

**Engineering Fracture Mechanics**  
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**Module No. # 06**  
**Lecture No. # 27**  
**SIF for Surface Cracks**

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**Embedded elliptical flaw**

SIF for Various Geometries and Loading

$$I_2 = \int_0^{\pi/2} \left( 1 - \frac{c^2 - a^2}{c^2} \sin^2 \theta \right)^{1/2} d\theta$$

$$K_1 = \frac{\sigma \sqrt{\pi a}}{I_2} \left[ \sin^2 \theta + \left( \frac{a}{c} \right)^2 \cos^2 \theta \right]^{1/4}$$

Note:  
 Solution for embedded elliptical flaw is used for modeling surface cracks.  
 Here,  $I_2$  is elliptical integral

Note:  $a$  is used for minor axis!

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We were discussing in the last few classes, the evaluation of stress intensity factors for variety of geometries; initially, we looked at through the thickness cracks, then we moved on to embedded cracks. So, in the case of an embedded elliptical flaw, what do you find? The expression of K is given in this fashion and it varies as a function of theta. What this implies is along the boundary of the crack front, the stress intensity factors keeps changing and you locate the point corresponding to a given theta like this.

You put an angle theta, it hits the circle from there you drop a line and you have this point located on the ellipse; for this point on the ellipse, you get K 1 equal so much and theta is defined like this and you also have the quantity I 2 expressed in this fashion.

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ENGINEERING FRACTURE MECHANICS

SIF for Various Geometries and Loading

### Surface Cracks

- Surface crack initiates at one face of the specimen, but does not go all the way to the other face like through the thickness crack.
- Surface cracks are usually modeled as semi-ellipses in the literature of fracture mechanics.
- If the crack happens to be at a corner of the specimen it is usually modeled as a quarter-elliptical-crack.

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And what we will now see is from elliptical flaw, how do we graduate to surface flaws and this is where we had looked at in the previous class, how you will model the surface cracks. When you have a surface crack, the surface crack can be idealized as a semi-elliptical flaw to start with.

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Suppose I have a corner crack, the corner crack can be idealized as a quarter elliptic; when I have a flaw like this, this could be idealized as the semi-elliptical shape and a corner crack can be idealized as quarter of an ellipse.

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### Semi-elliptical surface crack

Surface crack is modeled as a half ellipse with minor axis into the thickness direction.

$a$

$2c$

$$K_1 = \frac{1.12\sigma\sqrt{\pi a}}{I_2} \left[ \sin^2 \theta + \left( \frac{a}{c} \right)^2 \cos^2 \theta \right]^{1/4}$$

Note:  
Note that  $2a$  is used for minor axis!  
Depth of the crack is more significant in fracture mechanics and 'a' has been used for this.

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So, from the elliptical solution, if you modify with simpler arguments at least for small sized cracks, these results could be comfortably utilized that is how people have proceeded. When I have a semi-elliptical surface crack, you have to remind yourself the notational change.

The length on the surface is taken as  $2c$ , that is the major axis of the elliptical flaw is taken as  $2c$  and the depth inside is taken as  $a$ ; if I have a full form of it will be  $2a$ ; it is a semi-elliptical flaw we are considering and you make a neat sketch of this and consider that this is an infinite object with a surface flaw.

You make a neat sketch, you have a uniformly distributed stress field and you have this length as  $2c$  and  $a$ , and we all know **when we have a semi** in a SEN specimen, you have a crack emanating from a free surface, so what we did? We simply modified that sigma as  $1.12\sigma$ .

So, what people have initially have attempted to find the stress intensity factor value is, they utilized the similar approach. Based on the elliptical solution, you have the expression for stress intensity factor for a surface flaw given like this,  $K_1$  equal to  $1.12$  time sigma root by  $a$ ; this is the only change; you divide by  $I_2$  and put this expression in terms of theta, sin square theta plus  $a$  by  $c$  whole square cos square theta whole power 1 by 4.

So, what we have done is, whatever the knowledge that we have gained in analyzing SEN specimen, we have extrapolated, combined the result from an embedded elliptical flaw.

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**Semi-elliptical surface crack** ....Contd

At the extreme end of the minor axis at B ( $\theta = 90^\circ$ ), SIF is

$$K_1^{90} = \frac{1.12\sigma\sqrt{\pi a}}{I_2}$$

At the extreme end of the major axis at A ( $\theta = 0^\circ$ ), SIF is

$$K_1^0 = \frac{1.12\sigma\sqrt{\pi a} \left(\frac{a}{c}\right)^{1/2}}{I_2}$$

The segment of crack tip which is deep inside the material possesses higher SIF.

Crack tends to grow deeper than sideways on the surface.

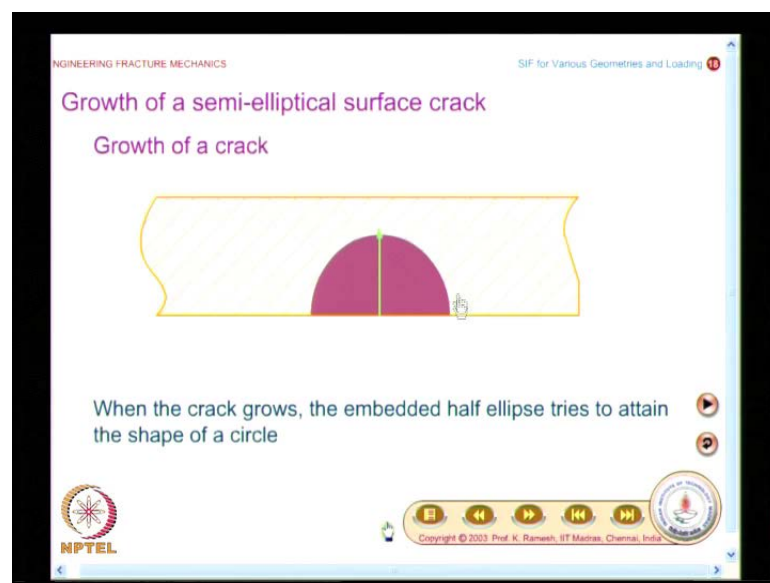
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As we have modeled the surface crack as a semi-ellipse, we now say change sigma to 1.12 sigma and we will again look at similar results like what we had seen in the embedded flaw. The stress intensity factor does not remain constant, but varies on the periphery of the crack front and we will see on the major axis and at the ends of the minor axis, how these values turn out to be. At the extreme end of the minor axis, that is, theta equal to ninety degrees, the stress intensity factor is given as  $K_{190}$  that is equal to  $1.12 \sigma \sqrt{\pi a}$  divided by  $I_2$  and this happens to be the maximum value of stress intensity factor on the crack front. And suppose theta equal to 0 degrees that is at the end of the major axis, the value of stress intensity factor is given as  $1.12 \sigma \sqrt{\pi a}$  divided by  $I_2$  multiplied by  $a$  by  $c$  whole power half .

