Rocket Propulsion

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Lecture No. # 14
Unconventional Nozzles and Problems in Nozzles

Well, good morning. In the last class, we talk in terms of the contour nozzles. The shape of the contour nozzle is in the form of a bell. Initially, you expand, the flow is larger and you compress the flow later on, such that you get a very small value of divergence angle, let say 2 to 5 degrees. Initially, you expand the flow at a larger angle say between 20 to 50 degrees and this shape of this contour is something like a parabola. You you fit in something like a second order parabola, such that you can get this point and this point into this particular parabola.

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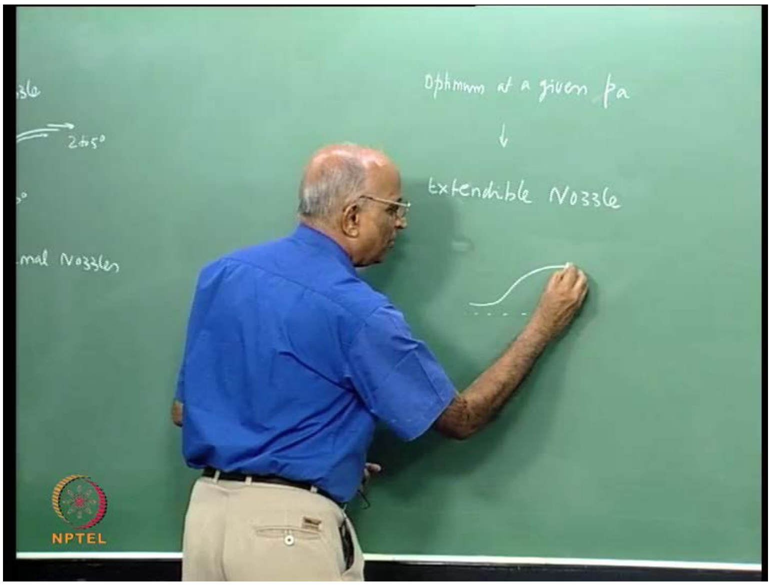

This is how a contour nozzle looks like. In today's class, let us look at some unconventional nozzles and say, are there any better ways of having nozzles other than, let say conical nozzle, or contour nozzle.

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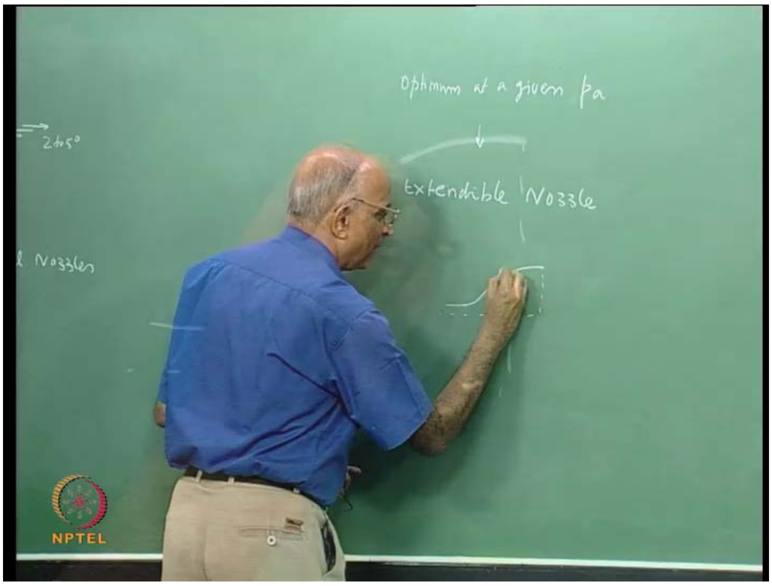

Also, may be make mind set little more clear, what should be the future work which we must do on nozzles. I think that, may be another 10 minutes and then, I will work out some examples. You know, first I can tell myself, well, a nozzle operates at its optimum at a given value of ambient pressure. In other words, if I want to make a nozzle, I design it for, let say 15 kilometres. Start operating it from 0 kilometres and the rocket goes up to 30 kilometres. Initially, 0 to 15, it is not optimum. At 15, it is optimum and beyond 15 again, it is under expanded and not optimum. Therefore, can I have something like, let us say a nozzle, which I can have something like an extendable nozzle.

Let us, let me give you an example. Suppose, I have a nozzle something like this and this nozzle is operating at lower altitudes or higher pressures. Now, I want to, I store, maybe I have to expand it out somewhere and again make it higher. Therefore, I have, I initially store something like this. I bring it over here and I lock something over here. This initial nozzle operates at low altitude.

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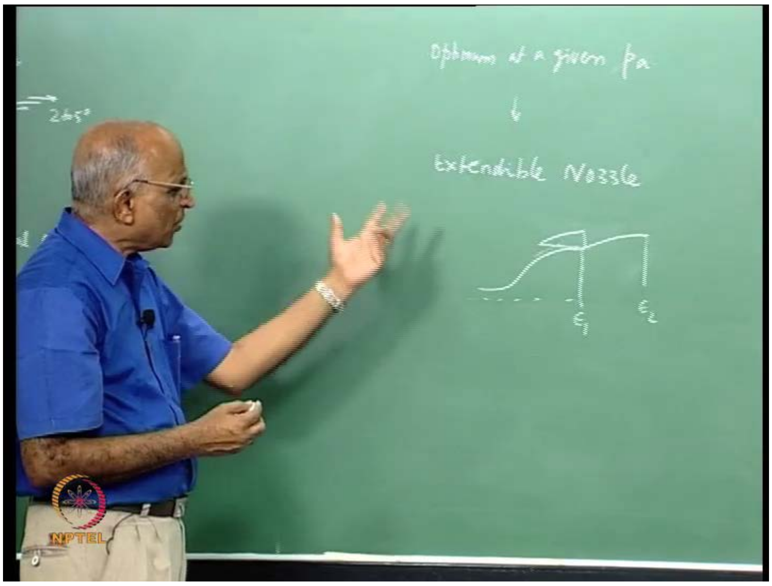

The moment the nozzle of goes to higher altitudes through some mechanism, I shift this over here and then, may be make it little go still higher. That means, this becomes my initial low area ration nozzle and corresponding to this, I have a higher area ration nozzle. That means, I extend the nozzle during the flight and during the lower altitude, I use the smaller area ration nozzle. Then, I store this on top of this and when it reaches particular altitude, I push it over here. The nozzle length increases and the area ratio increases and this is what we call as an extendable nozzle. This was tried in a flight but, it does not, is not used extensively. It is not used at all even though it has been tried. We call it as extendable nozzle. If, instead of having an extendable nozzle, the other alternative is, why not we have a double bell nozzle or something like a dual bell nozzle?

In other words, I have a nozzle like this and I want to increase the area ratio still further. What I do is, I put something like a step over here and then, I continue this nozzle profile like this. Now, this is my centre line. Now, what is going to happen? At lower altitudes, the flow expands to the ambient pressure and flow separates and flows over here. At higher altitude, because the pressure is very much low, much higher than the ambient pressure, the flow reattaches here. Therefore, I can get area ratio epsilon 1 and area ratio epsilon 2 over here. This is known as a dual bell nozzle. In fact, this month’s issue of AI journal has a paper on this dual bell nozzle looking at the optimum conditions. That is, still works are going on this.

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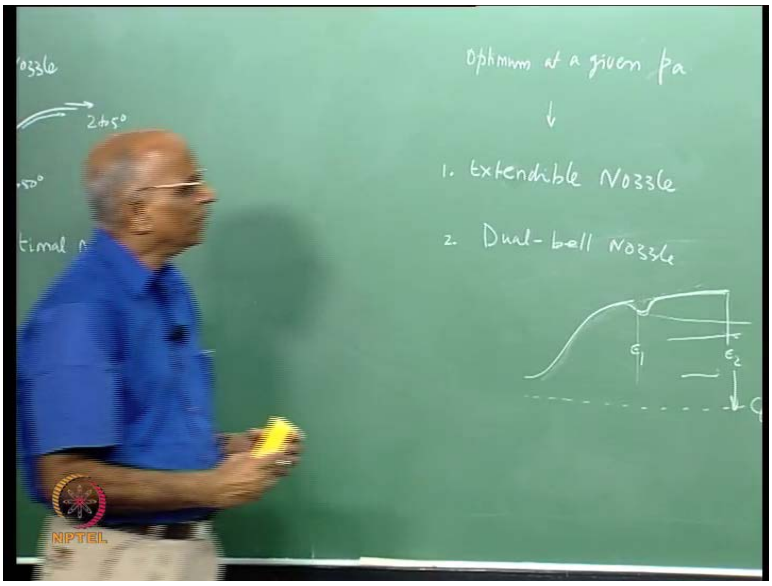

Therefore, second we say unconventional. First, extendable nozzle and second is a dual bell nozzle and third, we say radial nozzle. Radial flow nozzle, what do I mean? Something like, what I say is an expansion deflection nozzle. Let let me put one throat over here. Let me say, this may convergent and I have a divergent like this. This is my centre line over here. I put a throat over here and I put a block over here. Something like a plug over here. I allow the flow to take place and what happens? The flow is guided by the contour wall over here at the centre, it is not guided. Therefore, I have something like an expansion wave. It is able to adapt to different altitudes.

