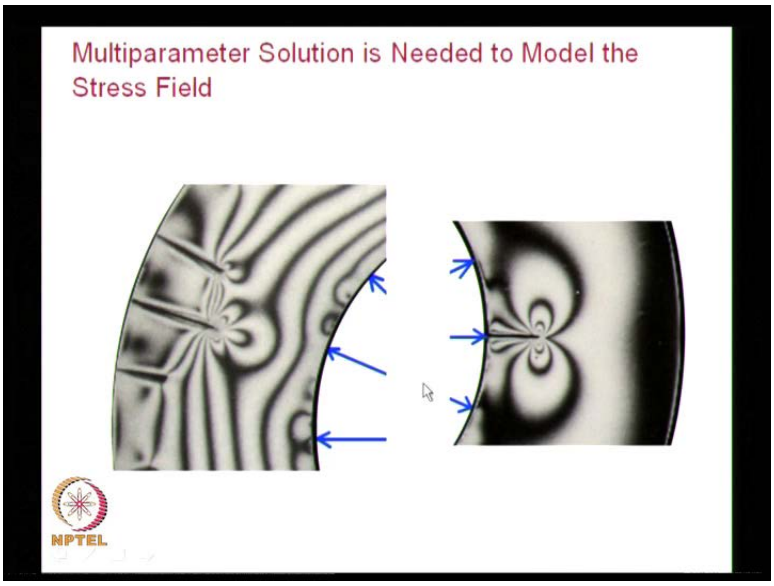
It is backward and it is becoming straight near the crack and what I find here? In this case as well as in this case, I could interpret the experimental data by having just an additional term introduced by Irwin.

(Refer Slide Time: 36:06)



Suppose I go to a real situation like a crack emanating from outer boundary of a pressurized vessel, I have a frontal loop.

(Refer Slide Time: 36:25)



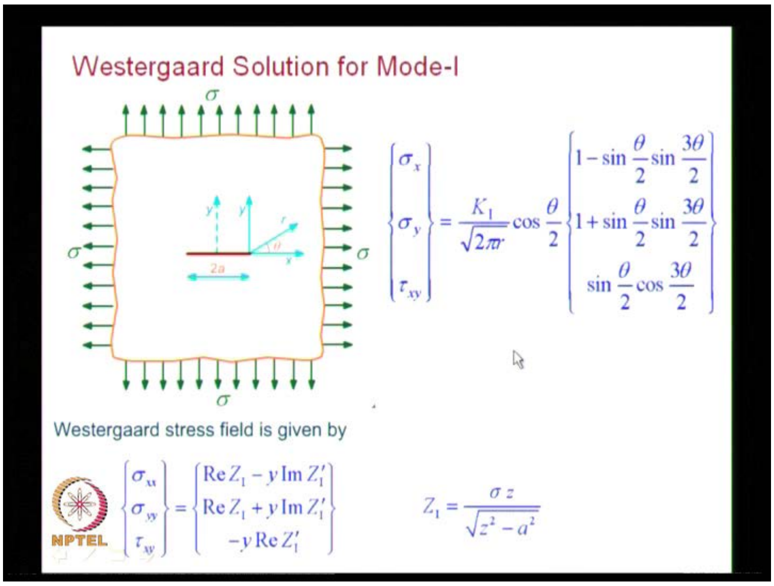
So, i have variation of fringe order along the crack axis, which is shown as the blower. On the other hand, if I have a crack from the inner boundary, I have a situation that these fringes are tilted backward and these fringes are tilted forward. So, I have a combination of backward and forward tilted fringes, even in that case, I need to go for multiparameter solution.

That is why I have shown both of them in one slide; I have crack emanating from outer boundary; I have crack emanating from inner boundary. Though I could get some value of k by processing only the forward tilted fringes which are closer to the crack-tip, a reasonably accurate solution is possible only when I have modelled all the aspects of the fringe pattern.

So, that cannot be modelled by the Westergaard neither by Irwin's modification. So, you have to go in for multiparameter solution, which you develop in the next class also, then we would see that solution would be quite useful.

Sir when you have explained the Westergaard solution, you mention that it is applicable only for a biaxial state of state of stress and not for uniaxial loading; however, it is used to represent the stress field in uniaxial case 2. So, could you please explain it again, also will stress in horizontal direction play any role when crack advances?

