**Engineering Fracture Mechanics**
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**Module No. # 02**
**Lecture No. # 09**
**Fracture Strength by Griffith**

In the last class, we had developed the equations for strain energy stored in slender members subjected to axial load, torsion, then bending. The expressions you had already learnt in a course in strength of materials. Nevertheless, we looked at what is the nature of those expressions, and in fact towards the end of the chapter on energy release rate, we would use them to find out the energy release rate.

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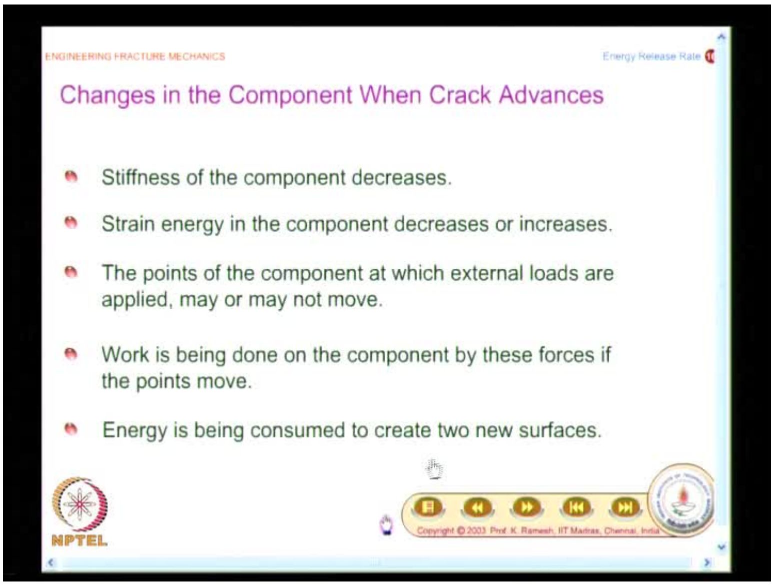


We would take up simple problems, where the crack is long enough, so the portions of the object can be considered as combination of slender members, the crack put demarcate the components as slender members, and we would use the expression for energy stored because of the external loading and evaluate the energy release rate from fracture mechanics point of view. And we have also looked at and asked the question what are the changes in the component when crack advances.

One obvious change all of us could recognize easily is, stiffness of the component decreases, and then we moved on to look at strain energy in the component decreases or increases. This, also we had a look at the problem of double cantilever specimen, and we were able to see for two extremum conditions of constant load and constant displacement, we would look at them again. And the other issue what we will have to keep in mind is, when you are having a general situation, the points of the component at which external loads are applied may or may not move.

So, in one case we had looked at constant load, where the points were moving. We have also looked at fixed grip, the points will not move and it is obvious, work is being done on the component if the points move, and another important aspect what you will have to have to keep in mind is, energy is being consumed to create two new surfaces.

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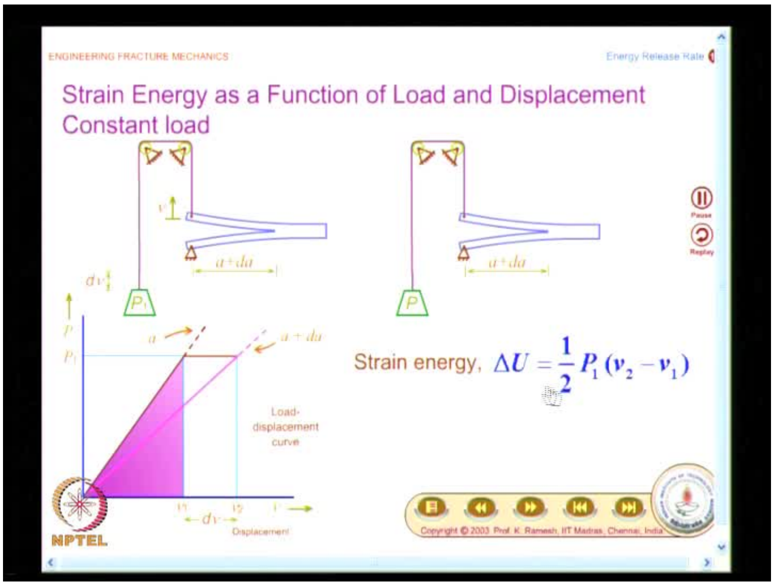

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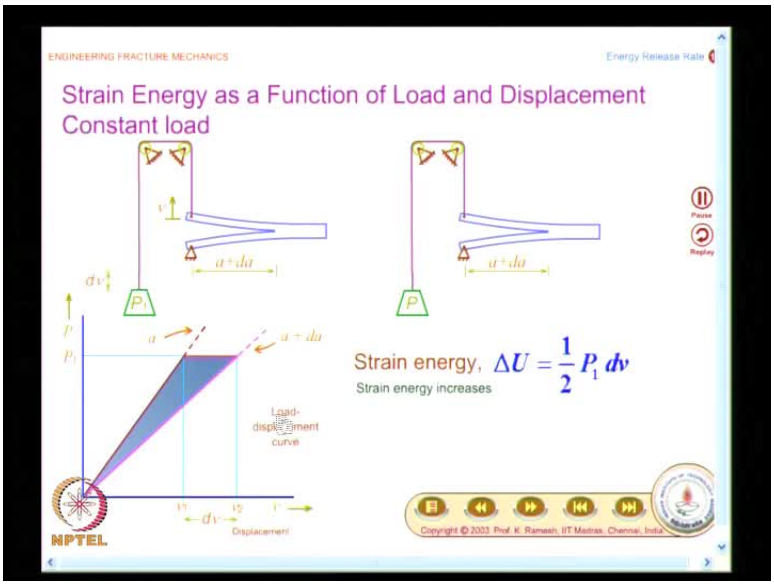


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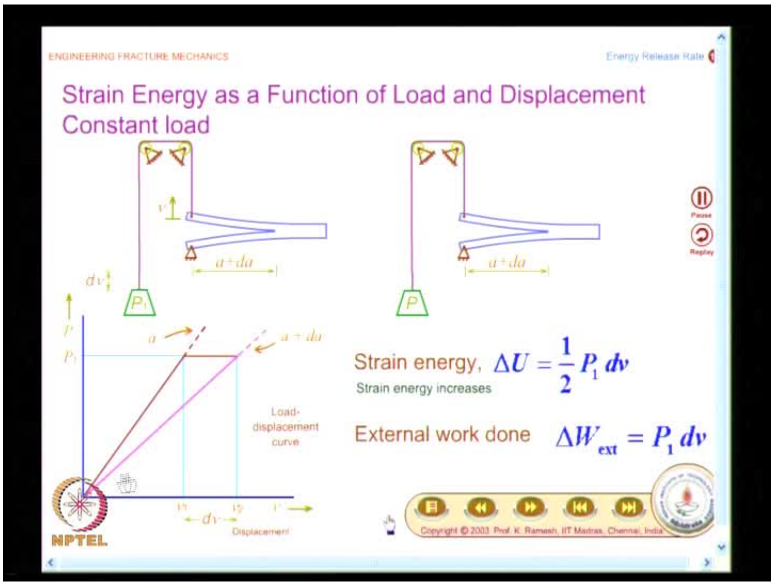


This is the key statement in the Griffith's approach to fracture mechanics, he recognized the formation of two new surfaces requires energy; there is inherent resistance persisting in the material and formation of crack requires two new surfaces to be formed. They consume energy and this energy has to come from some source, and for the purpose of analysis, we have looked at two extremum conditions: one is the constant load, we will look at the animation, we had seen that in detail in the last class, just observe it, as the crack is long the stiffness is reduced, and you find when the load has reached the value of P1 crack has advanced. And we calculate what a change in strain energy is and you find the change in strain energy is nothing but this triangle. And here, the strain energy increases when the crack advances and you should also note that, the load has moved a finite distance, dv. What is the external work done that is the rectangular area, what you find here and what is the energy available for crack growth that is a triangular area. And it is simple arithmetic that would tell you that, the work done is twice the strain energy available for the formation of two new crack surfaces and in this case, the strain energy increases, because of external work done.