Therefore, by putting this, I can make this particular nozzle operate at different altitudes. In other words, I have the outer wall, which guides the flow. Inner part is free. It can adapt different altitudes and therefore, this is known as an expanded ED nozzle or expanded deflection nozzle. See, what we have to do is, we want the nozzle to operate at different altitudes.

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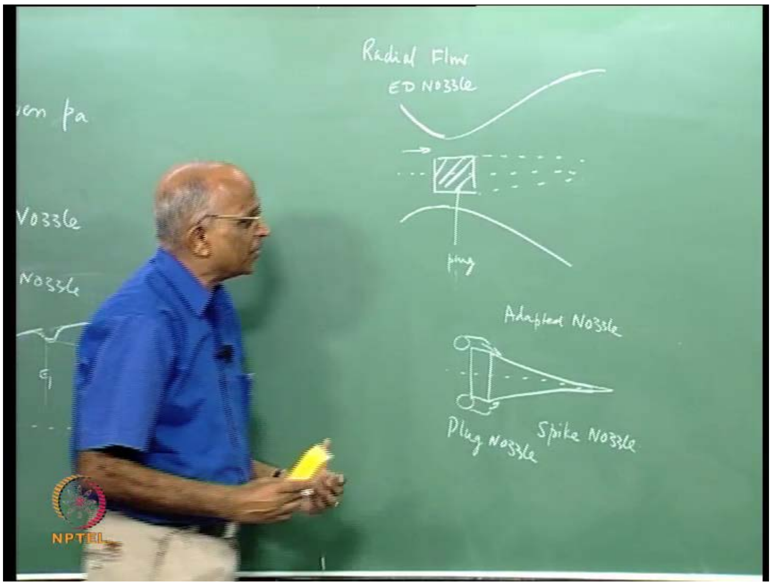

Therefore, what we have done is, we have done with a plug and we allow the contour to change the pressure of the outer flow. But, inner flow we keep free, such that, the inner flow can expand and do this, the expansion deflection nozzle. As an extension of this, we could also have a nozzle instead of having plug here and allowing the outer thing to flow across a nozzle. I can have a nozzle and I can have this plug in the form of, let say, a contour over here. Centre contour like this and what I do is, I bring the flow over here, from the chamber. In other words, I have a annular chamber over here and I allow the flow. I allow the flow to come across this. I allow the inner contour instead of the outer contour to balance the flow. Keep the outer opens such that, it is again a case of an adapted nozzle. It can adapt to any condition. I could have it in the form of a spike, in which case, I call it as a spike nozzle or simply, I call it as a plug. Because, I put this plug in this, I call it as a plug nozzle. I could still have some variation.

This particular plug which is at the centre, I could terminate it a little earlier. Somewhere here, I do not allow the total spike. In other words, I have a primary flow along this and I have the shock waves here and the secondary flow here. I have recirculation and a base pressure here and this becomes what we call as the aero spike nozzle. Let let me make these things very clear. What we are saying, the combustion is happening over here; an annular combustion chamber or the combustion chamber over here. Somewhere over here, combustion is taking place and I push the flow along this and I allow the spike to regulate it.

Instead of having an outer boundary which regulates the pressure, I have an inner boundary which corresponds to a spike, which we call as a plug nozzle or a spike nozzle. Of course, this is a plug and the same plug which we used in the case of a contour nozzle, allow the contour to deflect the flow. Here, it was free such that, these are something which we can call as nozzle which can adapt to the ambient pressure.

Aerospike again, we are having additional thrust coming from this which we call as a Aerospike. But, why should we always think in terms of a cylinder or a bell or something like that. Why not open out the bell. Make it something like linear. If I have, not a cylinder, but I stretch out and I call it as a linear nozzle. What do I mean by a linear nozzle? Well, I open out the bell something like this and I have this as a contour. Now, I allow the flow, this is the ram as it were and I allow the gases to come along this and it is guided by this particular ram.

In other words, I have some something like this. Let say, over here, it could be a shape of something like this. Flow comes along this, guides along this and comes out and similarly, flow comes along this and this becomes something like a shape, which is linear or this could be one part of an aeroplane, like let say a wing or a fuselage.

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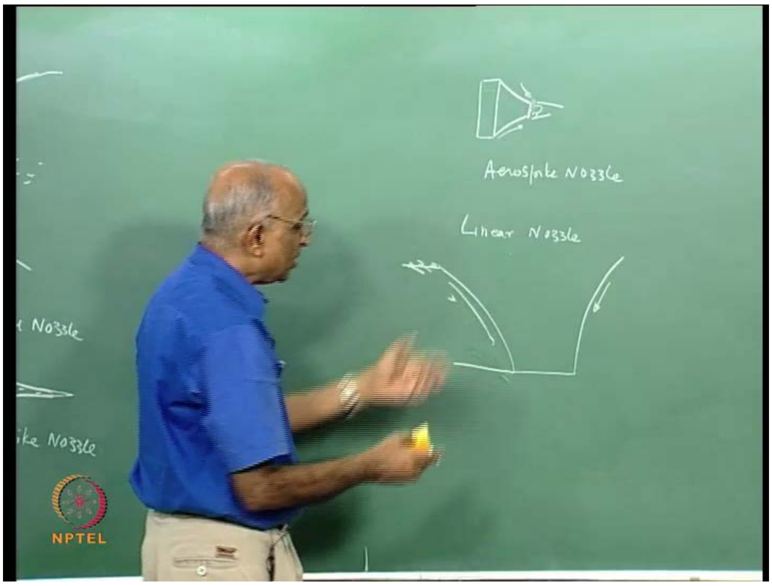

I allow the gas to come along it and flow along this and these are certainly known as linear nozzle. This has been used in aircraft industry for the space plane. Let let me not dwell on these things because, these these follow the same principle what we have discussed so far.

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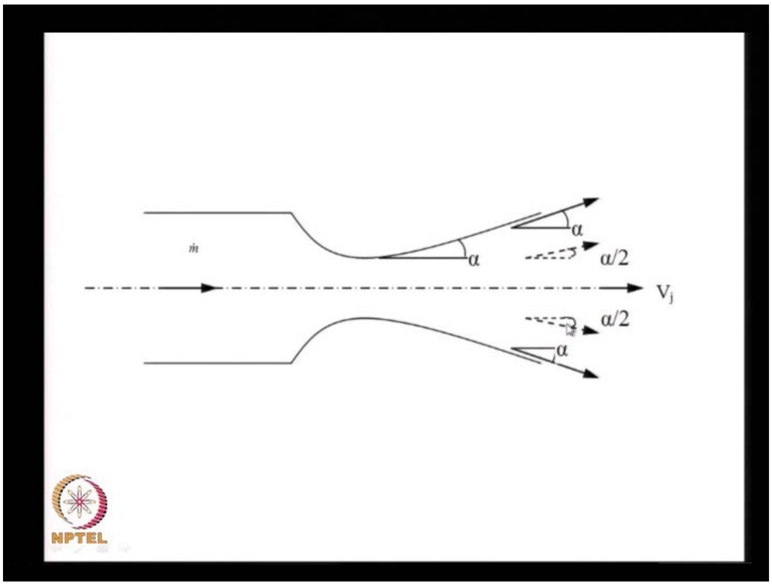

I will not get into detail but, I will just summarize it through series of, may be slides over here. In the first slide here, I will show the divergence losses, may be, alpha and how we got the divergence losses coefficient. This was the value of delta, which we decided as percentage loss in thrust versus the angle alpha.

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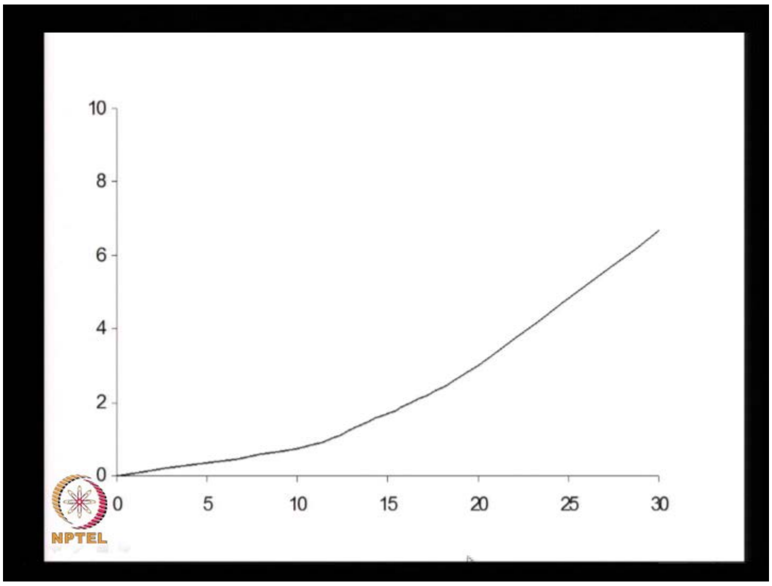

We derived this and we told ourselves, the nozzle is around 15 degrees or so for a conical nozzle. We talked in terms of a contour nozzle and we told ourselves instead of having a cone, I initially expand the flow to alpha i.

