Engineering Fracture Mechanics

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Module No. # 06

Lecture No. # 29

Irwin's Model

In this class, we will try to look at the approximate shape of plastic zone, followed by Irwin’s model in finding out the extension of crack length from the plasticity point of view. He would try to do a redistribution calculation; we will look that in detail, but for the purpose of shape of the plastic zone.

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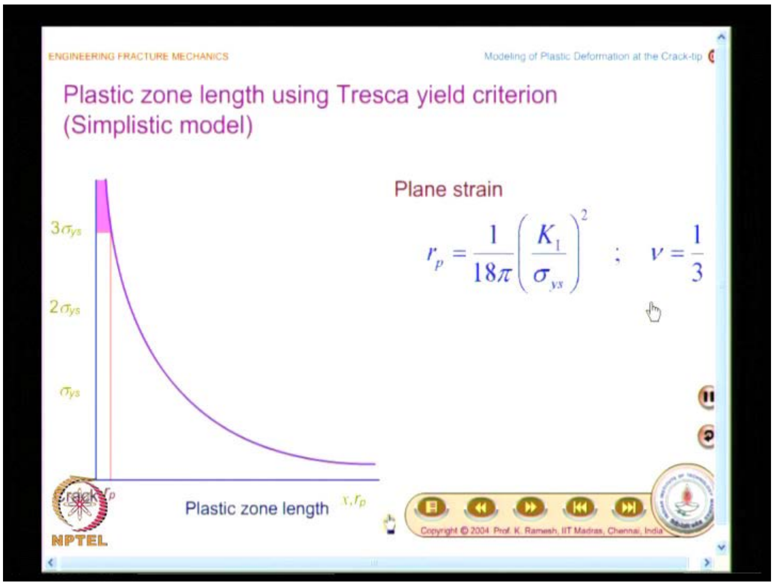

What we are going to do is; we are going to use a very simple model, and we will first look at how to get the plastic zone length along the crack axis? And if you look at when the crack is sharp, both sigma x and sigma y varies like this and your Tresca yield criterion, because your other stress is zero, that directly gives you what is the value of the individual stresses, then yielding takes place. What we are going to do is; what is the value of the sigma y stress in plane strain? Suppose, I have Poisson ratio is 1 by 3, in plane strain the maximum stress value would be as high as 3 times the yield strength. So, what we will do in the simplistic model is, just pick out that point from this graph and say that is the length of plastic zone ahead of the crack.

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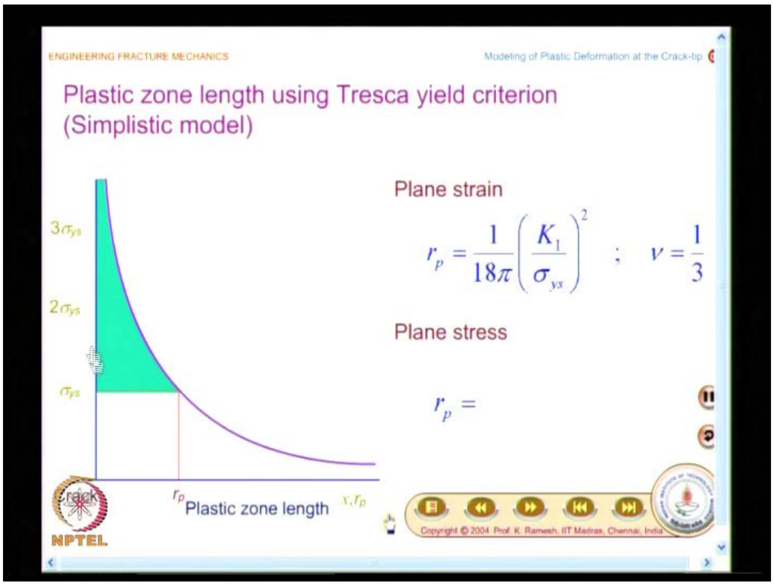

So, the approach is simple and straight forward but it is not strictly correct. So, you have this as 3 sigma y as an in plane strain, and this distance you call it as a plastic zone length with the symbol r p. And in plane strain the expression takes this from, r p equal to 1 by 18 pi, K 1 by sigma ys whole squared, and this is obtained by using nu equal to 1 by 3. And in plane stress you know, the yield condition is satisfied when sigma y reaches sigma ys, in the case of plane stress it is sigma ys, in the case of plane strain when nu equal to 1 by 3, it is 3 times sigma ys.

Though the approach is very simple, it brings out a very important pictorial representation that, the plastic zone length in the case of plane stresses is much larger than what you have in plane strain so; this is the advantage you get from a simplistic model.

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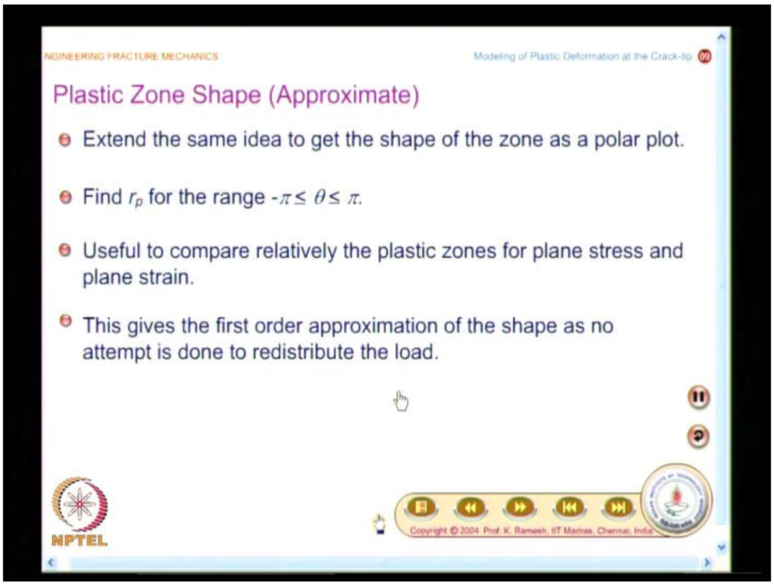

This we have looked at what happen ahead of the crack length, express it as a function of theta you get a shape, which is very approximate, in fact I had asked the students to plot this for mode 1, mode 2 and mode 3 in the last class. I am sure many of you may not have done it so, we will have a look at those shapes.

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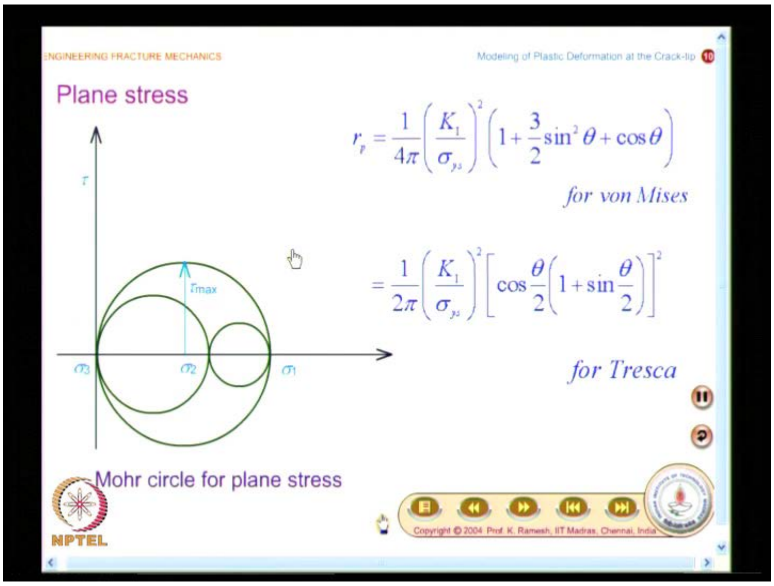