You know this expression is very significant; it carries quite a lot of information. First thing you have to recognize when you have a surface flaw, stress intensity factor does not remain constant on the crack front, and it changes from point to point. So, what does it imply? The crack can have a preferred way of extension.

We saw in the case of an elliptical flaw, the elliptical flaw which is embedded would try to become a circular flaw, because that is how the stress intensity factors were very. So, yes similar thing also can happen in the case of a surface flaw and this also helps to appreciate the concept of leak before fracture. You have stress intensity factor higher at this point; so the crack will try to penetrate inside, rather than on side base. The segment of crack-tip which is deep inside the material possesses higher stress intensity factor and in view of this, the crack tends to grow deeper than sideways on the surface.

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So, when it grows deeper, it is beneficial for us. Only based on such observations, people have also brought in a criterion call leak before break, which is achievable; this is how a material will respond. If you adjust your design parameter suitably, you can ensure that leak always occurs before break, because it is possible people have also evolved this as a criterion and we will see a nice animation which gives you this pictorially.

So, what we are actually saying is a crack, which is a surface crack like this would proceed in this fashion. You know, whatever the flaw that you have here, it will proceed in such a fashion that it becomes a circle and you have to note down many things in this. What we want is, if the crack grows like this and touches the surface, if I have a pressure vessel, the pressure vessel would start leaking; when it starts leaking what you can do? First of all you can go and find out something is gone wrong and you have to go and

correct it; other way of have looking at is, when the pressure vessel is leaking automatically the loads acting on it comes down.

So, it is also a protection **in that** from that point of view. So, when the load acting on it comes down, it is good; however, you could also have crack propagated because of a water hammer, in that case the pressure would be higher than the original pressure. So, when you are designing, you have to ensure both the possibilities, pressure coming down or pressure going up also; pressure going up has to be looked at much more carefully.

And another aspect also you can learn from this; if you go and look at the literature, we would also discuss about this towards the end of this class. When I have a crack front like this, in the zones which are closer to the front of the crack front here away from the edges, this is more or less in a plane strain type of situation; you have a tri-axiality constrained predominantly present and you would use  $K_{Ic}$  as a fracture toughness for this, which is the plain strain fracture toughness. And I had mentioned without showing a graph that the fracture toughness is a function of the thickness; in plane stress, fracture toughness is higher; in plane strain, fracture toughness is the least.

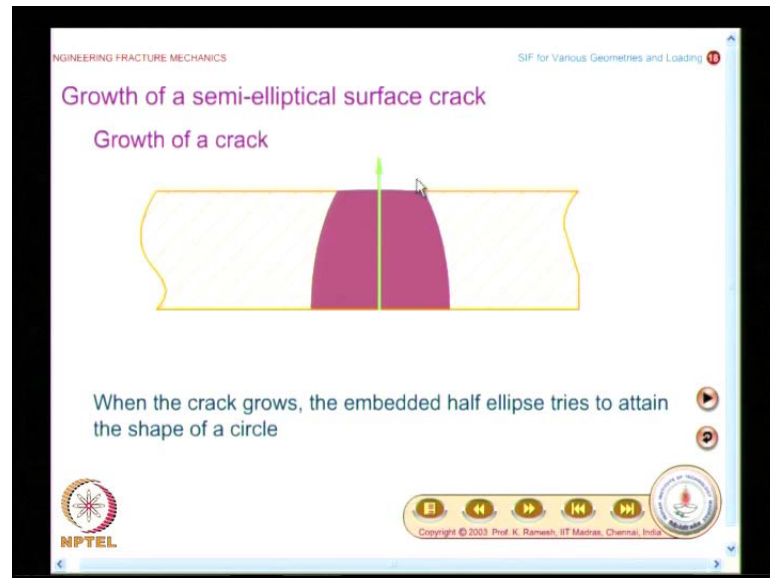
In the case of a surface flaw what happens, you have a nice combination, stress intensity factor is higher in this zone of the crack front; not only this, the fracture toughness that is to be used for propagation of this, has to be plane strain fracture toughness which is usually less than plane stress fracture toughness.

Suppose the crack has become fully grown, then you can consider this thickness as idealizing a plane stress situation. When the crack has to move sideways, we would essentially use plane stress fracture toughness and plane strain fracture toughness less than plane stress, so you have a good combination that the crack will open up and then touch the other end, then it will start propagating sideways.

So, what are the stages in this? See you need to have a crack developed on the surface, then crack has a growth face by some growth mechanism, then it reaches instability and what way it happens? It penetrates first through the thickness, because your fracture toughness is also lower and also your stress intensity factors are higher; once it has reached the other surface, it is not that it starts proceeding on sideways immediately; if it proceeds immediately, then you have no hole. In the lake before break criterion what you

want to ensure is you want to have a healthy gap between the crack moving front and then moving sideways. That is why you alter the design parameters in such a manner that you have sufficient time to go and attend on to it, let us look at this.

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So, what do we want is, it has to reach this stage and then allow the fluid to escape. So, you have a possibility to do some kind of a repair or stop the plan, do some corrective measures. On sideways, you will have a growth face followed by instability; it is not that it becomes unstable and then penetrates through the thickness and move sideways, if that happens one after another immediately, then you do not have that leak before break criterion satisfied. In a leak before break criterion, you try to ensure that you have sufficient margin of time for you to attend on to it and it is doable. The way you have looked at stress intensity factor variation over the crack front and also the fracture properties what you find is, leak before break criterion, if you really play with the design parameter is doable.

You know this knowledge you would not have got when we were discussing only the through the thickness crack; in a through the thickness crack what you saw? On the crack front, stress intensity factor remain constant; once you come to elliptical flaw, you find the stress intensity factor varies on the crack front and you have a favorable situation in the case of a pressure vessel piping, where you can really achieve leak before break provided, your design is properly check for.



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ENGINEERING FRACTURE MECHANICS SIF for Various Geometries and Loading

### Shallow surface crack

- For a very shallow crack ( $a/c$ ),  $I_2$  is close to unity and SIF at  $\theta = 90^\circ$  becomes

$$K_I^{90} = 1.12\sigma\sqrt{\pi a}$$

Back free-surface correction

which is same as the result of a through-the-thickness-edge crack of length  $a$ .

- Therefore, a shallow crack is equivalent to a through-the-thickness edge crack of length  $a$ .

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And people also have looked at, suppose I have a shallow crack, in a shallow crack  $I_2$  is close to unity and SIF at ninety degree becomes simply  $1.12\sigma\sqrt{\pi a}$  and we have already seen that what you have here is because of a free surface; you call this as a back free-surface correction.

Because we are going to see shortly a front free-surface correction, keeping that aspect in mind, you call this as a back free-surface correction. Suppose I have a crack front, I can have a front surface as well as the back surface. If I am going to apply only one correction factor, I would say free-surface correction factor, because I have a separate correction factor for back free and front face you call it different.