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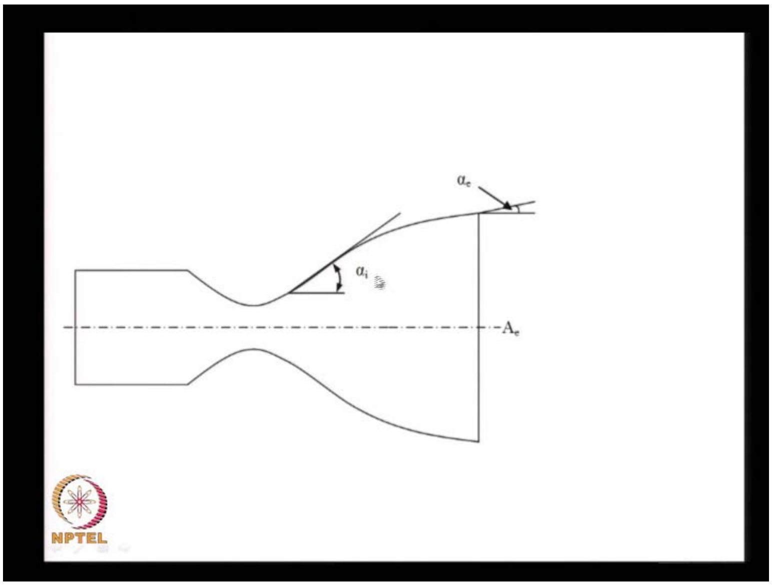

alpha i is between 20 to 50 degrees and then, bring it back. I have a low angle over here at the exit of something like 2 to 5 degrees or so. Smaller the angle at the exit, smaller is my loss, what I have. This is the extendable segment.

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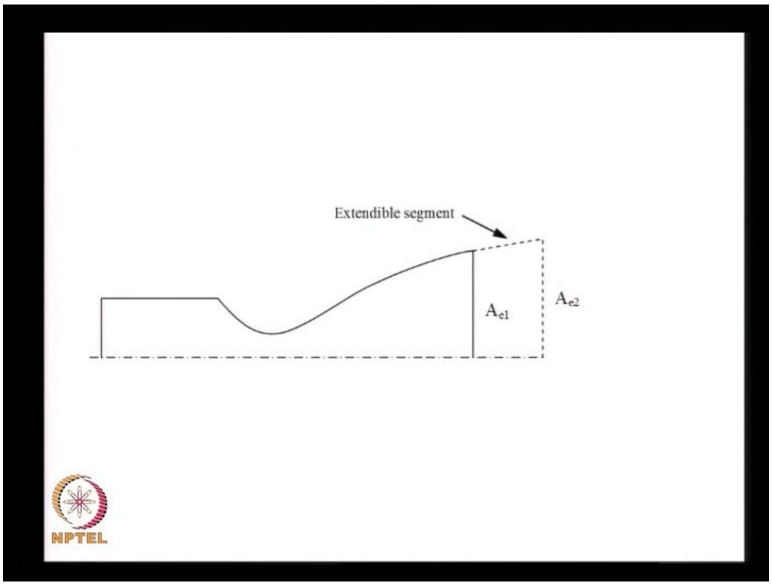

Initially, I have a small area ratio corresponding to A e1. I store this on top of this and when I want to fly at higher altitude, I push it out and I get a larger value of this over here.

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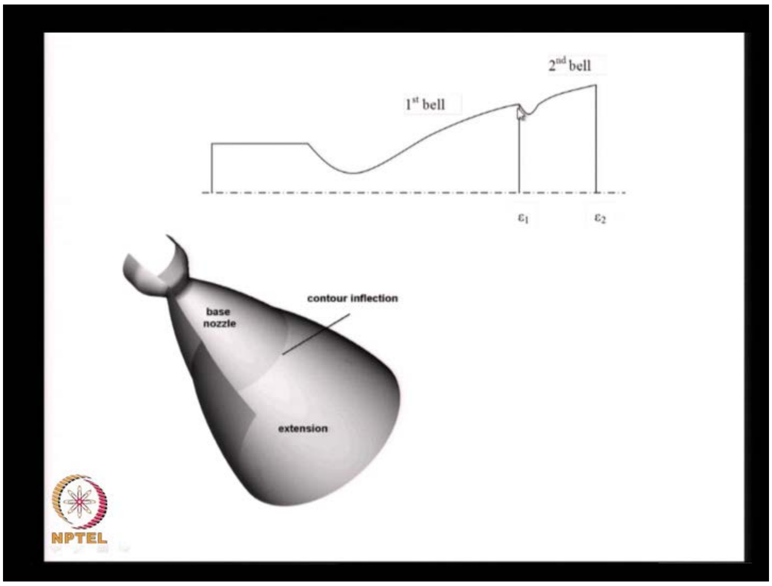

This is a dual bell nozzle. As you see, this is the contour. I give a step over here and you see the step over here and this is adaptable to different altitudes. As I told you, there was a paper this month in a I a, which deals with these nozzles.

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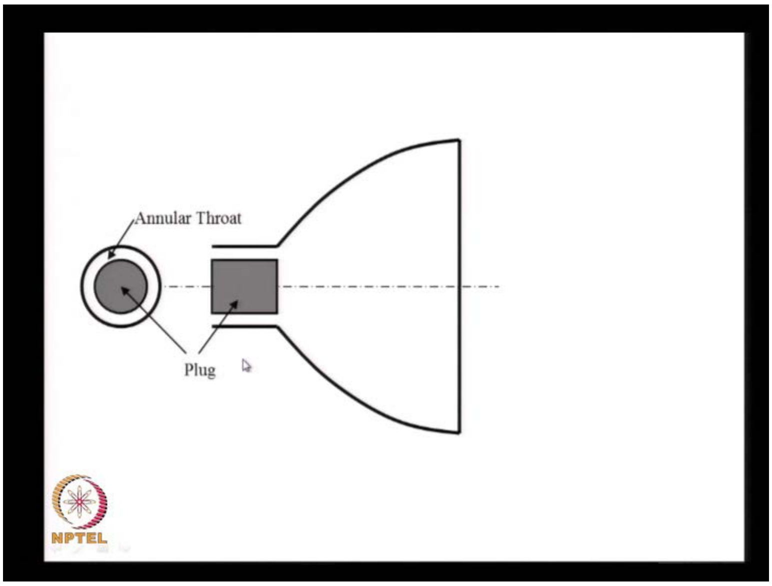

Those who are interested should go through it. This is a plug nozzle. I put a plug here and I have an outer surface, which guides the flow. The inner surface is free therefore, it can adapt to the flow where it where. What you have is an annular throat, instead of having a diametrical throat over here.

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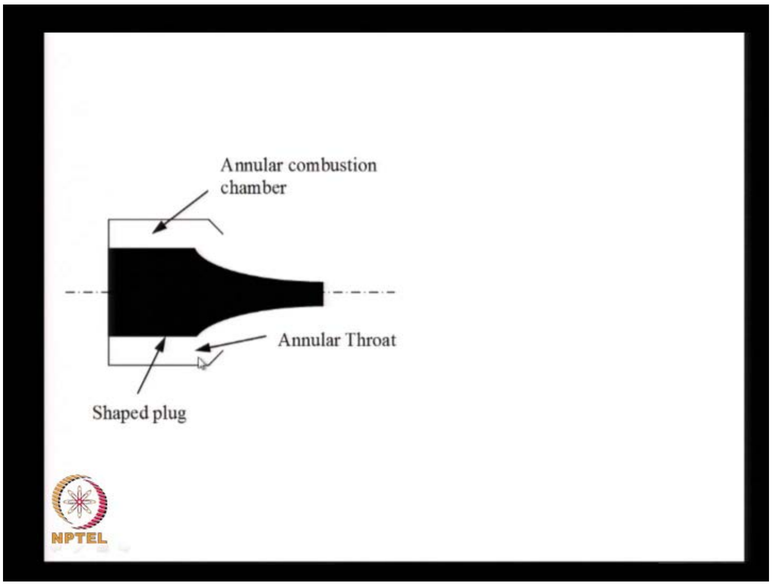

This is what we say is a plug nozzle or a spike nozzle. I have a spike over here. The flow comes. The flow in these cases, you know, you generate it in an annular chamber instead of having a chamber over here. I push the flow onto these inner surface and inner surface guides the flow. Outer surface is free; therefore, I can have any type of expansion what I want.

