Rocket Propulsion
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Lecture No. # 12
Characteristic Velocity and Thrust Coefficient

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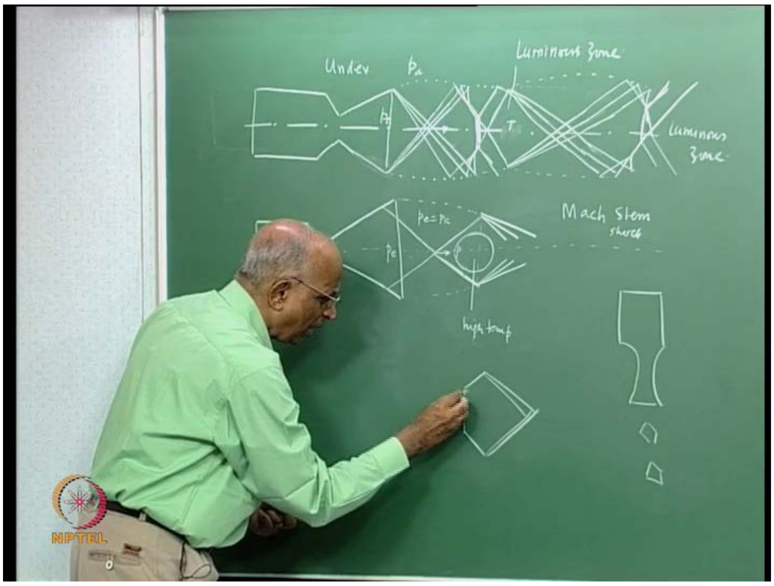

Good morning. We will continue with what we were doing with a nozzle area ratio, I re plotted the figure which we had in the last class. We had one case, we had the under expanded nozzle. In other words, the exit pressure here was lower than the once higher than the ambient pressure. So, is the flow continues to expand, how does the flow expand when its mix out it, a you have series of rare fraction fans and when these rare fraction fans hits the boundary, they are reflected as oblique shock waves, and these oblique shock waves when they when they are, when they interact with boundary, I get rare fraction fans which again forms oblique shock waves and this wave continues.

Therefore, when I look at an under expanded jet as it were, I start to put a high value of p e, I get first a set of expansion fans which are followed by a oblique shock waves and I behind the oblique shock waves. Since, I have higher compression, I get something like a higher temperature, and this higher temperature region shows as a lightly anything which is at a higher temperature emits more light and this is shown as luminous zone. And this again I have rare fraction fan, again compression fan, again I will have another luminous zone over here. Similarly, if I have an over expanded nozzle, the value of pressure at the exit is lower in this case than the ambient pressure.

Therefore, I form something like a shock wave which matches the pressure and therefore, to be able to get the value p e- equal to p a-, I have something like an oblique shock wave. At these oblique shock waves they continue to interact and I have now something like an oblique shock wave, but after the oblique shock wave, because I need to have the velocity at the center which is still going straight and therefore, here the pressure exceeds the ambient pressure.

And then I have another expansion, another high pressure region and therefore, in this particular zone, I have something like a high temperature region, a high pressure a high temperature region and high pressure, a high temperature region gives me, some light. Therefore, what is happening in an under expanded nozzle, I have a set of initial rare fraction fans, followed by oblique shock wave. In the case of over expanded nozzle, I have oblique shock waves which thereafter result in the expansion fan. Therefore, there is a distinct change in the clue, if I consider a rocket moving around.

If the expansion is not there, maybe I should get some bright spots and these bright spots are due to the shock waves and these are known as shock diamond. I get this both for, the under expanded flow as well as the over expanded flow. The only thing is that there is a phase difference, if I have over expanded flow, the shock diamond comes much of here, because I do not initially get the expansion fan, I start with the oblique shock wave, it happens much nearer whereas, if I have under expanded nozzle, it is shifted out a little later. Therefore, at trained I, if I look at a nozzle and that is where I again show those two pictures which we had earlier.

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See in this case, we had the nozzle, we had the oblique shock wave, we would have got some shock diamond is a little later as it shown here. But in this particular test some cooling of the (( )) was done. Therefore, it was not visible; this is a test were you have the exhaust of the nozzle and you have one shock diamond shock timer shock timer and it keeps on going in an in visit flow, you could have infinite number of these diamonds. But with viscosity, there is some dissipation and that is where I showed this particular slide which shows this flight a SR 71 in which case may be you do see over here from the exhaust, your one shock diamond second one, third one, fourth one and keep on going. The other engine keeps on producing this.

Therefore, the shock diamond are the reflection of under expansion and over expansion in along so. You know it is very nice to see some of these pictures and try to conjuncts what is really happening. Therefore, but in practice there is another problem we have, whenever I have this oblique shocks you know when the oblique shock interact very often you get something like a Mach shock, a (( )) shock is formed.

And I have instead of having a regular reflection like this, I have Mach reflection and therefore, shock is found like this. Therefore, this shock wave is known as a Mach shock wave. That means, I have an incident shock, I have a reflective shock, I have spot shock over here. Similarly, I have the incident shock over here, reflective shock and so on. This the shock pattern therefore, continually changes and you have a shock structure forming over here, similarly I will have something like a shock structure which is getting formed like this. That is the simplified picture (( )) little complicated, because I have something like a Mach stem shock which is formed.

And that is, how you see the diamonds in particular pictures? The diamond pattern is actually something like this therefore, this how the pattern looks like. Normally, I would have expected may be incident waves like this, what we said was the diamond pattern is something like this and then you have the reflective shock goes, but very often you have a mass stem shock and I do not observe this reflection pattern over here. I will have a shock like this and then it gives rise to reflection over here.

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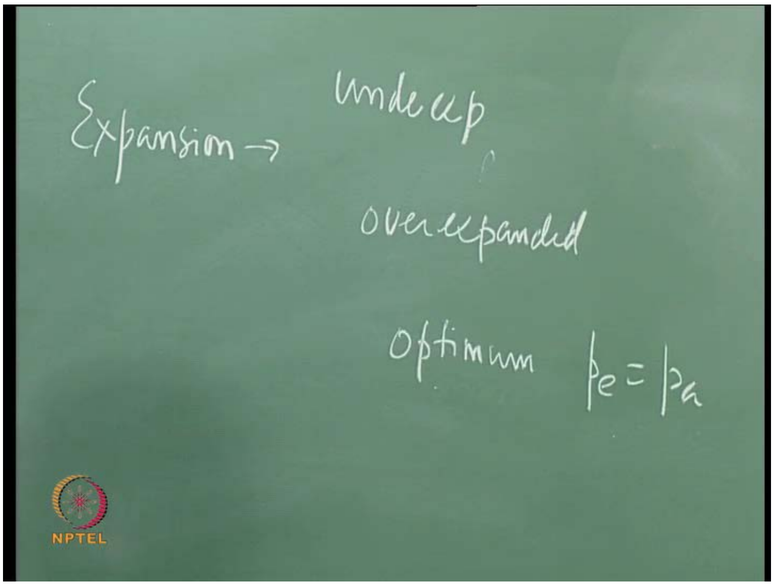

And I get a diamond (( )) anyway it is of interest to see that whenever I have a expansion of a nozzle, the expansion could either be under expanded or it could be over expanded or it could also be something like an optimum expansion. Optimum is when the exit pressure of the nozzle is equal to ambient pressure in which case it goes straight as it well. See there are some problems we have with a under expanded and over expanded and we told ourselves.

