Rocket Propulsion

Prof. K. Ramamurthi

Department Of Mechanical Engineering

Indian Institute of Technology, Madras

Lecture No. # 10

Nozzle Shape

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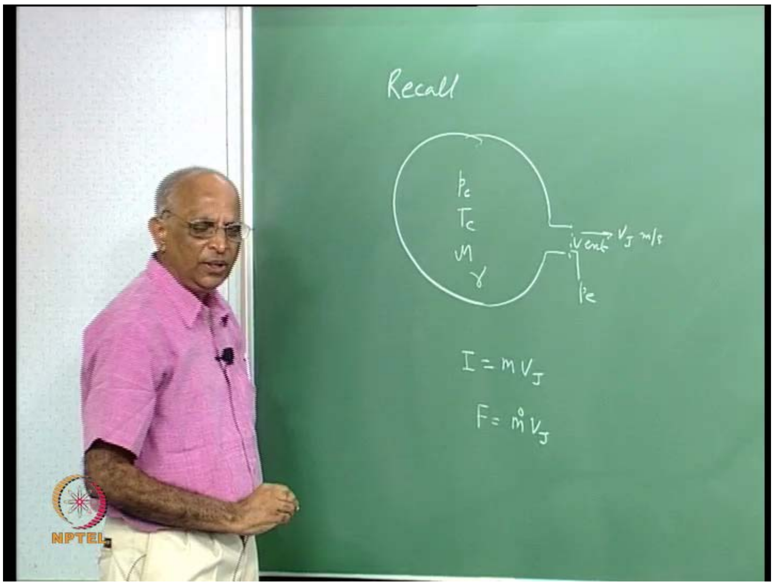

Well good morning, will continue with rocket nozzle today. If we recall what we did in the last class, we told ourselves if I have a chamber, let say any chamber, and this has a gas at a pressure p c, temperature T c, the molecular mass of this gas is m gram per mole. And I sort of expand this gas out through a small opening, which we called as a vent; we were able to calculate the exhaust jet velocity in meters per second. We found out the condition for which V J is quite large we told ourselves, well the temperature must be large, the value of the chamber pressure must be large, and the molecular mass must be small. We also examine the variation with respect to gamma, told ourselves gamma is not very influential, especially when p c is small or the ratio, if I say the exit pressure is p over here, when the value of p by p c is somewhat high or p c value is small compare to p e, then gamma is not influential; this is what we showed last time through the expressions which we derived.

Now, in a whenever we have a rocket, we told ourselves, well impulse is equal to mass, which is rejected into the jet velocity or rather we say that the force is equal to d by dt of I, which is equal to m dot into V J. Therefore, we are interested in a high value of this velocity with which the mass effluxes leaves the chamber.

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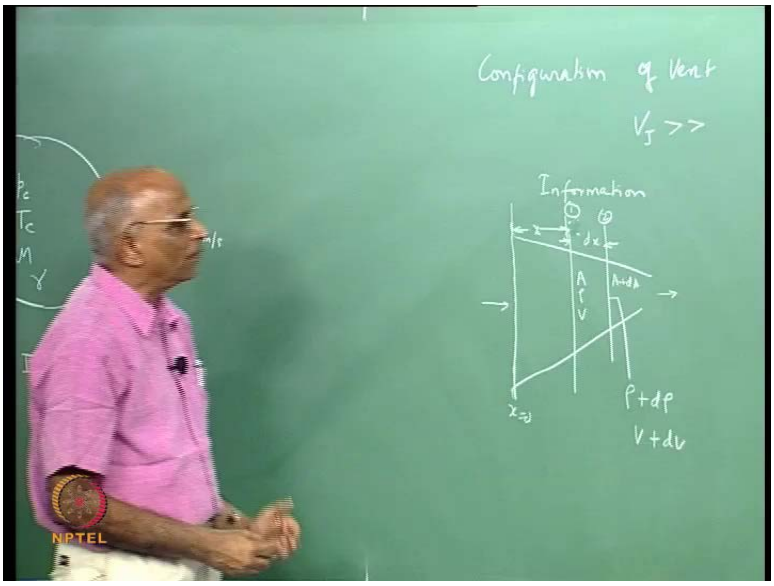

Therefore, the question is there anything like a configuration or shape of vent, which can give me a high value of jet velocity. And this is what I get started with, then we will go back and try to find out, what are the conditions for this high jet velocity. We will have to look at matching of the exit pressure with respect to the chamber pressure and that is what I proposed to do in today class. We will also physically try to understand, is there anything like some information transfer between the outside and inside of a chamber. Let's get started with this background. Let us first say, well I have a nozzle and this nozzle I am interested in the shape, therefore I say, well let me have shape like this. The shape is such that at a distance, let us say this is x is equal to 0, at a distance x from the initial origin; that means, this is x over here. Let say the area of the vent is A; let the density of flow through this be rho, let the velocity of flow be V, at this particular section x at a distance 0 from this reference plane, which is over here. Let us consider the variation in the properties at a distance x plus dx.

The area is different from A, because I want to find out the configuration of this particular vent which gives me the maximum V J; therefore I would be able to derive that I say, let the area at dx be A plus dA, let the density at this section be rho plus d rho and the velocity at this section V plus dv. My main aim is to find out what must be the shape, such that I get a high value of V J, therefore I just look at these two sections; let say section one, at which the area is a density is rho, velocity is V. at section two, dx away from this at which the area is A plus dA, small change in area, I think according to this figure dA should be negative, but I just have a general thing A plus dA, let the density be rho plus d rho, the velocities V plus dv. Therefore, now I say that the flow is constant, in other words whatever comes here is this I consider as steady flow through this particular opening or vent and when I consider as steady flow, I have the mass flow rate m dot in kg per second is equal to rho A V.

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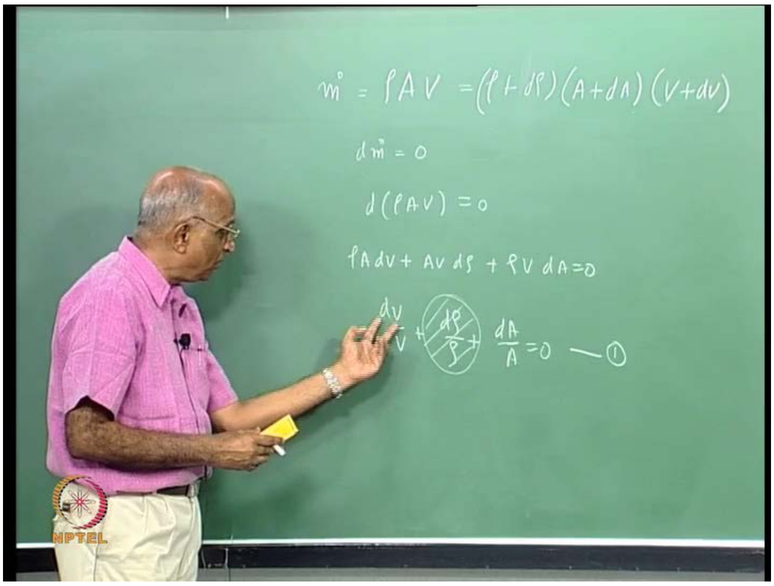

through the section one and the same thing flows through the section two, which is equal to rho plus d rho, that is density at the section two into A plus dA into V plus dv; and this is mass balance equation. Since the flow is steady, the same mass flow rate flows through this section and this section. Let solve this equation and what do we get? Mass flow is a constant or rather d m dot must be 0, because mass flow is constant. Therefore, I get the expression d of rho A V is equal to 0. And therefore, now if I expand it out I get rho A dv plus I have A V d rho plus rho V dA is equal to 0.