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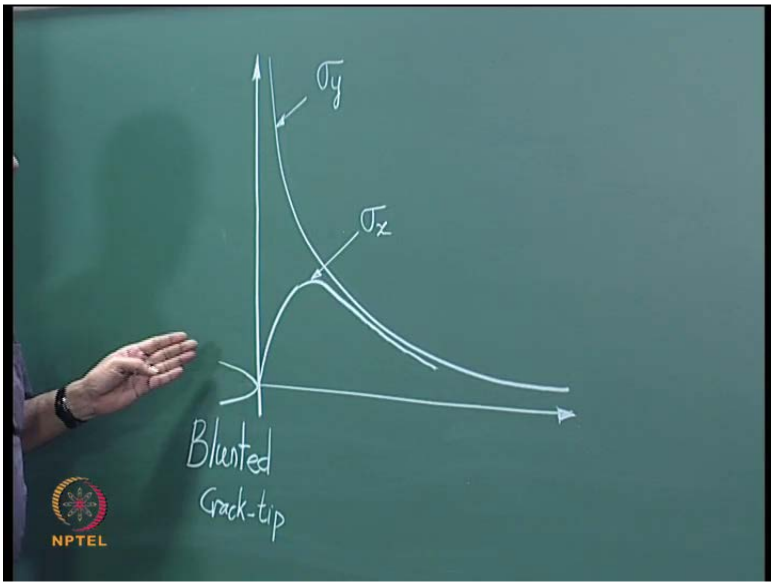

An approach like this, gives the first order approximation of the shape, because we do not make any attempt to redistribute the load. And for the case of plane stress we have looked at the Mohr circle like this, I have sigma 1, which is nothing but your sigma y and sigma 2 is your sigma x, and sigma 3 is zero, this you have to recognize in the case of plane stress. And what you are really looking at is, see you are looking at a case when the crack-tip is blend, the moment you are looking at plastic zone we will also have to recognize the crack-tip will become blend, in view of this, because of the free surface the sigma x will be zero at the crack-tip and it reaches a maximum at slight distance away from the crack-tip.

So, a sigma x variation will be like this and your sigma y variation will be like this. So, in this case what happens? You have sigma y as well as sigma x both are positive, and you have to recognize sigma z 0 or sigma 3 0, the yielding is dictated by your sigma y. And this is what is depicted in the Mohr circle here, you have to take care of the zero value, this is very, you should not ignore this. Otherwise, you would make a wrong judgment that your maximum shear stress is only this, if you consider only sigma x and sigma y.

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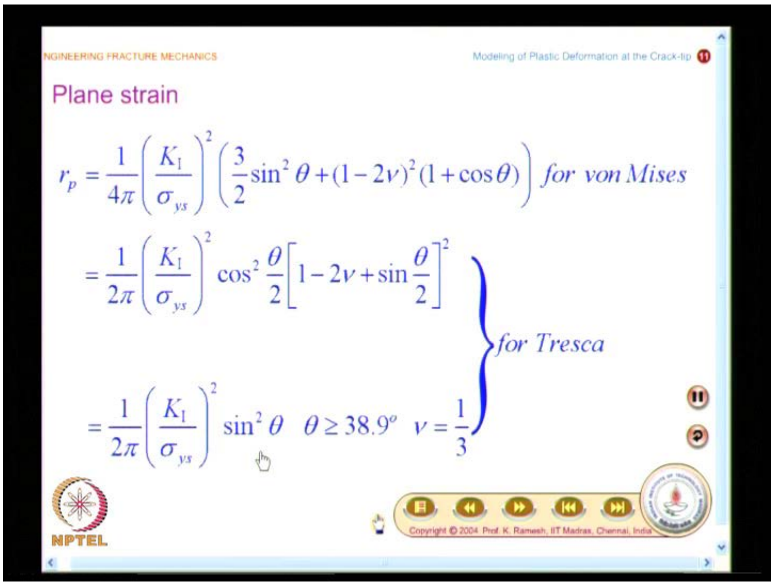

Sigma y will be like this and sigma x, and you should not take this, you should recognize the zero value of the other principle stress, and maximum shear stress is this is the value. And when you write this expression of r p as a function of theta it takes a form like this, and when you go to Tresca, the expression is different. If you plot this expression as a polar plot by varying theta and marking the values of r, you would be able to get an approximate shape of plastic zone at the crack-tip. What we will look at is? We look at for plane stress as well as for plane strains situation.

For the plane strain situation, the expression for r p turns out to be 1 by 4 pi multiplied by K 1 by sigma ys whole square, multiplied by 3 by 2 sin square theta plus 1 minus 2 nu whole squared multiplied by 1 plus cos theta, and this for Von Mises. And for Tresca the expression is given for different extent of theta so, you have this as 1 by 2 pi, K 1 by sigma ys whole square, cos square theta by 2 multiplied by 1 minus 2 nu plus sin theta whole square. And for theta greater than 38.9 degrees and nu equal to 1 by 3, you have to use this expression, this is 1 by 2 pi, multiplied by K 1 by sigma ys whole square sin square theta.

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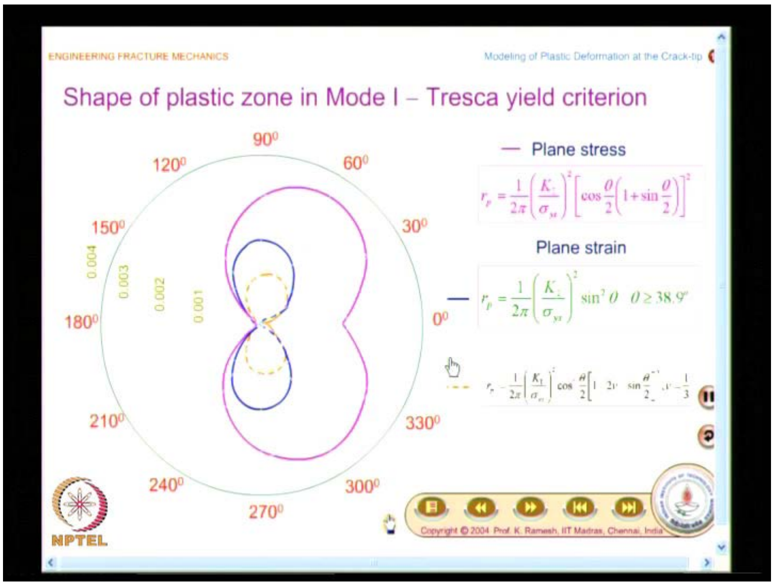

So, what we are going to look at now is, rather than looking at these as expressions, if you plot them, that gives you some kind of an insight. And what we will do is, we will do a similar exercise for mode 1, mode 2 and mode 3, and look at what are the shapes of the plastic zone, but you have to keep in at the back of your mind these are very approximate shapes, and the polar plot is dawn like this.

So, what is done here is, this is the crack-tip, I have radial lines drawn for various angles, make a neat sketch of this, you have an expression for the plastic zone length r p as a function of theta. So, mark this and join them as a curve, and this is obtained for plane stress situation by Tresca yield criterion, you have already written down this expression and you will have to draw this sketch. This is for plane strain, this expression also have written. And this expression also you have already written, the only thing what you have to do is you have to draw this sketches so, I have this for plane stress where the plastic zone size is very large, and when you come to plastic zone for plane strain, you have a loop here like this, only this blue and continues red line you draw it, the dotted line you do not have to draw and this is how the shape is.