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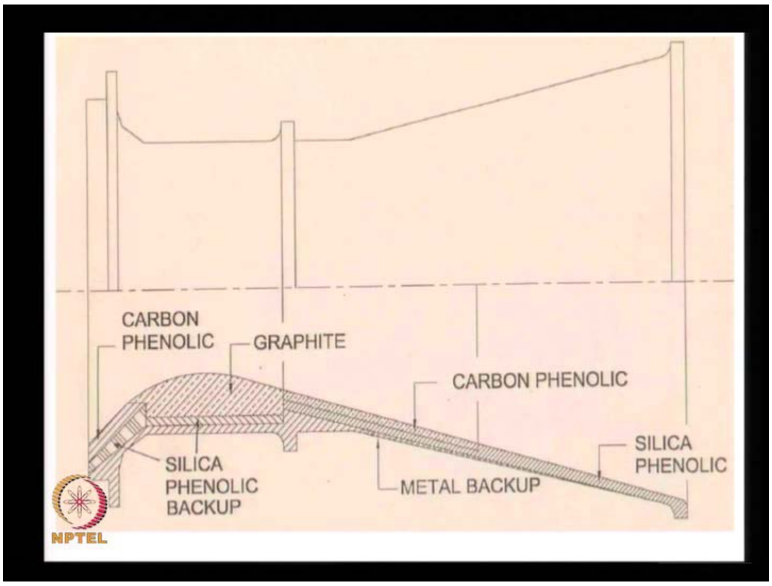

It is still not used in practice. This summarizes what we learnt about nozzles. It is a conical nozzle and the nozzle runs hot. Therefore, to protect the nozzle, I give insulation on the inner surface. This is the conical nozzle. I give something like a carbon, which can take a lot of heat. I give some something known as ablative materials. I will come back to it, when I deal with cooling of rockets. I will get back to this slide a little later in the course. But, this is how a practical nozzle looks like. This is the outer surface and this is the inner wall of this.

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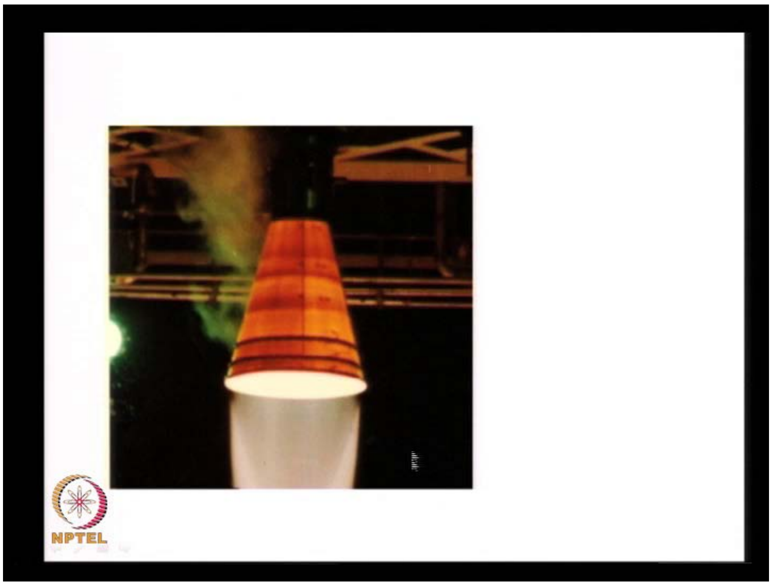

I repeat this. A nozzle is firing for a certain amount of time. This is the conical nozzle. You see, that the nozzle runs red hot and you have the flow taking place. This is where we said, I have the waves and all that and we talked in terms of shock diamonds.

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Well, with that we close the portion on nozzle. But, it will be incomplete without doing one or two problems. In the next, matter of 25 minutes, I will do one of the problems and the problems which I do, is related to this particular vehicle. This vehicle is known as Saturn 5 vehicle. Let us put it down. Let us do this problem. See, Saturn 5 vehicle was used to put first men on the moon. Therefore, you have Saturn 5 launch vehicle, which puts the Apollo capsule carrying three men on to the moon. This is, by far the biggest rocket ever made in the history of rockets. It is the most powerful rocket and what does it consist of?

If you go back and look at this particular slide, the the first stage of the rocket, somewhere over here, consists of five rockets clustered together. That means, each one of these rockets is known as F 1 rockets. It consists of five F rockets clustered together. The second, it uses kerosene and oxygen as fuel. Kerosene as fuel and liquid oxygen as oxidizer. The second stage consists of five engines again. It is known as J 2 engines. I will get back into the details of this. 5 J 2 engines clustered together and this uses liquid hydrogen and liquid oxygen. The third stage consists of one single J 2 engine. Therefore, what is it we are talking of? The Saturn 5 rocket consists of the first stage, which consists of, may be fives and these are 5 F 1 engines of 5 F 1 rockets together. The second stage similarly, consists of a cluster of 5 J 2 rockets. You know, these are all name of the rocket. J 2 rocket uses liquid oxygen and liquid hydrogen as fuel. This uses kerosene and liquid oxygen as fuel. On the third stage, you have a single J 2 rocket and on top of this sit the particular capsule, which is the Apollo capsule, where the three people who travelled to the moon sit over here.

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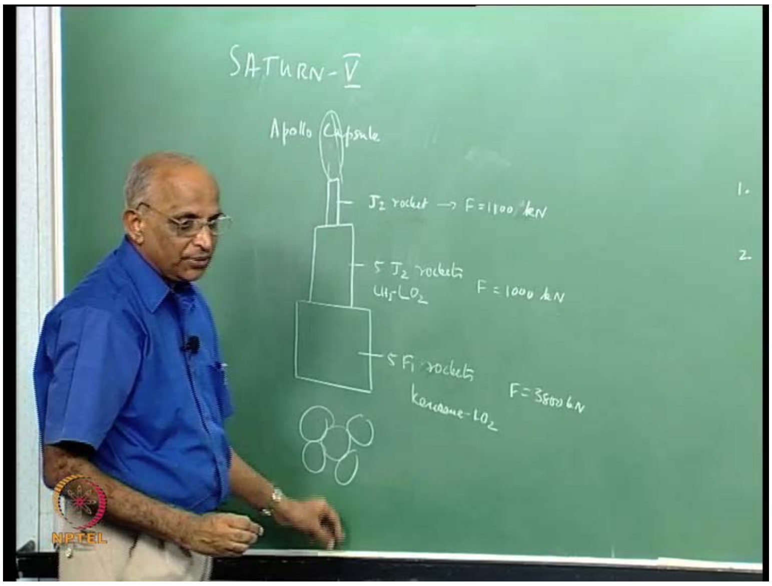

We would like to do an example. Let us take an example of F 1 rocket. Let before that, let us put some numbers down. J 2 rocket has a thrust of something like 1000 Newton, 1100 Newton. Each of the J 2 rockets here, mind you, the same J 2 rockets when used for the second stage, has a thrust of 1000, I am sorry, kilo Newton. Each of the F 1 rockets has a thrust of something like 3800 kilo Newton. Let me make sure about the numbers. This has something like, you know, when I look at the force, please be very clear that this has the thrust of something like 110 ton thrust. Because kilo Newton, therefore, we are talking of 10 Newton is equal to 1 kilogram. Therefore, we are talking of a huge force here. Therefore, why is it that the same engine when used in second stage produces less thrust than when using in the third stage? Altitude. That means, higher the altitude, I get more specific impulse and therefore, I have thrust. Therefore, let us take the example of one F 1 rocket. Out of all these five, let us do the nozzle problem related to, let us say one F 1 rocket. The problem I consider is, the thrust of this rocket is equal to 3800, is that what, no my numbers are not correct. This this gives me a much higher value. 6800, I am sorry.

This is 1000 and this is 100. Each one has a thrust of 6800 kilo Newton. The mass flow rate through the nozzle is equal to 2600 kilogram per second. These are typical numbers, you know. We should keep in mind that we are not talking of 1 kilogram per second. We are talking of something like, almost 3 tons of propellants going through the nozzle. The area of the nozzle is equal to 0.65 meter square. That means, if I put it in terms of diameters, well, a man can easily stand at the throat or go through the throat of this nozzle.

The molecular mass of gases which are passing through the nozzle is equal to 22 grams per mole. The temperature of the combustion products in the chamber is equal to 3300 Kelvin and the chamber pressure is equal to 6.65 m p a. That means, something like 66 bar. This is little below the standard pressure of 7 m p a, which we are talking of.

Mind you, this rocket was developed in the period 1962 and we had the moon mission by 69. Therefore, we are talking of an old rocket. But, mind you, it is still the most powerful rocket ever developed in the history of rockets and that is where I thought, maybe we should do a problem on this.

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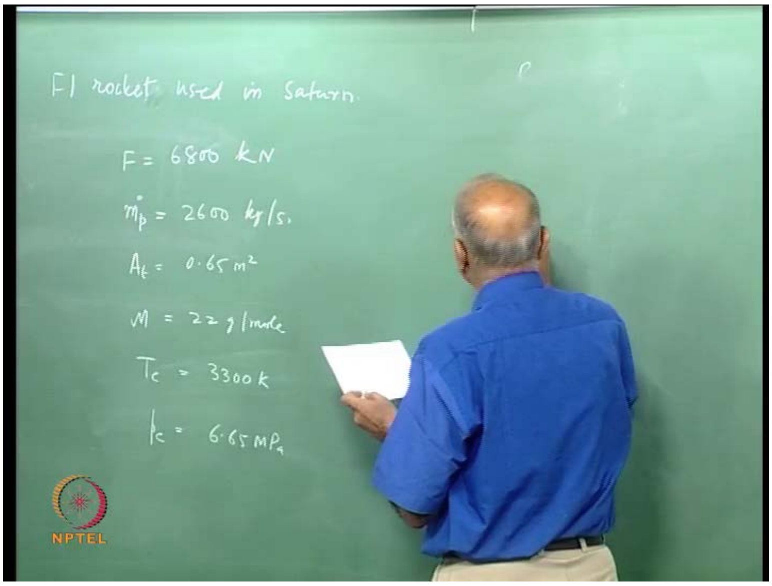

Now, I want to find out for this F 1 rocket, the value of eta c star, the value of I s p and the value of thrust correction factor zeta F. Let us do it. You know, because the nozzle is there and the area is there, and you know how much propellants is burnt per second.