Or rather I divide this entire equation by rho A V and I get dv by V plus d rho by rho plus I get dA by A is equal to 0 and this becomes my mass balance equation in the differential form. I could have derive this expression by saying rho A V is a constant, therefore logarithm of rho A V is constant and differentiating this would have given me dv by V plus d rho by rho plus dA by a is equal to 0; which we call as the continuity or the mass balance equation. So, why did I have derive this. I want to it find out what is the change in area, which will give me high value of velocity and therefore, my aim is to relate dv by V with dA by A. Unfortunately, I am left with d rho by rho and I get d rho by rho, because the flow, I assume is compressible, the density is changing; and therefore, I am left with one term here, d rho by rho, which I should express every in terms of dv by V or dA by A to be able to find the dependence of velocity on the change in area. Therefore to do that I again go back and ask myself, can I write one more equation; let say the momentum equation. What does the momentum equation tell me? The momentum equation tells me, the rate of change of momentum must be equal to the impressed pressure or impressed force.

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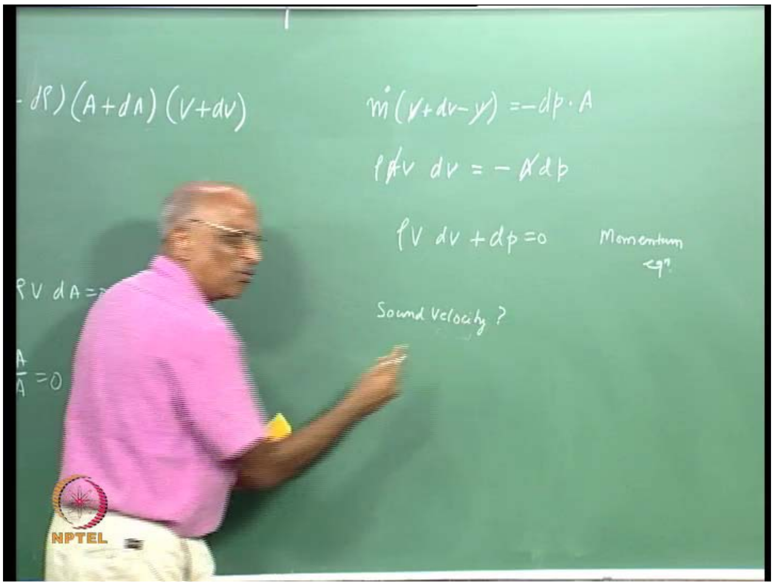

Therefore, let us write the momentum equation, the mass flow rate is m dot, the change in velocity is V, rate of change of momentum is, no the change of velocity is from V to V plus dv, that means dv minus V, that means m into V plus dv minus V is the rate of change of momentum.

And this is balanced by the force and what is the force what I get? I have the change in pressure across is dp and the area is A, I find dp is higher therefore, the force acts in the direction of change of momentum and therefore, I have mass into change of momentum is equal to the force. And what is m? m is equal to rho A V into V and V get cancel, dv is equal to minus A of d p, A and A get cancel and therefore, I get rho V dv is equal to or plus dp is equal to 0, which becomes my momentum equation or pressure balance equation.

See we must be able to generally derive the momentum equation by just applying the Newton's second law. Seeing well, I have a mass here which flows across here, the change in velocity is dv, the rate of change of momentum is m dot into dv and that you have the pressure force, which acts on the mean area into dp is the change in is the force and therefore, this is the force balance equation. Therefore, now I find say I wanted to get rid of d rho, but now I have got an expression in terms of d p. So, how do I still get rid of this term in some way and to able to do that we have to look at little bit on sound velocity. What is this sound velocity? You know it is very central to gas dynamics. The notion of sound is central to gas dynamics for the following reason, like for instance I talk to you, when I talk to you the sound waves travel, we say at the speed of sound; and we say that the speed of sound.

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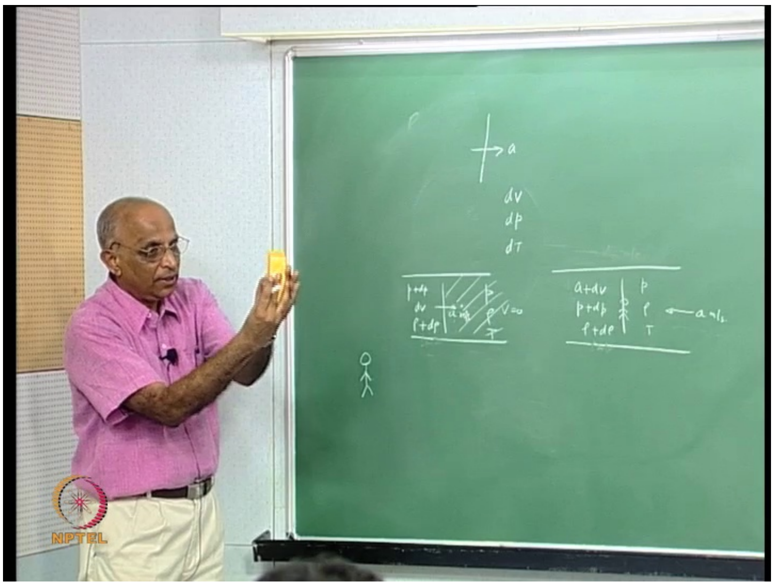

Let me erase this now. I have a sound wave, which is travelling at a speed of a; and what you say when the sound wave, you hear the sound wave, you get a little bit of pressure fret of bastion which by which may be ear response. How you get some velocity fret of bastion? May be, I get some velocity fret of bastion, I get some pressure fret of bastion, maybe I could even get a very small temperature fret of bastion. If the sound intensity is large, may be the temperature could be high; therefore, let us take look at what this sound wave means and how do I introduce sound wave into that equation. And I towards the end of this course, we will try to see the importance of sound wave when I talk of the flow.

Now, let get started. Let us imagine I have a pipe here and I say a sound wave propagates in this pipe. I take the same medium, I take some medium of a gas, let the pressure of this medium be p, let the density of this medium be rho be rho and the temperature of this medium be T. The medium, let it be stagnant like the room temperature over room over here; that means, the velocity here is equal to 0. In other words, I am just looking at the sound wave propagating in a stationary medium; that means, velocity is 0, of pressure p, density rho and temperature T. And what happens when the sound wave propagates through the medium? Let say this sound wave, it has come here, it increases the pressure little bit by p plus dp. Initially, velocity is 0, I could have a velocity dv, it frets of it. I have our density rho plus d rho and temperature should could also change.

Now, I want to write an equation for this particular medium. Now, what is happening, I am standing over here, this is my frame of reference. I am watching the sound wave go by, the sound wave processes this medium, which is initially stationary, increases the pressure by dp, increases the velocity from 0 to dv of the particles, may be the density changes from rho plus d rho and that is what I am watching. And for me, to write the equations standing here to see the wave and what is the velocity of sound I say; I say that the sound velocity is a meters per second, I just presume establishing at a constant speed of a meters per second. How is very difficult for me, because this is moving to write the equations of motion. And therefore, I transform this into the frame of reference, instead of me standing here and watching the wave go by, I position myself on the wave and now I can find out what is the difference over here, because nothing is happening.

And if I stand on the wave and I moving along with the wave, I see the gas coming towards me with a speed a meters per second, because I have told that the wave moves with a velocity a. And now, the conditions here are p, rho perhaps T over here, velocity is a now. And I had when the velocity was 0, I had dv therefore, the velocity here is a plus dv the pressure is still the same p plus dp, the density is rho plus d rho. I think this transformations are important in the sense you know, I initially watched the sound wave go by, I am talking, I see the sound wave going. But if I stand on the wave, what is that I see? Since I am moving, I put myself on the wave therefore, the thing is coming over here and I have the changes happening over here. Now, I write the equations for this, what will be my equations which I will get?