And how does this look like? We look at this, something similar to your isochromatics. Yes, straight, but it have to be forward tilted, and you have along the crack axis there is some extent of plastic zone is available though, this is approximate shape this gives you an insight that plastic zone size is very small in the case of plane strain, quite large in the case of plane stress.

And you know, when you change the yield criteria then also there would be some small change in the shape, it is unavoidable. We have got this for Tresca yield criterion.

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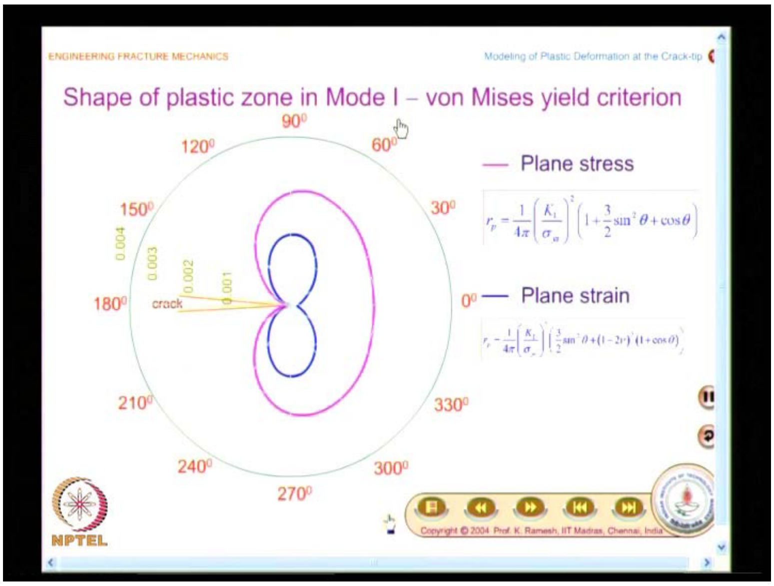

Now, we will go and see how the plastic zone shapes differ, when I use the Von Mises criteria and this is for mode 1 situation. This is for plane stress, you have the expressions you do not have to write them, but draw the sketch, and this is for plane strain the expression you have already written. And here again you have a visual representation that, the plane strain plastic zone is very small and this is also very similar to your shape of the isochromatics, there are similarities between the two.

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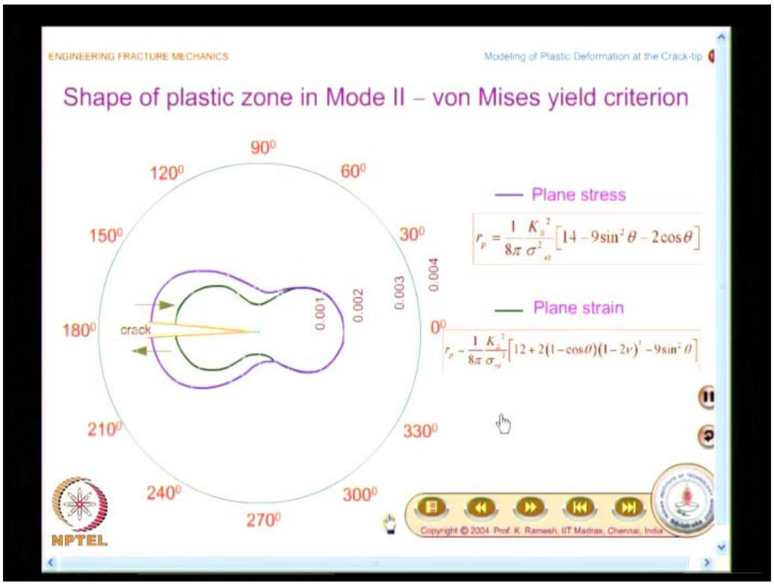

So, make a neat sketch of this, and we would have similar plots drawn for mode 2 as well as mode 3. When I go to mode 2 situations, I am not drawing it for both the yield criteria I will draw it for Von Mises yield criteria, this is how the zones appear like this, I think you need to write this expression, r p is given as 1 by 8 pi K 2 squared divided by sigma squared by s, multiplied by 14 minus 9 sin square theta minus 2 cos theta. And when they change it to plane strain situation the expression turn out to be like this, r p equal to 1 by 8 pi K 2 squared by sigma ys squared, multiplied by 12 plus 2 into 1 minus cos theta, multiplied by 1 minus 2 nu whole square minus 9 sin square theta.

See, that precaution you have a complicated expression, any times when you come across a complicated expression you also attach that, this is a correct expression, do not have that kind of a mental block, this is a very crude representation of plastic zone shape, you will be able to appreciate this only when you look at the Irwin's method of calculation, where he also considers redistribution of load that to only along the crack axis that itself is the big exercise.

So, unless you go to experimental methods or numerical exhaustive techniques you will not be able to get the shape for a generic situation nevertheless, these kind of plots give you certain kind of understanding on the relative sizes for plane strain and plane stress. And this also you can notice, how does the shape look like? I have already shown you mode 2 isochromatics, a similar to mode 2 isochromatics.

So, that is one way of comparing the shapes. And the difference between plane stress and plane strain is not that significant in the case of mode 2, in the case of mode 1 the sizes where quite different for the two cases. And what you need to draw is, just draw the shape, you do not have to draw these polar plot the way it is drawn, this is to aid your understanding how this graph is drawn, you collect several points for various values of theta and join them as a smooth curve, for your purpose you just draw these two shapes.

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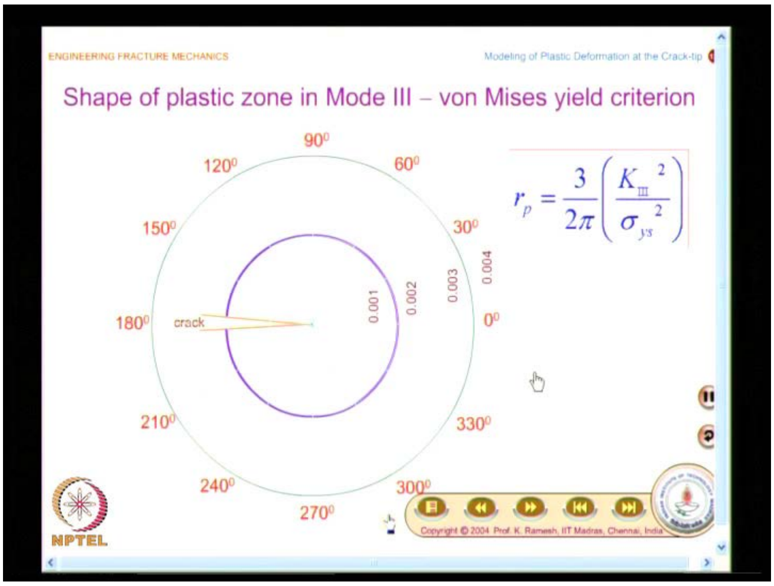

Now, we will go and see, what is the type of shape in mode 3? You find a very simple graph, you get r p equal to 3 by 2 pi K 3 squared divided by sigma ys squared. See only in the case of mode 3 loading the plastic zone shape is circle, in other cases the shape is far different, and in many books if you come across, whenever they come across the plastic zone they will simply put a circle. That kind of utilization came into existence, because they have looked at the shape is circular in the case of mode 3, which is extrapolated to for mode 1 also, which is not strictly correct. You will have to look at the actual shape, we will also look at the actual shape as seen in experiments or as done by complicated numerical analysis.

Have you seen the fringes for this case? Photoelasticity would is not applicable for mode 3 loading situation, because it is out of plane loading. Photoelasticity is not applicable, only for in plane loading you would be in a position to do so, photoelasticity is applicable only for mode 1, mode 2, and combination of mode 1 and mode 2, you cannot analyze mode 3 problem from photoelastic analysis.