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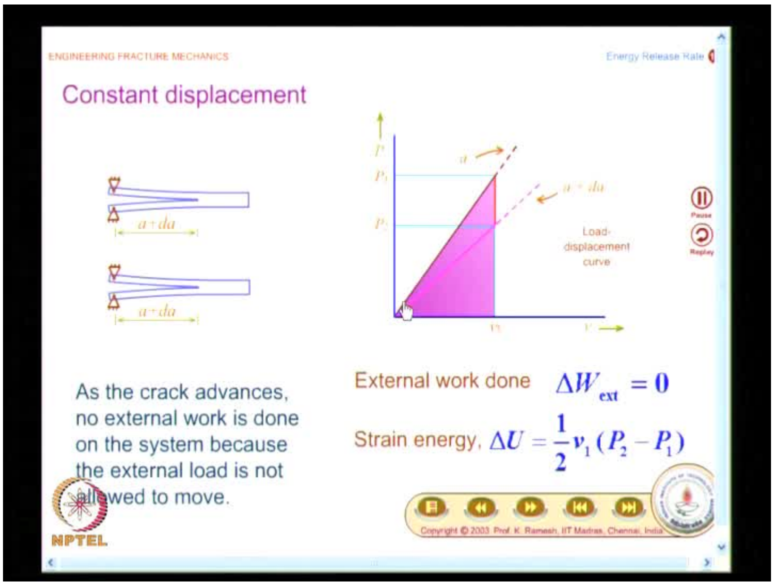


And we have also looked at the other case of constant displacement and here again you find, when the crack is longer the stiffness of the component decreases, so you have the line corresponding to a plus delta a is below the case for the crack length a. Watch the animation as the crack advances under constant displacement, and you find the work done by the external load is 0 because the points do not move. And what is the strain energy change? This is the final strain energy, this is the initial strain energy, so the change is what you find here, and in this case the strain energy decreases (Refer Slide Time: 05:40).

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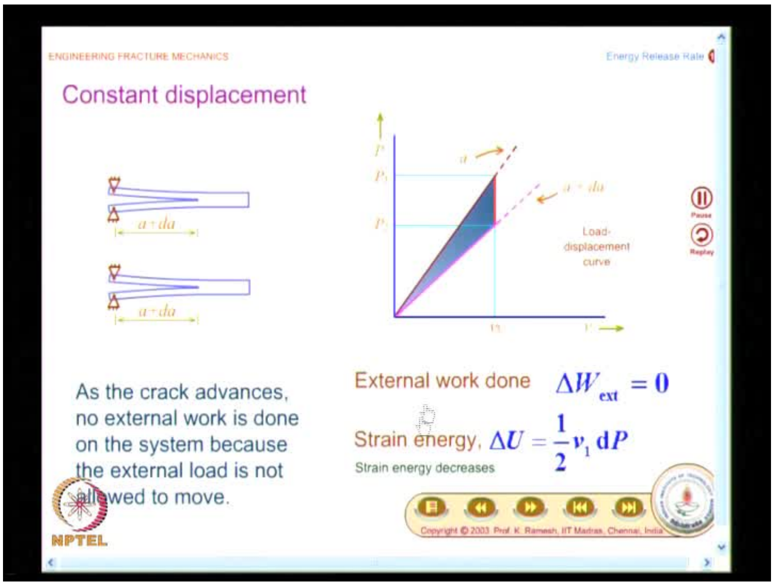


So, you have to keep this in mind, I had also mention the use of constant load or constant displacement is convenient from the point of developing the mathematics and also convenient to interpret the results from an experiment. I can also calculate the strain energy release rate, once we learn it in this chapter from a graphical approach, and we have also made a statement in the last class, that in a generic loading you have a combination of constant load and constant displacement.

So, it becomes convenient for you to have the results as a ready recover, what happens in constant load, and what happens in constant displacement, and what is a challenging question, that we have now is, suppose I have a component with the crack how to find out the strain energy in the presence of a crack, and what did Griffith do? Griffith took the problem of a central crack following the example of a plate with the circular hole, a plate with the elliptical hole; he considered the problem of an infinite plate with the central crack subjected to uniaxial tension.

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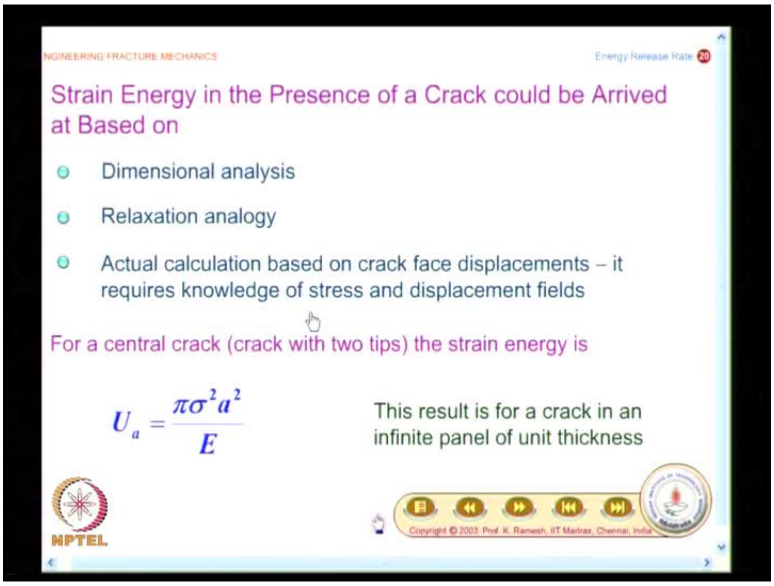


Now, the question is how to find out the strain energy in the presence of a crack; one simple approach could be use dimensional analysis, a second possibility is go for a relaxation analogy, and the third possibility is perform actual calculations based on crack face displacements, that would be the most accurate way to find out the strain energy in the presence of a crack. But right now, we do not have the necessary quantities to do that kind of a calculation, if you have to do that, we need to know the knowledge of stress and displacement fields in the vicinity of the crack-tip.