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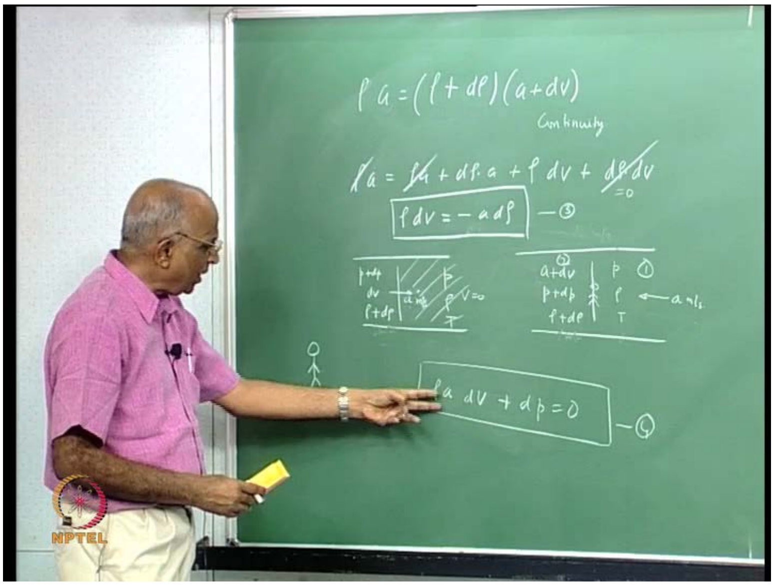

I have over here, the on the right hand side and disturb medium, this is the disturb medium, I stand over here watch the fun, I am writing the equation, I get rho a is the rho a into cross sectional area, I take the same because it really the does not matter and rho a is equal to rho a into a and a here cancels, rho plus d rho into a plus dv, this becomes my mass balance or continuity equation. You see the similarity of this equation to this particular equation, where in you have rho A V here because rodent variable area, I just wanted find out how the sound wave travels and therefore, I write this. Let simplify, I have rho a is equal to rho a plus d rho into a plus rho into dv plus, I have d rho into dv. But mind you we are talking of sound waves; sound wave is travelling at a speed a meters per second.

We also know that the pressure fret of bastions dv is small, the density fret of bastions d rho is small, therefore I can for all practical purposes say this is equal to 0. And therefore, what is it I get from this equation? I get rho a rho a get cancel, I get the value of rho dv is equal to minus a d rho, this is the, this is from the continuity or mass balance equation across a sound wave, as I set on this sound wave and I see the medium processing. I now write the momentum equation, what should be the momentum equation? Well, I am standing here, I see the change of momentum, that means, I see rho a is max flux which is coming over here and what is happening? I have the velocity changing from a to a plus dv rho a dv is the value of the rate of change of momentum; and that is balanced by the change in pressure is equal to 0, which by observations I write. I do not need to again go back and derive it, this is rate of change of momentum, this is the pressure, change of momentum per unit area therefore, force per unit area is 0 and this becomes my momentum equation.

Now, I look at this equation, which I call as equation three, because I have already derive the continuity equation which was one and I have the mass balance or momentum equation let us call it as equation two. Now, I look at the momentum equation, I call as three over here; three is the mass balance equation, four is the momentum equation. Therefore, if I have solve these two equations together, I have rho a rho dv from this equation let us put it together.

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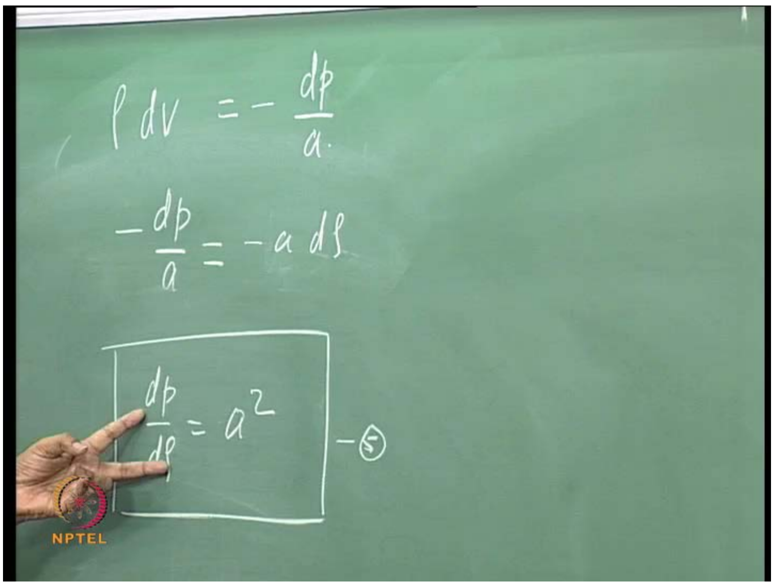

Rho dv is equal to minus dp by a; therefore, if I substitute rho dv over here, what is it I get? Minus dp by a is equal to minus a d rho and what does it give me? It gives me the dp by d rho is equal to a square. That means, the velocity of Sound Square is equal to the ratio between the fresh of fret of bastion to the density fret of bastion in the wave. That is when I am talking to you all, the type of fret of bastions which I get in pressure and sound and density behind the wave, the ratio of that is equal to the sound velocity square. Now I get this, maybe I call it as equation five. Let us come back and deal with this little later, because we should not focus, the lose focus of the problem. because I wanted to get rid of d rho by rho and therefore, if I have to write d rho is equal to dp by a square, I am going get from this equation I get dv by V is equal to plus d rho by rho, d rho I take here therefore, I get dp by a square and d rho is equal to dp by a square I have one over rho plus dA by A is equal to 0.

Now, I go back I am say you know well I know dp is equal rho V dv and therefore, if I have to bring substitute the value of dp as minus rho V dv what is it I get? I get the value as dv by V plus now I get minus, because I have minus rho V dv and rho and rho gets cancel therefore, I get V and on top that is dp is equal to V dv by V dv rho and rho get cancel V by a square into I get the value of dv plus the last term that is equal to dA by A is equal to 0. What is it have done. We substituted the value of d rho by rho and go in terms of dp by a square into one over rho, because we found the dp by d rho is equal to a square and therefore, this became we substitution. And then we wanted to write get rid of dp in this and therefore, we use the momentum equation, which we wrote as minus rho dv. So, therefore, dv; that means, minus rho dv; that means, minus V rho and rho get cancel dv by a square, that means, minus V dv by is what we derive earlier. please check if it is alright.

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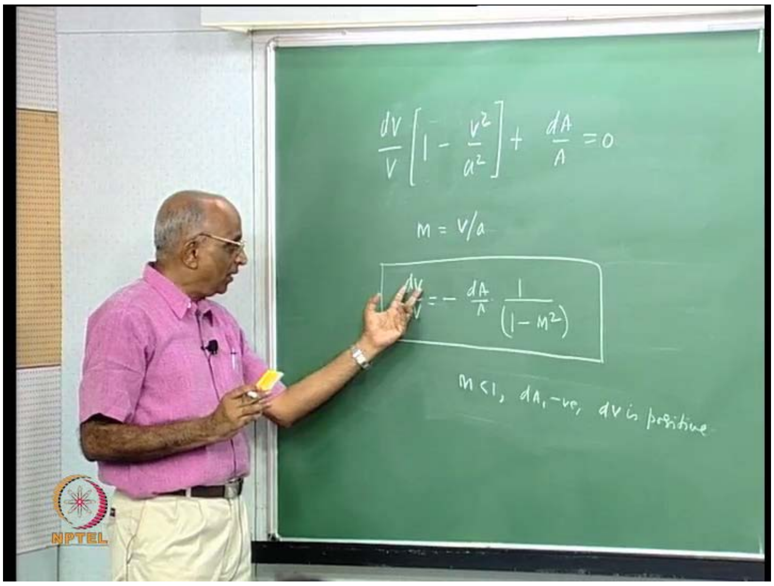

Let us simplified and write it down over here. Therefore, I get dv by V into one plus. now I get an expression, I have I am I took V outside at the denominator therefore, I get V square on top divided by a square, because I still have a square plus dA by A is equal to 0. Or rather, now I just blindly say let me call, mock number M as equal to the velocity of the medium divided by the sound velocity, I will come back to the physical significance of this I will tell later; and therefore, I can write dv by V is equal to minus dA by A into one over I get, yes I have minus here. lets go back and look at this, we had one, we had minus their therefore, we had one minus V square over here, because what happened was we had substituted dp and that dp was minus there, therefore, we had minus V dv p, please careful about this signs.