So, one way of making our knowledge in a comparative sense, a shallow crack is equivalent to a through-the-thickness edge crack of length  $a$ . And we have always said edge crack is more dangerous than a center crack; not only this, we will also see later though you compare a surface crack with a through-the-thickness crack. On the crack front in the through-the-thickness crack in thin plates, you use plane stress only, whenever you have a surface crack even in a thin plate on the crack front because of triaxiality constrained, you will have to use plane strain fracture toughness which is lower than that.



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**Plastic zone correction**

Plastic deformation makes the crack behave as if it is longer than its actual physical size.

$$K_I = \frac{1.12\sigma\sqrt{\pi(a+r_p^*)}}{I_2} \left[ \sin^2 \theta + \left(\frac{a}{c}\right)^2 \cos^2 \theta \right]^{1/4}$$
$$r_p^* = \frac{K_I^2}{4\pi\sqrt{2}\sigma_{ys}^2}$$

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So, in a sense the surface crack is always more dangerous than a through-the-thickness edge crack, that you will have to keep in mind; though the stress intensity factor is a similar, but the fracture toughness we would use differently for a surface crack. And you know in the next chapter, we would discuss in detail how to calculate the plastic zone and as surface cracks are very important, people thought in what ways you could improve the results that we have obtained for surface cracks.

So, people felt when you are applying for ductile materials, it is also desirable to do some connection factor for development of plastic zone and one of the simplest ways to handle this is, when you have a crack length of  $a$ , you take an additional small length which is calculated by a certain procedure, we would see in the next chapter. We would just look at the result, instead of using crack length as  $a$ , you will use a modified crack length and that is what is mentioned here, plastic deformation makes the crack behave as if, it is longer than the actual physical size.

So, you are going to have what is known as  $r_p^*$ , the rest of the expression is same as what we had seen when we had put this factor 1.12. And the  $r_p^*$  this is a historical value; this was reported by Irwin in 1960. In the next chapter, we would see better estimates of this additional length. So, you must take this more from a historical perspective. So, I have  $r_p^*$  is given as  $K_I^2$  divide by  $4\pi\sqrt{2}$  multiplied by

$\sigma_{ys}$ , where  $\sigma_{ys}$  is the yield strength of the material and I want you to write down this expression.

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**Plastic zone correction**

Plastic deformation makes the crack behave as if it is longer than its actual physical size.

$$K_I = \frac{1.12\sigma\sqrt{\pi a}}{\sqrt{I_2^2 - 0.212\sigma^2/\sigma_{ys}^2}} \left[ \sin^2 \theta + \left(\frac{a}{c}\right)^2 \cos^2 \theta \right]^{1/4}$$

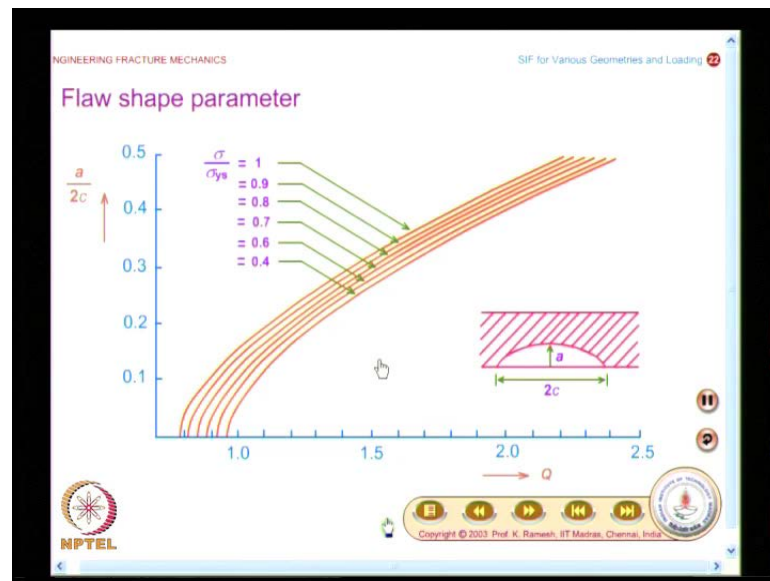
$$K_{I_{\max}} = 1.12\sigma\sqrt{\pi \frac{a}{Q}}$$

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You know in those days people have only slide rule and things of that nature, so people always wanted to have whatever the final result in the form of a graph and we find that people have simplified this and we would also have a look at it. So, when you write the expression for these additional crack length and simplify, the expression for  $K_I$  changes to  $1.12\sigma\sqrt{\pi a}$  divided by square root of  $I_2^2$  square minus  $0.212\sigma^2$  divided by  $\sigma_{ys}^2$  whole multiplied by  $\sin^2 \theta$  plus  $\frac{a}{c}$  whole square  $\cos^2 \theta$  whole power  $1/4$ . And this is represented differently for convenience sake and we are really talking about the maximum stress intensity factor that occurs at the minor axis of the surface flaw that is given as  $1.12\sigma\sqrt{\pi a}$  divided by  $q$ .

And you have graphs available from which you can get the value of  $q$  and when you are handling ductile materials, you will have to worry what is the applied stress in comparison to the yield stress that is also a factor. When you are really looking at plastic zone correction, what is a level of stress I have applied to the object in comparison to yield strength also plays a rule; if the stresses are closer to yield strength, then the correction factor is having a more influence; if you are operating at a lower stress, then the correction factor as a marginal influence.

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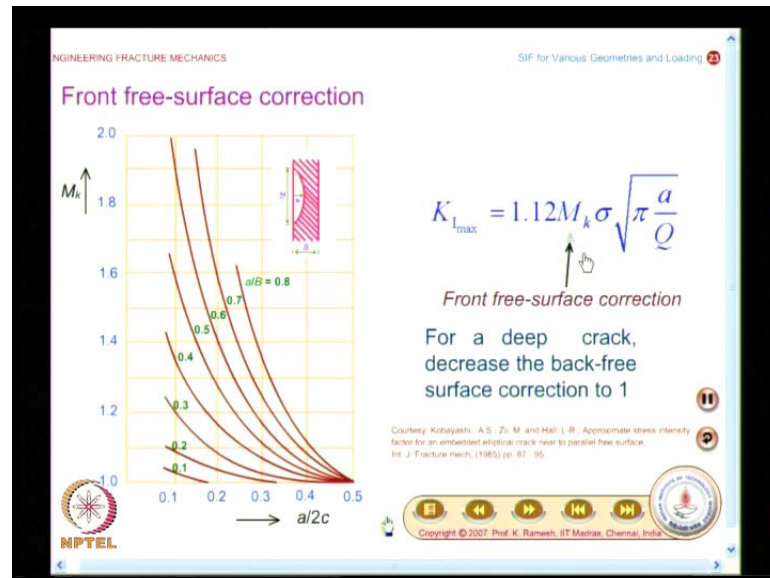
So that is also becoming one of the important parameters. So, in order to evaluate the value of maximum stress intensity factor, people have provided graphs, these are very popular graphs. The surface flaws are so important and when you look at the literature you have this as a flaw shape parameter, they have a very characteristic appearance and on the x axis you have the value of q; on the y axis, if you look at the crack, the length is given as  $2c$  and the depth which in the material is given as  $a$ .