So, we have not seen the shape of the isochromatics there for you to compare nevertheless, you have to keep in mind when you see a circle plastic zone is circular only in the case of mode 3, many books they simply put a plastic zone even for mode 1 as a circle.

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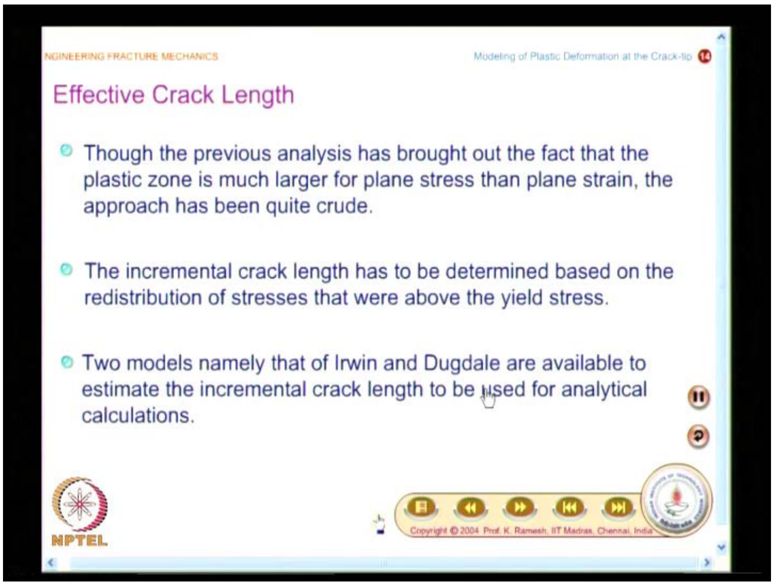

Now, what we will do is, we have already said that, we are going to model the plastic zone from further fracture calculation, by defining what is an effective crack length. The previous analysis has brought out the fact that, the plastic zone is much larger for plane stress than plane strain. However, the approach has been quite crude.

The incremental crack length has to be determined based on the redistribution of stresses that were above the yield stress, if you do not do that, the estimation of incremental crack length would be erroneous. And if you look at the literature, two models exist: one model was proposed by Irwin and another model was proposed by Dugdale. See, you will have to see the distinction, it is only a model it is not a theory, the problem is very complex so they have modeled it in this fashion, this is valid under certain restrictions you have to take it that way.

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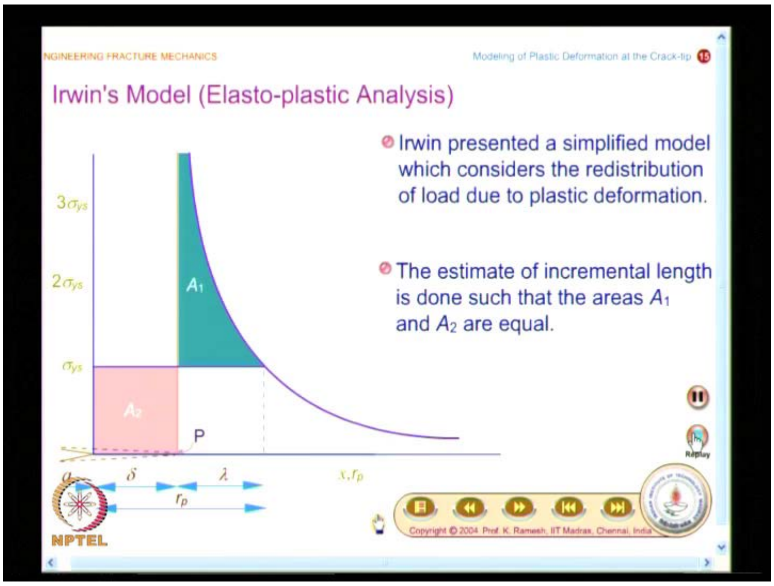

And, because the problem is complex, whatever the value of incremental crack length that you would get from Irwin and Dugdale models would be different, we have to anticipate that, they will not be identical, they will be different. And what is said Irwin has done in his analysis, you have to look at the animation very carefully, and this is considered as elasto-plastic analysis, because we do some kind of redistribution of load, and he definitely presented a simplified model, which considers the redistribution of load due to plastic differentiation.

And how the model is developed? You make a sketch of this, I have a crack-tip and you have on the x axis the distance x as well as the length of the plastic zone r p, then you have stresses on the y axis sigma ys, 2 sigma ys, and 3 sigma ys. And what was done in a simplistic model? The simplistic model, you simply mark this and set that, this is the size of the plastic zone whereas, this stretch of material was supporting a load like this. Where would the load go? You cannot simply knock off, when you are drawing a graph on the board you can simply erase it and then show the graph can be thought of like this, but what happens the materiel? When you say this stresses reach a value of sigma ys in essence, what Irwin argued was the extent of plastic zone would become longer for redistribution of load to take place.

And look at the animation very carefully, and that physics is illustrated here. So, you have this, since this has to be taken care of by the neighboring material, you will have a larger plastic zone ahead of the crack-tip. And you will have to find out the areas marked as A 2 and A 1 such that, they are equal. Is a physics clear? If you want, I will again do redo the animation. You just observe the animation, Irwin presented a simplified model, and the model is like this, you have the stresses shown like this.

Since, this cannot be cut off just like that what we have done the simplified model this, has to be supported by the neighboring material so, instead of only this zone subjected to this loading, because of yield stress it will extend and will occupy, and we would find out this extension in such a manner that the area A 2 equal to A 1.

So, as part of the calculation we will have to find out, what is delta? As well as, what is lambda? The whole length is the plastic zone length, it is very simple if you just follow the physics, translate it into mathematics and you will be able to get the expressions for delta as well as lambda.

So, you make an attempt to look at the redistribution of load, which was not done the simplistic model and imagine this itself is a big circus, along the crack axis, if you have to find out the shape of the plastic zone, this kind of redistribution should be looked at for every angle, which is not possible from your hand calculation, we will have to depend on a computer to do it.

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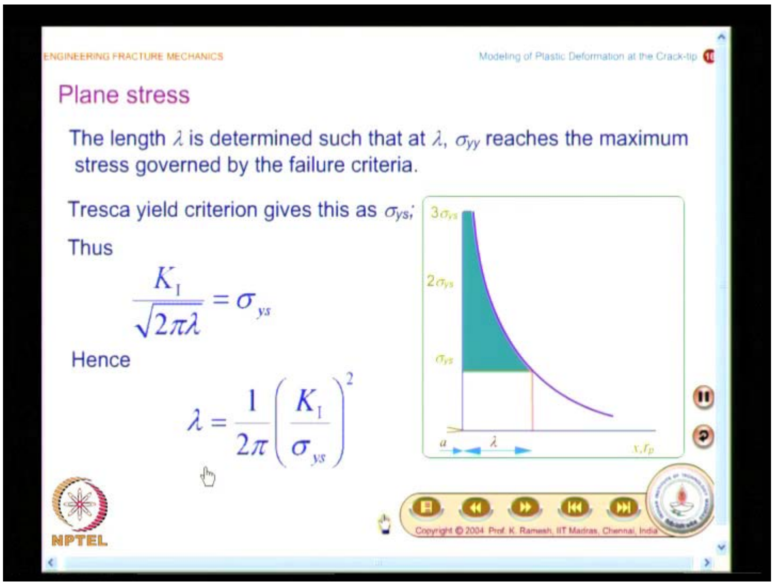

We will start our discussion from our simplistic approach, see first of all you will have to know, what is the stress that need to be redistributed? This is the area above the yield stress that has to be redistributed.