Therefore this is a expression what we get. Now, let in a whenever we derive an expression, we must look at the, what is happening behind it. Therefore, what we find? If dA is negative like what we have drawn here, what did we draw? We say area is decreasing as x progressing, if dA is negative, this becomes positive. And if we say M is less than one, when dA is negative, then dm is or dv is positive. What is it? I am telling you, I want the velocity to increase as the flow progresses, if for velocity to progress, if n is less than one, I get this to be a positive number, it is still minus of this; therefore, unless I have area which is decreasing as x proceeds, I cannot have a positive value.

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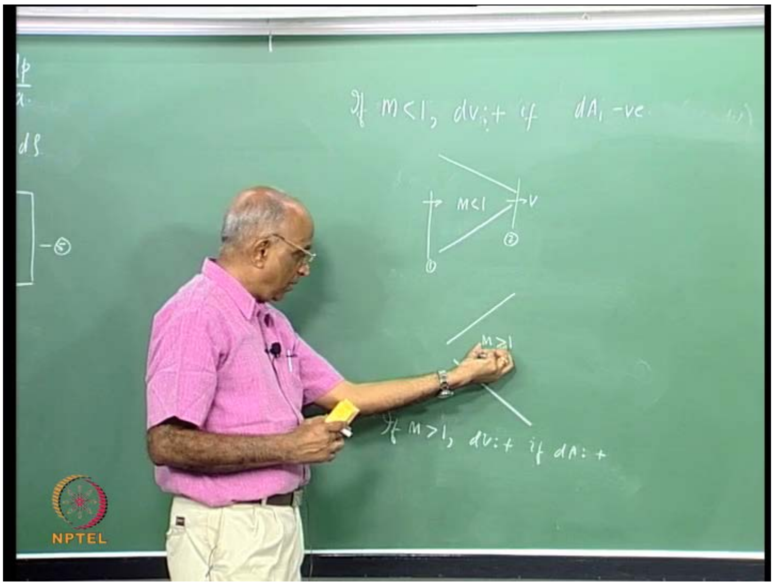

In other words, I tell myself flow will accelerate or flow will increase in velocity, if mock number is less than one, dv will be positive, if we have dA to be negative. In other words, all what we are saying is the cross sectional area, if I have a subsonic or mock number less than one, then I should have something like this for velocity here at section two to be greater than at section one. That means, here it enters at lower velocity dv dv gets increase I have high velocity over here. That means, for the case of a subsonic flow or a flow for which mock number is less than one, flow will accelerate in converging passage.

On the contrary, using the same set of arguments, if I have a case where in the flow takes place in diverging configuration, then if I have mock number greater than one, then what happens? Mock number is greater than one this becomes negative, negative and negative gets cancel, therefore, dv by V goes as positive of dA by A. In other words, the flow accelerates, if mock number is greater than one, dv is positive, if dv is positive. See through the simple argument of looking at the mass balance and the momentum equation, we are able to come to conjugature that may be in a converging section velocity will increase only if mock number is less than one. Let us if I have a diverging section and if mock number is greater than one, then only the flow velocity will increase.

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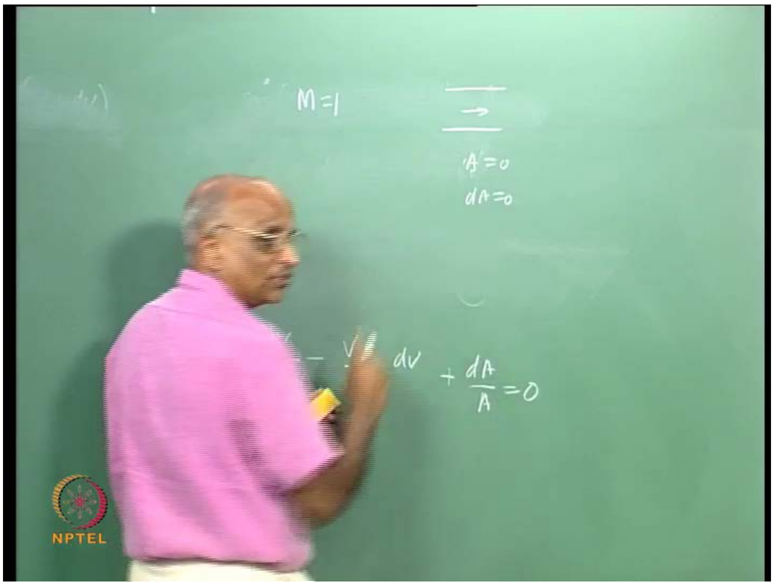

What will happen if the mock number is one? If mock number is one, the equations sort of breaks down. In fact, I find mock number is one, then I have one over 0 and unless I have dA by A equal to 0, this equation cannot predict anything at all. Therefore I tell myself if mock number is one, maybe I must have a constant area; that means, A is constant or rather dA must be equal to 0 for flow to take place, therefore all what we have done.

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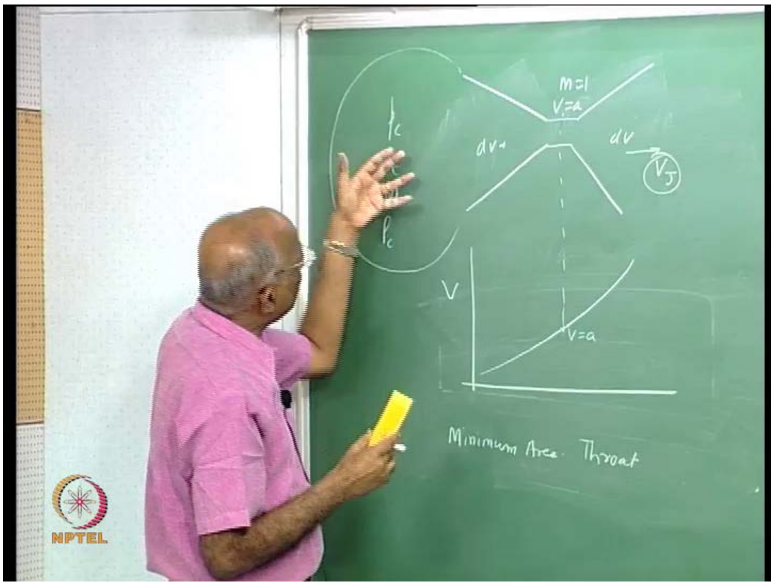

So far is we looked at the continuity and momentum equations, and found that in a vent or in a small opening, we should have initially the flow should come like this, it should come to a value of m is equal to one. And then if you pass through the divergent, such that dv is positive here, I have dv is positive and therefore, I can have acceleration of flow. And therefore, along this particular length if I plot the velocity V be the velocity will keep increasing, and at over here the velocity must be equal to the sound velocity. Therefore, for to get in order to get a high value of jet velocity, what we require is we must a minimum area and this minimum area like my throat, you know which is minimum area, I call it as throat. Therefore, we start with a large area, we converge it, we increase the velocity to a value. Velocity is equal to the sound speed at this section is equal to mock one and there after the mock number is greater than one and the flow increase the flow velocity increases and therefore, I can have high value of jet velocity.

Therefore the configuration of the vent should be a convergent followed by a constant section which I call as throat followed by divergent, if I have to get a high value of V J. supposing by chance, when I am doing all this, my mass flow rate like I consider the balloon which had a pressure of P c I had, T c I had, molecular mass and rho c over here, if it is such that the mass flow rate through this is less than mock equal to one, then what is going to happen?