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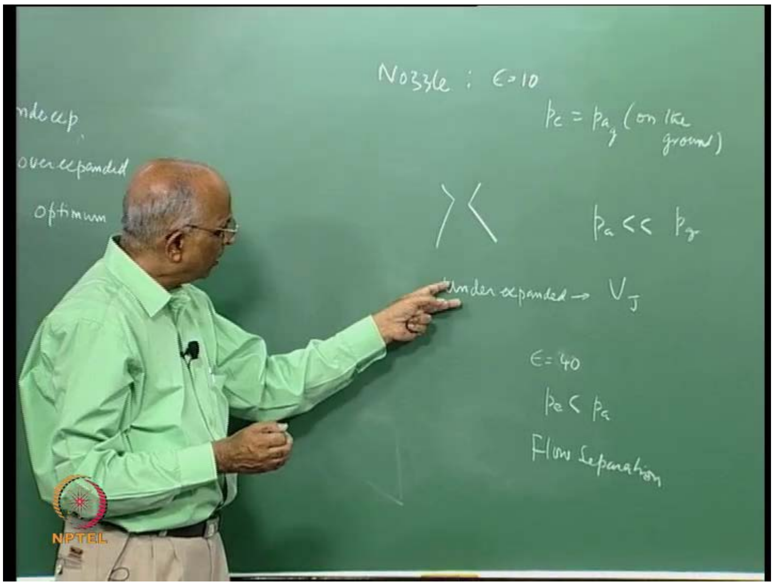

When supposing, I was to have a rocket nozzle with let us say, an area ratio of let us say ten or so, which gives an exit pressure let us say, equal to the ambient pressure on the ground and in this particular nozzle where to fly at an altitude. Where in p a- is at a high altitude less than the value on the ground. Let us say, p a- on the ground then, what is going to happen? My exit pressure has reduced; the nozzle becomes under expanded. And therefore, I will not be able to get a high value of V J which would have been possible, had I got a high value of expansion ratio. Whereas on the other hand, if I have a nozzle on the ground, I have an area ratio of let us say forty which gives me an exit pressure which is less than the ambient pressure on the ground, because now the nozzle is over expanded. Then in this case, I will have flow separation which is again not desirable, because I have shocks and I have as asymmetric flow separation or I have side waves on the nozzle.

Therefore, we find that in general. We cannot always have, we cannot live with under expansion, and because I lose jet velocity, but at the same time, I need to have, I cannot have over expansion because; I cannot deal with flow separation also. Therefore, there are some problems in nozzles.

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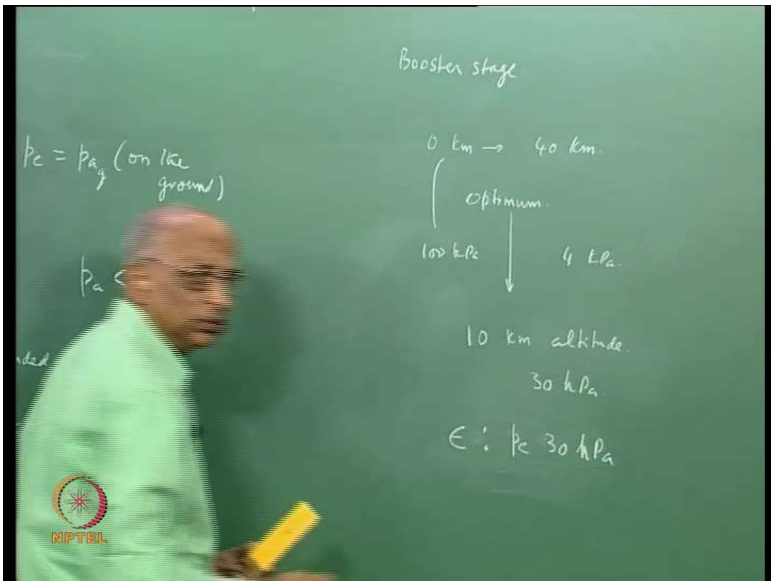

And therefore, let us tell ourselves, supposing I have a let us say booster stage of a rocket and we define booster stage as a ground stage which has to fly like let us say between zero kilometer to something say like up to forty kilometers or so. Then I cannot have a nozzle which will be perfect or optimum for all these conditions. In other words, at zero kilometers I have hundred kilopascal pressures at forty to fifty kilometer, maybe my pressure may be something like four or five kilopascals might be lower. Therefore, my ambient pressure is decreasing and therefore, it is not right for me, I cannot have a single nozzle, I cannot have a rocket with a single value of epsilon or expansion ratio which will fly throughout this altitude.

Therefore, I make the nozzle such that for an intermediate altitude let us say between these two maybe I have to fly forty kilometers. I design my nozzle for ten kilometer altitude in which case, the pressure may be somewhat different from hundred and four may be its something like nearer to let us say thirty kilo Pascal; that means, my exit pressure of the nozzle my area ratio is such that the exit pressure of the nozzle gives me a value p e- a value of 30 kilo Pascal.

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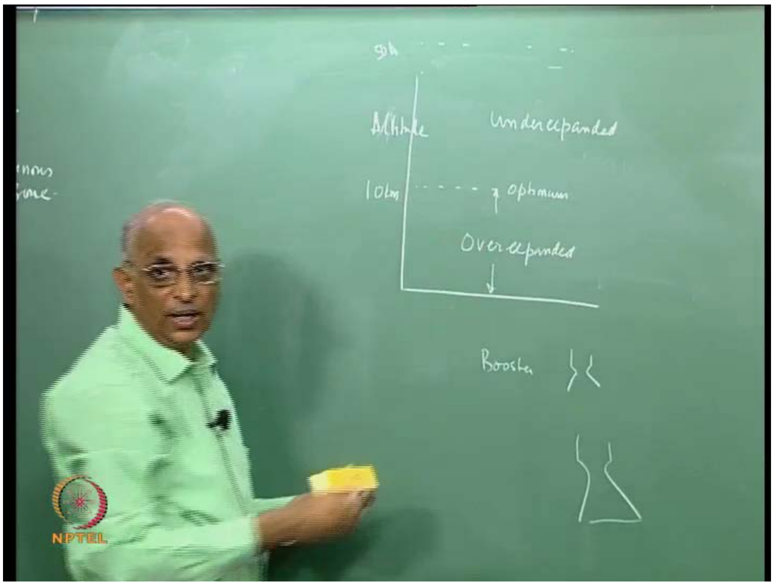

Now, this particular nozzle when the rocket takes off, what will happen? Is it over expanded, under expanded exactly. Now let us plot it the figure over here, I have the altitude and we say this design altitude is ten kilometers. The rocket has to operate up to an altitude let us say of fifty kilometers. This is where I have the optimum nozzle therefore, initially the nozzle performs over expanded mode and thereafter, it performs in under expanded by having a rocket performing in an under expanded mode.

I am not getting the high value of jet velocity which I could have got instead of choosing a value of epsilon which gives me ten kilometer. Had I chosen a value of epsilon which was corresponding to fifty kilometers, I would get a much V J. Therefore, I am not, but then I am getting not high V J. Then I am cannot also afford to over expand my nozzle and get into some side thrust problems and it is always it is always that a given nozzle operates either at a under expanded mode or an over expanded mode, but we try to decrease it and as you have seen in the earlier figures what I have shown a booster stage has a small nozzle. Whereas, if it is that rocket is going to fire at higher altitude, I will design a larger nozzle such that, it is more that is the optimum shifts to higher altitude and this something which we are going to keep in mind. I think something more under expanded and over expanded nozzles.

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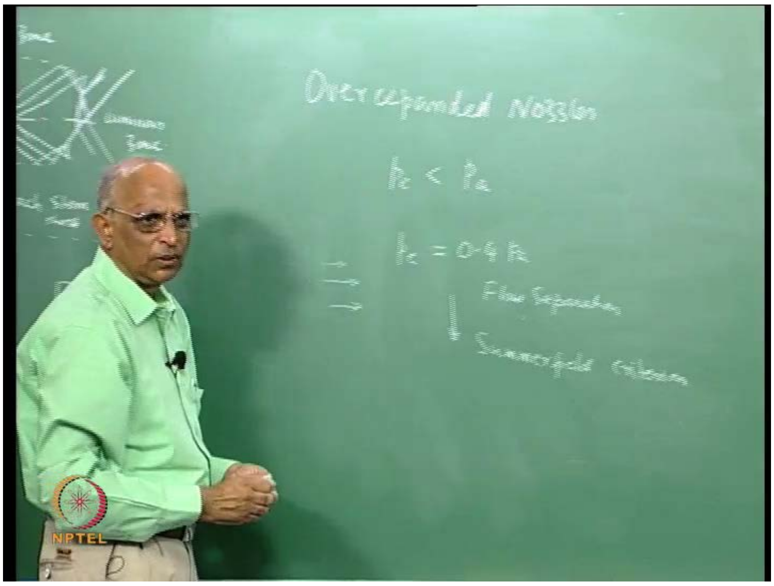

We told ourselves that the nozzle over expanded that means, p e- is less than the value of p a- this is the condition. But in general, gases flowing at high velocity of nozzle in the divergent and therefore, we have lot of this inertial force as it well. And in practice, when I have the exit pressure is equal to something like 0.4 times the ambient pressure. Then only I have this problem of flow separation and shock formation even though, ideally I said that p e- and ambient pressure. In practice experiment has shown because of the inertia of the forces which are already available, the rocket nozzle can operate at a much lower value of p e- compared to p a- and this value is 0.4 times. This condition was devised by Summerfield and it is known as Summerfield criterion. What is the implication of this?