So,  $a/2c$  is plotted on the y axis and this you do it for different values of  $\sigma_{\max}/\sigma_{ys}$ ; here it is equal to 1 and I would have graphs drawn for other cases also and I would like you to make a neat sketch of this. So, the flaw shape parameter is a very important when you are handling surface flaws and what you find here? If I remember  $K$  equal to  $\sigma \sqrt{a}$  by putting 1.12 or putting  $q$ , you are able to graduate from through-the-thickness crack, central crack to edge crack, from edge crack to surface crack all that have similar appearance; we will also have a summary of this at the end of this class if possible and for shallow surface cracks all this discussion is valid.

I would have the graph drawn for other values of  $\sigma_y$ ,  $\sigma_{ys}$  and this is the very characteristic shape of the flaw shape parameter; even when you look at this kind of a graph, even though if it is not label, you should be able to identify that this is the graph for flaw shape parameter. So, from the graphical approach you can find out  $q$ ;  $\sigma_s$

easy to know from the way you have applied the load. So, you can find out the stress intensity factor that is the maximum value for the surface flaw.

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And I had already mentioned, people also thought once you have the result how to improve its accuracy. One way of doing it is bring in plastic zone correction; other aspect is, when I have a crack, you just make a observation of this, may be I could enlarge this, I have a crack here; I have the thickness  $b$ , this becomes the back free-surface and this becomes the front free-surface; if the crack is shallow which is closer to this surface, a back free-surface correction is alright.

People thought they would analyze initially shallow cracks, but you will also have to look at what happens to cracks which are growing in the thickness direction.

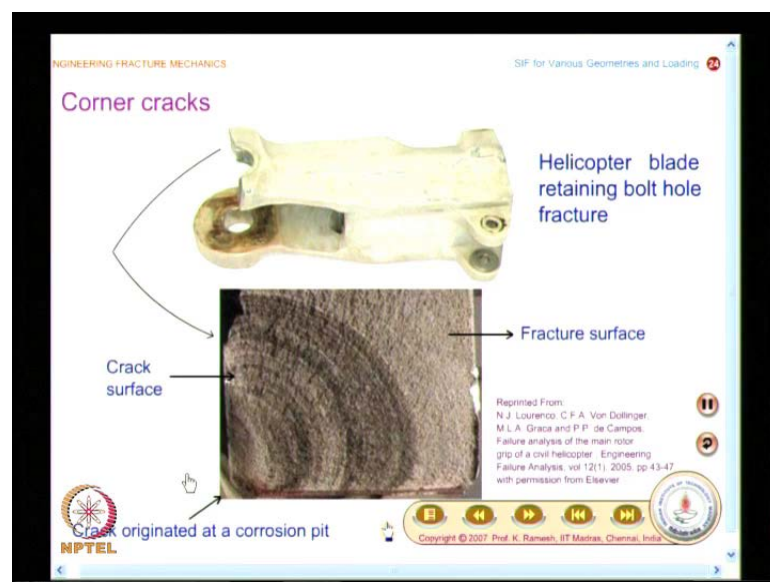
So, when they grow, you have an influence of this surface also comes into the picture that is called a front free-surface and that is given by this simple  $M_k$ . So, I have 1.12 multiplied by  $M_k$  sigma root of  $\pi a$  divided by  $q$ . So, this is an improvement over the result what you have got for the surface crack. So, you have these graphs also available; you have these graphs plotted for  $a$  by  $2c$  as well as  $a$  by  $b$  as a parameter.

For different values of  $a$  by  $b$ , that is, different values of the depth of the crack with respect to the thickness, you have these values given; make a neat sketch of this; it give you it gives you an appreciation how the value of the front free-surface correction

changes. And for a deep crack what you do? For a deep crack, decrease the back-free surface correction to 1, because the front free-surface correction factor becomes more predominant that is what you find as  $a/b$  increases, the value of  $M_k$  also increases and  $a/b$  is small, it is closer to 1; when  $a/b$  is larger, you find this is becoming larger and larger and this is the result by Ze and Hall Kobayashi and there team. So, this is one more improvement on finding out stress intensity factor for a surface crack. You know we are going to look at quite a few improvements.

Right now what we have look at is, we have taken the solution of an **elliptical flaw** embedded elliptical flaw, from the embedded elliptical flaw we have constructed the solution for surface flaw; we have looked at a semi-elliptical crack.

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Suppose I look at a corner crack, how do I modify this solution? So, first we look at certain simplifications, later on we look at exhausted solution. And we have already noted a corner crack appearing in a helicopter blade retaining bolt hole fracture; we had seen it in the context of delineating a difference between the crack surface and the fracture surface and you have this as the shape of the crack and what do you notice immediately? I have a free-surface on this edge as well as the free-surface on this edge and we have learned a recipe; when I have a free-surface, I should do something about it; the stress intensity factor would be higher; instead of one free-surface, I have two free-surfaces.

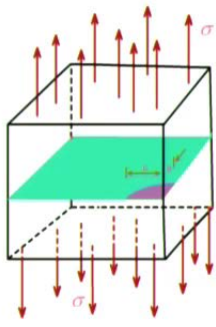
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ENGINEERING FRACTURE MECHANICS

SIF for Various Geometries and Loading

### SIF for quarter elliptical crack

- Quarter-elliptical crack has two free surfaces – the back-free surface correction should be applied twice!
- Instead of  $1.12 \times 1.12$  use  $1.2$