So, I will have to know what is this lambda, this calculation is very simple to your very similar to your simplicity calculation, we simply say K 1 by root of 2 pi lambda equal to sigma ys, that is how we locate this point.

So, this also gives you a definition of what is K 1 in terms of lambda, which we would use it later in the derivation, this is very similar to your simplistic approach, from this I can find out what is lambda. Lambda is given as 1 by 2 pi multiplied by K 1 by sigma ys whole square, what we say here is, this estimation of plastic zone length along the crack axis is not strictly correct. So, we have to go and try to redistribute the load and make the calculation.

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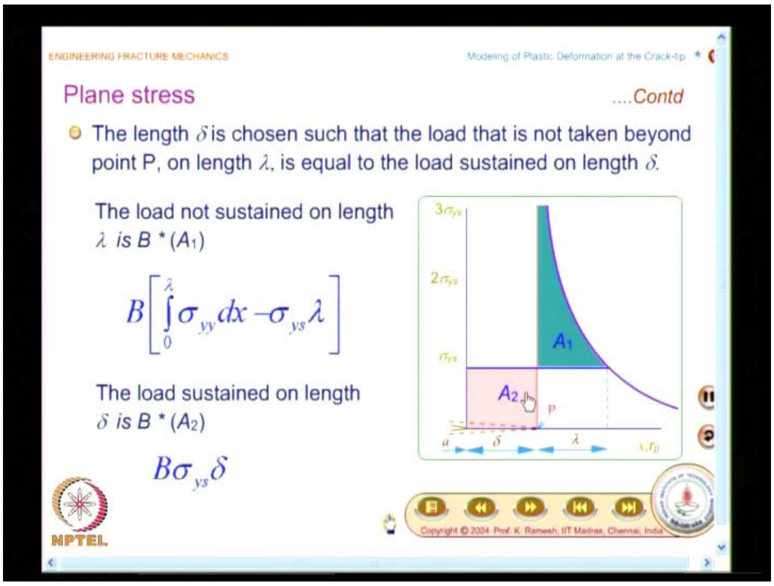

We have already seen the sketch, based on that sketch I can make a statement that length delta is chosen such that; the load that is not taken beyond point P on length lambda is equal to the load sustained on length one.

What is the load not sustained on length lambda? What is shown as area A 1 is not sustained on length lambda, and I have taken a plate of thickness B, and the expression is written like this, it is integral delta l to lambda sigma yy dx minus sigma ys lambda.

See just, because it is written like this I have been saying many times, you have to be alert, is it right? Are the limits are put correctly, this is understandable you have sigma ys into lambda is understandable, that refers to this area, but it is not from delta to lambda you are actually doing it from, you are only calculating this, So, you are doing it, you have to do it from 0 to lambda, because my interest is to find out this area, for that area it is going from 0 to lambda and that is what we are looking at.

So, we have an expression for A 1, and the load sustained on length delta is B times the area A 2. So, I have this as B times sigma ys multiplied by delta, say it will also have to caution you here on symbolism, I have used delta in these derivations as the extension of the original crack A by a fictitious length delta. For all my future calculation, I will use the length of the crackers A plus delta, by delta as a symbol for advance studies when you go to EPFM, it is used for crack-tip opening displacement.

So, we have also seen crack opening displacement expression, the crack opening displacement talks about what way the crack opens up when you have a loading apply, we have seen that it opens up like an ellipse. Once you go for elasto-plastic analysis, people define crack-tip opening displacement, that comes from Irwin's model or Dugdale's model, we are taking a crack length as a plus delta so, we will look at in those context delta will be used for C T O T.

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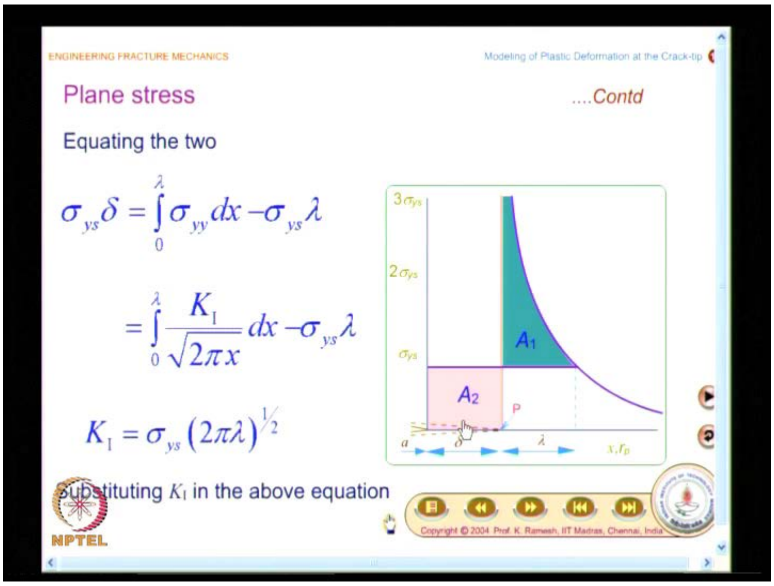

So, depending on the context attach a meaning to the symbol delta. And we are discussing the plane stress situation and we have the equate the two, and this animation is repeated here for your convenience.

So, sigma ys multiplied by delta is equal to, integral 0 to lambda sigma y by dx, minus sigma ys lambda. And I can replace sigma yy in terms of K 1 by root of 2 pi x, multiplied by dx, what is there in this expression, minus sigma ys lambda. We have already looked at how to express K 1 in terms of lambda, just go back to the expression and I can write K 1 as sigma ys multiplied by root of 2 pi lambda or 2 pi lambda whole power half.

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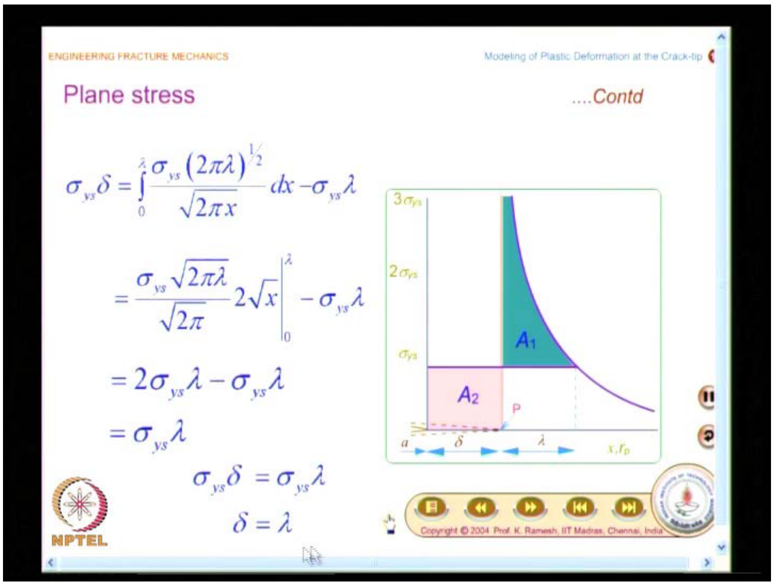

And you substitute this expression for K 1 in the above equation, and simplify. What we are really looking at is, we have to estimate what is delta. So, what you have here? I have sigma ys delta equal to integral 0 to lambda, sigma ys multiplied by 2 pi lambda whole power half, divided by root of 2 pi x dx, this is very simple to integrate, there is no difficulty at all.