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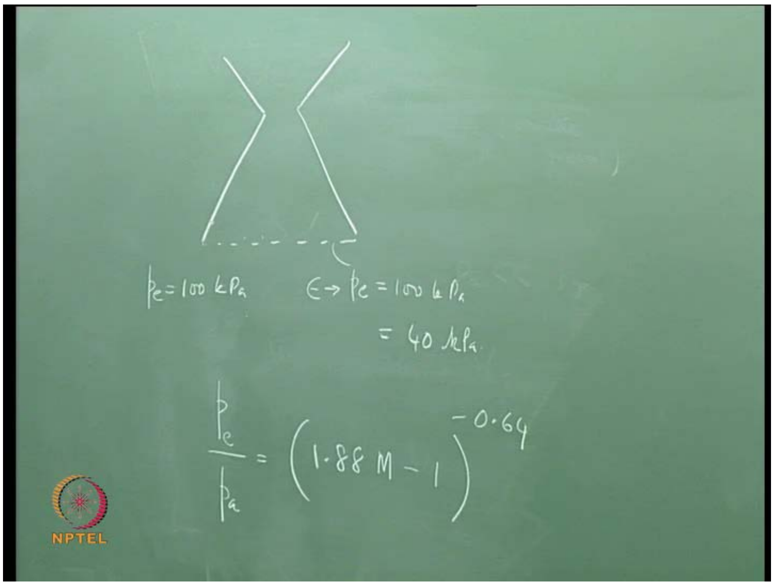

Supposing I have to make a nozzle perform on the ground that means, I have something like a nozzle. Let us now plot the shape of the nozzle convergent followed by divergent, then my ambient pressure is equal to let us say hundred kilo Pascal on the ground.

Then I do not need to have the nozzle, the exit pressure that is the area ratio such that the exit pressure is equal to hundred kilo Pascal. But it is sufficient for me I can have a give a much larger nozzle such that the exit pressure is equal to much lower as forty kilo Pascal and this is based on experiment, because you find that the initial inertial field helps to delay the force of inertia. Having said that it is also necessary that, this is also not perfectly in fact a local Mach number at the zone of flow separation affects it. And the value is in fact given by the value of p by p a- is equal to something like 1.88 into the Mach number at the plane of exit or wherever you want flow separation, to the power minus 0.64. In fact based on experiments, it is not only the inertial force which delays into a constant value of 0.4 times the ambient pressure, but it depends on the local Mach number at which the flow separation takes place and this is the value of the exit pressure for which flow separation takes place, this is again based on experiments.

Therefore, I hope by now we get a feel for a nozzle, we find that the nozzle area ratio depends on the ambient pressure. Ambient pressure keeps varying in the flight and therefore, we need variable area ratio nozzles. The moment we talk of area variable area nozzles, it is also necessary for us to consider the problem of under expansion and over expansion by under expansion, we lose performance by over expansion. We get side loads which are harmful. Having seen that let us go to the next phase wherein, we also would like to know a little bit more about, what is the thrust? Or what is force generated by the rocket? What is it we have considered?

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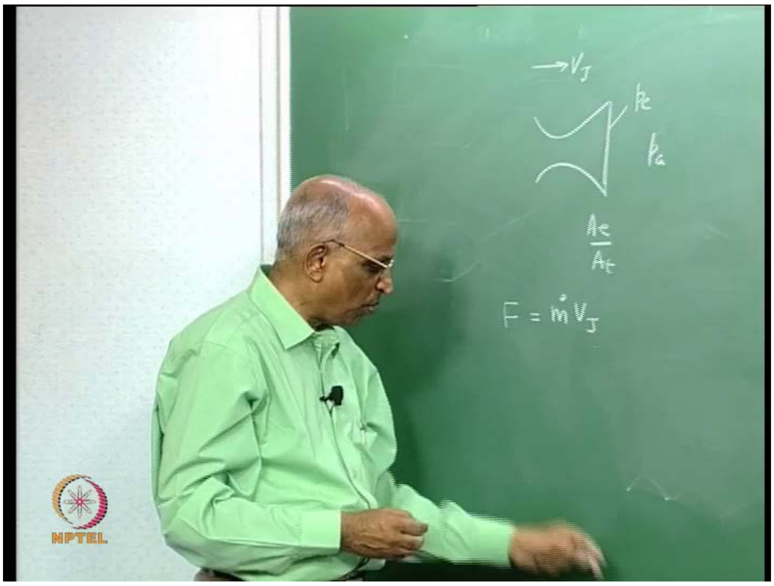

So, far we consider the jet velocity as a function of the chamber properties; we said a convergent divergent nozzle is required; we look at the area ratio A e- by A t- of the nozzle. We talk in terms of the under expanded, over expanded and optimum nozzles by now. We are clear about this, but now the question is, what is the thrust generated by nozzle? What is the thrust generated your momentum thrust m dot into v j-. But we are telling ourselves each time, that depending on the nozzle area ratio, the exit pressure p e- could be different from ambient pressure p a-.

Therefore, the thrust could come from pressure also, because I am not able to fully expand this. Therefore, I have some pressure also coming therefore, let us write some expression for the force developed by a rocket, which we know is equal to rate of change of momentum.

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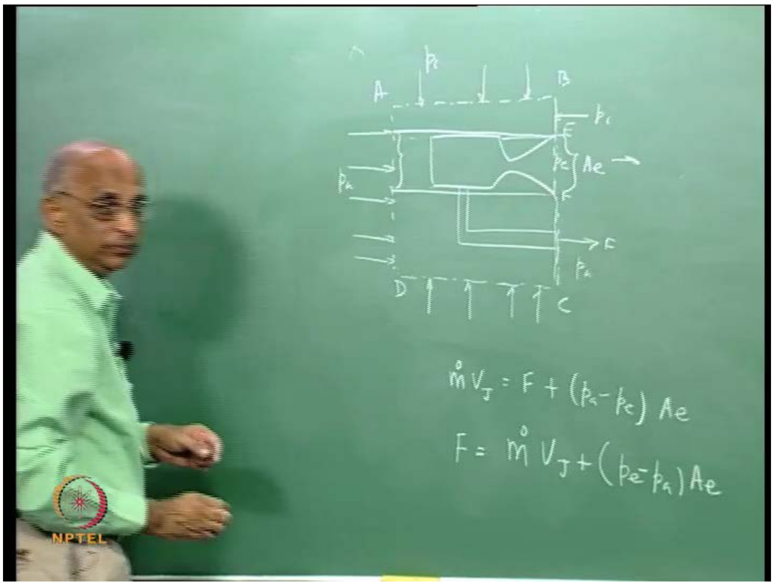

Let us put the diagram down, we again start from basics. Let us imagine, I have a rocket here, I have a nozzle here let the nozzle exit pressure p e-. Now what I do is, I want to find out what is the force is generated? Therefore, I clamp the rocket on the ground and I attach it to a fixture over here. I hold it on such that I hold it on with a force f, such that the rocket is stationery. Now because, I want to find out the force which rocket develops? Because my exit pressure is not balanced by the ambient pressure therefore, I keep this rocket in a position like this. Then I make a control volume diagram about this.