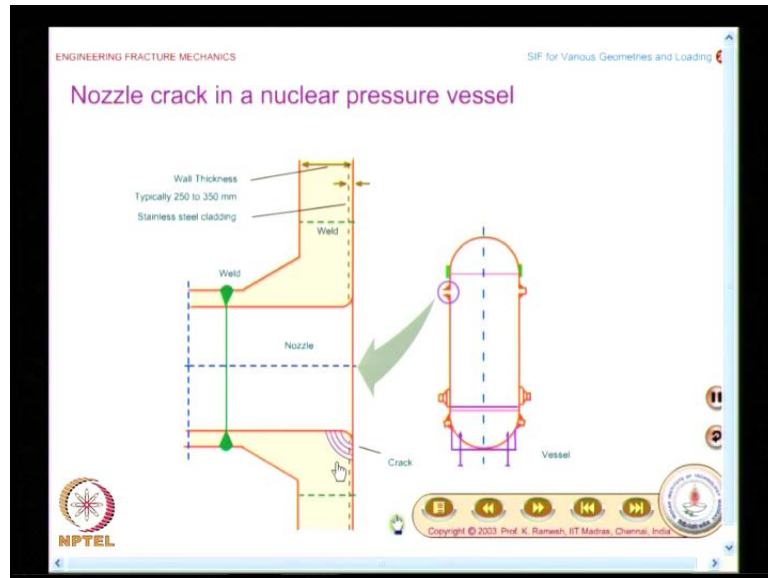
$$K_I = 1.2\sigma\sqrt{\pi a}$$


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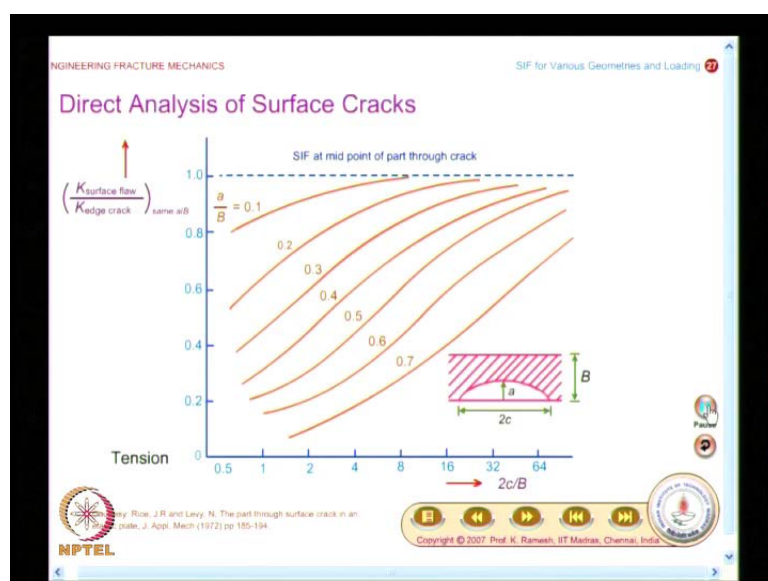
So, what people have done? People have done, you have to multiply 1.12 into 1.12 instead of using this, they have approximated this as 1.2; I have one free-surface on this side; I have another free-surface on this side. So, for this case, you have the stress intensity factor given as  $K_I = 1.2\sigma\sqrt{\pi a}$ . See these are all trying to get solution based on certain knowledge that we have developed on the basis of through-the-thickness crack; this may be quite good to start with, but you could not end the course with that; you should also know exact results for it or methodology to find out in a critical situation, how to evaluate them as accurately as possible, that also we look at it.

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And in reality, corner cracks are very important. You know when you have a nuclear pressure vessel like this, you know the thickness of the shell is very high, it is about 250 to 300 millimeter thick, that means, it is a very thick steel plates; this is with a steel cladding that is what is shown here and you can have, when you have the nozzles from the corner, you could have corner cracks developed and these could be essentially quarter elliptical crack that kind of modeling is justified.

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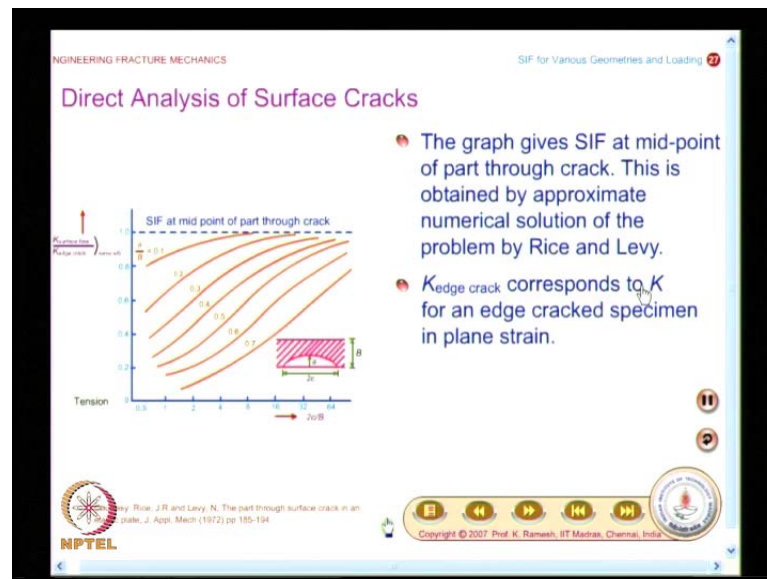
So, knowledge of stress intensity factor in a corner crack or a surface crack, they are all very important from practical design point of view. What we have now looked at is, if I have a shallow surface crack or a shallow corner crack, we know what is the way to get the stress intensity factor. And you know, people have also provided results in the literature on direct analysis of surface cracks, such results are also available and in those days people, were comfortable with graphs.

So, graphical results are available please make a neat sketch of this, you have the geometric parameters of the crack is shown here and on the x axis, you plot  $2c$  divided by  $b$ , where  $b$  is the thickness and  $2c$  is the length of the crack and on the y axis what is plotted is the ratio of stress intensity factor. For the surface flaw divided by stress intensity factor for edge crack, that means, through-the-thickness crack and this is given for a particular  $a$  by  $b$  ratio.

So, when  $a$  keeps changing, the crack is growing interior to the material; so I have this graph for  $a$  by  $b$  equal to 0.1 and this is plotted for  $a$  by  $b$  equal to 0.2 and what you find is as  $2c/b$  is increased, the stress intensity factor for a surface flaw approaches the stress intensity factor of the edge crack. So, what you find here is the crack should be long enough, so that stress intensity factor approaches that of the corresponding edge crack, this is one aspect of the story.

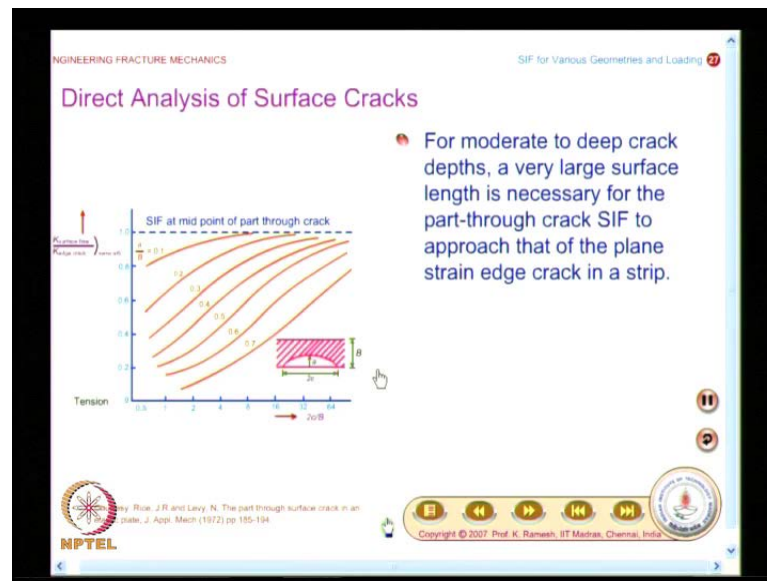
The other aspect of the story what I said? Whenever you have a surface flaw, for you to determine crack instability, you will have to use the plane strain fracture toughness. So, that way surface cracks are always more dangerous than edge cracks. And you have this set of graphs drawn for a tensile load and you have this plotted for various values of  $a$  by  $b$ , please make a reasonable sketch of these diagrams; these graphs are drawn for  $a$  by  $b$  equal to 0.1, 0.2, 0.3, 0.4, 0.5, 0.6 and 0.7.