Once we take a crack-tip stress and displacement fields, we would develop displacement as well as stress field then come back and do these calculations again, and satisfy our self that, we could find out the strain energy in the presence of crack, from a mathematical stand point accurately. For the current discussion, we will look at for a central crack, and keep this information important, crack with two tips this is also very settle point, but equally important. The strain energy is given as U suffix a equal to pi sigma squared a squared divided by E.

This is for a central crack and once you say central crack it is crack with two tips, and whatever the result that you have seen here is for a infinite panel of unit thickness, so it is multiplied by unity, and if you look at from dimensional point of view, this satisfies the expression for energy, and the suffix a denotes strain energy in the presence of a crack.

Now, what we will do is, we will look at a relaxation analogy to satisfy our self how this expression could be viewed at but ultimately, you will be able to convince yourself well, only when you do the actual calculation, this we will postpone it for the moment, nevertheless we will have a look at it.

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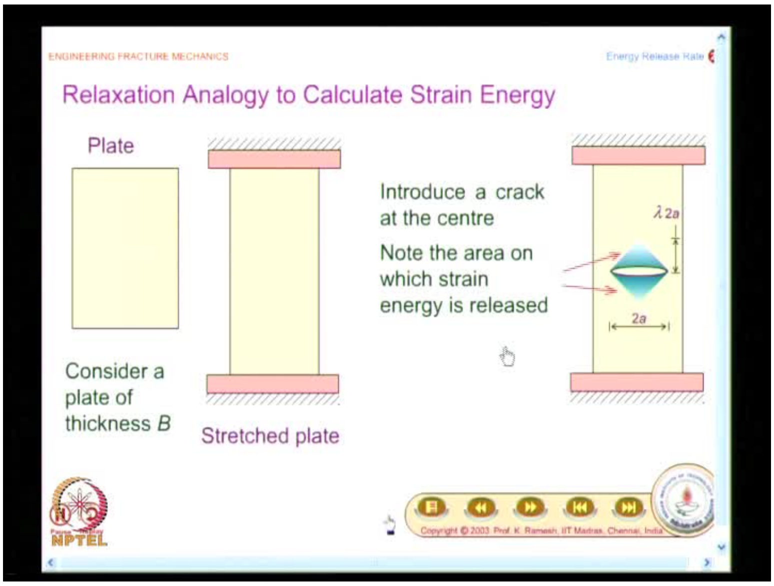


Now, let me look at the relaxation analogy and what I do here, I take a plate of thickness B, because I want to introduce in fracture mechanics literature rightly or wrongly, they have use the symbol capital B to denote the thickness. For the purpose of your understanding you could consider that as a rubber plate, you know you could take a cycle tube and then cut it and then make a sheet out of it, and what do you do is, you take it and stretch it, and then clamp it rigidly.

Then, what I do is introduce a crack at the center, remind you whatever we discuss it is meant for a brittle material, but for visualization purpose, it is convenient for you to take a rubber sheet and then look at what will happen. Suppose, I introduce the crack what do you anticipate up to some extent, the crack will remain stable beyond a length, the crack will propagate and the strip will separate itself.

This is you can visualize physically for the formation of crack, you need to have surfaces to be formed and energy is required to form the surfaces; this energy would have come from the strain energy of the system. And what is the kind of situation we are analyzing? We are looking at fixed grips, so there is no external work done.

We will just look at what way the crack is put and for the formation of these two surfaces some energy would have come out of the strain energy of the stretched plate, and we will have to calculate this quantum of energy. We will make a simple calculation and to make our life simple, we consider a triangular area in the neighborhood of the crack is what is releasing the energy to form the two new surfaces, it could be any shape. This is just to illustrate a simple calculation methodology to find out strain energy in the presence of a crack.

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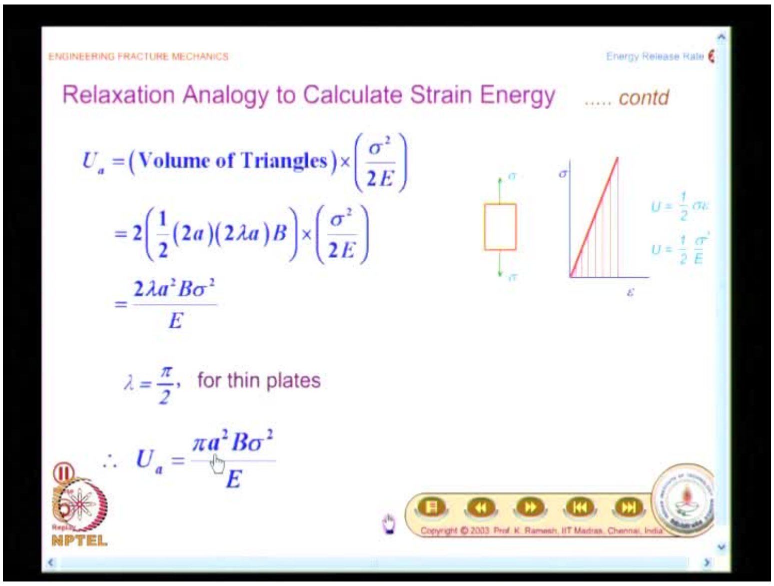


We would find out the calculations, we have already looked at the form of the strain energy in the earlier examples, where sigma squared by 2 into young's modulus, this is multiplied by the volume, and here volume of the triangles. I have two triangles, so I have two here, half BH, we have assume the height as lambda times a, and lambda is used just as a proportionality constant here, it is not denoting a wavelength, because we are accustom to seeing lambda as wavelength, this is just used here as a proportionality constant and when you multiply this, you get an expression 2 lambda a squared B sigma squared divided by E, see we are only doing a relaxation analogy, the idea is to feel that strain energy would get released from the system given to the cracks to form to illustrate that we have looked at relaxation.

We can be satisfied only when you do the mathematical calculation completely on the displacements of the crack phases, as well as the stress feel associated with it, that would require some more time for us to do that. Right now, we only look at an analogy and we also bring in one more aspect, we have taken a thin plate, we are really considering a plane stress situation and for plane stress situation, lambda is taken to be pi by 2.

This also will get clarified, when we actually do the calculations based on the displacements and the stress field. So, when I do this, I get this as U a equal to pi a squared B sigma squared divided by E. The earlier expression, we saw was for a plate of unit thickness, now you have a thickness of B and what do you see here, the strain energy in the presence of a crack is related to a square.

So, when the crack length changes, the energy would change as a parabola, second degree curve. And now we will look at for the formation of two new surfaces what way we require the energy, we look at these two natures we can understand, there could be a crack growth phase and a catastrophic crack propagation, we would try to plot them.