When you integrate this, you get this as sigma ys square root of 2 pi lambda divided by square root of 2 pi, multiplied by 2 times root of x in the limits 0 to lambda. So, on substitution of these limits and on simplification you get this as 2 sigma ys lambda, and we already have sigma ys lambda does not here. So, on subtraction you get sigma ys delta equal to sigma ys lambda. So, this gives you a final expression delta equal to lambda.

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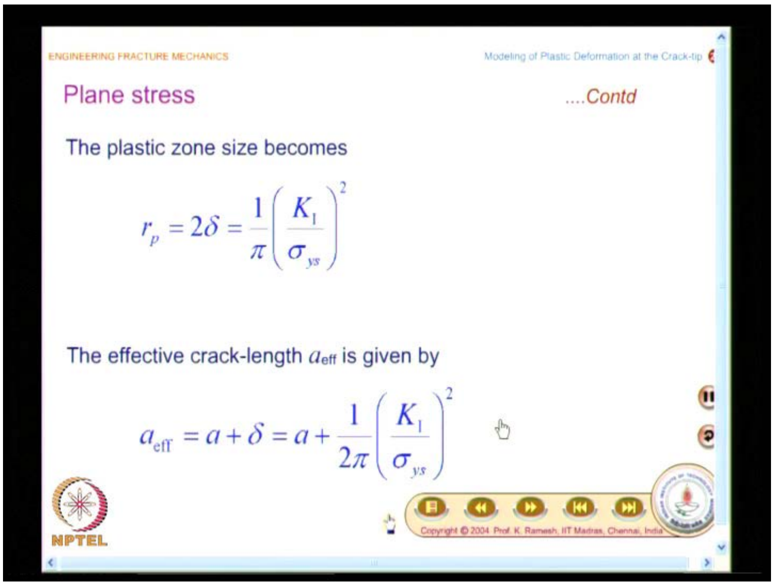

So, what you are really looking at is, you estimated lambda based on simplistic calculation, and you find the actual plastic zone, because of redistribution is much larger in fact, it is twice that length. Then we also have expressions for plastic zone size so, the r p becomes, r p equal to 2 delta equal to 1 by pi K 1 divided by sigma ys whole square.

And what is the effective crack-length? From Irwin's approach the effective crack-length is taken as, a plus delta and delta is 1 by 2 pi multiplied by K 1 by sigma ys whole squared. See, in fact these are models, the moment I go to Dugdale's calculation he would take the entire plastic zone length as the effective crack-length, that is why he said these are all not theories, but they are models which are applicable for certain kind of situations, when they satisfy the approximations.

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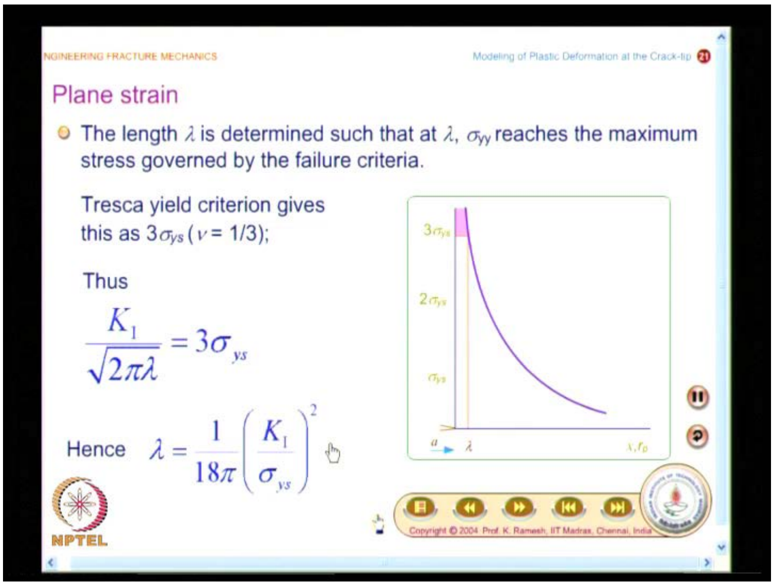

And you will also have to keep in mind that delta has a different symbolic representation when you go to EPFM, where it refers to CTOD rather than extension of crack length like this, we have looked at for plane stress now, let us see how do we estimate the plastic zone length in the case of plane strain?

Here again you will look at the simplistic model, you have to find out the length lambda. And what we did. we simply took the stress at the crack-tip is 3 times sigma ys when nu equal to 1 by 3, while doing this calculation we have ignored blunting of crack-tip, in reality when you are looking at plastic zone correction blunting of crack-tip also takes place, you cannot ignore that.

So, what we will do is, without blunting what is the kind of result that we get? With blunting what is that we have to modify? If we use this as 3 sigma ys considering that the crack-tip is sharp, I get K 1 divided by root of 2 pi lambda equal to 3 times sigma ys. Hence, lambda becomes 1 by 18 pi, K 1 by sigma ys whole squared. See, this is not going to be correct, because we want to look at certain final aspects, we are saying there is plastic deformation at the crack tip and we want to find out modification of crack-length by an incremental amount, and in the process you get values without considering blunting of the crack-tip, would not be the right approach.

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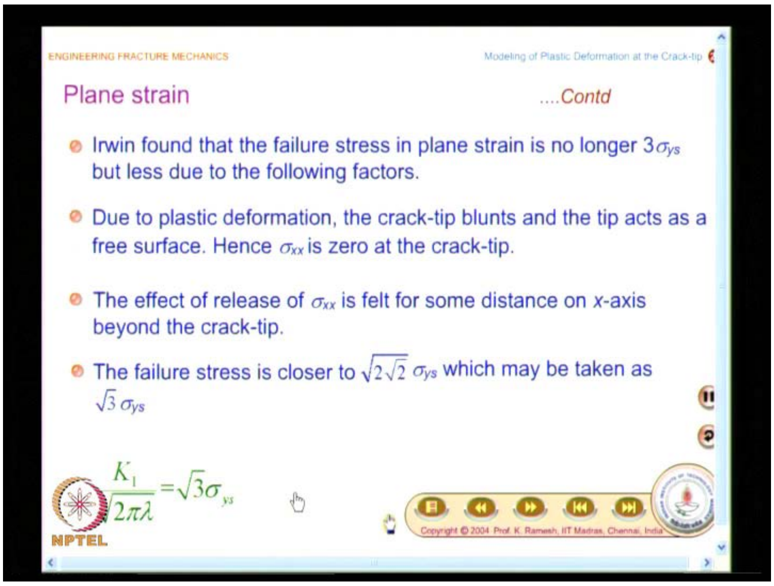

When there is no blunting in plane strain the sigma y, the maximum stress reaches the value of 3 sigma ys, which we find is not strictly correct, because of blunting, this was pointed out by Irwin. It is no longer 3 sigma ys was his argument, and what are the factors? Due to plastic deformation the crack-tip blunts and the tip acts as a free surface, I have already noted this down in earlier part of the lecture so, what you have is sigma xx is 0 at the crack-tip.

So, sigma xx is not same as sigma yy and the effect of release of xx is felt for some distance on x axis beyond the crack-tip. So, when you get into the yield criteria, the maximum stress I mean, the value of stress corresponding to, yielding to take place will not be 3 sigma ys, it would be different than, that some estimate needs be done. Irwin made estimate, what you have done was the failure stress is closer to square root of 2 root 2 sigma ys, which could be simplified and taken as square root of 3 sigma ys.

So, if you really look back and see what we were really saying is, when the crack- tip is sharp your simple calculation show, that sigma y would be 3 times sigma ys. If you consider the crack-tip to be blunt, you have to modify that as root 3 times sigma ys, this is an accepted practice many other discussions and fracture mechanics we will use this value.