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And these are results reported by Rice and Levy and whatever I had said is mentioned here; the graph gives stress intensity factor at mid-point of part through crack and for this part through crack, these results are obtained based on the approximate numerical solution of the problem by Rice and Levy; whatever the numerical results that they have got, they are summarized and plotted as the graph for others use. And the edge crack corresponds to stress intensity factor for an edge crack specimen in plane strain they have also noted. Because I said for a surface flaw when you have the maximum stress intensity factor, we are going to use plane strain fracture toughness; keeping that issue in mind, they have compare the stress intensity factor of a surface flaw to an edge crack in plane strain that is far enough.

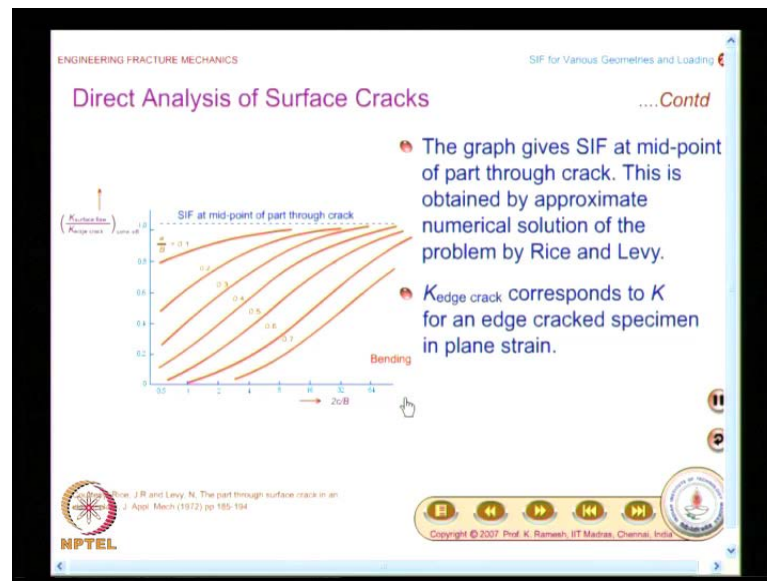
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So, that is what is summarized here and the observation I had mentioned that the crack has to be long enough when the depth is more so that the stress intensity factor approaches that of the edge crack that is what is summarized here. For moderate to deep crack depths, a very large surface length is necessary for the part-through crack stress intensity factor approach that of the plane strain edge crack in a strip.

See if you look at in service, you have tension as well as bending. In fact, the result given by Irwin was not really recognizing the bending loads, they were working on strip subjected to tension, instead of a center crack or an edge crack, and they have analyzed how you can handle a surface crack. But in reality, you will also have bending loads; when people perform a numerical analysis, they could comfortably change the loading and also get the results in the presence of bending how the stress intensity factor varies.

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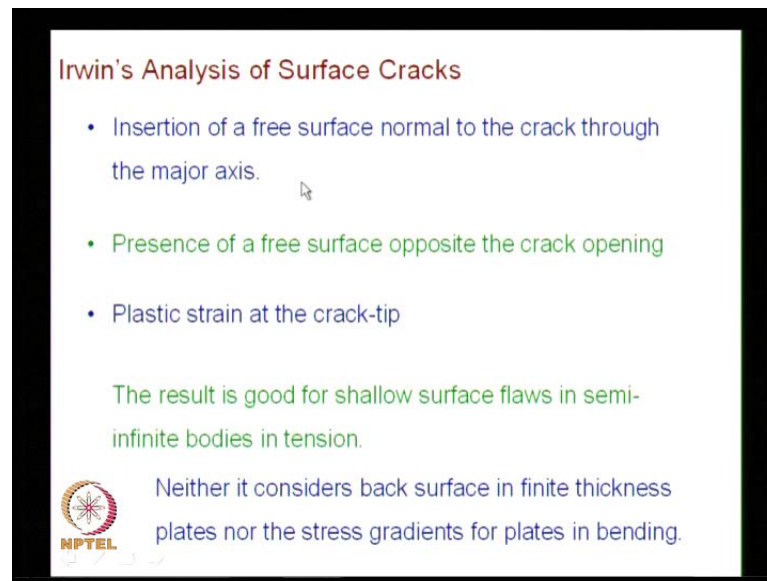


So, when you have these kinds of a numerical result, they are also available for tension as well as bending. The graphs are available; so make a sketch of this; it is again very similar to what we have got for tension, but in detail it is different. The feature is similar, but if you really calculate the values, they would be different; because the loading is bending the graph is very similar. On the x axis, you have  $2c$  by  $b$ ; on the y axis, you have a normalized the value  $K$  of surface flaw is divided by  $K$  of edge crack or the same value of  $a$  by  $b$ .

And these graphs are drawn for  $a$  by  $b$  equal to 0.1, 0.2, 0.3, 0.4 as well as 0.5. See what you will have to notice is, you have graphs available for capital  $q$ , with that also you can calculate the value of stress intensity factor for a surface flaw or you directly look at a graph and find out the stress intensity factor for a surface flaw in comparison to the edge crack that is the way people have looked at it.

What you find here is you also have the solution available for bending; you have the solution separately available for bending and tension. But what you will have to keep in mind is all these are steps towards getting a solution for a surface crack; they are not comprehensive enough. From industry point of view surface cracks are the more dangerous, so more attention was paid and people have develop better solutions such solutions are also available now.

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**Irwin's Analysis of Surface Cracks**

- Insertion of a free surface normal to the crack through the major axis.
- Presence of a free surface opposite the crack opening
- Plastic strain at the crack-tip

The result is good for shallow surface flaws in semi-infinite bodies in tension.

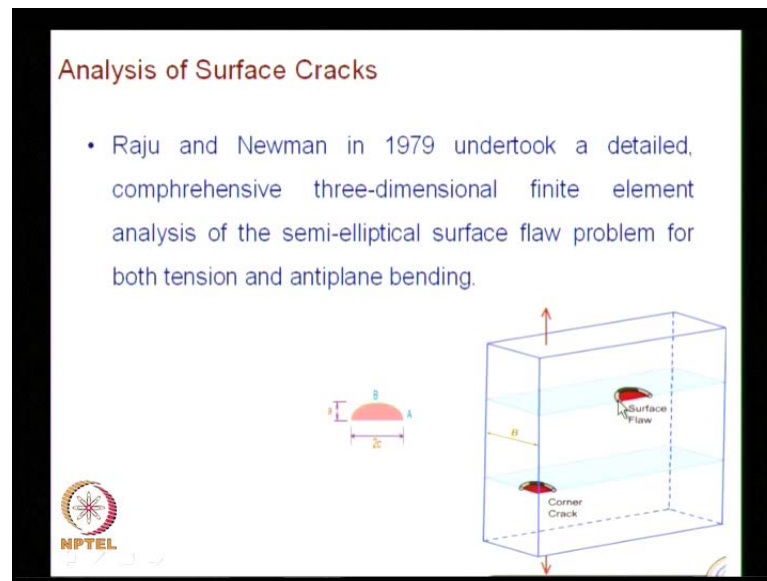
Neither it considers back surface in finite thickness plates nor the stress gradients for plates in bending.