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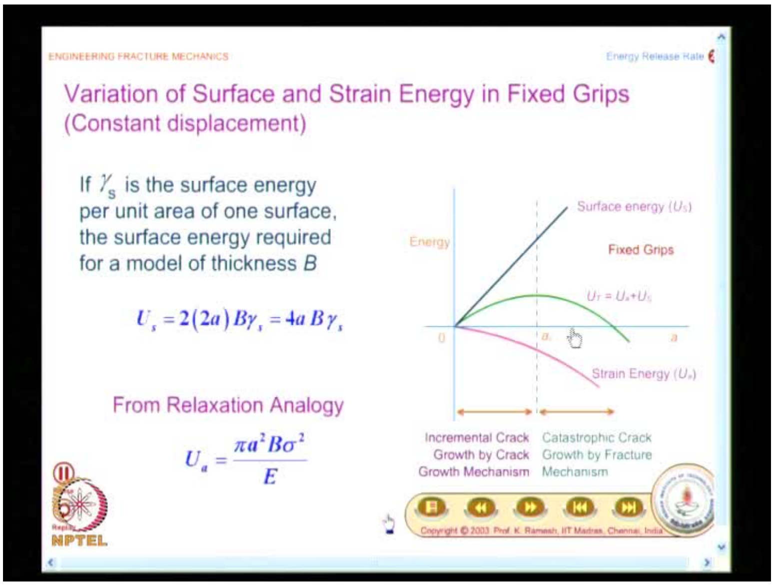


So, this is what I have here, what is a variation of surface and strain energy in fixed grips, you should not forget this, we are really looking at fixed grips; fixed grips means constant displacement, and we have gamma s is the surface energy per unit area of one surface. Then the surface energy required for a model of thickness B is 2 into 2a into B into gamma s and this we denote as U suffix s, and if you look at this final expression, it turns out to be 4a into B gamma s, it is a linear function of the crack length, as the crack length changes this would vary linearly and we would try to plot this variation in a graph, where the X axis is taken as the length of the crack, and Y axis is taken as a energy this would be obviously a straight line, we will plot that.

So, I have this as a function of crack length you find that this as a straight line, this denotes the surface energy, we have already got the result of what is the strain energy in the presence of a crack, which is a second degree curve. You have already got that as U a equal to pi a squared B sigma squared divided by E.

This is plotted in this fashion please note that, this is only a schematic it is not drawn with actual values, it is only a schematic to illustrate the nature of the surface energy, the nature of the strain energy. Suppose, I find out what is the total that would change in this fashion that would change like this, the slope would become 0 at a particular point, that demarcates the region from stable crack growth to fracture. You get the critical crack length from that graph this will denote the critical crack length, so you have a region where it is incremental crack growth by crack growth mechanism, and in this zone it is catastrophic crack growth by fracture mechanism.

This is given pictorially, you know we would like to have this quantitatively, so we will look at it from a mathematical stand point, but I would like you to make a neat sketch of this diagram, this is very illustrative of what is really happening, so you are able to find out a critical crack length beyond which the crack would propagate faster, and what we are really looking at is the incremental change in crack length to occur, so we are really looking at that the incremental energy requirement should be satisfied. Mathematically, how will you write it? The change of strain energy in the presence of a crack should be equal to change of surface energy with respect to the crack.

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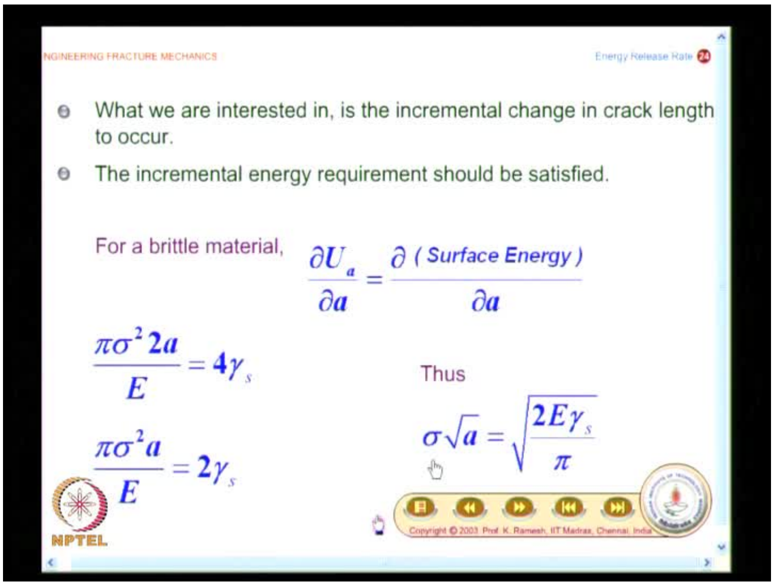


So, dou U a divided by dou a equal to dou of surface energy divided by dou a, this is the kernel of a Griffith's approach to fracture mechanics. He said, the formation of two new surfaces requires energy and this energy since we are looking at fixed grips comes from the strain energy.

When this is satisfied, the crack can grow and now you have to be very careful. See you may be tempted to differentiate with respect to dou a, what it really denotes is length of the crack, in the numbers that we have got, the expressions that we have got whether you would differentiate with respect to a or differentiate with respect to 2a, because you are going to do it on the left and right hand side, the end result would be same.

If, you differentiate it with respect to a, you will get this as pi sigma squared 2a divided by E equal to 4 gamma s. If you differentiate with respect to 2a, you will get this as pi sigma squared a divided by E equal to 2 gamma s. In fact, if you simplify this, you will come to this step, but do not differentiate with respect to a because I have already emphasized he considered the problem of a central crack the moment you look at a central crack you should recognize, that it has two crack-tips. If you have to interpret these equations properly, you have to differentiate with respect to 2a, so the caution is do not do it like this, do it like this, even in the definition of energy release rate, we will bring in the energy per crack-tip that is very important.

So, we get the final expression as pi sigma squared a divided by E equal to 2 gamma s, this could be rewritten as sigma root a equal to 2E gamma s divided by pi, a very very important expression and symbolic step by Griffith in advance in fracture mechanics, because he has been able to find out the fracture strength in the presence of a crack, you could also look at this expression differently as sigma equal to root of 2E gamma s by pi a.

I have taken out a in this left hand side, so it appears as sigma root a, and you know once you develop an expression like this it has to be experimentally justified, otherwise you know you will not be satisfied with the mathematical development. What is the kind of experiment with Griffith performed to establish this? Griffith carried out a series of experiments on glass tubes and spherical vessels subjected to internal pressure, p.