Now I want to find out this force, I find that everywhere on the on the walls of the control diagram, that is this imaginary lines, what I am drawing now? Let us call it as A B line, B C line, C D line, the nozzle is E F over here. I find all along this, the pressures are same, may be pressure is acting over here; pressure ambient pressure is also acting over here. Similarly, ambient pressure p a- is acting over here; ambient pressure p a- is also acting in this stretch. It is also acting in this stretch over here. Therefore, the only place where I have a difference in pressure is in this region. That means, sorry about this over this particular area, I have ambient pressure p a- is acting; over here, p e- is acting. Now let us write the force equation for this system over here. I get force or I have a momentum which is coming or what is the momentum that, I get m dot into V J is the momentum thrust whatever, I am getting and that is balanced by the value of force plus p a- minus p e- over .

If I say the area ratio or the exit area is A e- into p e- over here. p a- minus p e-, this is the unbalanced force in the actual direction. Because in all other cases, this pressure balances this pressure balances this part over here; this part balances this part and therefore, only over A e- I have an unbalanced pressure p e- into p a- in this direction p a into p e- in this direction. Therefore, this force or rather I get the net thrust is equal to m dot into V J plus p e- minus p a- into exit area therefore, now I have slightly modified my first of the thrust equation which originally we considered the exit pressure to be same as p a-.

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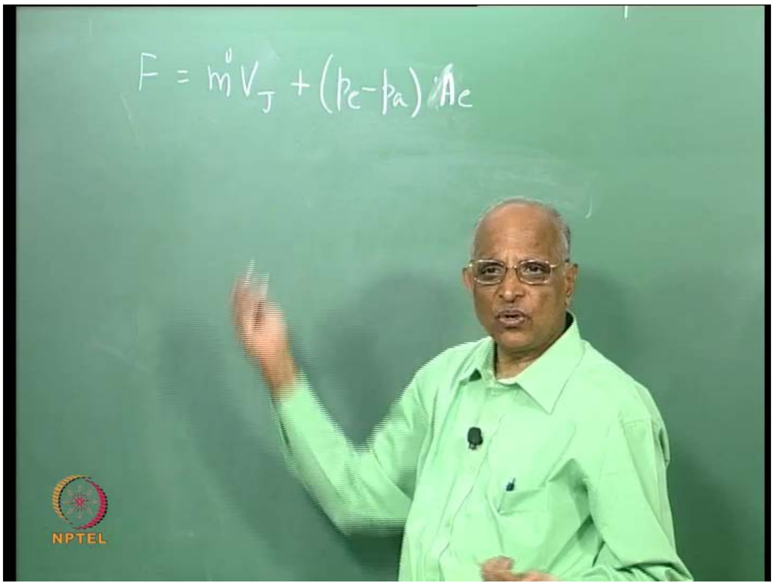

And we got an expression as F is equal to m dot V J. Now we tell ourselves well have to add the pressure term p e- minus p a- into the value of the exit area. Well, I want to get the force of the thrust developed by the rocket therefore; I need the expression for m dot V J. We have already derived; we told ourselves to gamma minus (( )) into one minus p e- by p c- into the power gamma minus one therefore, if I can derive an expression for m dot, I can find out the forces which are located. Let us do that now. I can be a little fast, because we have been doing these things for the last three or four classes.

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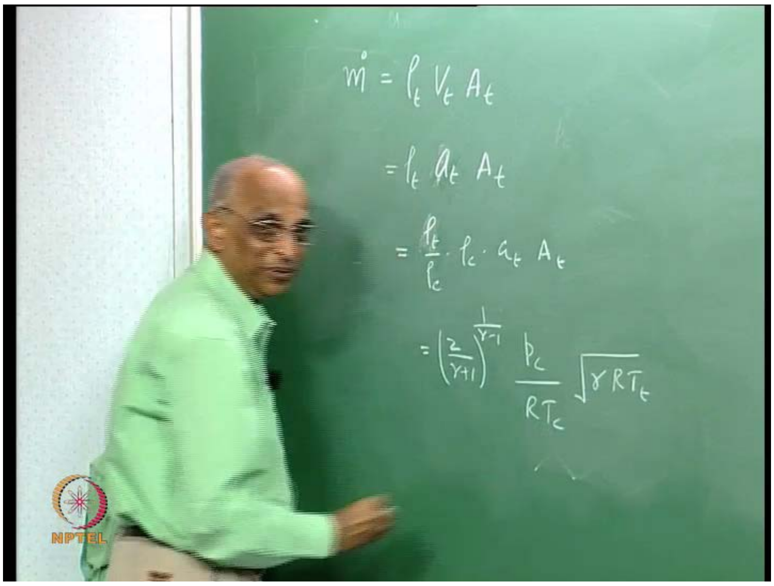

m dot is equal to rho t, V t- into A t- and is better I use the reference as A t- because, that is very standard, that is the throat and whatever beneath I need the throat. Therefore, now I say V t- and V t- is equal to V J, rho t, and no V t- is equal to A t- sound speed at divided by into A t- over here. Now can I have simplify can I write it in a form which is easy to do. Let us do that, I have rho t, by rho c, into rho c into A t- into the throat area is sound speed. Now I have rho t, by rho c and rho t, by rho c, we already know is equal to 2 over gamma plus 1 divided by 1 over gamma minus one, all right, no please, let us keep trying in the last class, we derived the expression for rho t by rho c and p t- by p c- and T t- by T c- and we got his expression now, what is rho c, p c- by R T c , what is A t- sound speed under root gamma, R T t- local condition and A t- over here.

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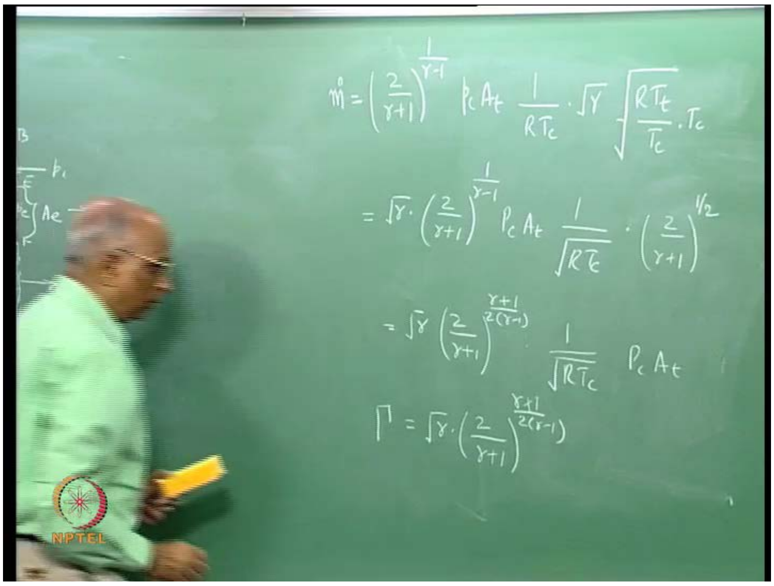

Therefore, what is this expression translated therefore, m dot in mass flow rate is equal to 2 over gamma plus 1 to power 1 over gamma minus 1 p c-, I take A t- over here together. Now I have 1 over R T c into under root gamma, somewhere here, let us put it into under root R T t. Now I find, you know I have T t- here, T c- here. But this is fully here, supposing I was to get a value here. Supposing I were to say, I multiply this one by T c- and T c- here, the expression still the same and now if I were to use this, what is it? I get now under root gamma into 2 over gamma plus 1 divided by 1 over divided by 1 over gamma minus 1 into p c-, A t- into 1 over under root R T c into the value of under root T t- by T c- have I missed out anything? I take R T c- under root outside and it gives me under root R T c. I am left with T t- by T c- here and the remaining things are the same. Is it all right, If I were to simplify this term, I keep this term as it is.

Now instead of T t- by T c-, I can write it as, 2 over gamma plus 1 to the power half T t- by T c- is 2 over gamma plus 1 and it is under root and now I m about to combine the whole thing. I tell this is equal to under root gamma into 2 over gamma plus 1 to the power 1 over gamma plus 1 half is equal to gamma plus 1, that is 1 over gamma minus 1, plus half, 2 plus gamma minus 1, gamma plus 1 divided by 2 into gamma minus 1 into 1 over R T c under root is what I get into p c-, A t- and this gives me you know this is all the function of gamma. Let me define under root gamma into 2 over gamma plus 1 in to the power gamma plus 1 divided by 2 gamma minus 1 as, capital gamma is equal to under root gamma divided by 2 into gamma plus 1 divided by gamma plus 1 divided by 2 gamma minus 1.