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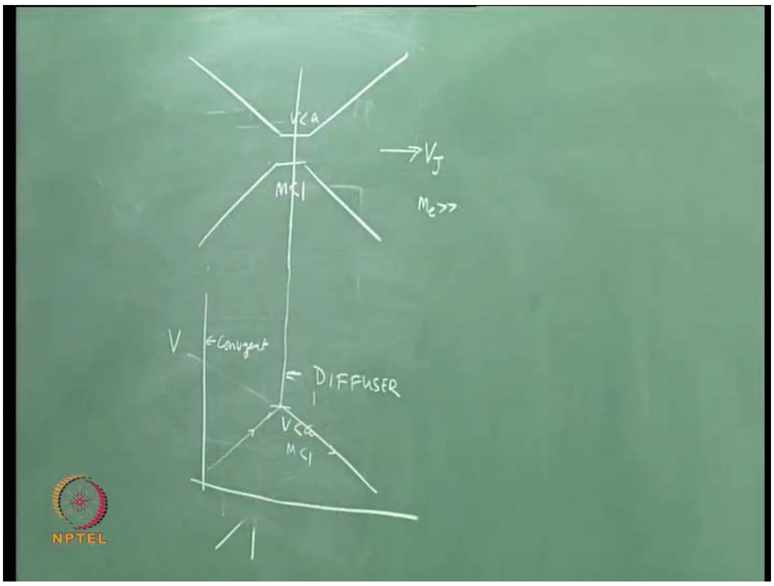

Let us say I have the same configuration, in other words I have the convergent followed by the throat, followed by the divergent, and if by chance, my velocity is less than a here; that means, mock number is less than one at the throat section, then what is going to happen? Well, I now plot velocity as a function of distance, the velocity keeps increasing up to this place, but I find that V is still less than the sound speed here or rather the mock number here is less than one. And therefore, the mock number here is less than one and therefore, the velocity drops further. That means, in the convergent the velocity increases in the divergent velocity decreases; and this portion divergent is what we call as diffuser. A contraption, which increases the velocity and enhances the pressure, is what we call as a diffuser and a contraption which increases the velocity is what we call as a convergent for the nozzle section. And this total is what we call as a nozzle and therefore, if I can have mock number one at the throat and then I have a convergent followed by divergent, I can get a high value of jet velocity; that means, mock number at the exit will be a large value.

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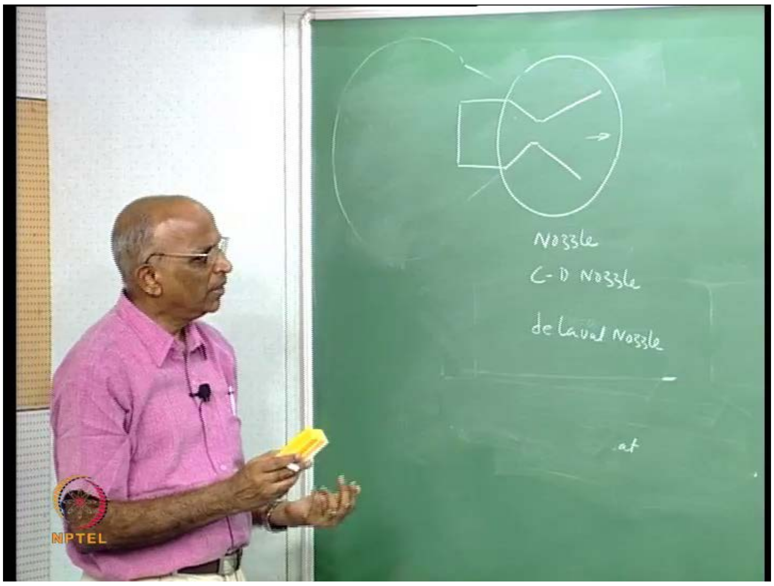

See therefore, from the simple writing simple expression of continuity and momentum. we tell ourselves, well if I have a rocket chamber, it is necessary for me to have chamber and I should have a convergent section, followed by a throat section, followed by a divergent section and this is what gives me a high velocity and this is what I call as a nozzle; which I find is a convergent divergent nozzle, which in some text books is also written as de Laval Nozzle.

But let us be very clear, if by chance I do not get m is equal to one at the throat, well the buildup of velocity in the convergent is a lost in the divergent and the velocity drops. And this is the configuration of a nozzle and we find that all nozzles, in order to give me a high jet velocity should be convergent divergent nozzle. Let us go through the exercise, because this is central to having a high jet velocity. Let us find out the variations of parameters across a convergent divergent nozzle.

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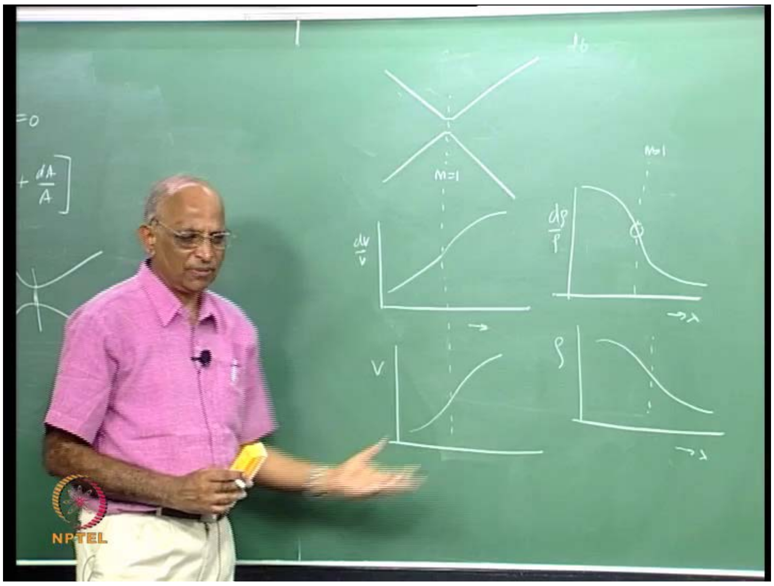

Let me first sketch a nozzle here, convergent may be a throat, may be a divergent and let us assume that at the, I have such a flow rate that I have mock number is equal to one at the throat. Now, let us find out what is a change in dv by V along the length, we just now found that dv by V increases up to the throat.

And then it further increases over here, therefore, dv by V, because mock number is one and characteristic of the equation changes m square minus one becomes negative and therefore, dv by V is like this. If dv by V is like this, well V by the value of velocity would also keep changing and what I get as may be the velocity increases like this. If the velocity increases, what is going to the happen to the pressure or the density. lets plot a few more parameters now, instead of dv by V, I am going plot, let us say the variation of pressure that is let us say dp by p. How should it look like? How should the variation or I know the velocity, I want to find up what the pressure variations or let let’s find out first one the let say the variation of d rho by rho, because I be already had an expression, which we derive lets write that expression again.

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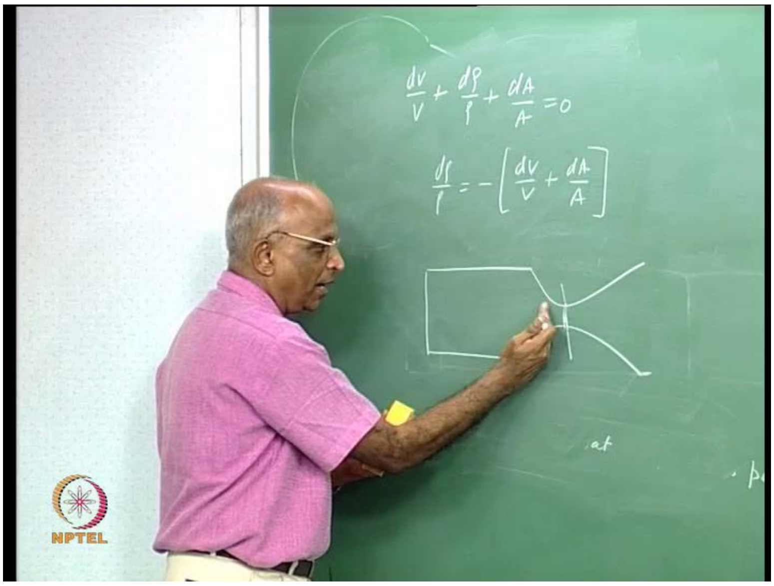

We had the expression dv by V plus d rho by rho plus dA by A is equal to 0 or rather we have d rho by rho is equal to minus of dv by V plus dA by A. I know the value of dv by V increases like this therefore, d rho by rho will be negative and dA by A initially is negative. In other words, if I say this is my throat region; that means, I have mock is equal to one or here, this is my length x along the nozzle which I am considering I find therefore, the density will keep falling. And what happens at the throat region? At the throat region, I had one minus m square at the in the term over here therefore, I have something like a step gradient in velocity, I have very step region here and d rho by rho comes over here, I think this is important.