And we will just review what was the Irwin's analysis of surface cracks. The moment you have a surface crack, you have a free surface normal to the crack through the major axis that was recognized by him, then you have a presence of a free surface opposite the crack opening, that is, the front free surface, then you have plastic strain at the crack-tip. Irwin has taken care of this plastic strain at the crack-tip as well as the back free-surface. And we have also mentioned the result is good for shallow surface flaws in semi-infinite bodies in tension.

And what was the defect of Irwin's analysis? It does not consider the back free surface in finite thickness plates. But in our discussion, we had seen a solution by Rice and others where you have the front free surface correction, I think it is by Kobayashi, front free-surface correction factor also we have seen how we can modify. If you look at Irwin's analysis, he was not able to get those results. Definitely Irwin's result does not consider the stress gradients for plates in bending; from actual service point of view, we need to get solution for this situation also. Once the problem is important, you know people pay attention however, expensive they do a detailed analysis.

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And this was done by Raju and Newman in 1979. See imagine, whatever the Irwin's contribution, he played with all those tricks in 1960's; so you should appreciate.

The initial stage of fracture mechanics, where people were groping in the dark, Irwin definitely showed the way how to handle graduate from simpler problems to more challenging complex problems. The finite element technique and also the computers you had only after 1960.

So, when numerical techniques were available in 1979, Raju and Newman undertook a comprehensive three-dimensional finite element analysis of the semi-elliptical surface flaw problem for both tension and antiplane bending. See I have always mentioned, many of the solutions that are available as empirical relation have a origin from a numerical analysis, whatever the numerical analysis that they get, they try to represent it in the form an empirical relation so that for other configurations also people can use it.

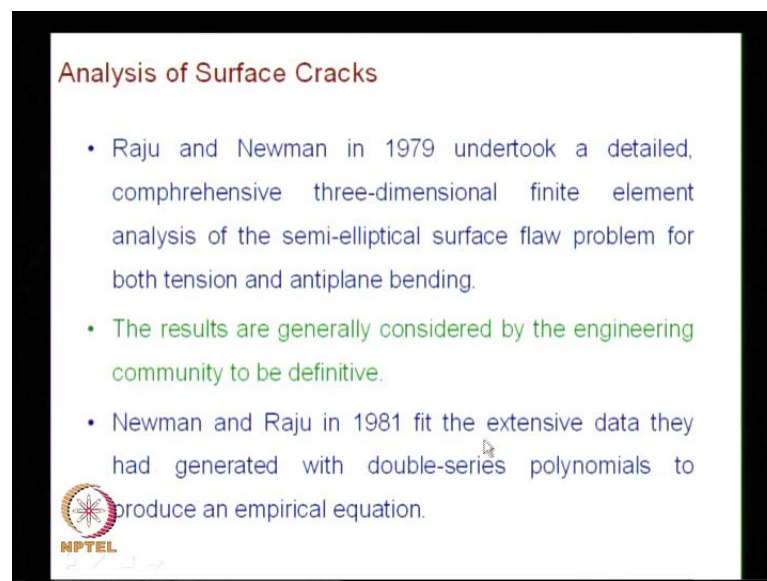
The defect in numerical analysis, for each configuration, you have to run the software and get the result. But if you put it an empirical form intelligently for a variety of similar configurations, it is possible to extrapolate the results. And if you look at the literature, the result was not brought post process immediately; it took sometimes for them to post process. And what this diagram gives is, they have analyzed a semi-elliptical surface

flaw and I just want to remind you that, how you identify the crack length as  $2c$  and depth as  $a$ .

And also you could see this portion is shaded, we will see that reason possibly in a next class. You have high triaxial zone here, we would discuss that is the reason why I said a surface crack use a plane strain fracture toughness for all the instability calculation, though we compare a surface crack to an edge crack from stress intensity factor point of view, though they are comparable.


If you compare the fracture toughness to be used, you use lower fracture toughness for as surface crack. So, surface cracks are always more dangerous than through the thickness crack, that you have to keep in mind.

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**Analysis of Surface Cracks**

- Raju and Newman in 1979 undertook a detailed, comprehensive three-dimensional finite element analysis of the semi-elliptical surface flaw problem for both tension and antiplane bending.
- The results are generally considered by the engineering community to be definitive.
- Newman and Raju in 1981 fit the extensive data they had generated with double-series polynomials to produce an empirical equation.

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And whatever the results that were obtained by Raju and Newman were considered by the engineering community to be definitive. People put lot of faith on his results that is how you have the results. And in 1981, two years later, the numerical solution, they had taken the trouble of fitting the extensive data by double series polynomials to produce an empirical equation.

So, what you will have to keep in mind is, if you do not have any result, you play with this factor 1.12, front free-surface, back free-surface and then, go for a corner crack as 1.12 multiplied by 1 by 1.2 all that circus you do. But if a really in the business and you



want to know the exact result, you go to the solution of Newman and Raju, you have those empirical relations available. And I am going to present that empirical set of relation, you have to write long expressions be prepared for that.

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Empirical Equation by Newman and Raju - 1981

$$K_I = (\sigma_t + H\sigma_b) \sqrt{\frac{\pi a}{Q}} F\left(\frac{a}{B}, \frac{a}{c}, \frac{c}{W}, \theta\right)$$


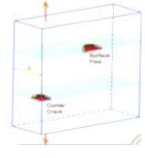
$\sigma_t$  is the remotely applied tensile stress and  $\sigma_b$  is the maximum fibre stress due to the bending moment  $M_b$

Empirical expression for Q (Rawe)

$$Q = I_2^2 = 1 + 1.464 \left(\frac{c}{a}\right)^{1.65} \quad \text{for } \frac{a}{c} \geq 1$$

Maximum error is 0.13 % for all values of a/c

$\sigma_b = \frac{6M}{WB^3}$

And the expression is like this,  $K_I$  equal to  $\sigma_t + H\sigma_b$ , where  $\sigma_t$  is the stress due to tension and  $\sigma_b$  is the bending stress multiplied by square root of  $\pi a$  divided by  $Q$  and you have a function  $F$ , which is a function of  $a/B$  ratio,  $a/c$  ratio,  $c/W$  ratio as well as  $\theta$ .