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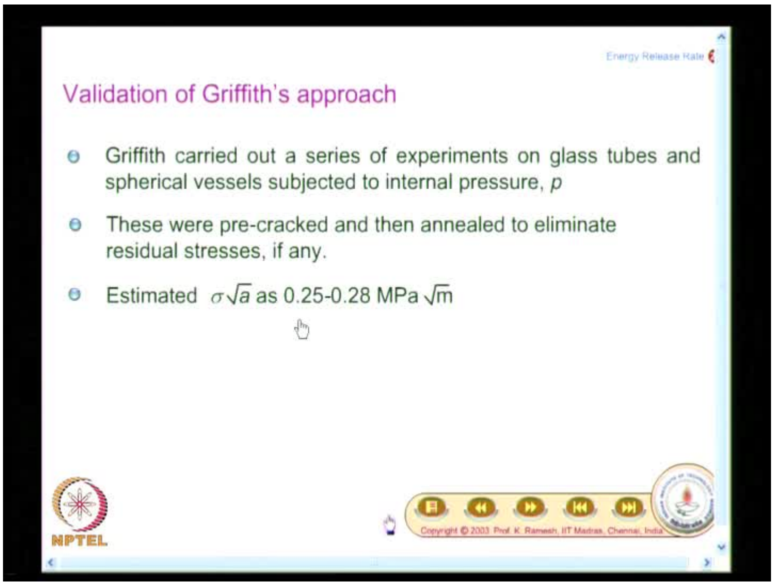


And you should keep in mind almost all the work of Griffith was confined to brittle solids, so the energy release rate concept was originally developed for ideally brittle materials, later on this was extended by Irwin and Arowan for high strength ductile solids, until we do that all our discussion confined to brittle materials. What is the advantage of a brittle material? We have already seen, that crack could heal in a brittle material such things have been observed, we have also seen one such example and the advantage is when you are approaching from the energy point of view, you are looking at a conservative system, so energy method is easily applicable. When you say the glass tubes and spheres, they were pre-cracked and then annealed to eliminate residual stresses, if any.

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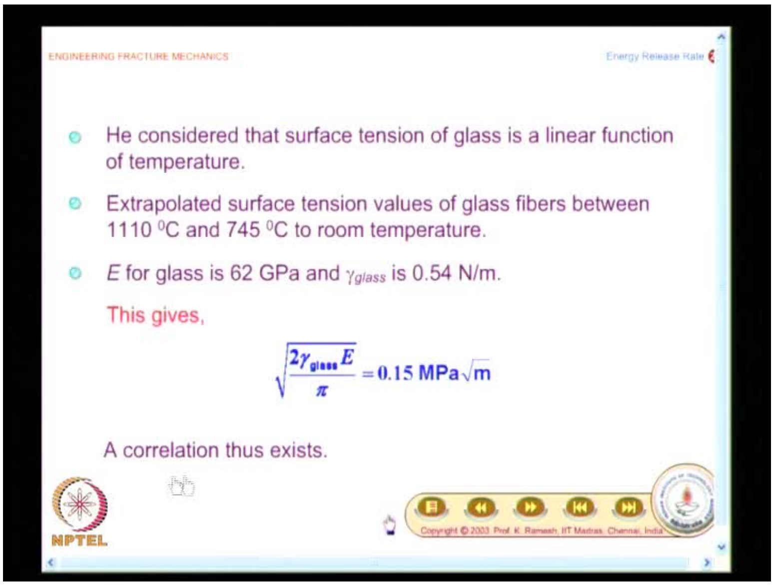


So, you had taken sufficient precautions to performing the experiment, and he increase the stresses, and he estimated what is the value of sigma root a, which has found to be in the range of 0.25 to 0.28 MPa root meter have a look at this value he has got some value in the range of 0.25 to 0.28 and this he has got the left hand side expression, sigma root a has been able to get it. For you to get the right hand side expression you need to get the value of surface energy; surface energy in glass has to be estimated you know you will have to congratulate Griffith for having taken the paints to perform such difficult experimentation.

He considered, that surface tension of glass is a linear function of temperature good approximation you know, this is how engineers always approximate, we are very close lines we always want to had take anything as linear, if it satisfies our requirement we leave it at that, we want to avoid as much of nonlinearity as possible and what he did, he extrapolated surface tension values of glass fibers between 1110 degree centigrade and 745 degree centigrade to room temperature, from that exercise he could estimate the surface energy of glass as 0.54 Newton per meter and E for glass is about 62 GPa, and when you want to evaluate this expression, root of 2 gamma glass into E divided by pi gives you 0.15 MPa root meter.

Is it same as what he had got in his experiments, it is not same when students perform experiments these days and compare it with theory for fear of experiment being rejected, they report only a difference in the second decimal place. I have been mentioning, when you are performing an experiment you have to learned to report the results that you have got faithfully experimental is have to be extremely honest, because the values may carry more information than what you had originally thought of.

We have seen, in the case of fatigue experiment lot of scatter that is a well performed experiment. So, do not think that when you compare theory and experiment they will be very close unless, you are taking a benchmark problem, and theory is very well understood, and also the experiment is very carefully performed you will not have that kind of a correlation. Nevertheless, if you look at the order of magnitudes in this case, a correlation definitely exists.

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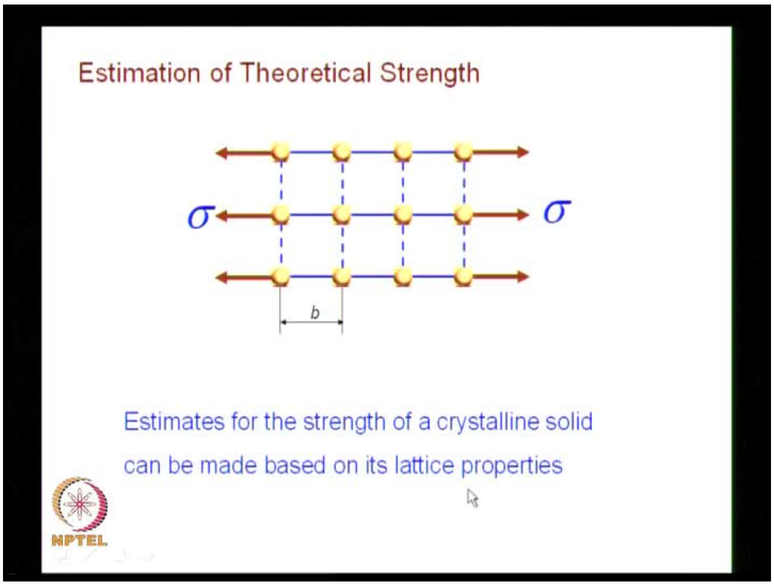


This gives you a confidence the way you have developed the expressions, sigma root a equal to 2 gamma s E divided by pi, it is a meaningful step, but we will also look at in what context Griffith had really approach this problem, he had not in his mind that there is fracture mechanics that he has to develop, he had not approach the problem from that perspective, his perspective was different. You know scientists were really looking at how to estimate strength of the material from a theoretical stand point.

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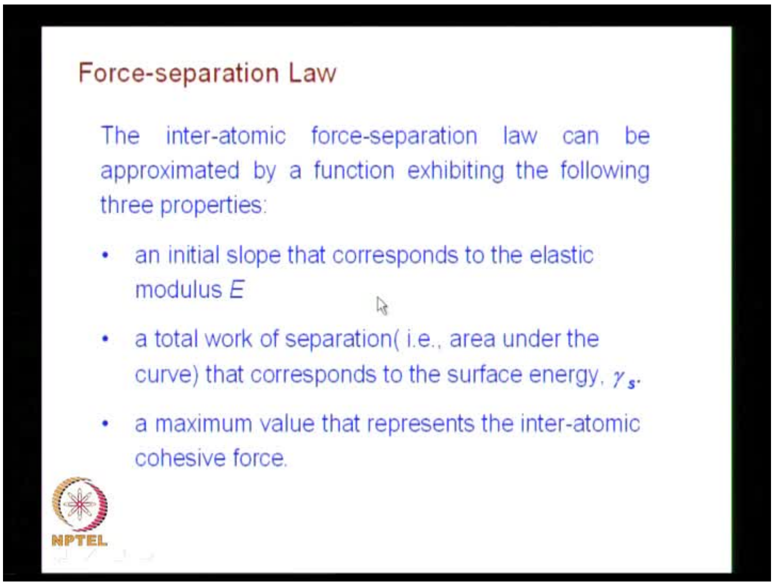


So, you have an atomistic model you say it is a crystalline solid make a sketch of it, and you have this equilibrium position is given by the lattice spacing b. And you want to find out, at what value of this stress would there be separation. So, the idea is, estimate the strength of a crystalline solid based on its lattice properties. And for this, you need to go for a force-separation law and how this is point, the inter-atomic force separation law can be approximated by a function exhibiting the following three properties.