In other words, the density keeps decreasing at M is equal to one that is a rapid change in d rho further and further d rho by rho changes and this region at the throat is a region of rapid density change. In fact, we will find and when we get into this problem of combustion instability, we will find that when we have a rocket nozzle, let say have a rocket I have convergent divergent section like this. The rapid decrease in density or rapid change in density over here acts as a it is a solid, there is a very rapid change therefore, it tends to be as a closed end and not as an open end. We will come back to this and this is important. That means, at the throat portion, you have somewhat a large change in this and therefore, if I now plot my density as a function of x here, I have the throat here well the density keeps decreasing. Therefore, for our convergent divergent nozzle, for which M is equal to one at the throat velocity change increases, velocity increases, density change decreases and the density decreases.

Now, there are two other parameters which are left, what is going to happen to the temperature? What is happen going to happen to the pressure? We know any way made an assumption when we talked in terms of the vent, we told ourselves the flow through the vent is adiabatic and is reversible.

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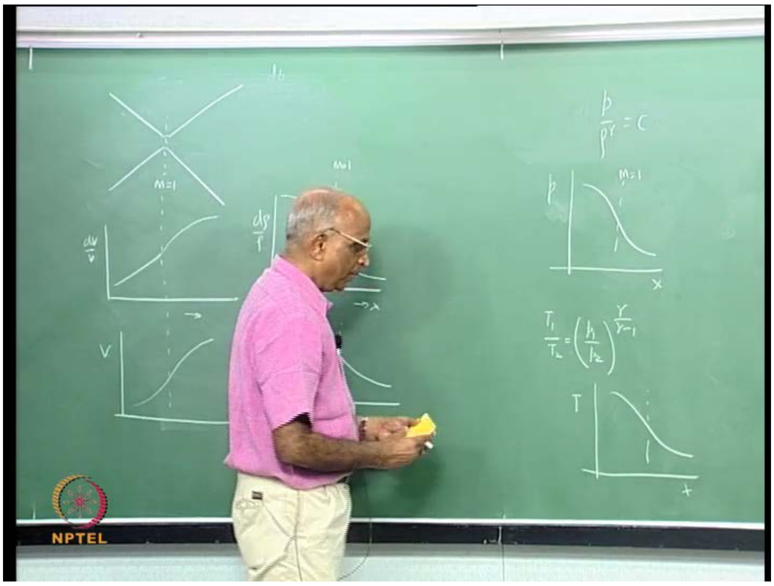

Therefore, p by rho to the power gamma is a constant. I know density changes and therefore, the pressure if I have the throat at M is equal to one, well the pressure should also decrease this will be my variation of pressure with respect to x.

And we also had derived an expression, we said the temperature one divided by temperature two is equal to P 1 by P 2 divided by we had gamma by gamma minus one. Or rather, P 1 by P 2 was T 1 by T 2 raised for gamma minus one by gamma and therefore, the temperature, if I have the throat here should also decrease and this will be by temperature variations with respect to the distance x.

See all what we are telling is the pressure decreases in a nozzle monotonically, the density decreases in a nozzle with rapid change in density appearing at the throat, well the velocity increases; this is all what we and this is something which all of us know.

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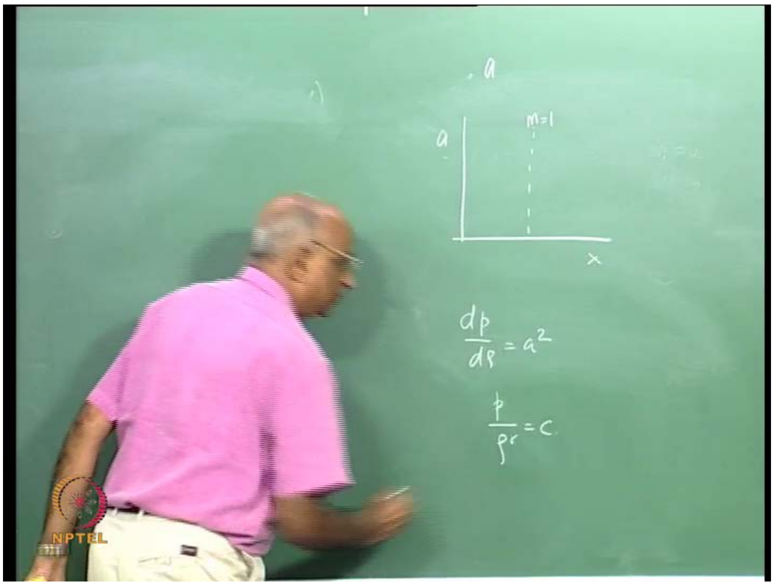

Now, let us put some other parameters. If I want to plot the sound velocity variations what will it look like? In other words, I have the expression a, I want to plot how the sound velocity will vary along the length and I tell myself at the throat the mock number is one. We again go through the expression what we derived today; we told ourselves dp by d rho is equal to a square. Now we told that the nozzle flow is isentropic that is adiabatic and reversible therefore, we have p by rho to the power gamma is a constant. Iherefore, what is a square?

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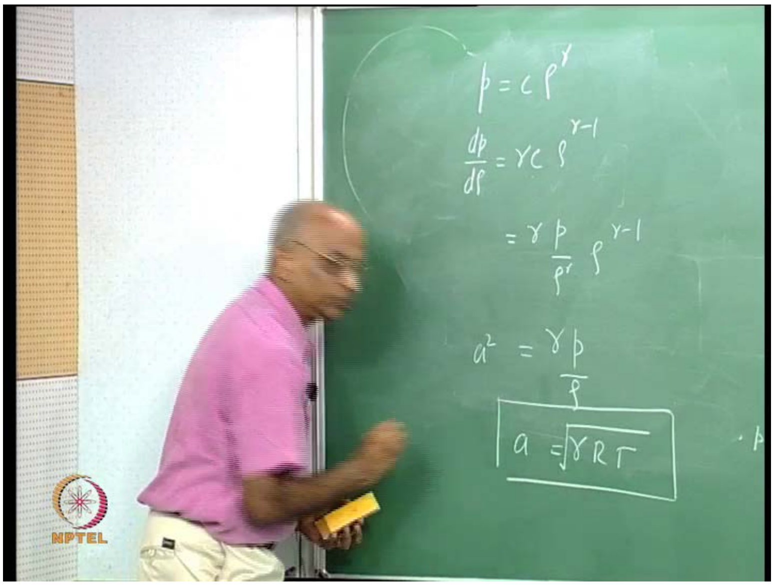

Let write term p is equal to a constant rho to the power gamma. Therefore, dp by d rho is equal to gamma c into rho to the power gamma minus one. And therefore, now if I want write the value of c as equal to gamma p rho to the power gamma into rho to the power gamma minus one or this is equal to gamma p by rho. Therefore, we find that the sound speed is equal to gamma p by rho and for perfect gas or for an ideal gas p is equal rho r p or this is equal to gamma R T or sound speed is equal to under root gamma R T.

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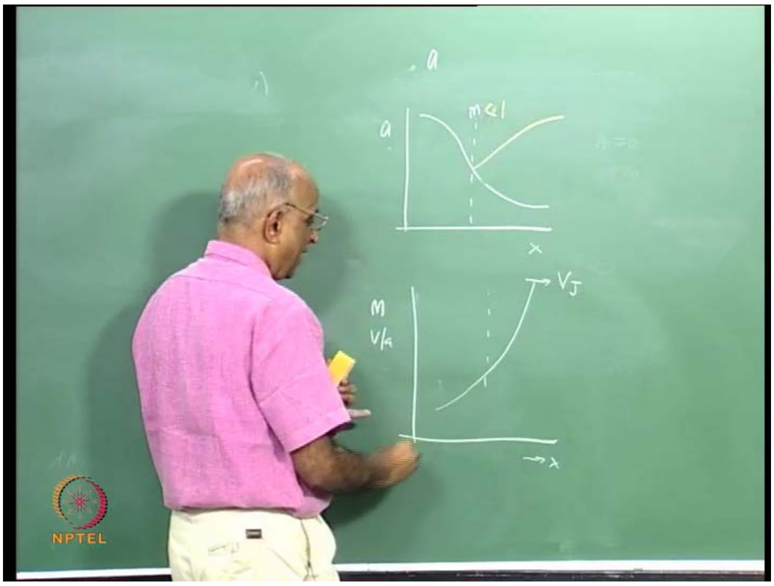

Therefore what is it I find for sound speed variations, I will find. Well the sound speed starts with a high value of sound speed it keeps on coming and it comes like this. one last parameter, I can still think of is the mock number at variations along the I know the velocity variation which I had over here velocity changes along the x over here in this particular form, I also find that the sound speed keeps coming down; and therefore, if I find out the mock number variations as a function of x, what is it I get? This is numerator is increasing, denominator is decreasing, because mock number is equal to V by a, and therefore, may be this is my plane, well is going to come like this I am going to like this.