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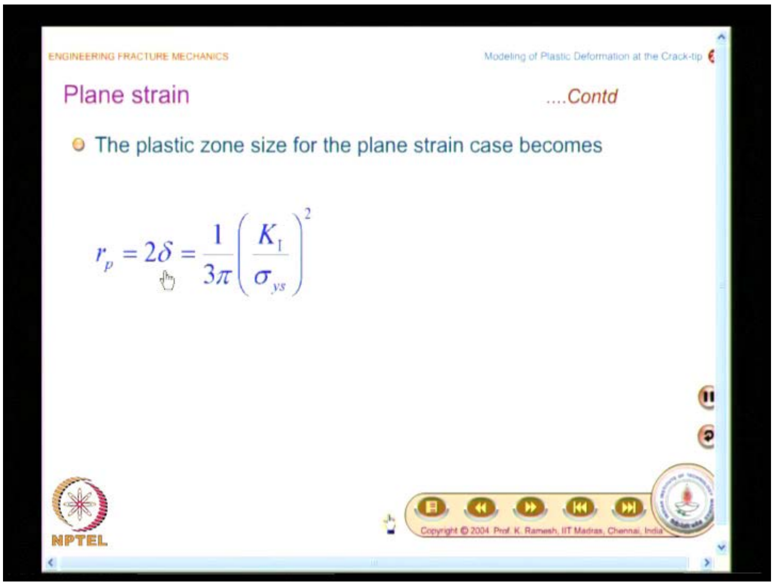

So, I get K 1 by root of 2 pi lambda as root of 3 sigma ys. See, the moment you find out lambda, when you look at the redistribution of load, we have already establish lambda equal to delta similar expression, you will get for plane strain also from that you can write, what is the size of the plastic zone as well as, what is the effective crack-length. The plastic zone size for the plane strain case becomes r p equal to 2 times delta that is equal to 1 by 3 pi multiplied by K 1 by sigma ys whole squared. So, if I look at the effective crack-length, I would have to get delta which would be 1 by 6 pi K 1 by sigma ys whole square.

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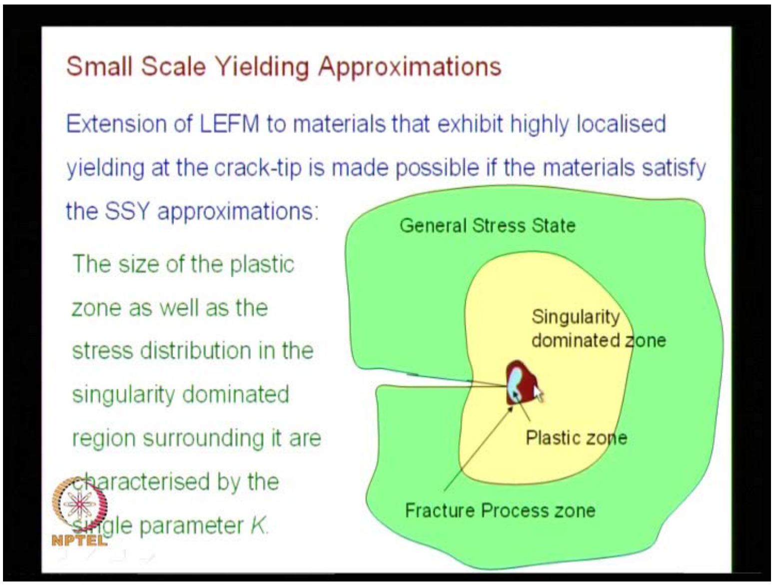

Say now, what we will have to look at is, what do you understand as small scale yielding approximations, because this is the key idea that is useful in applying LEFM so, if we want to extend LEFM to materials, that exhibit highly localized yielding at the crack-tip, the materials need to satisfy small scale yielding approximations and what do we mean by that?

We will see three aspects of it this: the first aspect is the size of the plastic zone, as well as the stress distribution in the singularity dominated region surrounding it or characterized by the single parameter K.

Later on, we will look at EPFM, we will have a singularity dominated zone, then you have a j dominated zone and so on. So, in the case of LEFM when you are looking at SSY, we are looking at the situation like this, it is pictorially represented I have a crack and let us see that, this is a general stress state. And when you have SSY, what we are saying is near the vicinity of the crack there would be a zone, which is known as singularity dominated zone K is important, mind you, this is the highly enlarged picture for clarity it is drawn like this, make a neat sketch of this and within the singularity dominated zone I have what is known as the fracture process zone.

You know, these shapes are freely drawn there is no mathematics attached to it so, you have to take it as representative figures, in an actual problem these shapes have to be obtained by a detailed calculation, these are representative shapes so, what you are really looking at it, you may have general stress state prevalent everywhere, but near the crack-tip you can identify a zone which is singularity dominated, within which you have a fracture process zone, we will also see what is the fracture process zone later, within which you have the plastic zone. In fact the plastic zone I have given a shape which is very similar to what is seen in experiments when I am having a mode 1 situation that representation is given here.

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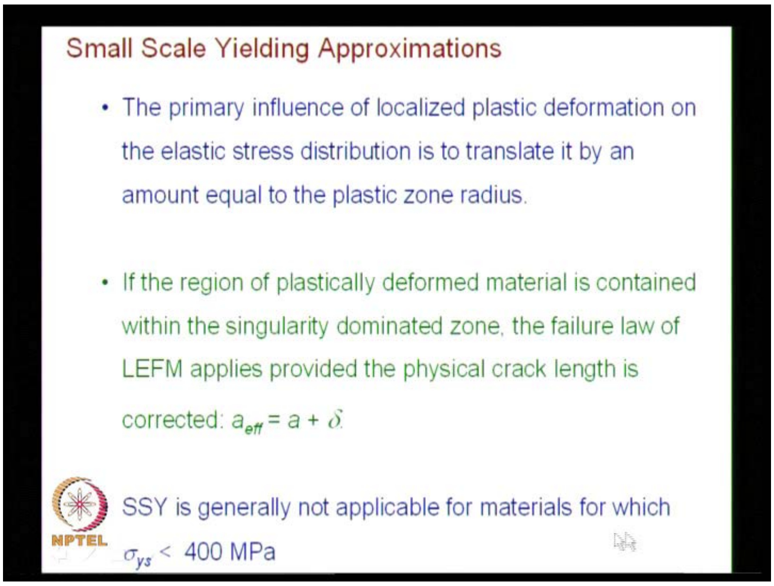

So, what you are really looking at it is, these zones are much smaller compare to the singularity dominated zone, that is the key point here. The primary influence of localized plastic deformation on the elastic stress distribution is to translate it by an amount, equal to the plastic zone radius.

In fact, when you looked at the development of Irwin’s model we shifted the graph to the right, something similar to that is being stated here. The effect of localized plastic deformation is to translate the elastic stress distribution by a small amount dictated by the plastic zone. And the third point is, if the region of plastically deformed material is contained within the singularity dominated zone, I can still apply the failure law of LEFM provided the physical crack length is corrected as a plus delta.

In fact, we have looked at while discussing surface cracks, Irwin has modified the length of the crack by a small increment and I pointed out, whatever the increment I have shown in that class was reported by Irwin in 1960, we would see improved calculation of this in the next chapter and we have seen those expressions given by Irwin in this chapter. So, you have to replace it with these expressions, when you do the calculation, but what it essentially says is, LEFM is still applicable provided I change the crack-length, a effective as a plus delta. And there is also another thumb rule that you have SSY is generally not applicable for materials, for which the yield strength is less than 400 mpa.