You know, I am going to present this without much of mathematics we will only look at the final results. In fact, people who work on cohesive zone modeling really play with different types of law, and then also incorporate it in the finite element modeling and analyze, how the plastic zone develops in an near the vicinity of the crack, they do such kind of activity but we will only look at salient features of it.

Our interest is to see, what kind of result people have got based on lattice property calculation. So, if I want to have the force-separation law I need to consider the law such that, the initial slope corresponds to the elastic modulus E of the material, whatever the material that you are looking forward to, and the second aspect is a total work of separation, that is area under the curve corresponds to the surface energy gamma s.

So, what you will have to look at is, people who are making theoretical calculation have already brought in, the role of surface energy in some other form. So, this kind of thinking was there at that time. So you need to have the force-separation law to have an initial slope that corresponds to the elastic modulus E.

Total work of separation that is the area under the curve corresponds to the surface energy gamma s, and finally, you should have the amplitude a maximum value that represents the inter-atomic cohesive force. So, you can obtain a force-separation law if it exhibits the following three properties.

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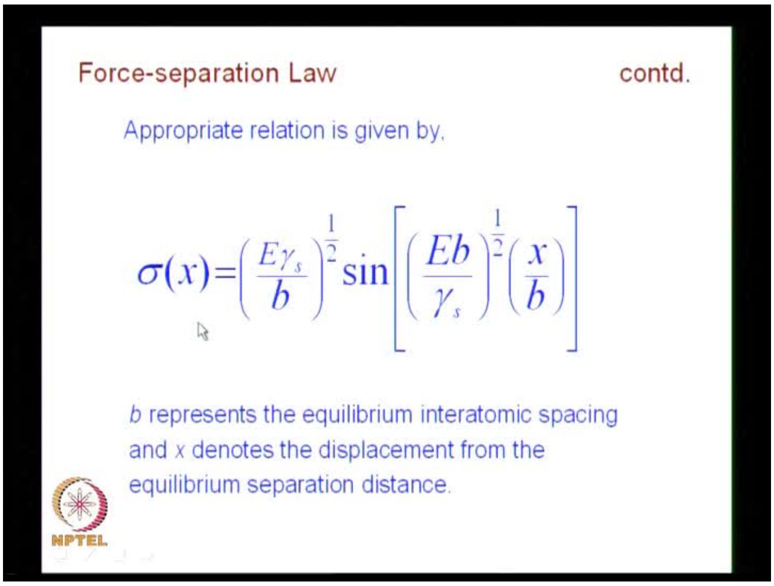


So, based on this the law appears in this fashion. Sigma function of x is given as E into gamma s divided by b whole power half multiplied by sin of E into b divided by gamma s whole power half into x by b, Where b represents the equilibrium inter-atomic spacing that is what you had seen in the sketch, and x denotes the displacement from the equilibrium separation distance.

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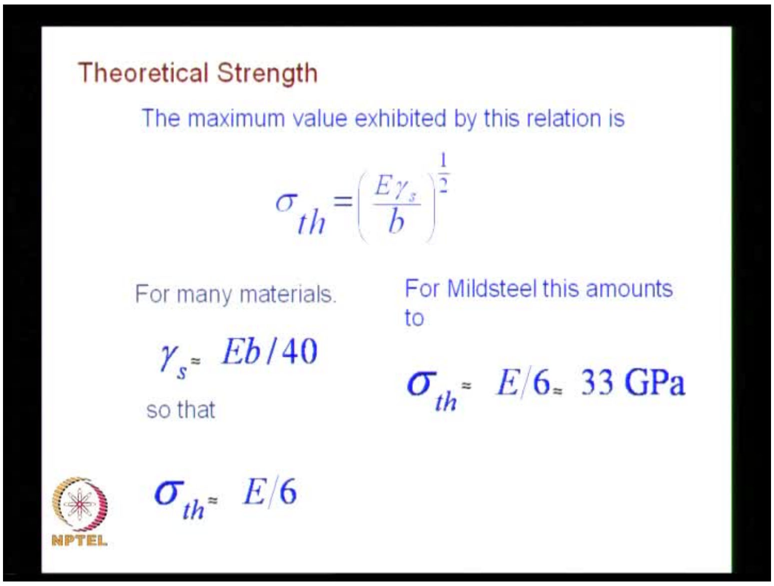


So, now we have an approximate relation which gives you the force necessary for separation, and what is the maximum value, is given by this; so the maximum value is nothing but the theoretical strength, the theoretical strength is given as E into gamma s divided by b whole power half. See we would be able to appreciate this only when we look at the actual values, from a lattice point of view you can calculate the theoretical strength, from experiments you find the actual strength displayed by the material and if we had understood the material behavior correctly my theoretical calculation should more or less coincide with experimental observation.

We will have to see, whether this is so or not, people have also found out what is the value of a gamma s for many materials it is found to be approximately Eb divided by 40, so when you substitute this into this expression, you get a value for theoretical strength. And what is the theoretical strength? Should be like E by 6 in fact it is very high, E by 6 is not a small value. We will look at for mild steel, because mild steel everybody knows although we may not apply fracture mechanics to mild steel to dispute the theoretical strength calculations are way off when you compare with experimental results, mild steel is a good example.

In the case of mild steel, you have young's modulus as 200 GPa, so 200 by 6 would be something like 33 GPa, mild steel is no where near this if you look at, its yield strength is something like 220 MPa or so, it is ultimate tensile strength would be something like 320 or 400 depends on the material composition, that is all in terms of megapascal whereas, theoretical strength is in terms of gigapascal, that means the theoretical strength calculations are way off why this is so.