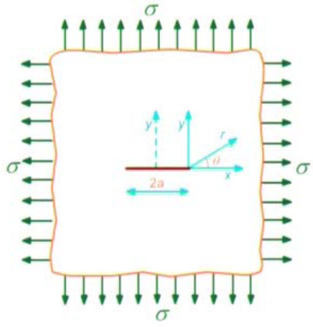
(Refer Slide Time: 37:59)



That's a good question. See, let us look at what we have discussed as Westergaard solution. If you look at the boundary conditions, we have clearly taken a biaxial loading situation and from that, we have got the Westergaard stress function and if you take the stress function in this form sigma z divide by root of z square minus a square, this is an exact solution, in the sense it can be considered as a closed form expression for the field. Mind you if you are not looked at generalized Westergaard, you would still not accept this as a closed form expression. See, what we have got here is, it definitely satisfies if I take the stress function in this fashion, it satisfies the stress free boundary of the crack as well as biaxial loading at infinity; this stress function takes care. But it also says tau x y equal to 0 along the crack axis, leaving that issue apart, it satisfies the boundary condition near the crack phases as well as what happens at infinite boundary.

See, in theory of elasticity, we are accustomed to looking at closed form expression that is the time where fracture mechanics was developed. What was the focus by Irwin? He said you do not worry about the crack, worry about the crack-tip; he shifted everything to the crack-tip.

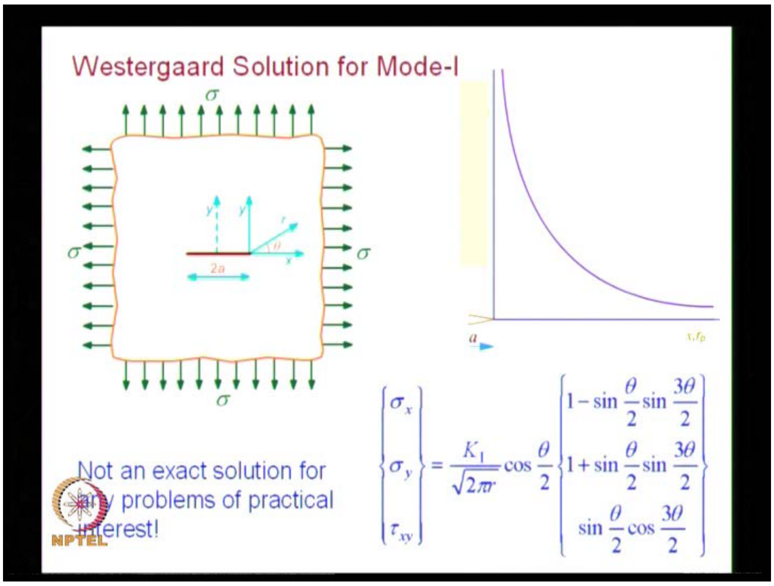
(Refer Slide Time: 40:04)



$$Z = \frac{K_I}{\sqrt{2\pi z_0}}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \begin{Bmatrix} 1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2} \\ 1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2} \\ \sin\frac{\theta}{2}\cos\frac{3\theta}{2} \end{Bmatrix}$$

(Refer Slide Time: 40:12)