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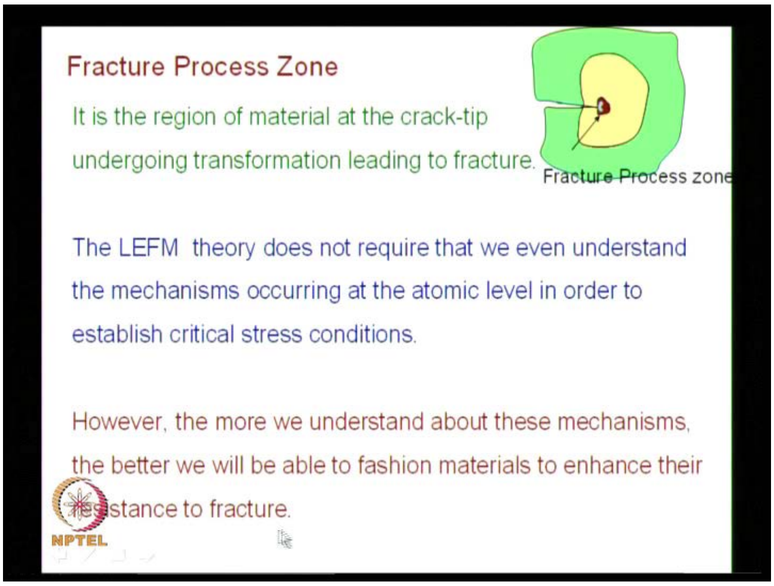

So, you have to apply some of these conditions only for high strength alloys, alloys which have yield strength greater than 400 mpa. And what is the fracture process zone? We have also seen that in the picture, it is a region of materiel at the crack-tip and undergoing transformation leading to fracture, to aid your visualization this picture is shown again, you do not have to sketch the picture, just observe that the fracture process zone is within the singularity dominated zone and if you relook at the LEFM theory it does not require that, we even understand the mechanisms occurring at the atomic level in order to establish critical stress conditions.

Having said that you should also realize however, the more we understand about these mechanisms, the better we will be able to fashion materials to enhance their resistance to fracture. From LEFM point of view it is not required for you to worry, but from fracture mechanics point of view and understanding will definitely help you to improve the material that you want to use for such applications. And people have indeed developed, there are certain alloys which would change their from body centered cubic type two face centered cubic, and thereby stiffen and they will able to with stand very high level of stresses.

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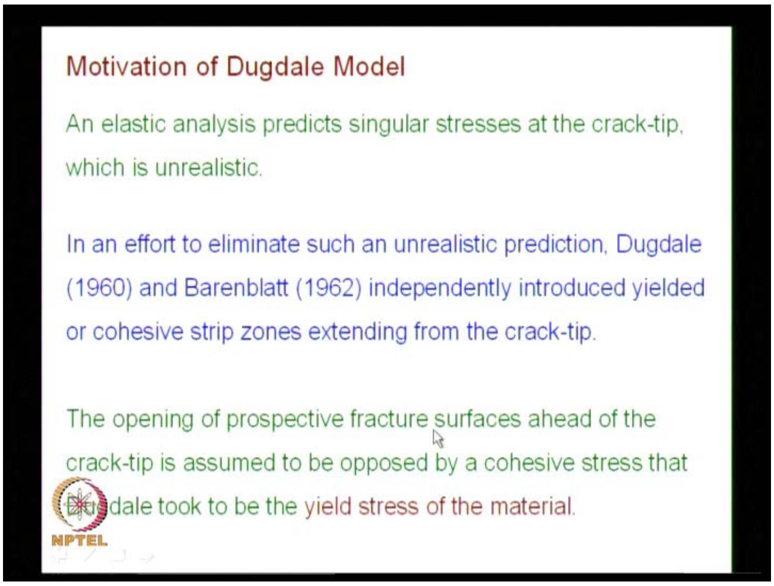

Now, what we will look at is, what is the motivation of Dugdale model? See, Irwin's model was very simple and straight forward, there has to be a need for another type of model and what way Dugdale have has approach. See, an elastic analysis predicts singular stresses at the crack –tip, which is definitely unrealistic, we did not have a choice so we were doing it not only that elastic analysis was very simple to do.

So, in order to capture some of these aspects Dugdale in 1960, and Barenblatt in 1962 independently introduced what are known as yielded or cohesive strip zones extending from the crack-tip. A very popular, yield strip model is a very popular model in fracture mechanics literature and this was advanced by Dugdale as well as Barenblatt.

The opening of prospective fracture surfaces ahead of the crack-tip is assumed to be opposed by a cohesive stress, and what did Dugdale do? He took that stress to be yield stress of the material, when we develop the Dugdale model we will also see he would make several assumptions, it is applicable for plane stress, and the material obeys elastic perfectly plastic, when all these conditions are satisfied his model reasonably predicts what happens at the crack-tip.

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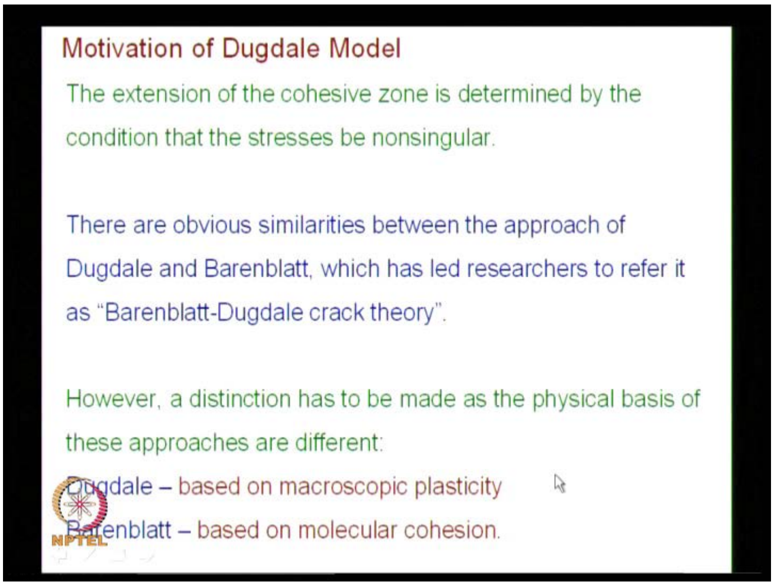

The extension of the cohesive zone is determine by the condition that, the stresses be nonsingular so, what I would appreciate is you go back and brush up whatever, the kind of stress intensity factor that we have developed for a crack subjected to symmetric loading. We will use that approach for us to find out the plastic zone length in Dugdale model, you need that expression. Go and brush up that, and come for the next class.

We will also have to note there are obvious similarities between the approach of Dugdale and Barenblatt. Which has led researchers to refer it as Barenblatt Dugdale crack theory, there is also an opinion people do not accept this, we will have to see, why?

Because you have to make a distinction on the physical basis that, they have used. The physical basis that, they have used in these two approaches of Dugdale and Barenblatt or different, you cannot argue that the final result are similar, both are same. Dugdale approach is based on macroscopic plasticity theory and Barenblatt is based on molecular cohesion.

So, in this class what we have looked at is, we have looked at approximate shapes of plastic zone at the crack-tip for mode 1, mode 2, and mode 3, and I pointed out only for the mode 3 situation, you have the plastic zone shape as circular. In some of the early books people have extrapolated the circular shape even for a mode 1 loading for discussion purposes, which is strictly not correct.

Even the approximate shape what we have got, is only an approximate, in that you have to keep in mind, because you have to do detailed calculation to get the shape, you have to do redistribution of load. And we have seen in the case of Irwin's methodology, how to find out the extension of plastic zone along the crack axis, based on that he also provided what way the original crack-length needs to be modified.

Then we moved on to understand what are SSY approximations and finally, we looked at what is the motivation of Dugdale's model, thank you.