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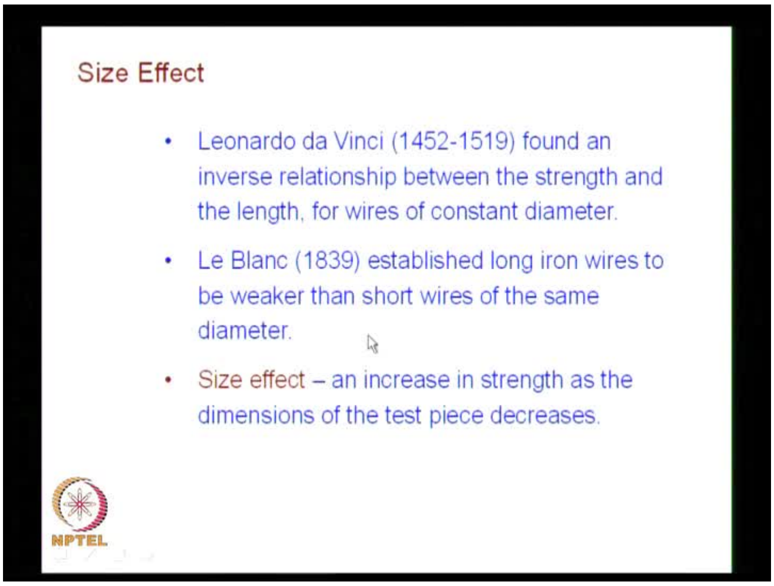


You know this is a puzzle, people wanted to resolve this puzzle for a very long time, and we will look at what are the kind of issues people based on experiments coin this as a size effect, they had no clue why the theoretical strength is far higher than what is observed in experiment. But in experiment they saw something very interesting, that can be traced way back to Leonardo da Vinci, he found an inverse relationship between the strength and the length for wires of constant diameter.

See, there are two issues that we will have to look, at in the case of conventional spring design if you look at the design books, you will find the yield strength for the same material when it is used as a wire is much higher than a bulk material and this is one aspect. There is another aspect, which was noted by da Vinci, what is that was the length if the length is shortened, the strength increases, it is an experimental observation.

See we need an explanation for this, remember experiment is truth if the experiment is carefully performed and the results are also honestly reported then, experiment is true. You have to find out reasons for such a behavior, and later you find there another person Le Blanc in 1839, he also established long iron wires to be weaker than short wires of the same diameter.

So, people are finding there is something to do with the size and they are generalized it as a size of it, and they reported an increase in strength as the dimensions of the test piece decreases, the two drastic conclusion. Now, this kind of a conclusion we cannot make it now, so in those days when they were struggling to find out, why the theoretical strength was not achieved in actual practice, but they observed when the size of the specimen decreases the strength increases. They have noted this, they needed an explanation for this and this was the aspect which Griffith was concerned about and he felt that, the size effect is actually a crack size length effect.

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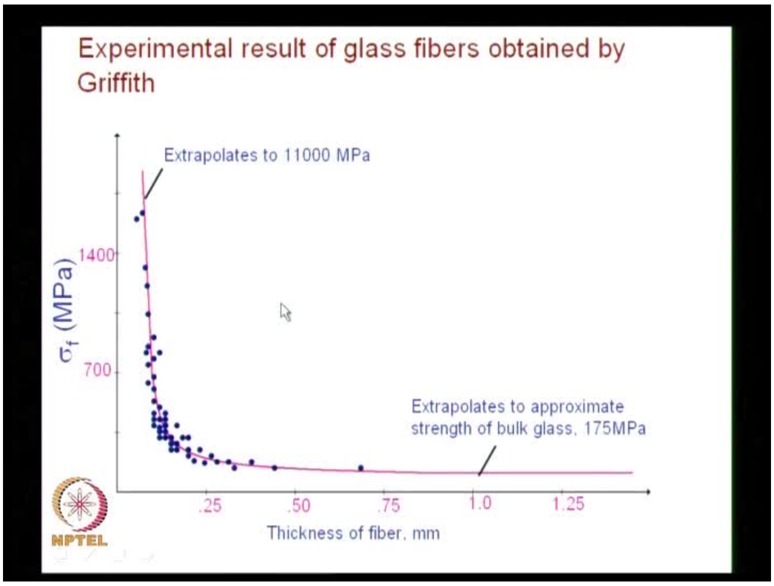


So, what he did was he performed result on glass fibers, carefully reduce their diameter and then he plotted thickness of the fiber versus the fracture strength and this is purely based on experiment, and this was exponentially decaying and it was asymptotically reaching the strength of bulk glass about 175 MPa.

When the thickness of the fiber is reduced, it was approaching 11000 MPa that is a theoretical strength of glass. And what is the difference? The difference between the two is, you have cracks of various dimensions exist, and if you draw the glass thinner and thinner the cracks diminish and glass becomes very strong. In fact, it is useful see, if you look at the development of composite it definitely hinges on this useful property, glass by itself is brittle, but glass fibers embedded in a resin works as a fiber glass composition very good, you have fiber glass boards, you have fiber glass mridangam and so on. So, you have a utility of glass in the fiber form better, when embedded in a resin.

So, what Griffith looked at was, in the case of glass fiber the presence of cracks have diminished strains, so he was able to provide a rational explanation why, when the size of the specimen diminishes, the strength increases. I would like you to make a neat sketch of this graph, it is very illustrative very important observation by Griffith, and you should also look at in those days when they were developing the theories if they had a doubt they had always verified by suitable experimentation.

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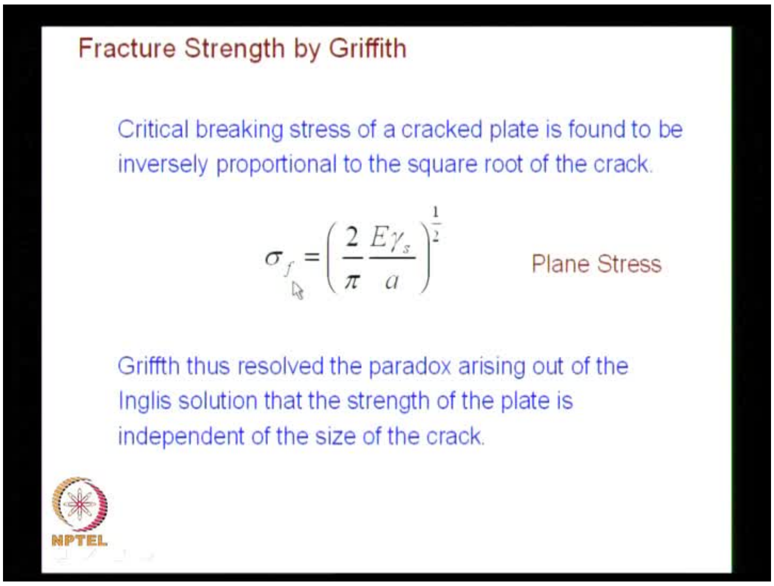


However, complex it may be, they have taken the trouble to do the experiment, because there was struggling to find answers for the questions that were burning at that time. And we have already developed this expression without getting into much of the mathematics.

The fractures strength by Griffith is as follows, it is 2 by pi E into gamma s divided by a whole power half and this is for a plane stress situation. And what is the other important aspect of this expression, see this expression can be looked at from various point of view.

One is he has related the fracture because of crack length, other way of looking at it is in the case of Inglis solution even a very small crack with very small external load, the stresses would be so high the specimen will break into pieces, but that is not so in actual practice.

So, that is a paradox the Inglis solution was useful, but utility of that solution is question, because people are unable to explain how actual structures reminds solid with cracks. So that paradox was resolved by Griffith at strength of the plate is independent of the size of the crack that was the immediate observation from Inglis solution, where as Griffith said the crack length does matter.