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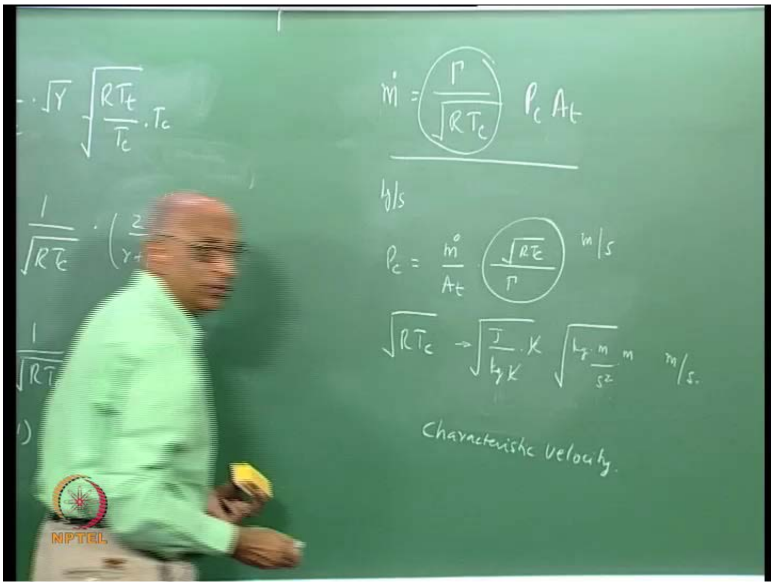

Excuse me, in which case the value of capital gamma becomes … or in terms of capital gamma, m dot is equal to capital gamma divided by, under root R T c into p c-, A t- this gives me my net flow rate as full. Is it alright? Now let find out, what is the mass flow rate through the nozzle? You find that for a given mass flow rate in kilograms per second, if I were to use this expression as something which transfers the mass flow rate into general pressure. I get something like this is something like a transfer function. What do you mean by transfer function? I get if I were to rearrange this equation, I get p c- is equal to m dot divided by A t- that is mass flux through the throat into 1 over gamma by R T c into or rather R T c divided by gamma, I take it upstairs, I can write this as, you know for a given mass flux through the nozzle this represents something like as a transfer function which will give me the chamber pressure; that means, R T c by gamma is something like a function which converts the mash flux into pressure and what is the what is the value? Let us see the unit of under root R T c, what is the unit of R T c? Joule per kilogram Kelvin into Kelvin under root, Joule is Newton meter.

And Newton meter is equal to kilogram meter per second square into meter Kelvin Kelvin gets cancelled therefore, you have meter per second therefore, you find that the value of, one over gamma under root R T c gamma is (( )) a function of gamma has units of velocity meter per second. This particular value under root R T c by gamma is what is spoken of as, characteristics velocity. It is unique of …

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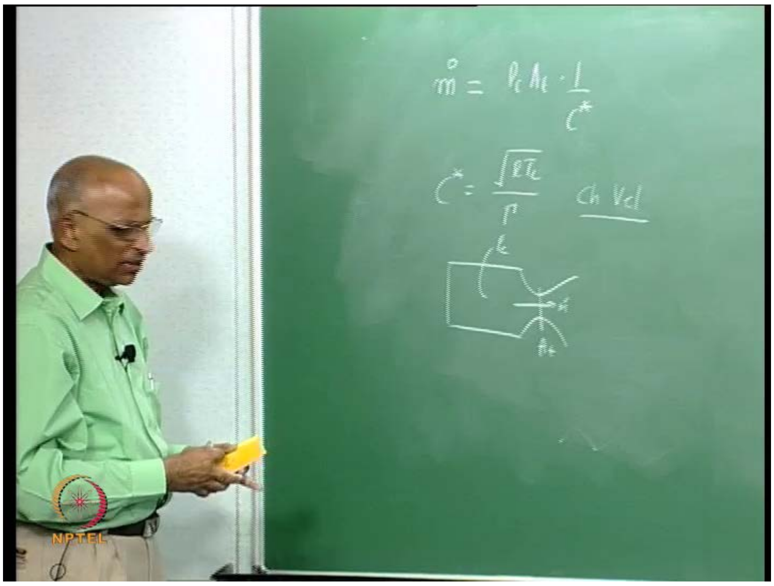

Therefore I can now write, my value of m dot is equal to, from that expression I get m dot is equal to p c-, A t- 1 over C star, where C star I define as a characteristic velocity which is equal to under root R T c by capital gamma. What did I do, m dot is equal to p c-, A t- into divided by the value of R under root R T c by gamma where this as a unit of velocity and therefore, I call it as characteristics velocity. Is it alright? We have just defining something, because we find that the unit is velocity. And therefore, given a rocket let say, I have a rocket chamber, I am told that the throat area is so much A t-. And if I know that the mass flow through the nozzle is so many kilograms per second, I can go back using this transfer function, find out what is the value of chamber pressure something like a transfer function which we call as characteristic velocity. It is an extremely important parameter, to characterize the mass generation rate of a problem. We will come back to this.

Therefore, what is it we ended up doing, we wanted to find out the value of m dot and the value of m dot, we said was equal to let us put it down. I have the value there. And therefore, I can write my force is equal to just write it down let us substitute the value of gamma under root gamma, 2 over gamma plus 1 divided by gamma plus 1 divided by two into gamma minus 1 into divided by under root R T c into p c- A t-, plus p e- , minus p a-, into A e-. This is the first generated A e- is the exit area which is the suffix e is it.

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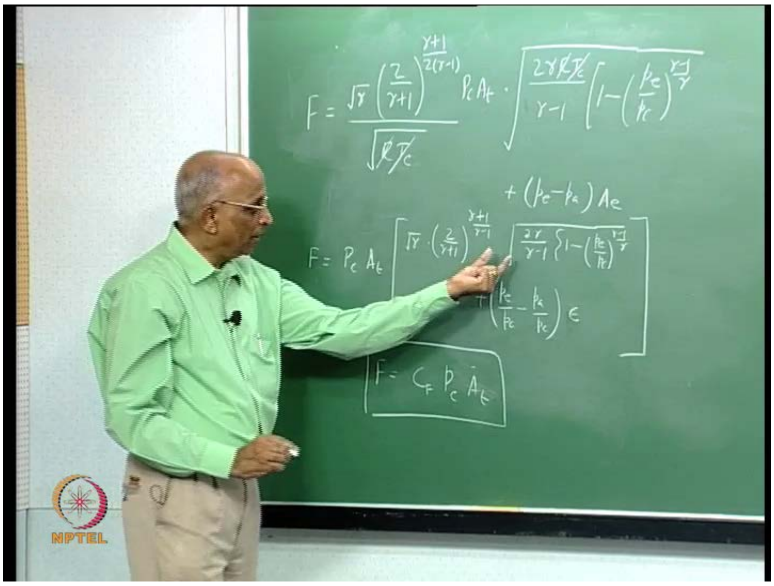

Let us simplify that expression and get our value for the first generated by a rocket or F is equal to under root gamma, again we have to take it (( )). Then we are totally done with that (2 over gamma plus1) p c- A t- .where did V J go? V J is here, what is V J, equal to - 2 gamma R T all right. Divided by no something has been lost in the mass itself R T c has forgotten. No please correct me, let us first find out, what is the value of m dot? m dot is equal to under root gamma, 2 over gamma plus 1 gamma plus 1 divided by 2 gamma minus 1 p c- A t- by under root R T c ,this is m dot. What is the value of V J? V J is equal to under root 2 gamma R T divided by, gamma minus 1 into 1 minus p e- by p c- to the power gamma minus 1 divided by gamma and now, you have, plus p e- , minus p a- divided by A e- divided by A e.

Therefore, now if I take T c- A e- outside and I simplify this whole expression again, can I cut off some number? I get R T c over here and this was equal to - 2 r chamber temperature T c- over here. Therefore, R gets cancelled and therefore, my expression for the thrust is equal to, p c- A t- into, now look at these numbers, you know you still carry under root gamma 2 over gamma plus one p c- A t- is taken outside, R T c is gone. I still have under root of 2 gamma by gamma minus 1 into 1 minus p e- by p c- to the power gamma minus 1 by gamma plus 1.