I just want to do this problem. Therefore, to be able to get the value of eta c star, I must get the value of c star, which is actual and I must also get the value of c star, which is ideal. The ratio of this is the c star efficiency. How do I get the ideal value? Well, we already know it is equal to under root r t c by capital gamma. The value of capital gamma, let us again substitute r is equal to r naught divided by the molecular mass divided by t c and capital gamma is equal to under root gamma 2 over gamma plus 1 into gamma plus 1 divided by 2 into gamma minus 1. It is also told to you, for a molecular mass of 22 gram per mole, the value of gamma for the gases is is equal to 1.22. Therefore, I substitute the value of gamma is equal to 1.22 and the value of capital gamma works out to be equal to 0.652 under root 2 by 1.22 plus 1 and 2.22 divided by 2 into 0.2 over here.

Therefore, I want the c star ideal. For c star ideal is equal to, now let us substitute the value, r naught universal gas constant 8.314 joule per mole Kelvin. I think, please write the units whenever we do a problem. The value of t c for this particular propellant combination or whatever is used is 3300. The value of the molecular mass is equal to 22 grams per mole but, I am talking in terms of joule which is related to kilogram. Therefore, I say 0.22 kilogram per mole.

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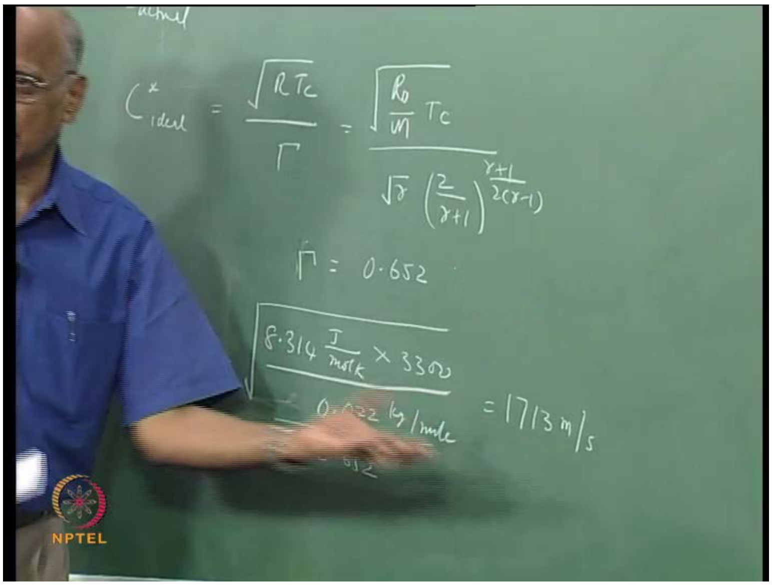

This is important. Many of us, mainly I find students just putting 22 here, which is not right because, when I say mole, I want the value of r in joule per kilogram Kelvin. Therefore, it must be kilogram per mole. The value of capital gamma, we already said is equal to 0.652 and this must be the numerator for this one. The value of c star ideal becomes 1713.

This is how we calculate the ideal velocity right, ideal c star. That means the capacity. Let us repeat to ourselves, the the capacity of kerosene and liquid oxygen to generate chamber pressure is given by c star ideal and this capacity is 1713 meters per second. Now, I want to get the value of c star. I have to calculate the actual value. How would I do it? I go back to look at the problem. The mass flow rate is given to me. The mass flow rate is given to me as 2600 is equal to 1 over c star into pressure. Pressure is given as, we see 6.65 into 10 to the power of 6 Pascal a t. a t is given as 0.65 and therefore, the value of c star actual, I calculate from this. This is 2600 kilogram per second. Yes, this c star will come out to be equal to this divided by 2600 and that will give me the value of 6.65 into 10 to the power of 6 into 0.65 divided by 2600 is equal to 1663 meters per second or rather the value of c star efficiency is therefore, equal to the actual value 1663 divided by 1713, which is equal to 0.97.

In fact, you find that the c star efficiency is quite high even for a rocket made in the year 1962. The present rockets like the space shuttle main engine, has a efficiency of c star of the order of 0.99. This is the way we are. There is hardly any room for improving the combustion any further. We have to understand that, when we do liquid propellant rockets, we will try to understand how come we get such values and what are the factors which governs it.

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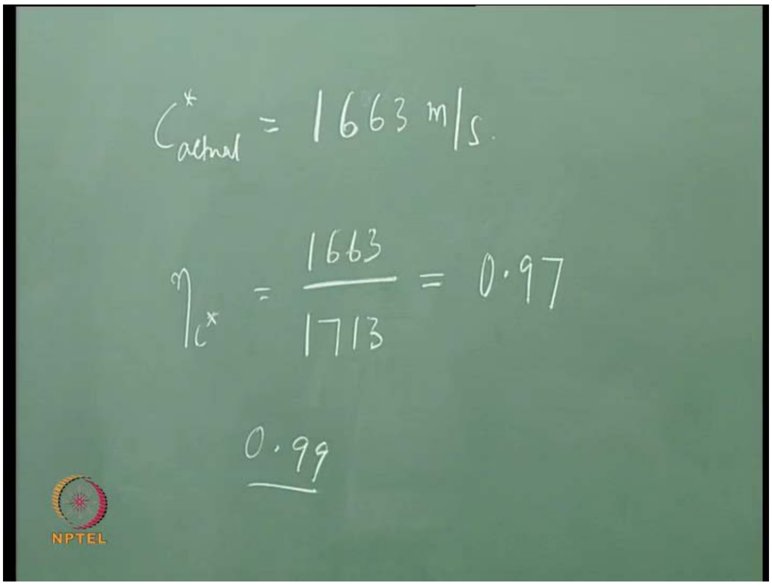

May be, we should use some of these tactics in the other propulsive devices also. Therefore, we have done one part of it, namely, what is the c star of this particular F 1 engine. The next one, I would like to understand, I want to find out what is the value of I s p. How do I do it? What is the specific impulse of this engine? Yes, I know the thrust is 6800 kilo Newton and I know the mass flow rate is 2600. Well, it is simple, is it not? F specific impulse is equal to I over m p which is equal to i over t divided by m p dot impulse specific I s p, which is equal to force divided by m dot p and which is equal to the value what you have, namely 6800 into 1000 Newton divided by 2600. The value of specific impulse will come out to be equal to what 2710. No, I think, I should have it somewhere. Well, whatever this number gives me, I think is less than 2710, which is, get this value. This is the value of I s p.

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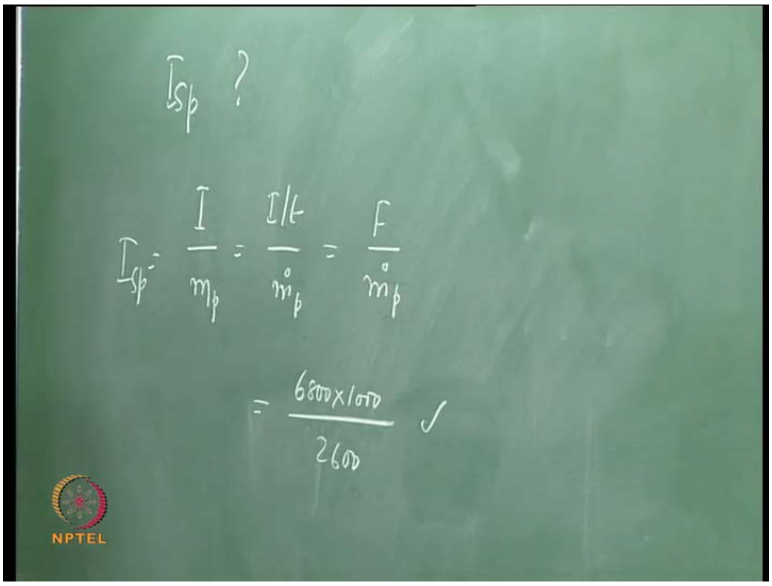

Now, I want to get the value of the thrust correction coefficient zeta F. For this, I need to do little more calculations. Let let me erase this part of the board. I get zeta F is equal to, I have to somehow relate the forces ideal to the actual. How do I get the ideal value?

This particular rocket, we tell ourselves that it develops a thrust of 6600 kilo Newton, when the exit pressure is also sea level or rather, when the value of p e of this rocket is equal to p a which is equal to 0.1 m p a because, it is tested. The test has been done at sea level, for which the exit pressure is equal to p a. We are assuming here that the nozzle exit pressure is equal to the ambient pressure.

Therefore, for this condition, the value of F ideal is equal to m dot into v J because, there is no pressure thrust coming. p e minus p a is 0 because, p e is equal to p a and how do I get the value of v J. We have, we know, we derive the expression v J square is equal to 2 of the enthalpy difference which came out to be equal to 2 gamma by gamma minus 1 r naught molecular weight t c into 1 minus p e by p c to the power gamma minus 1 by gamma, where p e is equal to the ambient sea level pressure. Put in the numbers, r naught, same thing, r naught is 8.314, m is equal to 0.022 and temperature is given 3300 and gamma is given 1.22.