I think this is fundamental to any nozzle flow, any expansion at be must remember this. Therefore, let me repeated again dv by V progressively increases when I have mock number one at the throat, velocity increases the density decreases rapid at the throat, pressure also decreases temperature decreases, the sound speed decreases along the line and therefore, the mock number increases along this. And we are interested in a high value of mock number or jet velocity at the exit.

If now I where to tell myself, if by chance the mock number at the throat is not one, that means, I have insufficient pressure over here to give high jet velocity equal to the sound speed; in other words, I now get let us say mock number is less than one, what is going to happen? dv by V is going to increase over here, it is still less than the sound speed that V is less than a and therefore, it falls down here; that means, my value is like this and comes down over here. Velocity increases up to the throat, but it is still the sound speed is less than the velocity is less than the sound speed and therefore, it comes over here. What happens to the density? Density falls up to the throat and increases back; therefore, I have a diffuser section here. Density again same picture and now then say mock number is less than one.

Now I say mock number is less than one, the pressure recovers here instead of falling it recovers over here, temperature recovers over here. If the mock number at the throat is less than one and similarly the sound speed falls up to the throat and recovers over here; that means, it now becomes less than one. And therefore, I have the same portion here, the mock number going up to this and coming over here, if the value of mock number at the throat is less than one.

I would request to you all to go back and study these figures again, because this is central. You know what we find is well I must have a convergent section followed by a divergent section and in between I must have a constant area section, which I call as a throat. Is this clear, I think this is Centro therefore, we now tell us ourselves.

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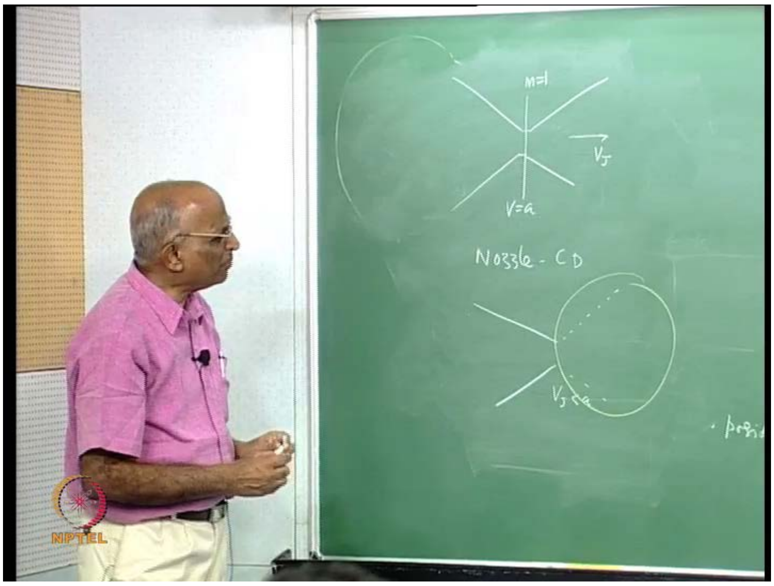

If want to have a high value of jet velocity, it is essential for me to now have a convergent followed by a throat section, followed by a divergent section, with the condition that mock number must be one of velocity must be equal to a sound speed at the throat. And then if I have this velocity keeps increasing, I can get a high jet velocity at the exit and this is what is central theory of a nozzle, which we call as a convergent divergent nozzle.

If I find that I have inadequate pressure that like for instance, I have a cold gas I have inadequate pressure and I cannot have mock number greater than sound speed or mock number equal to one at the throat or velocity is equal to sound speed, then its better my nozzle is like this, such that my jet velocity is still less than the sound speed. That means, if I have inadequate pressure or inadequate conditions here, such that I cannot get here then I must do away with the convergent section. Because if I have this divergent section over here, I am really loosing thing I am not really gaining thing. Therefore, whenever rocket nozzle has to be design, we have to ensure I have sound speed at the, that is the velocity of the flow at the throat must be equal to the sound speed.

Now, we ask myself I will ask myself one last question here, what is the significance of sound speed? Is there something very, very significant about a did.

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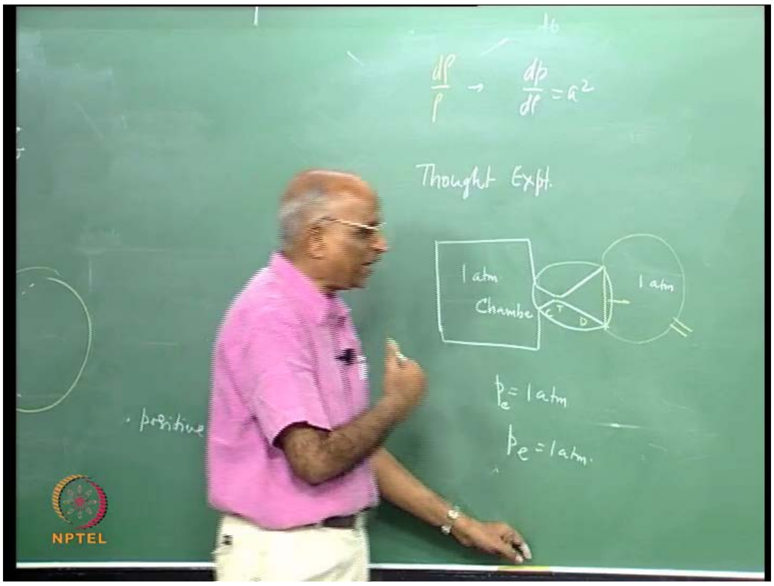

So, something you know I just introduce the sound speed through the equation and I told myself that in order to eliminate d rho by rho in that expression, I what I did was I try to put something like dp by d rho as equal to the sound speed square and I got the all they relations come, but does it have some implication. To be able do that, let’s do a simple experiment or let us say a thought experiment, what is this thought experiment? Let us say I have a chetank or a chamber something like this, it is an ambient pressure one atmosphere pressure, I am really not bothered about temperature at this point in time, maybe I attach a convergent divergent nozzle to it this is my thought; that means, a constant area section then let us draw it properly.

Now, what I do with this experiment which I am thinking of maybe I allow I collect all the air here, I put a pump here and I suck the air out of this particular nozzle. And what is the nozzle have this convergent, divergent thought nozzle and this particular contraption what I get, I attach it to the tank, which initially at atmospheric pressure here also the pressure is initially atmospheric; and then I start pumping out or start sucking the air out of this stand.

Now, what is going to happen? Now let lets plot lets, tell myself that the ambient pressure in the tank is initially one atmosphere, the pressure outside P e or we say this is tank this is the chamber therefore, I say P c is one atmosphere, let us say P e is also one atmosphere to begin. When the pressure here and pressure here are same, obviously, there is mass flow rate; therefore, let find out what happens, if I attach a pump over here and start sucking out the air from this chamber. You know this will tells us some clues on what really the sound speed, what have up it place or what are the useful things, we get we learn about sound speed.