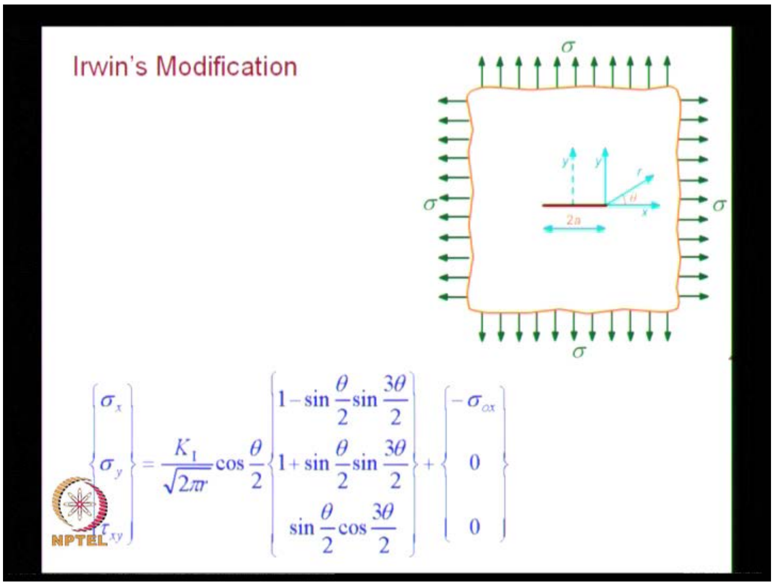


The moment you shift your origin to the crack-tip, you get a final expression like this; though you have selected this satisfying the outer boundary as well as the crack free surfaces finally, you shifted the origin to the crack-tip; you made a simplification that z naught is very small compare to the crack length, and finally, who arrived at this kind of stress field equations. And how do these stress field equation model the scenario? It is based on a stress function like this and if you look at, when r tends to 0, sigma x equal to sigma y and reaches infinity. In reality it would not reach infinity because of a plastic deformation, you will have some value dictated by whatever the plasticity condition that should do provide here.

When you go to infinity what happens? When you say r equal to infinity, it goes to 0, what does it mean? It is not an exact solution for any problem of practical interest, then why it is so important? See, what you will have to keep in mind is, the solution what we have got is a singular solution and this is valid in a close vicinity of the crack-tip; we are only looking at the remote loading.

The remote loading loses its significance as long as the crack faces open up. In the near vicinity, how near, that is problem dependent; we are not qualifying what is the value of that distance that is problem dependent. People have found out at singularity dominated zone, size varies from problem to problem. In that zone, this solution is reasonably accurate. So, if you are focussing your attention only in region close to this, as a singular solution people have used it interchangeably for uniaxial as well as biaxial. If you really make the fundamental question what happens when r tends to infinity, this solution says only zero.

(Refer Slide Time: 42:24)



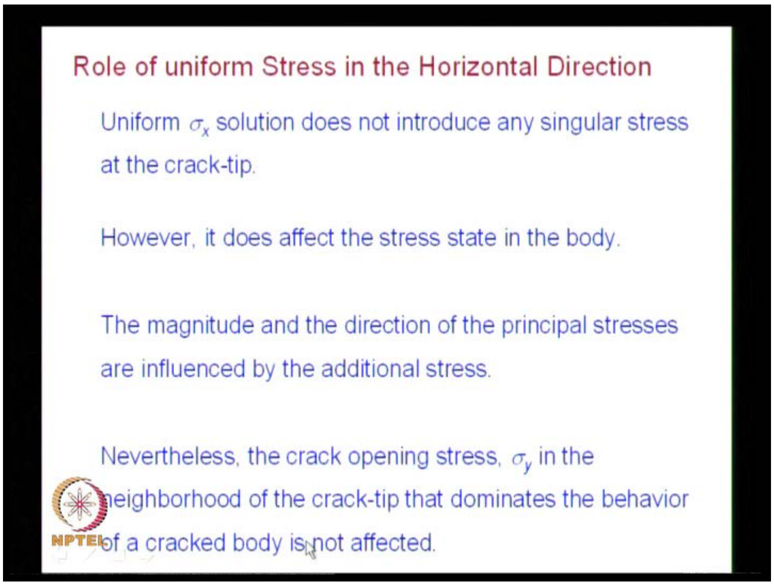
So, what you will have to keep in mind is, this is a near field equation; if you understand that this is near field not the closed form expression, then using this in an uniaxial situation is also reasonably okay. But people have made you know, certain observations which are quite not right. Irwin introduced a term minus sigma naught x and what we find in the literature? People initially reported when you say sigma naught x is equal to sigma when you put this at infinity, this goes to zero. So, this could be used for uniaxial loading situation, that interpretation is strictly not correct, because if you say r equal to infinity in this solution, there is no stress in the y direction.

What way you have to look at Irwin's modification is, from the experimental fringe pattern, find out sigma naught x as well as K 1 for a given problem and for all your fracture mechanics calculation use the value of K 1, which is obtained by solving both the parameters from the experimental fringe pattern. You know, once you look at the situation like this for finite body geometry, you will get a complicated fringe pattern; you will not stop at sigma naught x, we will also look at multi parameter solution, that way for all finite bodies you will be able to find out the solution; by calculating all the parameters involved.

So, why people have done is, in the initial development, people were trying to find out… They were actually grouping in the dark.

So, they have to get some kind of a solution and carry forward. So, if you focus your attention very close to the crack-tip, the distinction between a biaxial loading or uniaxial loading, no longer is so strong; I mean there is a distinction exist, but if unless you take a multiparameter solution, none of these solution are valid and that is the reason why you find now is, people also have looked at fracture theories involving higher order terms; people have used the second term which is called as t stress and let us look at what is the role of this stress, because that is the another part of the question that he has asked.