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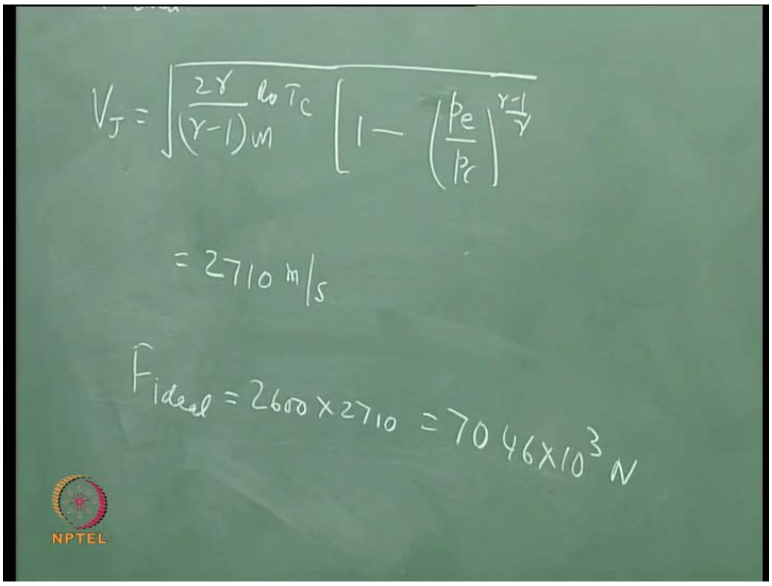

p e is equal to 0.1 m p a, p c is equal to the value what is given here that is 6.65 m p a. You substitute it and you get the v J is equal to 2710 meter per second. Therefore, what is the value of thrust? Thrust is equal to m dot 2600 kilogram per second multiplied by this. Therefore, the value of F ideal is equal to, multiply it m dot v J, which gives you the 2600, and let us write it down, into 2710, which is equal to 7046 into 10 to the power of 3 Newton.

What is the actual value of thrust? 680 kilo Newton. But, how do I get the value of zeta F? Therefore, my immediate reaction or anybody's immediate reaction would be to tell the value of zeta F. That is, the thrust correction factor is equal to F actual divided by F ideal. F actual is equal to, you get the value as 6800 is the actual value.

The ideal value is somehow larger, that is 7406 kilo Newton and therefore, you will tell me, that this value is equal to 0.965. This is what one expects. But actually, you know, we have, this is an actual rocket, in which we must also consider the effect of c star efficiency. We must also consider the effect of thrust correction factor. In other words, I s p at the actual thrust goes as eta c star into the thrust correction factor into the value of I s p, which is, could be ideal.

Therefore, if I were to correct for the c star effect, I should have zeta F is equal to 0.965 divided by the value of c star efficiency, which I got as 0.97. Rather, this works out to be 0.995, because this is only for the nozzle. Because, I looked at that total problem and the total problem gave me this and I have to isolate, what is a contribution of the nozzle. Therefore, the contribution of the nozzle is 0.995. Whereas, the contribution from the combustion or from the value of pressurization or c star is only 0.97 over here.

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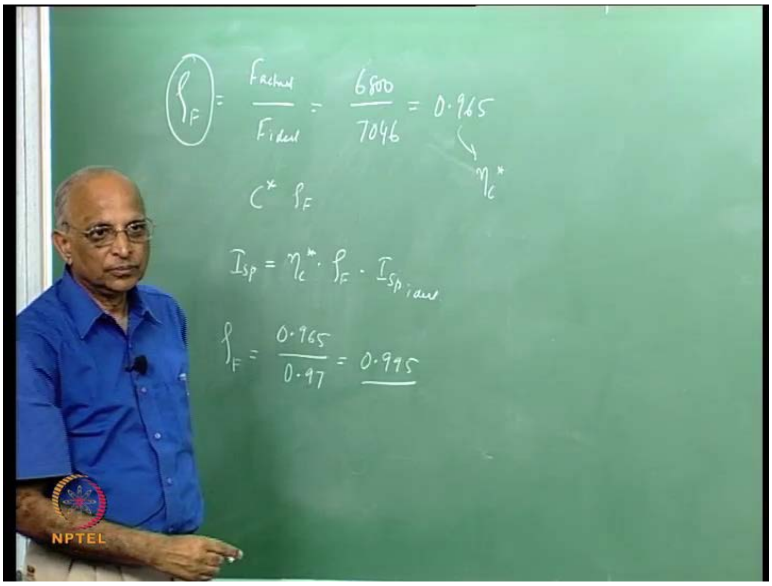

I think this is how we get the efficiencies. Well, I would be happy even if we put a number 0.965. But, let us keep in mind that 965 also include the value of c star efficiency. That is why, I had to remove it and that is where I got this particular number. Let us take one more problem that is problem of rocket flying at different altitudes. Let me pose this problem to you first. Yes, let us say a booster rocket flies between sea level 0 kilometres to 30 kilometres altitude and the chamber pressure of this rocket p c is given to be 7 m p a.

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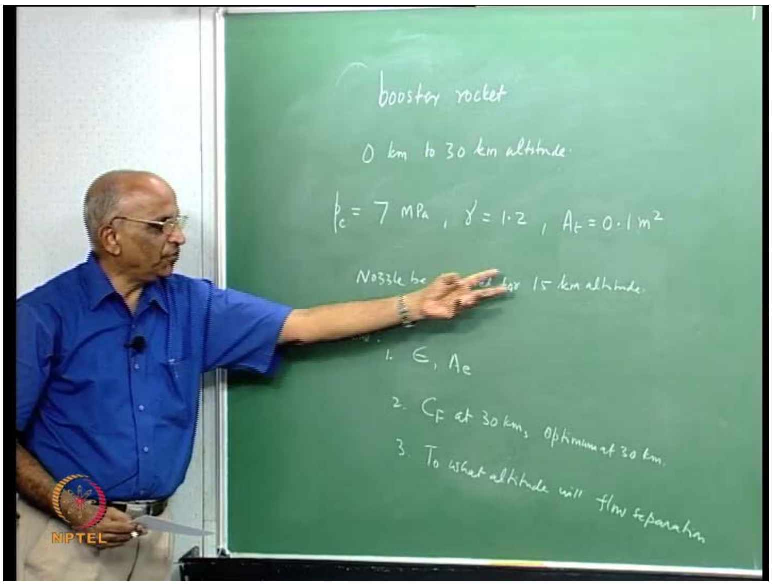

That is 70 bar and the value of the combustion products specific heat ratio is 1.2 and the throat area a t is equal to 0.1 meter square. Since, this rocket flies between 0 to 30 kilometre altitude. The designers felt that let the nozzle be designed for mean of the 2, say 15 kilometre altitude. Now, I want the following. Determine the nozzle expansion ratio, epsilon, and the value of a e. Second, the value of c F thrust coefficient at 30 kilometre altitude and what is the optimum at 30 kilometre altitude and third, I also want to know, till what height and till what altitude will flow separation occur. In other words, we assume it is a conical nozzle and it flies between 0 to 30 kilometres. The nozzle is designed for an altitude of 15 kilometres. I want to know till what height flow separation takes place in this conical nozzle. I also want to find out the area ratio, the area at the exit and the thrust coefficient at 30 kilometre, optimum value of thrust coefficient at 30 kilometres and to what altitude will flow separation takes place. Let us do this problem.

We need some data and the data which we normally use is something known as ICAO tables. You know, we we need to know the height in altitude versus the ambient pressure. This is given in the form of ICAO and is known as international civil aviation organisation. This gives some standards and they will tell you, well, if the altitude versus the pressure, ambient pressure in Newton per meter square if at sea level, the pressure in Newton per meter square is 101325 Newton per meter square. If the altitude is 4 meters height, the value is 61660. If the altitude is 8 kilometres, the value is 36651. You see, the value keeps decreasing. Let us put two or three more.

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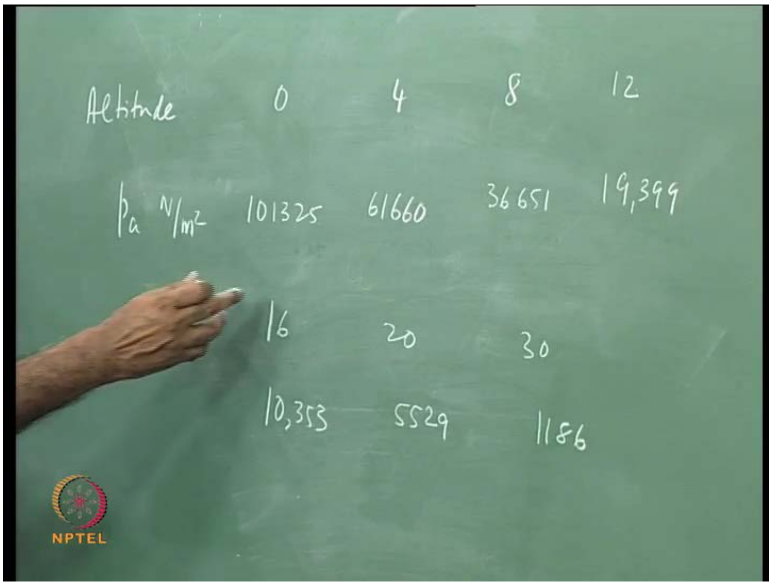

12 kilometres, the value is 19399 and if the altitude is 16 kilometres, it is 10353. If it is 20 kilometres, it is 10353. No, if it is 20, it is 5529 and if it is 30 kilometres, the value of pressure is equal to 1186. Since, I do not give the value at 15, let us assume that the nozzle is designed for 16 kilometre altitude. So that, the ambient pressure table is available to us. Therefore, now let let us try to do this three. We would like to first calculate the area ratio of the nozzle and the exit area. What do we tell? We say, well, the nozzle is designed for 16 kilometre altitude and therefore, for 16 kilometre altitude, we will have p e is equal to pa. Therefore, what should be the value of pe? For the nozzle? 10353, because the nozzle is designed for this particular altitude.