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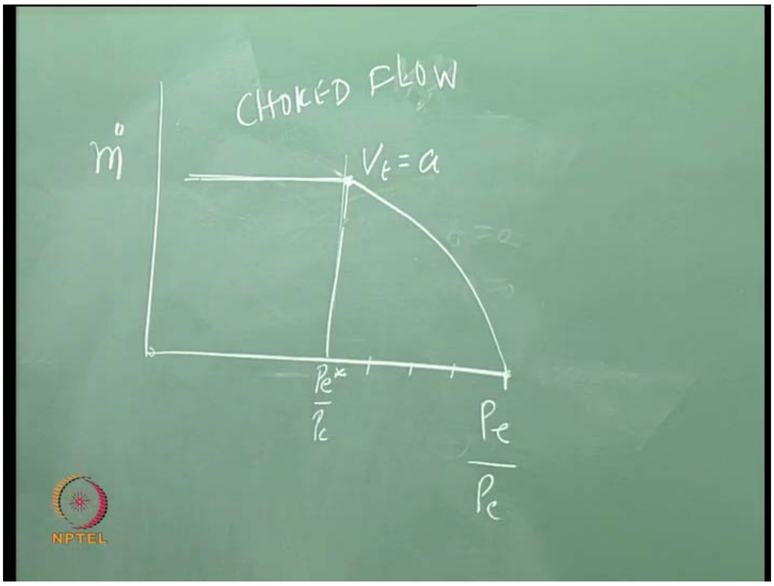

Let do this experiment, let say I now plot the mass flow rate through the nozzle, that means, the rate at which mass is getting sucked out as a function of let us say the value of P e that is the exit pressure to the chamber pressure.

We told ourselves, initially the pressure is one both are same therefore, the mass flow rate is 0; therefore, the flow rate is 0. Then what I do? I stop sucking of the air over here, I decrease the pressure over here; if I start decreasing, the mass flow rate will increase. That means, as I increase the or I decrease the value of P e, P c being the same. the mass flow rate will increase and therefore, do you think the as I keep on increasing the flow rate should keep on increasing or what should happen?

Let start thinking a little bit more we tell well, initially when the pressure outside is one atmosphere, pressure inside is one atmosphere; the velocity at throat is equal to 0. then I stop sucking out, as I keep decreasing the pressure here, V t will keep on increasing till it reaches a value V t is equal to a. In other words, let us just plot till that time, I keep on decreasing the value of p e, that means, I keep moving here, here, here; till the time I a stage has come at given value of P e, let us call it as star divided by P c, the value of velocity at the throat is equal to the sound speed a. Therefore, at that point in time what is happening?

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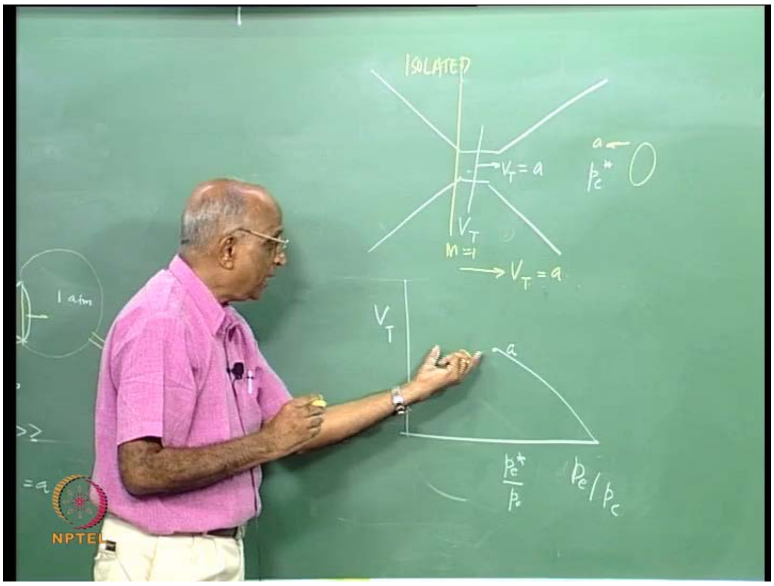

Let us sketch out the nozzle in a bigger way; I left this gap for this purpose. I have the convergent, the throat it divergent, let us plot the value of the velocity at the throat I call it as V T. What does happen? now for the same figure, which I have drawn there have P e by P c, as a function of V T, what I find is initially it is one velocity is 0 the velocity keeps increasing till the time for a given value of P e star by P c, the value here is equal to the sound speed at the throat.

What is the implication of this? Now, what I find is the velocity at this particular point for this condition, when I have reduce the pressure to P e star, the velocity at the throat is equal to the sound speed. Now, let us ask ourselves some foolish questions. We will ask ourselves this question, why is at the mass flow rate flows from this chamber to the outside? Because I suck something, when I suck something what happens? I reduce the pressure here and this information of a reduced pressure is transmitted through this over here and that is why mass flows, that means, I suck something the information is reach there and that is why something is flowing. Therefore, as I keep on decreasing the pressure, this tank senses that something some information it reaches about this downstream pressure being lower and therefore, the mass flows.

Now, when the stage comes, when the sound when the velocity at the throat is equal to V t and in how does information travel I talked to you it travels at the speed of sound. Therefore, any information here is travelling at the speed of sound and this sound tells me, tells this chamber that I want something more. But if the gas is not flowing at a velocity V T is equal to sound speed and now sound speed is travelling, it cannot travel, because it is the gases flowing here, the information is flowing against like it is the same speed and therefore, no information can flow here and therefore, this becomes isolated. This chamber over here cannot get any information of the reduce pressure here, when the sound speed is A, because information or disturbances travel at the speed of sound.

Therefore when mock number is equal to one at the throat, what is going to happen? You know I keep on decreasing, but the velocity here is the sound speed and it effectively isolates and therefore, the tank is unable to know that a low pressure exits here. And therefore, what is going to happen? The mass flow rate cannot increase any further and the mass flow rate, it is not able to communicate at all its totally disconnected and therefore, I have something like this. In other words, for when the downstream pressure is such that I get the velocity at the throat to be the sound speed, there after the mass flow rate is a constant. And that means, I even though I am trying to pull by information is not given to my tank it supplies this or rather I say that my throat is choked and this is known as choked flow. I think this concept comes from the sound speed and therefore, we tell ourselves well, when I have a convergent divergent nozzle at the throat when I have the value of mock number is equal to one, it corresponds to a choked flow.

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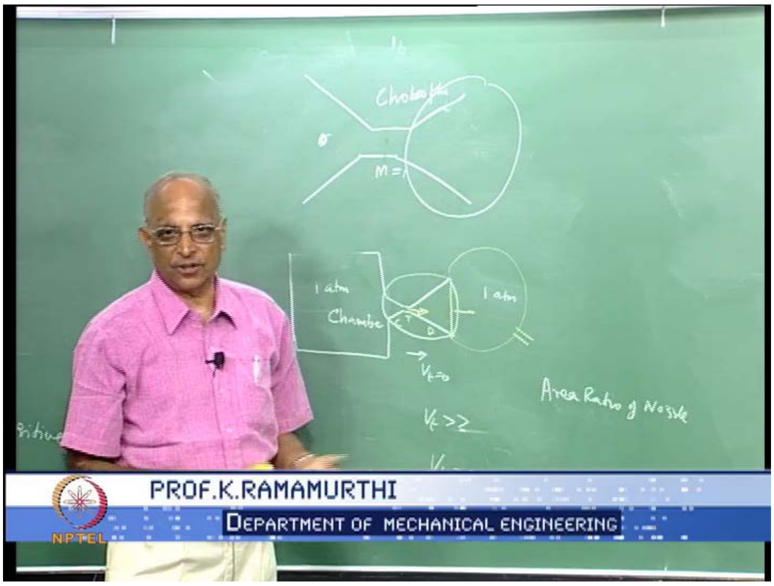

In other words, any downstream disturbances cannot really affect my flow here; in other words, the concept of a throat and choked flow is central to the nozzle, I will continue with this in the next class. In the next class, what we do is we will find out the value of the density corresponding to the choked flow, pressure corresponding to the choked flow and there after we will relate the area ratio of the nozzle of nozzle, because we are still talking in terms of pressure to the pressures and talk in terms of flow separation and other problems with a nozzle in the next class. Well thank you then I think.