(Refer Slide Time: 44:31)



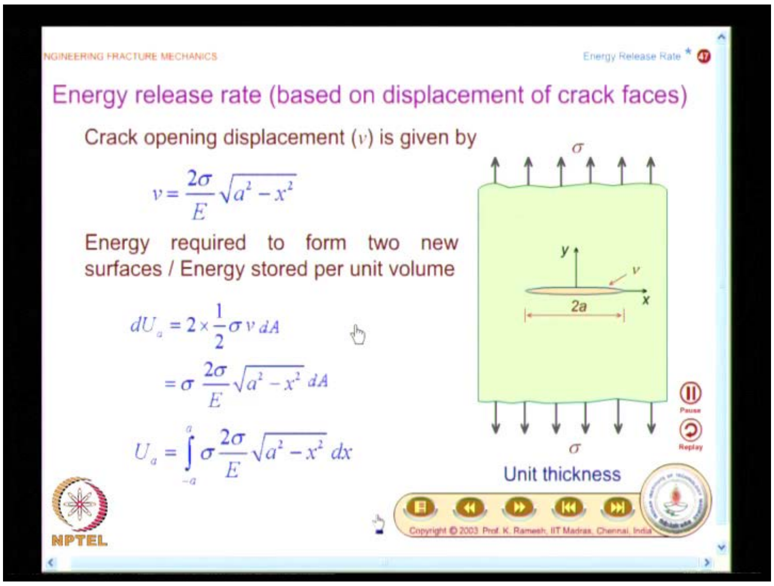
So, if you look at the addition of sigma naught x stress term, you have to notice this does not introduce any singular stress at the crack-tip; this is the observation number one. What is the observation number two? Definitely affect the stress state in the body, how does this do?

If you are going to calculate the magnitude and the direction of the principle stresses, they are influence by the additional stress; you cannot ignore it. See, we are going to discuss towards the end of this course, theories related to which way in the crack would propagate. If you look at only the singular solution, you will have in a case of mode one what is known as cell similar crack propagation; the crack was horizontal, it will propagate in horizontal direction, but people have to found turning of the crack also occurs in the case of mode. How do you go and explain it? Unless we bring in higher order theories, such things cannot be explained.

So, in initial stages of fracture mechanics development, if you are convey confining your attention only to the close vicinity of the crack-tip, use of the term sigma x really does not affect, because the crack opening stress sigma y the neighbourhood of the crack-tip the dominates a behaviour of a crack body is not effected.

So, from that point of view, it is justifiable; but you have to keep that, after looking at generalize Westergaard equation, none of the solutions are accurate; you have to go for multiparameter solution for any problem of practical interest.

(Refer Slide Time: 46:58)



Sir regarding the Westergaard solution for a crack opening displacement, it is u y is equals to 2 sigma by e root over a square minus x square, but this has been used in the problem for solving superimposition of two uniaxial stresses, how can this this be used?

You know, your question also borders on, why do we use a biaxial loading situation to uniaxial loading situation?

Let me put back; see, this representation comes from mainly because at the time when these theories are developed, people were looking at hole in a tension strip, elliptical hole in a tension strip and Griffith also developed his fracture strength based on uniaxial loading.

So, people are trying to find out in the presence of a crack, how to find out the energy release. They have to look at the crack plane displacement, so they simply took the crack plane displacement that was available, put it into this problem, and then, found an answer which was satisfying.

Now, after learning all of those development and also the development which are current, what you can really look at is, whatever the result that I have used, I can replace this diagram by a horizontal loading also, and then, apply this, the situation is same; whether it is a uniaxial loading or biaxial loading, we have used the biaxial loading result; only for a biaxial loading result, we know the definition of a k s sigma root by a, it is not a definition for uniaxial strictly.

So, from that point of view, we have not done any mistake. But if you look at any book on fracture mechanics which are currently available, all of them have this so call defect, which you have rightly pointed out that is what I would say, whenever they look at energy release rate based on crack plane displacement, you will only find uniaxial loading depict.

So, this also follows in the very close vicinity of the crack-tip as sigma y is not affected because of sigma x and you are really looking at v displacement; you are not looking at the other displacement component. So, people have used it, but whatever the result that we have got, that is g equal to k square by e that remains same.