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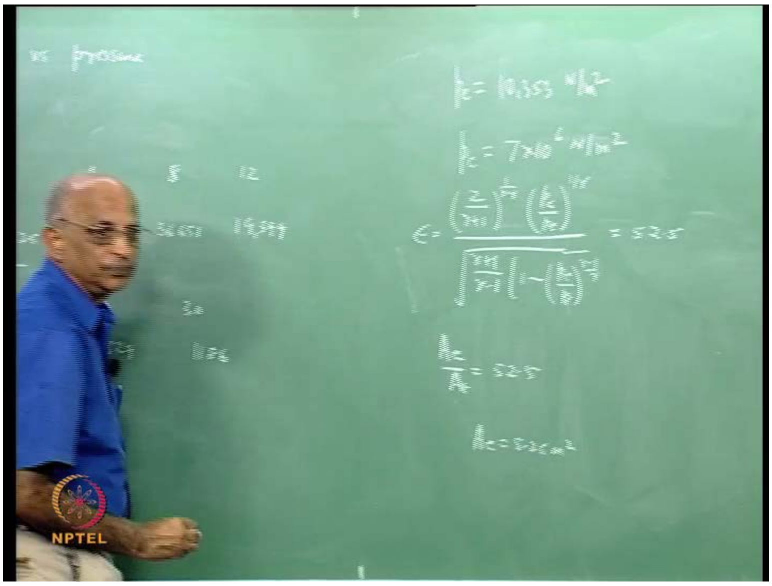

That means, we have 10353. Other data is given, what is the value of p c is equal to? Go back, it is 7 m p a. We say 7 into 10 to the power of 6 Newton per meter square. Gamma is given to you that is 1.2. Therefore, you you immediately write out the expression. For for the area ratio, what was the expression? Let us go back to your notes. Epsilon is equal to 2 over gamma plus 1. You all can easily derive it out. It is not anything difficult. I do not want us to memorize anything. p c by p e, 1 over gamma divided by under root gamma plus 1 gamma minus 1, one minus and you substitute the values, you get the value as equal to 52.5. Area ratio of the nozzle is therefore, 52.5. The value of the exit area a e by at is is equal to 52.5 or rather, at is given to you as 0.1 meter square. Therefore, the value of a e is 5.25 meter square. Is it alright? It is simple. You know, we we just and this is how we know rockets tend to be extremely simple.

In fact, I keep joking all the time, you know, when when we start doing something, you know in India, we still have not made good diesel engine or internal combustion engine or gas turbine engine. We have been taking time to do it and we have still not able to do. Whereas, rockets are very easy to do and that is why we see spectacular progress in making of rockets. It is quite simple. Therefore, you have a e is equal to 5.25 meter square. Therefore, let us go to the next part of the program. c F at 30 kilometres, how do I evaluate it? What will be involved in this? We had derived the expression for c F. Let us go back and take a look at it. c F is equal to 2 gamma square divided by gamma minus 1, 2 over gamma plus 1 divided by gamma plus 1 divided by gamma minus 1 into 1 minus p e by p c to the power gamma minus 1 by gamma, which is all under root plus p e by p c.

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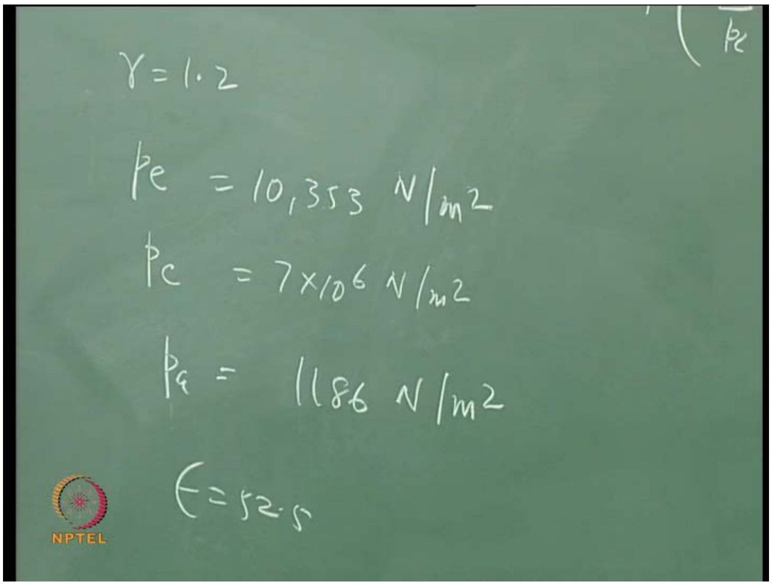

You will recall, we did this in the morning, p a by p c epsilon. Please check as to what are the values. I am interested at thrust coefficient at 30 kilometres. What are the values I put? You all must tell me. Well, you told we know gamma is equal to 1.2. That is given. What is the value of p e and what is the value of p c and what is the value of p a. Epsilon, we have already determined that it is equal to 52.5. What is the value of p e? Which value? Yes, the nozzle has been designed for 16 kilometre altitude and that is where the exit pressure should be. Because, it is now flying at a higher altitude but, exit pressure will not change. Therefore, p e is equal to 10353. Your answer is correct. p c, we know is 7 into 10 to the power 6 Newton per meter square. p a at the current altitude of 30 kilometres, 11806, you substitute it and you get the value of c F as equal to, I use the other side of the board that is 1.828 plus 0.0687 which is equal to 1.896.

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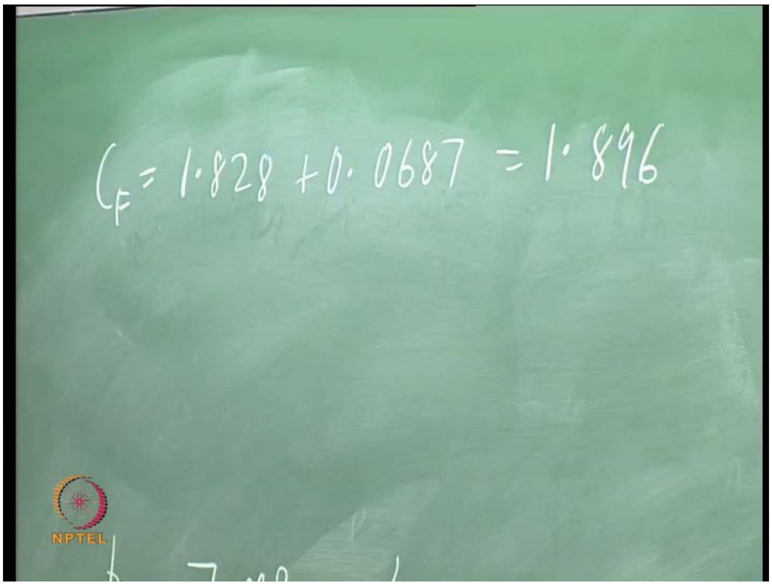

Please check these numbers. Now, what is the optimum value at 30 kilometres? Let let me go back. In this expression itself you can tell me that gamma is still the same and what will be the optimum value at 30 kilometre altitude.

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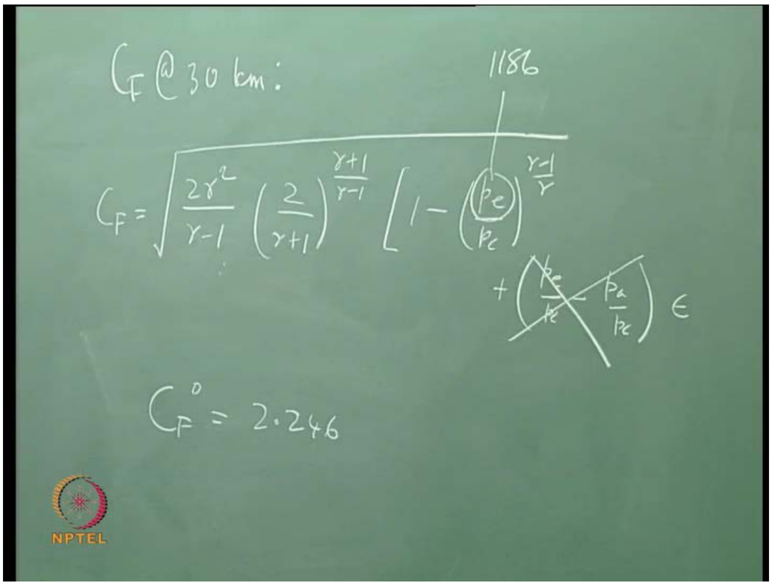

Well, p e is equal to p c. Therefore, optimum, therefore, this term will get knocked out and what will be the value of p, if it is optimum. Yes, let let us take a look at this table. Optimum at 30 kilometres, that means, I will have a nozzle which gives me this value, 1186. Therefore, I now change the value of p e equal to 1186 and this gets knocked out.