Now I divide it p e- by p c-, minus p a- by p c- to A e- by A t- is epsilon area ratio. A e- by A t- no let us, try to follow it, because we must understand each term properly and this whole term what I have here. I denote it by a coefficient C f- and I say the force generated by a rocket or thrust generated is equal to C f- into chamber pressure into A t- where C f- is under root gamma two over gamma plus one into a under root 2 gamma minus 1 to the power half into 1 minus this expression.

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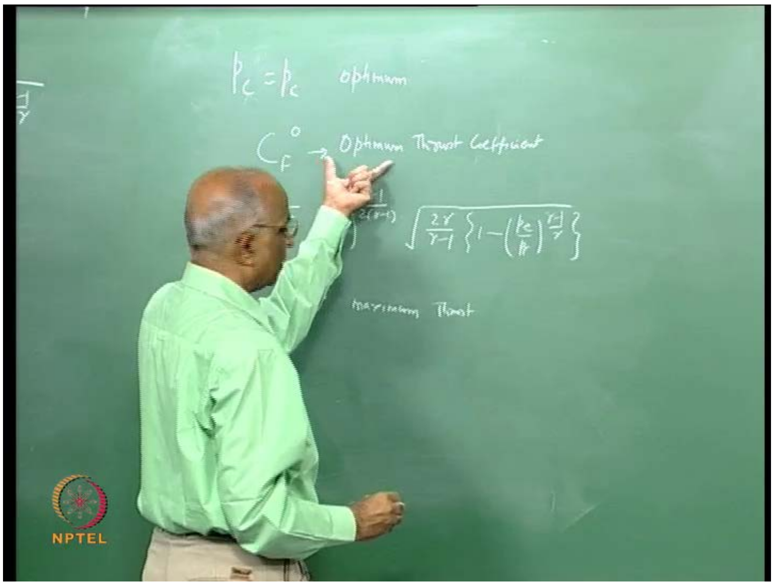

Let us therefore, write down the final expression for C f-, I think we should not write it again and again. We know what it consists of, I want to find out what will be the expression, and if we have p e- is equal to p c- an optimum. The value of C f- that is p e- is equal to p a- optimum. There in this case, this goes to zero and in fact when let us compare these two terms: first you know what will happen is, when the ambient pressure is equal to the exit pressure?

This term knocks out, but then you are increasing the value of p a. And therefore, the value of C f 0, corresponding to the value of p e- is equal to p a- will be maximum and that is denoted by C f- or rather I write C f 0- which corresponds to the optimum compare to C f-. I write it as C f 0- or let us put it C f 0- here is equal to, I just write the whole expression, other than this particular term which close to zero and I get C f 0- is equal to under root gamma 2 over gamma plus 1 gamma plus 1 divided by 2 gamma minus 1 into, I have the value of a under root two gamma minus 1 into 1 minus p e- by.

Whereas, if I have something like the exit pressure not being equal to the ambient pressure.

If I have something like an over expanded nozzle, I get a negative term. If I have excuse me optimum expansion in which p a- is equal to p a-, this becomes zero. if I have under expanded nozzle, I have positive thrust and in all cases when p e- is equal to p a-, I get the maximum thrust F. Because in one case, what is happening is the contribution from this increases? But the contribution from this decreases and only under the condition C f- is equal to C f 0-, I get the maximum thrust; that means, an adopted nozzle always gives me maximum thrust. Otherwise because of under expansion, I lose thrust. Otherwise it is the other way around therefore. Let us, now take a look at some of the results from this expression. You know, all we did was this is the value, we derived and this is the value, we get for the optimum value. I put gamma minus one outside over here and now, if I take a look at the optimum value of C F- which I call as thrust coefficient.

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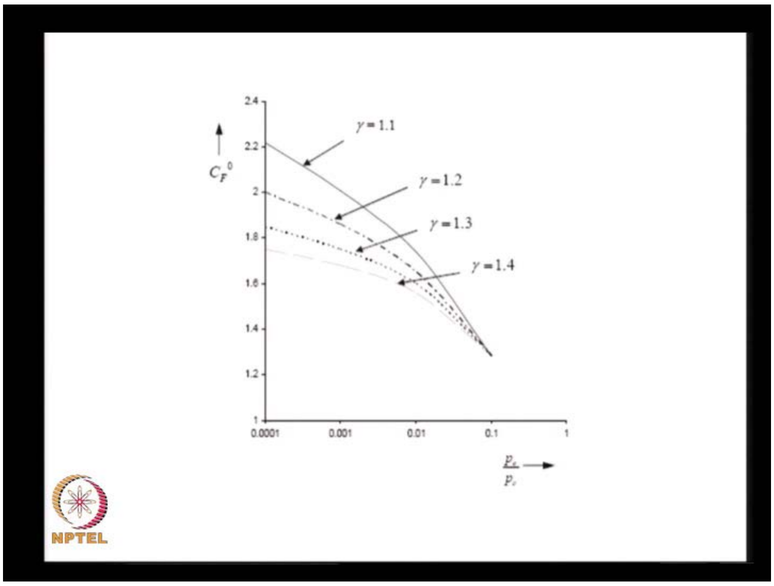

That is C F is thrust coefficient, C F dot is optimum thrust coefficient. What is it I find? As I increase the value, as I increase the value of p c- by p e- ; that means, I over here the value is p e- by p c-. That means I have higher expansion ratios, higher values of p c- over here, lower values of p c- over here. As I increase the value of p c- by p e- , I get a higher and higher value of C F and the values which are realizable by around two point four, two point five and we find that the lower value of gamma, gives me a higher value of C F than, a higher value of gamma; that means, the value of the thrust coefficient is sensitive to the value of gamma, in addition to being a function of p c- by p e- .

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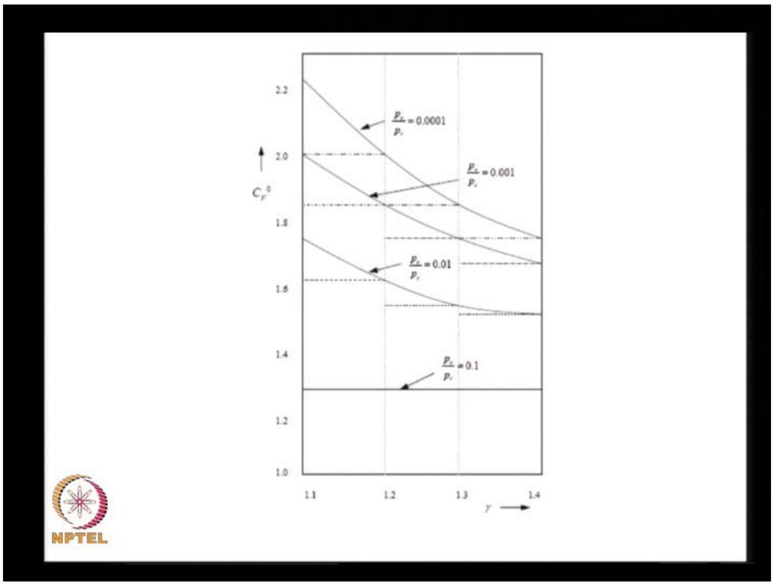

The same thing I plot on this particular diagram, wherein I plot the thrust coefficient as a function of gamma. I find that at load angle p c- by p e- gamma does not influence the first coefficient at all. Whereas, if I have a small value, that means I have p e- by p c- which is a very small number. The variation in gamma causes a considerable change. That means, a decreased value of gamma will give me, a higher value of C F.