I can find out using this expression fracture strength, as well as critical crack length for a given stress value, so this expression is very important. Now, we have looked at from very simplistic analysis that, you had got a energy from strain energy for the formation of two new surfaces, and we were able to get the expression for fracture strength.

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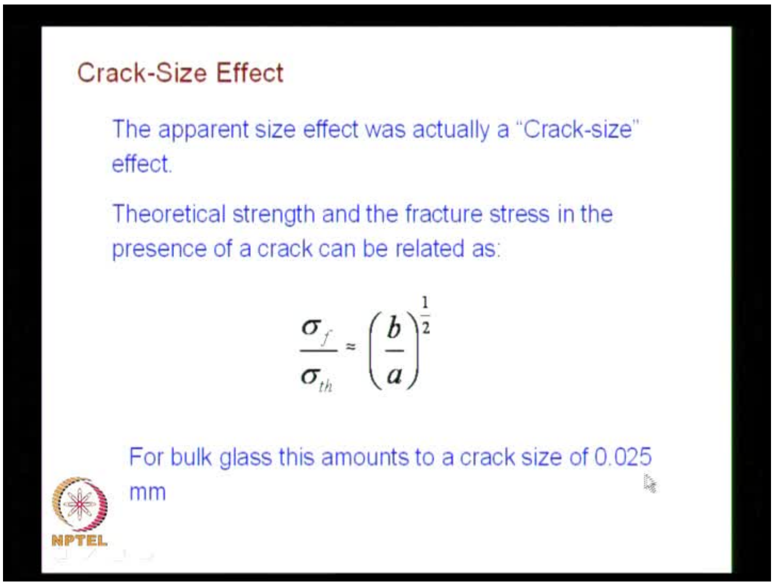


So, what Griffith concluded was, the apparent size effect was actually a crack-size effect. And if you look at the expression for theoretical strength as well as a fracture strength I can find out the ratio of sigma f divided by sigma th, that is approximately equal to b by a whole power half.

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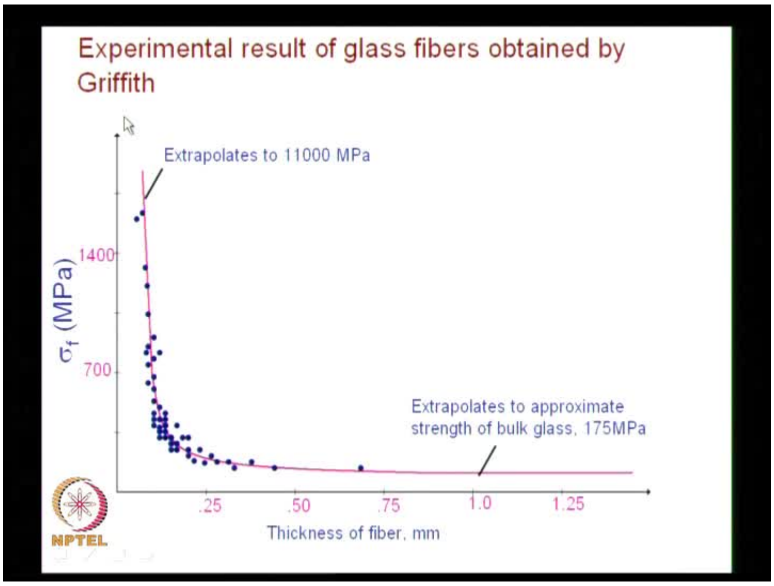


So, you can go and find out for a bulk glass what would be the size of the crack; the size of the crack is of the order of 0.025 millimeter because a bulk glass has this crack length its fracture strength is far below the theoretical strength. See, if you look at the graph we have noted that theoretical strength is around 11000 MPa, whereas bulk glass strength is around 175 MPa, because of the expression developed by Griffith. Now, we are able to say that, the crack length for the bulk glass would be around 0.025 millimeter. Now, you take a fiber of glass thinner than this, what happens when it is much below the size of that crack, you find the value approaches theoretical strength. Now, the story is complete. People were struggling why, when the size of the specimen decreases, strength increases.

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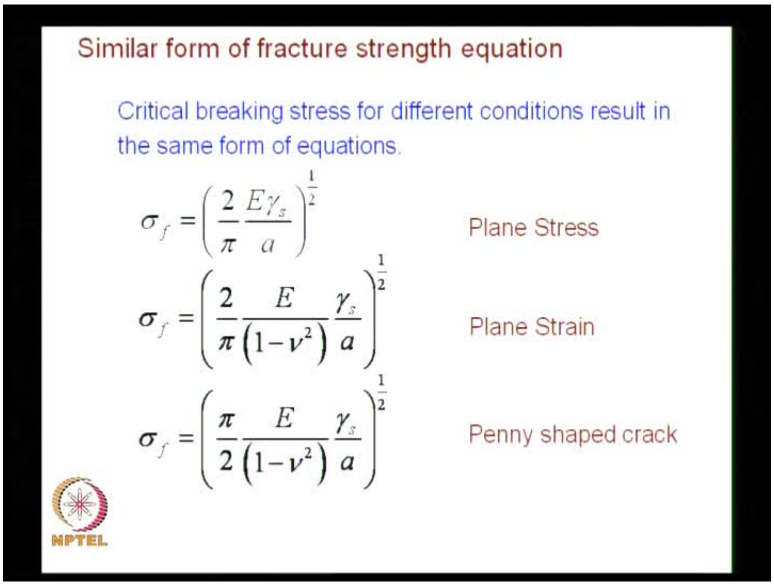


This was the problem with Griffith was focusing while developing is energy balance approach, he got the fracture strength and he could relate that length of the crack also plays a very important role. And people have worked on this kind of an approach and they have also found out the fracture strength for several other situations.

We have seen, fracture strength as 2 by pi into E into gamma s divided by a whole power half, for a plane stress situation. Suppose, I look for a plane strain situation how does the expression look like, there is only a small variation in this expression, what you find here is instead of young's modulus you get this as young's modulus divided by 1 minus nu square. In fact, you would come across this kind of modification in fracture mechanics, you will develop the expression for plane stress then, you can convert it to plane strain by changing young's modulus by young's modulus divided by 1 minus nu square, vice versa. We can do that and you get this for plane strain.

And people have also looked at another problem, you know when you have a three-dimensional solid, where you have a penny shaped crack the moment you have a penny shaped crack, people found the expression only in the numerical factor has changed the form remained same; you have young's modulus, you have surface energy divided by crack that remains same only the numerical factor was changing, and this is a very important observation utilized by Irwin to generalize crack problems.

So from the energy approach, Griffith identified formation of two new surfaces requires energy to be given from the strain energy of the system helped him to get the expression for fracture strength, which also explain the paradox of Inglis solution, also explained why actual strength of the material is far below the theoretical strength, and later on when people carried out for variety of practical situations, they found the similarity. So, it helps to develop fracture mechanics in the years to come, that is a very important contribution by Griffith mind you all his analysis were focus to ideally brittle solids you should never forget that, thank you.