The value of c F which now becomes c F dot, will now become something like 2.246. You see, the thrust coefficient is the ball part number of around 2 to 3. That number, therefore, what is the percentage reduction from optimum? That means, what has happened? You had a nozzle. I think I will erase this out now. You had a nozzle which was designed for 16 kilometres. You are operating it at 30 kilometres. If it was designed for 30 kilometres, it would have been optimum at 30 kilometres.

I have the value of c F dot is equal to, what I get is 2.246. But, since it is designed for a lower value of area ratio and I get the lower value of the pressure recovery as 0.0687 and I do not get this number to be high. I get the total number at as equal to 1.896.

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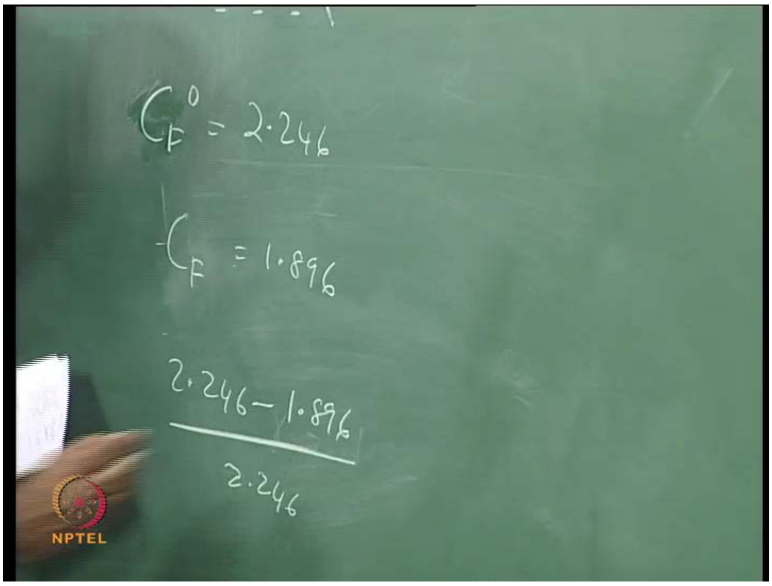

Therefore, I can tell that the percentage reduction from optimum is equal to 2.246 minus 1.896 divided by 2.246. In other words, I have something like 0.156 or something like 15.6 percent reduction from the optimum. Therefore, you see the importance. You know, if I have something like, I am not able to get the nozzle to expand, in fact, I am having an under expanded nozzle. That is why, I am losing something like 15.6 percent. Had my nozzle been designed for 30 kilometre height, I would have got, but then, I would have got a problem of over expansion or flow separation. Can we, therefore, go the next part of the problem and determine the altitude till which the nozzle is over expanded or the altitude till which the flow separation takes place. Now, we want to determine the altitude till which the nozzle is over expanded.

For this, we apply Somerfield criterion, which states that, when the exit pressure of the nozzle is less than or equal to 0.4 times the ambient pressure, then the flow is over expanded. This we looked at it. We also looked at a better criterion namely, involving mach number also.

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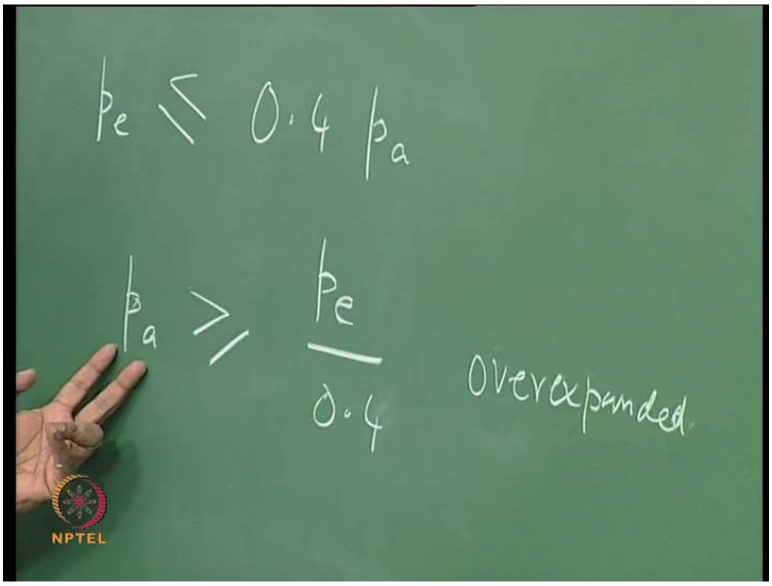

Let us use the Somerfield criterion. If we say this, when the ambient pressure p a is greater than or equal to p e divided by 0.4, then we can say when the ambient pressure is greater than or equal to the exit pressure divided by 0.4, then the nozzle is over expanded. Therefore, let us do this problem now. Let us examine the changes in the ambient pressure with respect to altitude. Let me plot the table again for this specific altitude of interest. We let us make a table of altitude in kilometre and the ambient pressure p a at this particular altitude in Pascal. Let us plot it for something like two or three altitudes for which we are interested.

At the altitude of 8 kilometres, the ambient pressure is 36651 Pascal. At the altitude of 12 kilometres, the value of the ambient pressure is now 19. It has reduced, because the altitude has gone up to 399 Pascal. At 16 kilometres, for which this particular nozzle is designed, the ambient pressure is 10353 Pascal. The question is, the nozzle is designed for 16 kilometres and therefore, the exit pressure of the nozzle is 10353 Pascal, we want to find out the condition at which, may be as the rocket goes up, the altitude at which the flow begins to separate or the nozzle gets to be over expanded nozzle. Therefore, we have to state here that flow separation, we have just written, p a must be greater than or equal to p e divided by 0.4 and p e, anyway, the nozzle is defined or defined or designed for an ambient pressure of or for a pressure of 10353 Pascal. Therefore, p a must be greater than or equal to 10353 divided by 0.4 and this is equal to 0.259 into 10 to the power of 5 Pascal. Now, the question is, when the ambient pressure is equal to or just greater than this value? When the flow separation starts or when the pressure is greater than this value, the flow separation starts and what is the corresponding altitude.

Now, when p a is equal to 259, it is somewhere between 8 and 12 kilometres. Therefore, we say at 8 kilometres, the ambient pressure is 36651 minus the value at 16 kilometres is 10353. But, we are interested in the altitude at 0.259 into the 10 to the power 5. Therefore, we have the value of of now, 2, 5 and then 900 minus the value at 16 kilometres, 10353 and this change in kilometre from 36 to 10 corresponded it to a value of something like 8 kilometres. Therefore, this is 8 and therefore, we have 8 kilometres plus their change is 25 to 10353 and therefore, the value is 8 plus this so much kilometres.

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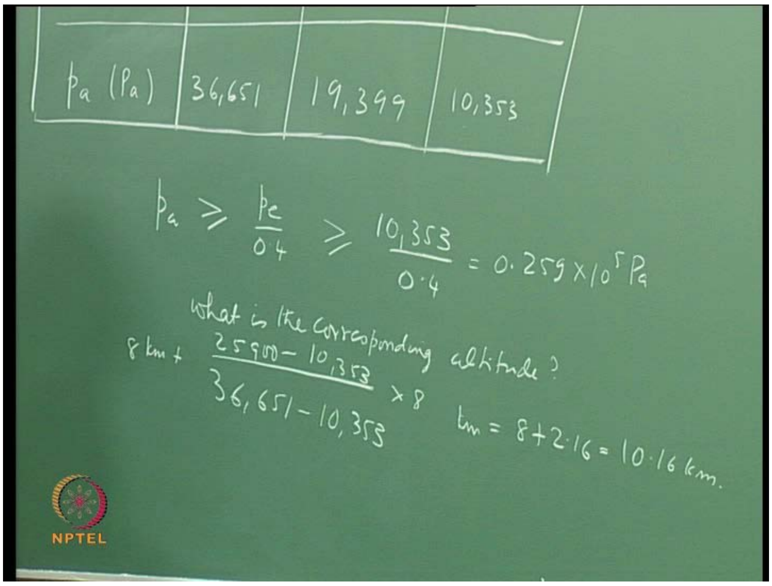

This works out to be something like 8 plus 2.16 which is equal to 10.16 kilometres. Therefore, this is the altitude at which flow separation starts or thereafter the nozzle is either runs full or is under expanded. This is all about nozzles. In the next class, we will start with chemical propellants. We will again keep it very very simple. In the sense, we will look at what are the requirements of chemical propellants and then see, what are the propellants that, we must use. I think, that is what we will do next. Well, thank you then.