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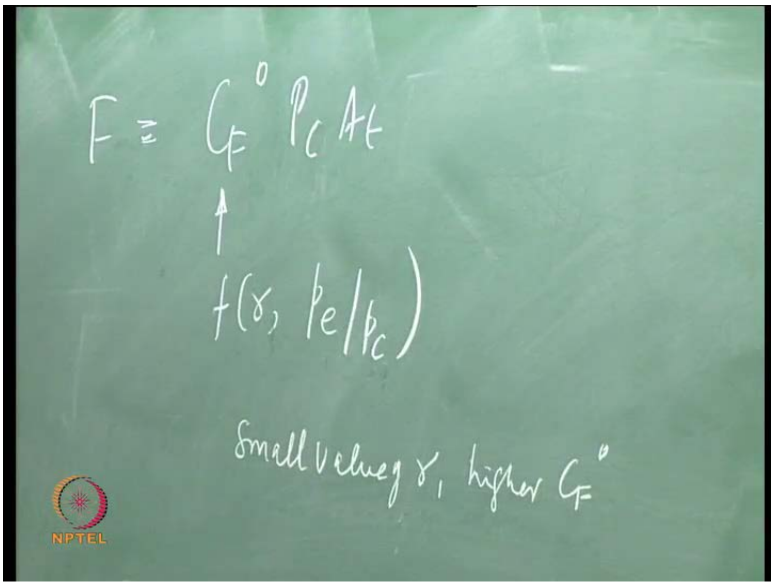

Therefore, what is it I have done? We look at the thrust coefficient optimum thrust, when you have a case of p e- equal to p c-, C F dot into p c- A t-. We find that the value of C F is also, function of gamma. In addition to being a function of p e- by p c-, a small value of p e- by p c- or a larger value of chamber pressure, gives me a higher thrust coefficient. This gamma depends on this value. if this value is not large, is not small, make for instance I have a low chamber pressure. Then gamma is not influential, but otherwise a small value of gamma will give me, a higher value of C F 0. This is all about thrust in a nozzle. How now what we have done? We have simplified the whole nozzle calculations and what is it we have done?

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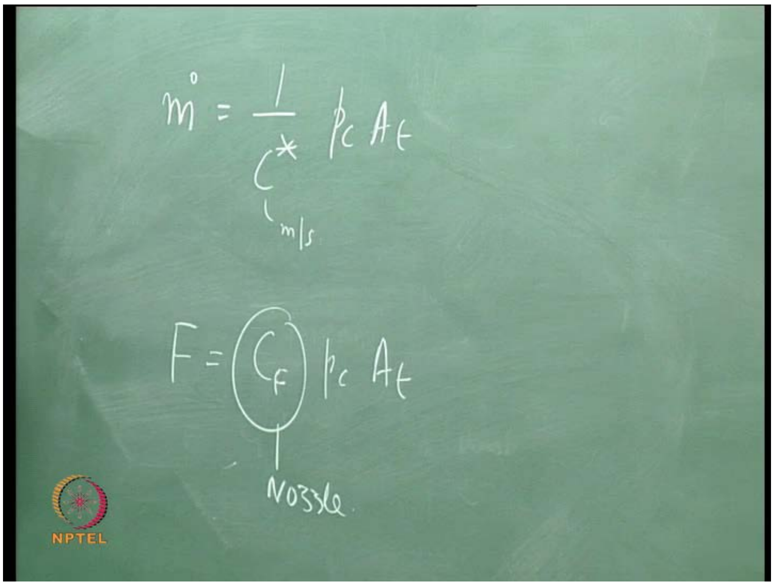

We told ourselves well, m dot - mass flow through a nozzle is equal to one over c star into chamber pressure into throat diameter, where c star is equal to under root R T c- by gamma which we called as characteristic velocity unit, is being meter per second. And we said, force in the nozzle is equal to C F, whether under expanded optimum or over expanded, is equal to p c- into A t-, two simple expressions. Therefore, whenever we make a nozzle, we go and evaluate the nozzle for C star. But C star does not come from the nozzle well, see why because we are talking of the transfer function between mass flow rate and the chamber pressure. Whereas, the thrust coefficient tells me what a nozzle is doing, it takes the chamber pressure multiplied by a coefficient, it gives you the force.

And that is why; we say this performance of a nozzle. I think I need to go a little deeper into this particular expression. Therefore, let us write out these two expression in a slightly different form and then, on this to obtain the next couple of minutes. And with that, we would have done some justice to the nozzle, but before that I would like to again emphasis, what it we have done is. So far we took a nozzle; we assumed the nozzle is adiabatic reversible flow, isentropic flow, and one dimensional flow. So, far it is all one dimensional.

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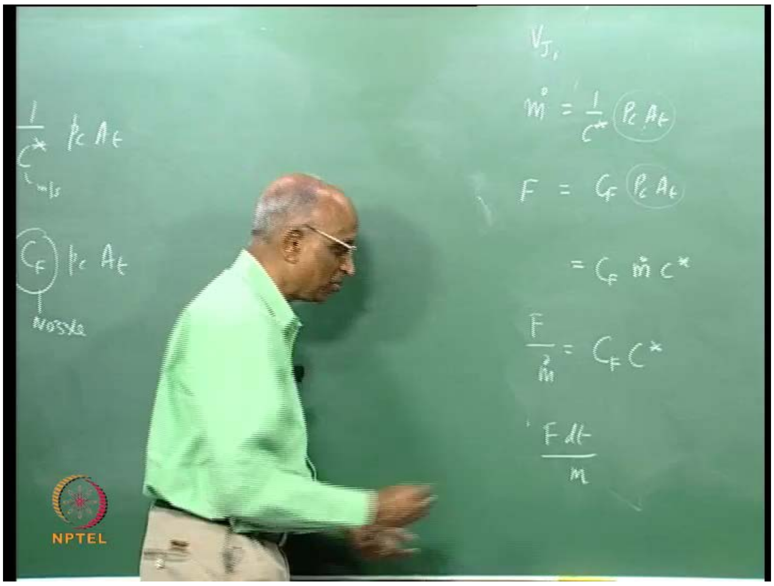

And for one dimensional flow, we drive all the expression for V J, we derive the expression for under expanded, over expanded, may be flow outside the nozzle and all that. And now, we find I can write m dot- is equal to 1 over c star into p c- A t-. We also derive the expression for the thrust form basis, we find it is equal to - coefficient into p c- into A t-. Now if I play with these two equations, I find over here p c- A t-, p c- A t- common. Therefore, can I somehow put this together? Can I substitute p c- A t- from mass equation into the force equation and if I were to do it, I get C F. I get p c- A t- is equal to m into c star. Therefore, I get here m into C star m dot into C star and now, I get the force first develop divided by m dot is equal to C F into c star .And what if F divided by m dot? What is the thrust per minute mass flow rate? That is we are talking of F d t impulse divided by mass of the propellant which is their, impulse per unit mass is a specific impulse ISP or rather what is it? We have got ISP is equal to - first coefficient into c square, but we have got an expression ISP out here.

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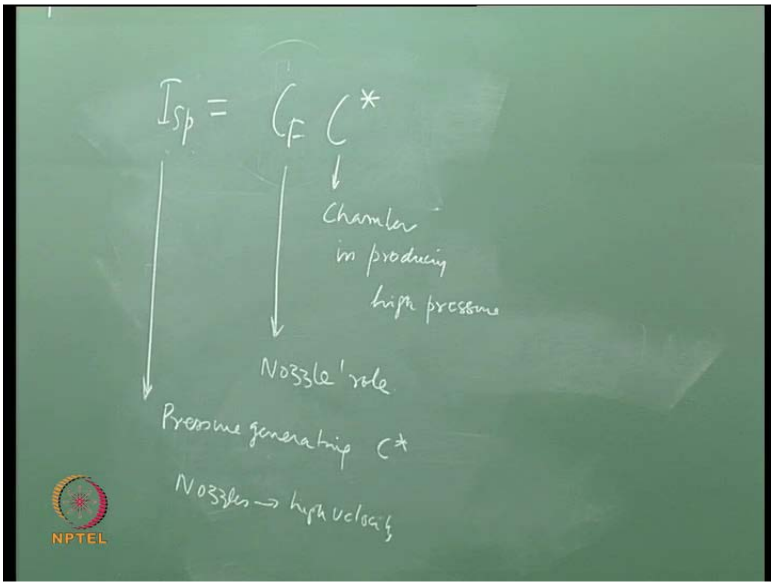

The expression for ISP was V J, what was V J? V J was, when the exit pressure was matching the ambient pressure that is when we derive the expression for, we got the value. Now you have the contribution coming from the exit pressure also. So and to some extent, what is happening we may not consider the flow in the nozzle?