So, that is the reason why I said in the class was, when you had a single crack extended by a little and close it, that is the best way to identify the interrelationship between k and g. This was only historical; finding out the energy release using this crack phase displacement was only historical note, and you can actually put horizontal stress component here; the result is valid and ultimate interrelationship between g and k still remains the same.

So, what you will have to keep in mind is, in the initial stages development, people are focusing only to the crack and they were used to looking at uniaxial because of the force of habit, they have also used it and that is the way I think we have to take it.

Can you allow one question?

Sir for deriving parameters like energy release rate and stress intensity factor, we have assumed a crack as through crack, but in many of the engineering problems, we encounter crack as embedded crack or cracks originating from surfaces, whether the derivations are applicable to embedded cracks also?

See, it is a very good and practical question, because in most of the engineering applications, the crack appears from surface or you have embedded cracks, this is the way you have and when you have this kind of situation, our ultimate objective is also to apply fracture mechanics to these difficult situations; we have to start somewhere. So, through cracks are easier to handle analytically.

So, we develop methodologies for through cracks and graduate to apply them for embedded cracks as well as to surface cracks. So, that is the way you have to look at it. The concepts will be applicable, but actual calculation would be different and in many cases you may have to depend on numerical approaches or experimental measurements for you solve the problem, because those geometries will become very complex and you have to dependent on sophisticated numerical code or experimental measurement for you to at this those issues.

We will definitely graduate to that; we will not stop with crack situations are load. Because even in your bending, I have been mentioning, you start from a constant bending moment, then we graduated to variable bending moment, constant shear, variable shear and I said that introduce certain difficulties, which we saw result from theory of elasticity, but the basic kernel is understood; in a bending situation, the stresses would vary linearly over the depth of the beam is a very key information.

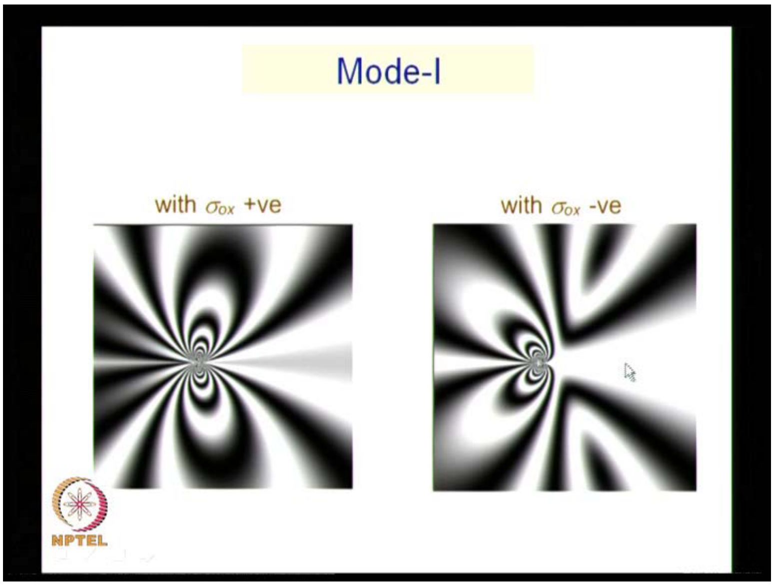
So, similarly in the case of a crack, what happens? When I have a through crack, I have a straight crack fringe.

We have understood stresses would be singular, but the singularity would remain same on the crack front. The moment I go for surface crack or embedded crack, I have a curved crack. So, the stress intensity factor would vary along the crack field. So, the problem is much more complex. So, you have to wait for the subject matter to be developed, we would definitely look at those issues as part of this course.

Whether the forward tilting indicates the crack advancing direction?

See, the answer is no.

(Refer Slide Time: 53:12)



I would show two situations; in the case of a SCN specimen, you find the fringes are forward tilted. I have a crack axis like this; this is actually the crack and this is the place, where the crack can advance and if you look at RDCB specimen, I have a crack here and crack can advance in this direction. In this case, the fringes are forward tilted; in this case, fringes are backward tilted, but in both the cases, the crack would advance in the forward direction.

Later on we will go and show that, it is indicative of the tri-axiality construct; it is nothing to do with crack propagation direction; it is dictated by tri-axiality construct. So, which you will have to wait for some time and appreciate.