So, we would see all these in the subsequent equations. What I mentioned is summarized here,  $\sigma_t$  is the remotely applied tensile stress and  $\sigma_b$  is the maximum fiber stress due to the bending moment  $M_b$ , where  $\sigma_b$  equal to  $6M$  divided by  $WB^3$ , where  $B$  is this thickness; I have  $B$  as this thickness; I have the surface crack and you also have expression for  $Q$ , when you are going for an empirical relation, people try to write that also and this expression for  $Q$  is credited to **1** Mr. Rawe and it is like this,  $Q$  equal to  $I_2^2$  that is equal to  $1 + 1.464$  multiplied by  $a/c$  whole power 1.65, for  $a/c$  less than equal to 1.

You know, you have to congratulate that team for providing an empirical solution for their numerical resource and empirical solution is very complex; it is not simple. There is no rational behind how they arrived at it; they have got the results and they were able to

quickly see a sort of evaluation of the results and then, they could pack it in a nice empirical fashion, how they did it? You have to go and ask them only.

The results are very... If you look at the expression, they are very long and complex; they have done a good job of putting that as empirical relation and Q becomes 1 plus 1.464 c by a whole power 1.65 for a by c greater than equal to 1. The next two three slides we deal only with this expansion of various terms. And what is mentioned also here is, if you evaluate Q from this, the error is within 0.13 percent for all values of a by c, that kind of an information is also provided.

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**Empirical Equation by Newman and Raju - 1981**

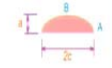
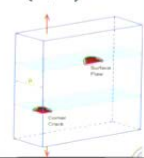
$$K_I = (\sigma_r + H\sigma_b) \sqrt{\frac{\pi a}{Q}} F\left(\frac{a}{B}, \frac{a}{c}, \frac{c}{W}, \theta\right)$$

The function F chosen by Newmann and Raju has the form

$$F = \left[ M_1 + M_2 \left(\frac{a}{B}\right)^2 + M_3 \left(\frac{a}{B}\right)^4 \right] f_\theta \cdot g \cdot f_W$$

Where

$$M_1 = 1.13 - 0.09 \left(\frac{a}{c}\right) \quad M_2 = -0.54 + \frac{0.89}{0.2 + (a/c)}$$

$$M_3 = 0.5 - \frac{1.0}{0.65 + (a/c)} + 14 \left(1.0 - \frac{a}{c}\right)^{24}$$



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Then the function chosen by Newman and Raju has a form like this, f equal to M 1 plus M 2 multiplied a by B whole square plus M 3 a by B whole power 4 multiplied by a function of theta and a function g and function of w.

We would see each one of them, that is the way you have to write the expressions and what I have for M 1, M 1 is given as 1.13 minus 0.09 multiplied by a by c and you have M 2 given as minus 0.54 plus 0.89 divided by 0.2 plus a by c, and M 3 is given as 0.5 minus 1.0 divided by 0.65 plus a by c plus 14 multiplied by 1.0 minus a by c whole power 24. I am sure you will agree the empirical relation is quite complex; how they arrived at in consolidating the result, you have to really thank them for providing this kind of a solution and this also emphasize at that even our students when they do their

work, they should report these in a convenient form report, whatever the results that they get in a convenient form for others to use.


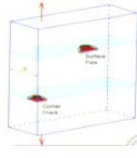

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**Empirical Equation by Newman and Raju - 1981**

$$F = \left[ M_1 + M_2 \left( \frac{a}{B} \right)^2 + M_3 \left( \frac{a}{B} \right)^4 \right] f_\theta \cdot g \cdot f_w$$

$$f_\theta = \left[ \left( \frac{a}{c} \right)^2 \cos^2 \theta + \sin^2 \theta \right]^{\frac{1}{4}}$$

$$g = 1 + \left[ 0.1 + 0.35 \left( \frac{a}{B} \right)^2 \right] (1 - \sin \theta)^2$$

$$f_w = \left[ \sec \left( \frac{\pi c}{W} \sqrt{\frac{a}{B}} \right) \right]^{\frac{1}{2}}$$




The function theta we are familiar with, we have been seeing it in many slides that is a by c square cos square theta plus sin square theta whole power 1 by 4, and g is 1 plus 0 point 1 plus 0.35 a by B whole square whole multiplied by 1 minus sin theta whole square and you have function of w is given as secant times pi c by W square root of a by B whole power half. See in this solution, you will find no correction for plasticity effects, because their analyses were elastic and this comes from the edge crack nomenclature; this comes from the embedded elliptical flaw.

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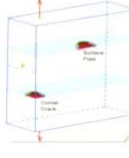
**Empirical Equation by Newman and Raju - 1981**


Using engineering judgement, Newman and Raju expressed the function  $H$  as  $K_I = (\sigma_t + H\sigma_o) \sqrt{\frac{\pi a}{Q}} F\left(\frac{a}{B}, \frac{a}{c}, \frac{c}{W}, \theta\right)$

$$H = H_1 + (H_2 - H_1) \sin^p \theta$$

$$p = 0.2 + \left(\frac{a}{c}\right) + 0.6 \left(\frac{a}{B}\right)$$

$$H_1 = 1 - 0.34 \left(\frac{a}{B}\right) - 0.11 \left(\frac{a}{c}\right) \left(\frac{a}{B}\right)$$

$$H_2 = 1 + G_1 \left(\frac{a}{B}\right) + G_2 \left(\frac{a}{b}\right)^2$$



 NPTEL

And it is very clearly mention in the literature, using engineering judgment Newman and Raju expressed the function  $h$  as that is a very important aspect; this function is not so simple.

So, they have reported  $H$  as  $H_1 + (H_2 - H_1) \sin^p \theta$  and you have values for everything given  $p$  equal to  $0.2 + \frac{a}{c} + 0.6 \frac{a}{B}$ . We will also have expressions for  $H_1$  and  $H_2$ ;  $H_1$  is given as  $1 - 0.34 \frac{a}{B} - 0.11 \frac{a}{c} \frac{a}{B}$  and you have  $H_2$  given as  $1 + G_1 \frac{a}{B} + G_2 \left(\frac{a}{b}\right)^2$ .

You know you have to define what is  $G_1$  and  $G_2$ ; I postpone it to the next class. We will have a look at what is  $G_1$ ,  $G_2$  and also look at a summary of the results, then we move on to the next chapter on plastic zone correction or plastic zone length calculation or plastic modeling of a crack-tip we will have a look at it. In this class essentially we looked at various methodologies to get the stress intensity factor for a surface crack.

We said it could be modeled as a semi-elliptical flaw; a corner crack model can be modeled as a quarter elliptical flaw and we also had a brief discussion on how leak before break is possible. Because of the way stress intensity factor and fracture toughness that needs to be used, for such calculations help us to do it that is why people

thought that by suitably designing your system, it is possible to provide a gap between leak and break.

Thank you.