Therefore, what is it telling you? It is telling you, ISP depends on the performance of the chamber, and what is a chamber do? Chamber in generating or producing high pressure and what is C F do? C F is in the nozzle which converts this high pressure into high velocity is the nozzle effectiveness or the nozzles role and therefore, C F is the figure merit of the nozzle, how it converts the high pressure into velocity

And c star is a figure of merit of the chamber. In that it will tell you, how high pressure is generated in nozzle? Because of some mass flow rate therefore, you have a composite index. Therefore, ISP is a product of pressure generating capacity in the rocket.

And how is pressure generated through c star? Whereas, once the pressure is generated the nozzle, helps you to generate the high velocity high jet velocity therefore, you have both the chamber capacity; that is a propellant to generate hot gas and C F which means the effectiveness of the nozzle. Therefore, ISP is equal to C F by C star, but you know what we have done? So, far is ideal one dimensional and all that therefore, it is necessary for me to go back, ask myself is everything cannot be ideal. Therefore, there has to be some something like some efficiency.

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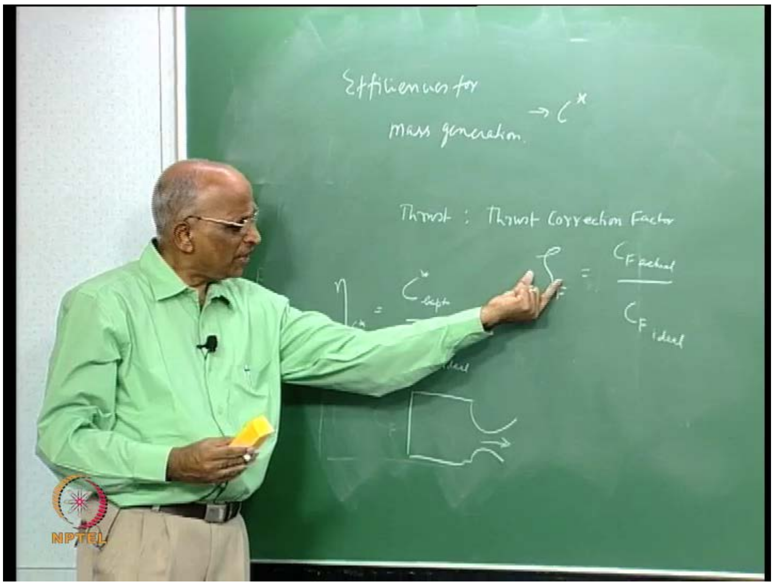

Therefore, can I talk in terms of efficiencies for mass generation and efficiencies for thrust. The mass generation, we decided in terms of C star therefore, if I have to put efficiency for mass generation, it is equal to C star which is experimentally observed, divided by the ideal value which we derive in this class and what is the ideal value under root R T c by capital gamma, and this is that.

Therefore, all what we do in an experiment, you take a rocket, you will find out, what is the rate mass is delivered out? Find out the ideal, and you say this is eta c star and we will find that the values are quite high of the order of 982 point 99. We will do some problems in this, how do you get the first value efficiency for thrust, you call it as thrust correction factor. And you denote it by zeta F, correction factor which is equal to, we say C F actually measured in a chamber divided by C F, whatever we calculated was ideal. And we use the ideal value, to find out the efficiency and again these efficiency are quite large for nozzles of the order of again point nine eight, point eight seven. Therefore, what we have done in today class is, we looked at the high temperature and high pressure gases generated by the propellant.

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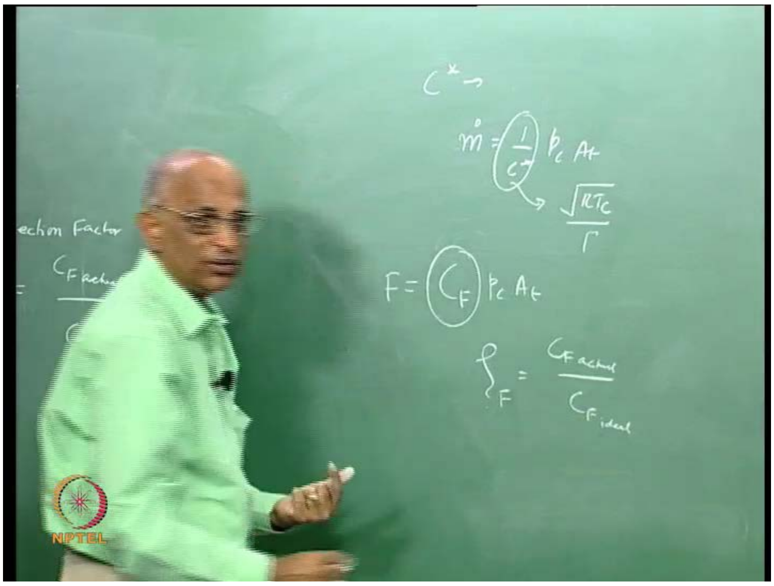

We expressed it in terms of C star; that means, C star tells you, what is the rate at which hot gases get generated from the propellant? And therefore, we wrote it as m dot is equal to 1 over C star into p c- A t-, where p c is the chamber, we call it as a transfer function. And the expression for this is extremely simple, is equal to R T c by the particular value of gamma.

We also talk in terms of the thrust coefficient which is used to generate or describe the thrust developed by a rocket, F is equal to p c- A t- and we find out the expression for C F, and also how C F varies? And we talk in terms of a thrust correction factor, which is equal to C F actual by C F which is calculated based on ideal, one dimensional.

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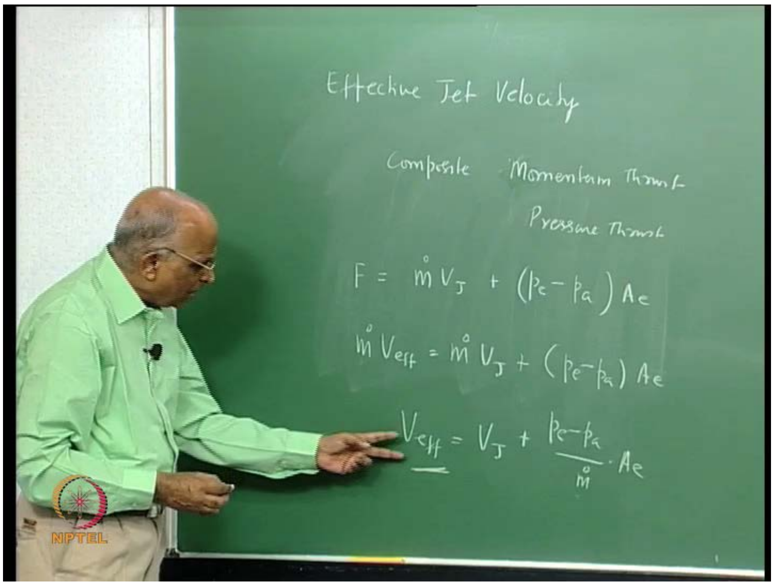

Sometimes an effective jet velocity is defined in the literature, to determine the composite of V jet or the velocity thrust or the momentum thrust and the pressure thrust. Let us examine, what is effective jet velocity is? Let say that the total thrust is given by the momentum thrust which is equal to m dot into V J and the pressure thrust is because at the nozzle exit, the pressure p e- is greater than p a- and into a e-; This becomes the pressure thrust. Now if I have to put this thrust in terms of an effective jet velocity, I say F is equal to m dot into v effective and this is equal to m dot into V J plus, you have the same thing p e- into p a- into area. And therefore, the effective jet velocity is equal to the jet velocity plus, you have p e- minus p a- divided by m dot into A e- and is this is defined as, effective jet velocity and when a nozzle is not adapted.

That means, the exit pressure is different from its ambient pressure at that particular altitude. The effective jet velocity is different from the jet velocity at the exit of the nozzle; this is too take care of the pressure force in addition to, the momentum thrust over here. We continue with nozzles in the next class, but in the next class well try to take a look how to contour a nozzle? And what are the approximations from the ideal cases, what you have studied? And this is what I am about to do in the next class well. Thank you.